27

THEORY OF THE VARIATION OF THE RESISTANCE OF A THERMIONIC VALVE WITH FREQUENCY

By S. R. KHASTGIR, D.Sc. (Edin.), Mem.I.R.E.

(Received for publication, April 24, 1941)

ABSTRACT Experiments of Mitra and Sil, performed some years ago, on the variation of internal resistance of a thermionic valve with frequency, showed that the internal resistance *decreased* gradually with the *decrease* of frequency in the high frequency range. With further *decrease* of frequency, however, they found that the internal resistance of the valve was *practically* constant. These measurements were recently extended to the lower frequency range by Kameswar Rao in this laboratory. Starting from a high frequency it was found, in substantial agreement with Mitra and Sil's results, that the internal resistance *decreased* steadily with the *decrease* of frequency, and that with further *decrease* of frequency, the internal resistance of the valve *increased* quite perceptively and steadily.

Mitra and Sil's theory of the variation of the resistance of a thermionic valve with frequency could very well explain the experimental results in the high frequency range but it cannot explain the observed *increase* of internal resistance of a valve with the *decrease* of frequency in the low frequency range.

In this paper an attempt is made to explain the observed variation of the internal resistance of a thermionic valve with frequency over a *wide* range of frequencies. While accepting Mitra and Sil's fundamental ideas as regards the conductivity of the valve due to the convection current, we have taken into account the conductivity arising from the displacement currents in interpreting the experimental results on the internal resistance of a thermionic valve.

An approximate mathematical theory of the variation of the internal resistance of a thermionic valve with the frequency of the field had been attempted by Hartshorn[1] some years ago. He showed that the internal resistance of a valve at very high frequencies when the displacement current would contribute almost entirely to the conductivity of the valve, should be slightly smaller than the value at very low frequencies, when the displacement current would be negligible and the voltage-gradient across the valve would be determined solely by the thermionic convection current. Later, Mitra and Sil[2] worked out a theory of the variation of the internal resistance of a thermionic valve with frequency and showed that their experimental results on the internal resistance of a valve were entirely at variance with Hartshorn's theory. Assuming a Maxwellian distribution of velocity of the electrons inside the valve they calculated

* Communicated by the Indian Physical Society.

the time of flight of the electrons making certain simplifying assumptions and approximately obtained the values of the conductivity of the valve for different frequencies. These values calculated on their theory agreed well with those obtained by them experimentally in the high frequency range. Some measurements of the internal resistance of a valve were subsequently carried out by Kameswar Rao[3] in this laboratory over a wide range of frequencies. Starting from a high frequency it was found, in substantial agreement with Mitra and Sil's results. that the internal resistance of a valve decreased gradually with the decrease of frequency. With further decrease of frequency, however, the internal resistance of the valve was found to increase gradually. This latter result of Rao which has also been recently confirmed by a different method cannot be explained according to Mitra and Sil's theory.

The object of the present paper is to explain the observed variation of the internal resistance of a thermionic valve with frequency over a wide range. While accepting Mitra and Sil's fundamental ideas as regards the conductivity of the valve due to the convection current, we have taken into account the conductivity arising from the displacement currents in interpreting the experimental results on the internal resistance of a thermionic valve.

Mitra and Sil assumed that the electrons emitted from the cathode have a Maxwellian distribution of velocity. On applying an alternating field, the electrons are set into oscillatory motion which is super-imposed on their original Maxwellian velocity. It is possible that under the influence of this field, for one half of the alternation, the electrons will be able to strike the anode of the valve giving up both their kinetic energy and charges. For just reaching the anode under the applied field of frequency $\left(\frac{1}{2T}\right)$ at the end of the interval T, there must be a critical velocity v_0 with which the electrons must be moving initially. All the electrons which have been moving with velocities greater than this critical velocity v_0 at the instant when the field has begun to act would also reach the anode within the interval T. The conductivity for a particular frequency $f=\frac{1}{2T}$ would then be proportional to $\left(\frac{n.T}{T}\right)$ where n is the number of electrons having velocities within the range v_0 to ∞ and reaching the anode surface within the time interval T.

Now with a Maxwellian velocity distribution, the number dN of electrons per unit volume having normal velocity components between v and $v+dv$ is given by

$$dN = N.\sqrt{\frac{m}{2\pi kT}}\,.\,e^{-\frac{mv^2}{2kT}},$$

where N is the number of electrons per c.c., T the temperature of the emitting cathode, k = Boltzmann's const. and m = the mass of an electron. Thus the number of electrons that escape from unit surface in unit time is

$$n = \int_{v=v_0}^{v=\infty} v . dN = N \sqrt{\frac{m}{2\pi kT}} \int_0^{\infty} v . e^{-\frac{mv^2}{2kT}} . dv$$

$$= N \sqrt{\frac{kT}{2\pi m}} . e^{-\frac{mv_0^2}{2kT}} . \qquad \dots \quad (1)$$

If x is the distance between the anode and the cathode and f the frequency of the applied field, then the initial velocity v_0 with which the electrons would move to reach the anode (after an interval of time corresponding to the frequency) would be given by

$$v_0 = 2xf.$$

Thus (1) can be written as

$$n = N \sqrt{\frac{kT}{2\pi m}} . e^{-\frac{2x^2 f^2}{kT}} . \qquad \dots \quad (2)$$

It is therefore evident from (2) that the conductivity of the valve would decrease gradually with the increase of frequency. This conclusion is based on the supposition that the electrons in the inter-electrode space move *only in the positive direction, i.e., towards the anode surface.* Considering the electrons which move inward towards the cathode and which may reach the anode under the influence of the positive field developed between the grid and the cathode due to the impressed *e.m.f.*, between the grid and the plate, Mitra and Sil found on *calculation* that for time-intervals greater than that corresponding to 36 metres, the number of electrons reaching the anode under the action of applied E.M.F. remained unaltered. Thus according to their calculation the conductivity should be independent of frequency for frequencies lower than that corresponding to 36 metres and it would decrease gradually with the increase of frequency *only in the range of the higher frequencies.*

This latter part of the calculation does not, however, apply to the experiments, where the anode is given a high voltage with respect to the cathode so that practically *all* the electrons move towards the anode. Theoretically, therefore, we would expect a continuous decrease of conductivity with the increase of frequency even for the lower frequencies. There is, however, an important factor which should be taken into consideration. In the Maxwellian distribution of velocity, there must be a lower limit u_0 which is the velocity component normal to the surface necessary for the electrons to escape from the cathode. In that case it is evident that the number of electrons reaching the anode will assume a constant value, when the frequency $(1/2T)$ is reduced to such an extent that the initial velocity of the electrons which would carry them to the anode in time

T becomes the same as u_0. According to this idea, therefore, the conductivity of the valve would remain constant for frequencies lower than a certain value corresponding to the limiting velocity u_0 of the electrons. This limiting velocity u_0 would be given by

$$\frac{e\phi}{300} = \tfrac{1}{2} m u_0^2$$

where ϕ is the work function (in volts) of the material of the cathode and e and m are the charge and mass of an electron. The frequency corresponding to the limiting velocity u_0 would then be given by

$$f_0 = \frac{1}{10x} \cdot \sqrt{\frac{e\phi}{6m}} \qquad \ldots \quad (3)$$

where the inter-electrode distance x is equal to $\left(u_0 \times \frac{1}{2f_0}\right)$.

The conductivity arising from the convection current only has been considered so far. We shall now determine the conductivity due to the displacement currents which can only be neglected for extremely low frequencies. If v is the sinusoidal potential difference across the anode and the cathode, and E the electric field intensity at a point in the electronic medium of dielectric constant ϵ inside the valve then the displacement current-density would be given by

$$i_d = \frac{\epsilon}{4\pi} \frac{dE}{dt} = -\frac{\epsilon}{4\pi} \frac{d}{dt}\left(\frac{dv}{dx}\right).$$

For low frequencies when all the electrons having velocities from u_0 to ∞ would reach the anode, there would be no space charge, so that the voltage-gradient at any instant would remain constant. We can therefore write

$$\frac{dv}{dx} = a. \sin \omega t,$$

where a is a constant and ω the angular frequency of the voltage.
Thus

$$i_d = \frac{\epsilon}{4\pi} \omega a \cos \omega t = A \cos \omega t \text{ where } A = \frac{\epsilon\omega . a}{4\pi}$$

Consider now a small cylindrical element of the current-path between the anode and the cathode. Let the length of the element parallel to the direction of the current be dl and the cross-section perpendicular to the current path be da. If σ_d is the specific conductivity due to the displacement currents, the power dissipation in this small element is given by

$$(i_d . da)^2 . \left(\frac{1}{\sigma_d} . \frac{dl}{da}\right) = \frac{i_d^2}{\sigma_d} (dl . da)$$

when averaged over one cycle, the power dissipation in the element is

$$dP = i_d^2 \cdot \frac{dl.\, da}{\sigma_d} = \frac{A^2}{2\sigma_d} \cdot (dl.\, da). \qquad \ldots \quad (4)$$

The power dissipation can also be written as

$$dP = \sigma_d \cdot \overline{E^2} \cdot dl.\, da = \sigma_d \cdot \frac{a^2}{2} \cdot (dl.\, da) \qquad \ldots \quad (5)$$

since $$E = -\frac{dv}{dx} = -a \sin \omega t.$$

Hence from (4) and (5)

$$\sigma_d = \frac{A}{a} = \frac{\epsilon \cdot \omega}{4\pi}. \qquad \ldots \quad (6)$$

We know, however, that the dielectric constant ϵ of the electronic medium increases with the increase of frequency. It was shown by Khastgir and Chowdhury[4] that the value of $(1-\epsilon)$ varied inversely as the square of frequency. Even if ϵ were regarded as constant, the conductivity σ_d due to the displacement currents would steadily increase with the increase of frequency. When the frequency exceeds the limit defined by (3), it should be remembered that the conductivity due to the convection current would no longer remain constant but would begin to decrease steadily with further increase of frequency. The experimental results on the conductivity of a valve for a wide range of frequencies can therefore be explained.

It will be interesting to obtain from (3) an approximate estimate of the frequency which corresponds to the limiting velocity u_0 of the electrons from the cathode. Let us take $\phi = 1$ volt for the oxide-coated tungsten cathode. If now the inter-electrode distance x be .3 cm, the frequency f_0 will be about .3 megacycles, *i.e.*, the wavelength will be about 30 metres. If $x = .5$ cm. the wavelength will be about 45 metres. Rao's experiments with a Telefunken valve showed that the turning point in the conductivity curve occurred at about 1600 *kc.*, *i.e.*, when the wavelength was about 180 metres. The observed turning point therefore appeared definitely at a lower frequency than what would be expected from (3). The equation $e\phi = \frac{1}{2}mu_0^2$ from which (3) is derived is, however, based on classical ideas. The results should be reviewed in the light of modern views of electron emission from metals.

DACCA UNIVERSITY

REFERENCES

1 L. Hartshorn, *Experimental Wireless and Wireless Engineer*, VIII Aug., 413 (1931).
2 Mitra and Sil, *Phil. Mag.*, xiii, 1081 (1932).
3 Kameswar Rao, *Ind. Jour. Physics*, June (1940).
4 Khastgir and Chowdhury, *Ind. Jour. Physics*, June (1940).