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ON THE EXPERIMENTAL INVESTIGATION OF NIGHT-TIME E ION-DENSITIES AND THEIR DETERMINATION BY THE APPLICATION OF CHAPMAN'S FORMULA

BY M. M. SENGUPTA

AND

S. K. DUTT

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ABSTRACT. The present work embodies results of experiments to verify whether the observed night-time E ion-density values in different seasons vary in accordance with Chapman's formula. It appears that Chapman's formula does not account wholly for the results of observation. This is also corroborated by the data obtained at Watheroo which is situated near about the same latitude as Patna but in the opposite hemisphere; the ionospheric conditions of these places have been taken to be the same for the purpose of comparison. It is suggested that the value of σ_0 in the Chapman's formula which is taken to be constant throughout for calculations, does not hold good at least during night-time and the discrepancy may perhaps be due to this.

Chapman's (1931) formula from which ion-density values of different layers of the ionosphere can be calculated is given by

$$\sigma_0 \frac{d\nu}{d\phi} + \nu^2 = \exp.\ [1 - z - \exp.\ (-z)\ f\,(\mathrm{R}, x)] = \mathrm{F}(z, x)\ \ldots\ (\text{day}) \qquad \ldots\ (1)$$

$$= 0\ \ldots\ (\text{night}) \qquad \ldots\ (2)$$

where z—a linear function of the height

$f(\mathrm{R}, x)$—a complicated function of a parameter R and angle x

h—height of the point of observation above the ground

θ—angular distance from the North pole (the Co-lat.)

δ—north declination of the sun

x—zenith distance of the sun at the point and time considered ($\cos x = \sin\delta\cos\theta + \cos\delta\sin\theta\cos\phi$)

t or ϕ—the local time reckoned from noon.

ϕ is in angular measure and t in seconds. Thus $t = 1.37 \times 10.^4\phi$.

n—ion-density at any time at height h

n_0—the steady value which n would attain at the level $h_0 (z=0)$ at the equator at midday (when $z=0$, $x=0$).

$$\nu = n/n_0$$

$$1/\sigma_0 = 1.37 \times 10^4 (\mathrm{I}_0 \alpha)^{\frac{1}{2}}$$

where α is the coefficient of recombination and I_0 is the production value corresponding to n_0; $n_0 = (\mathrm{I}_0/\alpha)^{\frac{1}{2}}$

Chapman shows that for a given angle x the height in terms of z at which the rate of ionization is greatest is given very nearly by $\log_e f(\mathrm{R}, x)$, so that if z_0 is the height corresponding to x_0, the value of x at noon, z_0 will be given by $z_0 = \log_e f(R, x)$. The above equation can then be written in the from

$$\sigma_0 \frac{dv}{d\phi} + v^2 = \frac{1}{f(R, x_0)} \exp. \left[1 - \frac{f(R, x)}{f(R, x_0)} \right] \quad \ldots \text{ (day)} \qquad \ldots \quad (3)$$
$$= 0 \qquad \text{(night)}.$$

In Table I the values of ν obtained by solving the differential equation (3) following the numerical method for latiude 24°N * at winter and summer solstices are given.** It may be noted here that for the purpose of calculation the value of σ_0 has been taken to be 1/25 as has been suggested by Champman (1931), and Best, Farmer and Ratcliffe (1938). It will be seen from the graph (Fig. 1) that the ratio of $\frac{\nu_{winter}}{\nu_{summer}}$ at any hour of the day and night is less than unity and indeed this should be so, as the formula involves the zenith angle x whose value at any place at any hour is less in summer months than in winter months, the solar radiation being the main cause of ionization

TABLE I

ϕ reckoned from noon	Hr. min.	ν winter solstice	ν summer solstice	$\frac{\nu \text{ winter solstice}}{\nu \text{ summer solstice}}$
0	12.00	.823	1.000	.823
.2	12.46	.816	.994	.821
.4	13.31	.786	.968	.812
.6	14.17	.726	.923	.786
8	15.02	.629	.852	.738
1.0	15.48	.472	.749	.630
1.1	16.11	.366	.679	.539
1.2	16.34	.254	.596	.426
1.3	16.57	.166	.498	.341
1.4	17.19	.116	.394	.294
1.6	18.05	.074	.182	.407
1.8	18.50	.054	.096	.563
2.0	19.36	.042	.065	.646
2.2	20.22	.035	.049	.714
2.4	21.07	.030	.039	.769
2.6	21.53	.026	.033	.788

As ν is proportional to the square of the critical frequency (f_c), $\frac{\nu_{winter}}{\nu_{summer}}$ is given by $\frac{f_c^2 \text{ (winter)}}{f_c^2 \text{ (summer)}}$. Hence experimental values of $\frac{\nu_{winter}}{\nu_{summer}}$ can be obtained from the results of systematic investigations of the ionosphere that have been carried out by the modern automatic multi-frequency equipment at Huancayo (Lat. 12° 2′ S. Long. 75° 20′ W.) and Watheroo (Lat. 30°19′ S. Long. 115°53′ E.). The Table II gives the values of $\frac{\nu_{winter}}{\nu_{summer}}$ calculated from the observed values of critical frequencies at Watheroo by Parkinson and Prior (1939).

*** The latitude 24°N has been chosen as this is the mean latitude between Patna and Calcutta**
**** The values of ν have been calculated in the same way as Millington (1932).**

TABLE II

(All the critical frequencies (Mc/s) are for the E layer at Watheroo (Lat. 30°19′ S, Long. 115°53′ E))

Hours	f_c winter June, 1939	f_c summer Dec., 1938	f_c^2 winter	f_c^2 summer	$\frac{\nu_W}{\nu_S} = \frac{f_c^2 \text{ winter}}{f_c^2 \text{ summer}}$
12	3.51 Mc/s	4.02 Mc/s	12.32	16.16	.76
13	3.44	3.96	12.83	15.68	.76
14	3.37	3.83	11.36	14.67	.78
15	3.05	3.80	9.30	14.44	.65
16	2.61	3.50	6.81	12.25	.56
17	1.91	3.10	3.65	9.61	.38
18	1.17	2.44	1.37	5.95	.23
19	1.00	1.46	1.00	2.13	.47
20	0.94	0.75	0.88	0.56	1.56
21	0.81	0.63	0.66	0.40	1.64
22	0.58	0.58	0.53	0.34	1.59
23	0.53	0.53	0.56	0.28	2.00

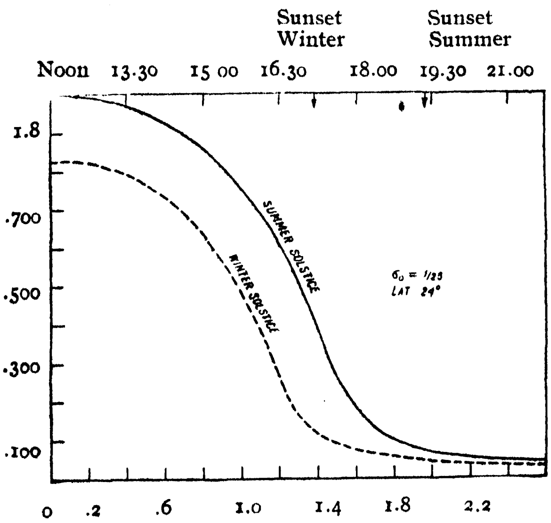


FIG. 1

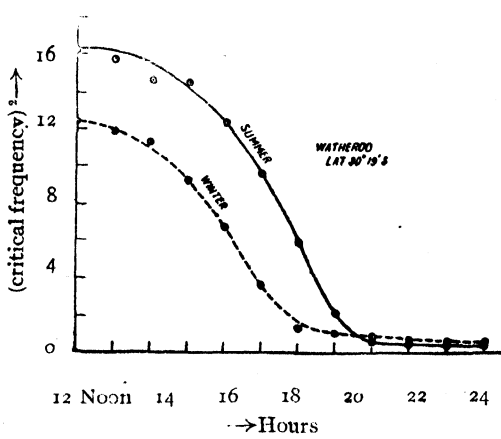


FIG. 2

It will be seen from the plot of f_c^2 against hour of the day (Fig. 2) that during daytime the ratio of ν_{winter} to ν_{summer} is less than unity, that is, quite in agreement with that indicated in theory; in night-time, however, the condition is reversed as shown by the intersection of the curves which is absent in the theoretical curves of Fig. 1.

The present work has been undertaken to verify whether the ion-density values vary with seasons in a similar way during night-time at a place having approximately the same latitude but in the opposite hemisphere. In his report Mitra (1935), has given data for such a place (Calcutta Lat. 22°31′ N, Long. 88°21′ E) and found fairly good agreement with theory of values of ν in summer only; but as no data for winter months are given in his paper, no definite conclusion can be drawn regarding the disagreement of the experimental results of night-time Watheroo from theory. Owing to difficulties arising out of war conditions it was not found possible for us to set up the necessary multifrequency transmiter required in the usual critical frequency method. We have, therefore, followed a differnt plan for our purpose.

EXPERIMENT.

In the method adopted by us the angles i of the downcoming waves of a particular frequency (810 kc/s) from a given station have been measured. Since the distance between the transmitting and receiving stations is known, the angle i_0 of the downcoming waves refracted back from the E layer (110 Km) for the given receiving station is also known. If the deflection is occurring in the E layer, the angle of the downcoming waves should be this particular value i_0.* The value of ion-density corresponding to any angle of reflection can be obtained from the application of the Appleton-Hartree formula and hence the value of N_{0E} corresponding to i_0 can be found, thus giving the lower limit of the ion-density which the E layer can have for the particular frequency during the period of observation; the ion-density, however, may have any value higher than this, but not less. For all other values less than i_0 the wave must have penetrated the E layer as is shown in Fig. 3 and deflection is no longer taking place from this layer but from a layer at a higher level where

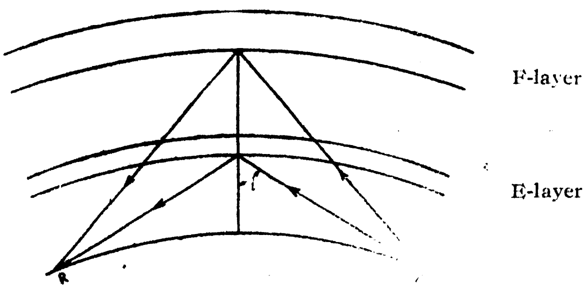


FIG. 3

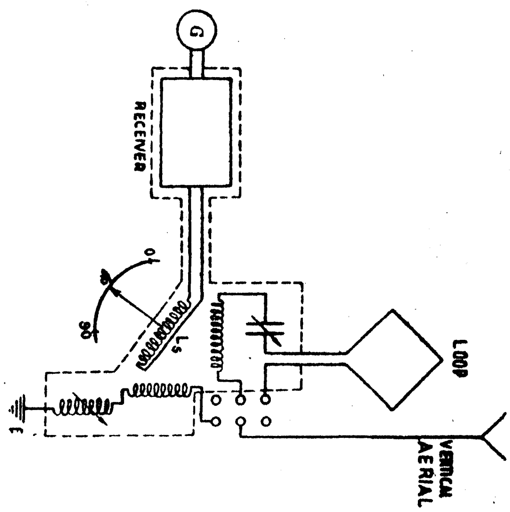


FIG. 4

the ion-density has a value corresponding to the observed i, in other words the value of the E ion-density must have fallen below the lower limiting value N_{0E}. Our method will, therefore, give whether the ion-density is greater or less than N_{0E} — the critical value of the E ion-density for the given stations and the frequency chosen; but since the variation in the ion-density can change the angle of the downcoming waves between two limiting values i_0 and i' (i' corresponding to next higher layer), a definite conclusion as to whether the ratio of the seasonal ion-densities is greater or less than unity is possible from a study of the variation of the angle of the downcoming waves.

The experimental arrangement is essentially the same as given by Smithrose and Barfield (1926). An outline of the arrangement is shown in Fig. 4.

* Slight variation of i_0 is possible, since there is variation in the height of the layer.

The angle (η) through which L_S is rotated is measured and from it the angle i of the down-coming waves is obtained from the relation $\sin i = \tan \eta$. The whole experiment was done in the most aperiodic conditions of the two aerials to minimise the effect of the coupled inductance due to the secondary circuit of the search coil L_S. It can be seen from an analysis that to attain this condition $\omega^2 M^2$ should be relatively small compared to the impedance of the aerial circuit for which the circuit must be as aperiodic as could be made without making the final galvanometer deflection small. In our case the deflections in the equality condition were never less than 6 cms. in spite of the fact that the aperiodicity was as great as 60 to 80% off the resonance. This ensured high accuracy of the whole experiment.

In our case the ground wave was found negligible after proper tests as done by Sengupta and Dutt (1941), and the formula $\frac{E_A}{E_L} = \sin i$

where E_A = e.m.f. induced in the vertical serial.

E_L = e.m.f. induced in the loop aerial.

i = angle of incidence with the vertical.

will hold good under all conditions as shown by Appleton (1927). The absence of the ground wave also simplified experimental arrangement and interpretation of results.

In order to minimise the antenna effect as much as possible, the dimensions of the loop were made purposely small. The loop was nearly 86 cms. square and the number of turns of wire was six. Tha effective height was about 100 cms. Proper tests were carried out to know definitely the amount of antenna effect and it was found negligible.

The receiver was properly shielded and was thoroughly tested for pick-up which was never more than a millimetre or two; as the readings were not usually less than 6 cms. the effect due to pick-up was negligible. The goniometer angle could be read accurately within half a degree and the individual readings varied by about $\pm 2^\circ$.

The possibility of laterally deviated indirect rays reaching Patna should not be excluded ; but it is easily seen that lateral deviation will produce large values of i and not small values. To determine the degree of lateral deviation observations were taken with another loop aerial. The arrangement of this loop was such that the loop could be rotated both about the vertical and horizontal axes. From the observations taken it was found that the plane of the down-coming waves was practically vertical, the deviation being generally within 5° and occasionally going up to 11°. The experiments of Smithrose and Barfield have shown that such lateral deviation is not very appreciable for conditions similar to those of this experiment for λ lying between 300 and 500 metres.

RESULTS

The results of our experiment given in Fig. 5 show the average of the observed values of the angle i during the period of observation (generally from 7 P.M. to 10 P.M. local time). It will be observed that during the winter months it is the E layer which is generally taking part in reflection ; this is, however, no longer true during the summer months, the layer being penetrated during this time.

In order to obtain the value of $\frac{\nu_{winter}}{\nu_{summer}} = \frac{n_w/n_0}{n_s/n_0} = \frac{n_w}{n_s}$ from our observations we have obtained the lower limiting values of n_s (summer ionic density) and n_w (winter ionic density) from the corresponding values of i averaged over each month by using the Appleton-Hartree formula as mentioned before. Taking into account the curvature of the earth and the great circle distance PC (Fig. 6) between the two stations Patna and Calcutta, the angle of incidence

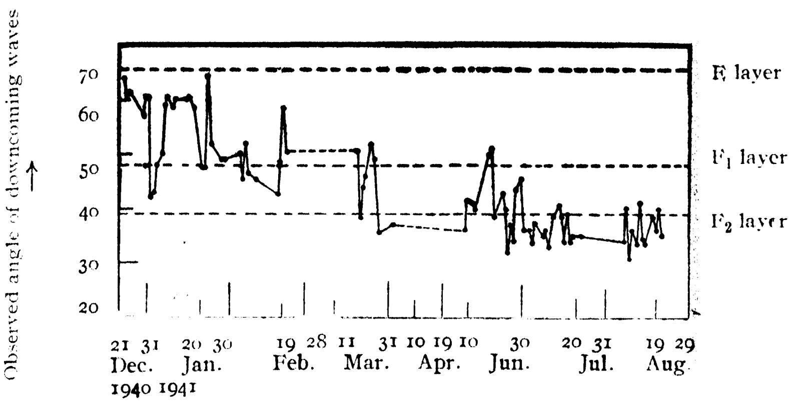


FIG. 5

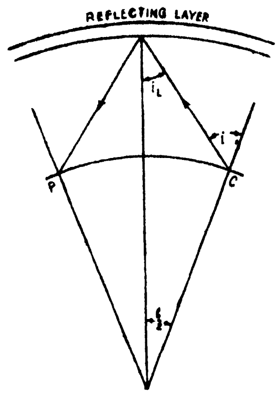


FIG. 6

at the reflecting layer is given by $i_L = i - b/2$

where b is obtained from the relation

$$\cos b = \cos\theta \cos\phi \cos\theta' \cos\phi' + \cos\theta \sin\phi \cos\theta' \sin\phi' + \sin\theta \sin\theta'$$

where θ, θ' and ϕ, ϕ' are the latitudes and longitudes of Patna and Calcutta respectively.

Patna	$\theta = 25°30'$N	$\phi = 85°15'$E
Calcutta	$\theta' = 22°34'$N	$\phi' = 88°55'$E

$b/2$ comes out to be $2°3' = 2.1$ approximately.

The value of N_{0E} has been calculated from the Appleton-Hartree formula, absorption and and friction having been neglected.

$$\text{Sin}^2\, i_L = \mu^2 = 1 - \frac{2p_0^2}{2p^2 - \frac{p^2 p^2_T}{p^2 - p_0^2} \mp \left[\frac{p_4 p^4_T}{(p^2-p_0^2)^2} + 4p^2p^2\right]^{\frac{1}{2}}}$$

where $p_0^2 = 4\pi Ne^2/m$

e, m = electronic charge and mass

N = number of equivalent electrons per c.c.

$p = 2\pi \times$ frequency of the wave

$p_T = eH_T/mc$, $p_L = eH_L/mc$.

H_T, H_L = Intensities of earth's magnetic field transverse and parallel to the direction of propagation of the waves

c = the velocity of light.

p_0^2 has been taken to be equal to $4\pi Ne^2/m$ and thus the Lorentz correction term has been eliminated in accordance with the reasons and arguments put forth by Darwin (1934), and supported by others like Mimno (1937).

Assuming symmetrical reflection taking place at the ionized layer at a point M (Fig. 7) midway between Patna and Calcutta on the great circle passing through the places.

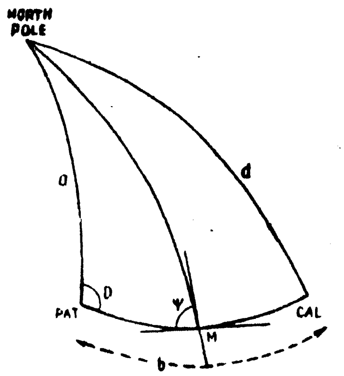
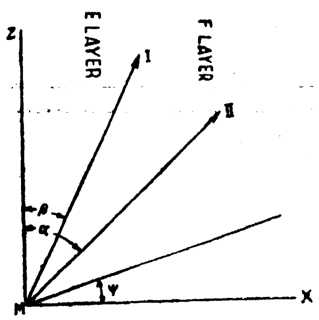


Fig. 7



MZ—is the vertical through M
I—Magnetic Field
II—Direction of propagation

Fig. 8

$$H_L = H(\sin \alpha \sin \beta \cos \psi + \cos \alpha \cos \beta)$$

and $$H_T = H\left[1 - (\sin \alpha \sin \beta \cos \psi + \cos \alpha \cos \beta)^2\right]^{\frac{1}{2}}$$

α and ψ being given by the equations

$$\tan \alpha = \frac{r \sin \frac{b}{2}}{r + h - r \cos \frac{b}{2}} \quad \ldots \quad \text{(from Fig. 6)}$$

and $\cos \frac{b}{2} \cos D = \sin \frac{b}{2} \cot \alpha - \sin D \cot \psi$ ... (from Fig. 7)

D is known from

$\cos d = \cos a \cos b + \sin a \sin b \cos D$ ... (from Fig. 8)

where r = radius of the earth

h = height of M (E layer)

$b = 4.1°$ (already found)

D = angle made by the meridian great circle at Patna, and the great circle passing through Patna and Calcutta (Fig. 7).

a = co-lat of Patna (= 64°30′)

d = co-lat of Calcutta (= 67°26′)

β = the complement of the dip at M

ψ = angle made by the great circles—one passing through Patna, M and Calcutta, and the other passing through M and the North Pole.

In all these calculations, the declination at M has been supposed to be zero, as those of Calcutta and Bombay being only 0°32′ E and 0°41′ E, any place lying between those two must have a small declination of this order. H and the dip for M have been taken to be the mean of the corresponding values for Patna and Calcutta. The values obtained are given in Table III.

TABLE III

α	β	ψ	D	H	H_L	H_T	N_{OE}
63°32′	54°	44°22′	134°58	.456	.356	.286	$.98 \times 10^4$

In the following Table IV the monthly averages of i and corresponding i_L obtained in our experiments during the period of observation are given.

TABLE IV

Season	Months	i average	i_L	E ion-density	n_ω/n_B	Remarks
Winter	December, 1940	61°.5	59°.4			Reflection from E Layer.
	January, 1941	53°.9	51°.8	$>.98 \times 10^4$		
	February	48°.9	46°.8			
	March	45°.1	43°.0			
	April	37°.0	34°.9		>1	
Summer	May	—	—			Penetration of E Layer.
	June	41°.0	38°.9	$>.98 \times 10^4$		
	July	36°.1	34°.0			
	August	36°.5	34°.4			

Assuming some variation in the height of the layer, i_L corresponding to E layer has been taken as lying between 63° and 51°; values below this must be taken as penetration of the E layer. It will be thus clear from the above Table that during winter months reflection is taking place from E layer whose ionic density is at least. 98×10^4. During summer, however, the reflections are definitely not from the E layer whose ionic density must have fallen below $.98 \times 10^4$ according to the results obtained. It is, therefore, obvious from our experimental results that ν_ω/ν_B must be greater than unity during night-time, thus corroborating the experimental results of Watheroo.

It may be observed from the above Table that the frequency 810 Kc/s used in our experiment is less than the critical frequency for E layer in winter and more than that in summer. Thus our experimental results are in full agreement with those obtained at Watheroo (given in Table II) by a different method using most modern equipment.

It is thus seen from our results and also corroborated by the data obtained at Watheroo that Chapman's formula does not account wholly for the conditions of the ionosphere. This discrepancy may perhaps be due to the fact that the value of σ_0 has been taken to be constant throughout. It seems to us that during winter night-time recombination is less than the summer nights. It may, therefore, be suggested that the value of σ_0 cannot be taken to be the same during all hours both in summer and winter for the calculation of night-time E ion-densities in accordance with Chapman's formula.

In conclusion we record our thanks to Prof. V. Rangacharya for his help in some of the mathematical calculations and Prof. K. Prosad, O.B.E., I.E.S., Principal, Science College, for the interest he has taken in our work.

PHYSICS DEPARTMENT,
SCIENCE COLLEGE, PATNA.

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