ON DEPENDENCE OF RESOLVING POWER OF PRISM, GRATING AND REFLECTING ECHELON ON STAGE OF RESOLUTION AND DETECTING INSTRUMENT

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ABSTRACT. The authors have discussed the variation of resolving power of prism, grating and reflecting echelon with the value chosen for I_{min}/I_{max} at limiting resolution, which is characterestic of the stage of resolution desired and the detecting instrument.

INTRODUCTION

Ditchburn (1930) has pointed out that the resolving power of an instrument depends upon the stage of resolution desired and the detecting instrument. The stage of resolution and the detecting instrument are characterised by the value of $C=I_{min}/I_{max}$ at limiting resolution, where I_{min} and I_{max} are the central minimum and maxima of the resultant intensity pattern of two lines to be resolved.

The values of C for three important stages of resolution, distinguished by Ditchburn, when the spectrogram is examined by a microphotometer are as follows :

Stage of resolution		C
Detection of inhomogeneity in radiation	...	0.98
Partial resolution (approximate measurement of wavelength separation)	...	0.8
Complete measurement (measurement of wavelength separation and relative intensities)	...	0.4

This communication discusses the dependence of resolving power of prism, grating and reflecting echelon, which is characterestic of the stage of resolution desired and the detecting instrument.

VARIATION OF RESOLVING POWER WITH C

The intensity pattern of a spectral line after diffraction by a grating is given by

$$I'=B\frac{\sin^2 N\beta}{\sin^2\beta} \tag{1}$$

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where N is the number of lines in the grating and $2\beta = 2\pi ve(\sin i - \sin\theta)$ is the phase difference between the rays diffracted by two adjacent elements of grating, where the symbols have usual meanings.

The maximum intensity, say I_0, is given by

$$I_0 = BN^2$$

and hence equation (1) may be expressed as

$$\frac{I'}{I_0} = \frac{\sin^2 N\beta}{N^2 \sin^2\beta} = \frac{\sin^2 x}{x^2} \qquad (2)$$

where $x = N\beta$ and $\beta \ll 1$.

Equation (2) also represents the intensity pattern in case of a prism if $x = \pi l v \sin\theta$, the symbols having usual meanings.

The quantity Δx is proportional to the angle between two close spectral lines and therefore we shall use Δx instead of $\Delta\theta$ to represent the latter in this investigation.

The intensity distribution of another spectral line separated by an angle $\Delta x = a$ is given by

$$\frac{I''}{I_0} = \frac{\sin^2(x-a)}{(x-a)^2} \qquad (3)$$

The resulting intensity pattern is given by

$$\frac{I}{I_0} = \frac{\sin^2 x}{x^2} + \frac{\sin^2(x-a)}{(x-a)^2} \qquad (4)$$

The central maxima $(x \approx 0)$ and minimum $(x \approx a/2)$ are given by

$$I_{max} \quad 1 + \frac{\sin^2 a}{a^2} \Big) \qquad (5)$$

and

$$\frac{I_{min}}{I_0} = \frac{2\sin^2(a/2)}{(a/2)^2} \qquad (6)$$

For limiting resolution

$$C = I_{min}/I_{max} \qquad \ldots (7)$$

$$= \frac{2\sin^2(a/2)}{(a/2)^2} \Big/ \Big(\quad \sin^2 a \qquad \ldots (8)$$

If the separation of two lines at limiting resolution is a it can be shown that the resolving power R is given by

$$R = kt\frac{d\mu}{d\lambda} \quad \text{for a prism} \qquad \ldots (9a)$$

and $R = kNn$ for a grating and for a reflecting echelon, ... (9b)

where $k = \pi/a$ ... (9c)

Table I gives the variation of k with C obtained by calculating both C and k for some values of a.

TABLE I

Variation of k with C

Serial no.	a in degrees	a in radians	C	k
1	150	$\frac{5}{6}\pi$	1.051	1.2
2	160	$\frac{8}{9}\pi$	98	1 125
3	165	$\frac{11}{12}\pi$	.9401	1.091
4	170	$\frac{17}{18}\pi$	.8982	1 059
5	175	$\frac{35}{36}\pi$	.8553	1.028
6	180	π	.8106	1.00
7	185	$\frac{37}{36}\pi$	.7647	.973
8	190	$\frac{19}{18}\pi$	.7199	.947
9	195	$\frac{13}{12}\pi$	.6747	.923
10	200	$\frac{10}{9}\pi$	.6306	.900
11	205	$\frac{41}{36}\pi$	.5866	.878
12	210	$\frac{7}{6}\pi$	.5451	.857
13	220	$\frac{43}{36}\pi$	.473	.837
14	225	$\frac{5}{4}\pi$	4640	.800
15	230	$\frac{23}{18}\pi$	.3931	.770

The variation of k with C is illustrated by the graph below:

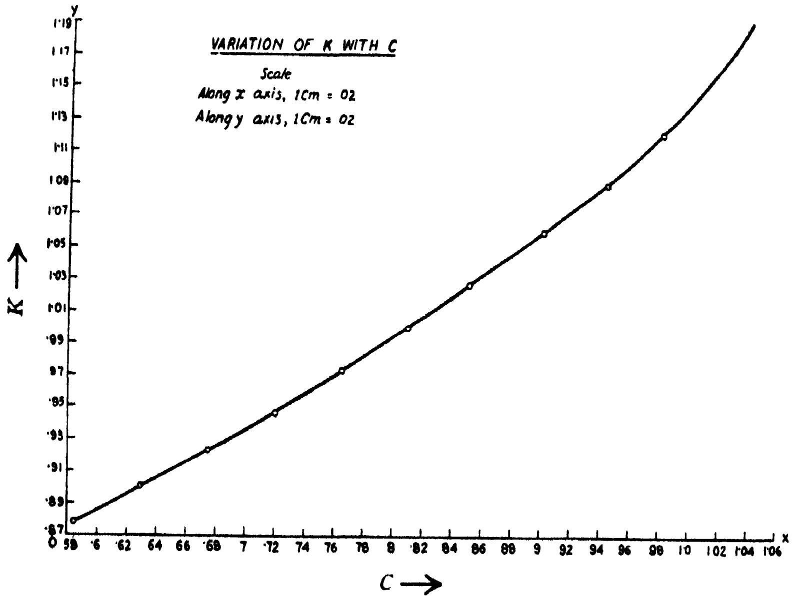

FIG. 1 Variation of k with C

The treatment for grating equation (9b) is also applicable to reflecting echelon for both have the similar intensity pattern.

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REFERENCE

Ditchburn, 1930, *Proc. Roy. Irish Acad.*, **39**, 58.