STUDIES IN ANTENNA RESISTANCE AND REACTANCE *

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ABSTRACT :—Measurements of antenna resistance and reactance were carried out by following Willans' double-beat method for a range of frequencies from about 330 kc/sec. to about 2300 kc/sec.

The theory of the method of measurements is described and the experimental results given in the paper. Measurements were made for three different types of receiving aerials *viz.*, (1) vertical aerial, (2) inverted L-aerial and (3) T-aerial, each being earthed. The effect of varying the length of the horizontal part of the last two aerials was also studied.

1. INTRODUCTION

The Antenna resistances and reactances of three different types of receiving aerials *viz.* (1) vertical aerial, (2) T-aerial and (3) inverted L-aerial were measured by Willans'[1] method for different radio frequencies within the range from about 2300 kc/sec. to about 330 kc/sec. and the results of these measurements are recorded in this paper. Willans' method had already been applied to a detailed investigation of the variation with frequency of the resistance and reactance of an earthed receiving aerial of approximate L-shape over a very wide range of frequencies by Wilmotte and Colebrook,[2] of the Radio Research Board, Great Britain. Antenna resistance measurements by other methods had also been carried out by many investigators, such as T. L. Eckersley,[3] Moullin,[4] Miller,[5] Smith-Rose and Colebrook[6] and others for different ranges of frequencies.

All the three types of receiving aerials employed in this investigation were grounded. In the case of the T-type and the L-type of aerials, measurements were also carried out to find the effect of the length of the horizontal parts of these aerials on the values of antenna resistance and reactance.

2. WILLANS' METHOD OF H. F. RESISTANCE MEASUREMENTS

The principle involved in the method of Willans is really an essential part of the method. The principle is the well known principle of transformer action

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e.g. when a current flows in a secondary circuit, the resistance and reactance of primary circuit apparently change.

Consider a secondary circuit having inductance L, resistance R and capacity C in the neighbourhood of an *oscillating* primary circuit having inductance L_1, resistance R_1 and capacity C_1. Let M be the mutual inductance between the primary and the secondary inductances and ω the angular frequency of the currents. If X_1, R_1 and X, R be the values of the primary and secondary reactance and resistance respectively without the secondary effect, then calling X' and R' the effective values of the primary reactance and resistance, we can write

$$R' = R + \gamma^2 R_1 \quad \ldots \quad (1)$$

and

$$X' = X - \gamma^2 X_1 \quad \ldots \quad (2)$$

where

$$\gamma^2 = \frac{\omega^2 M^2}{R^2 + X^2}$$

The change in the reactance means a change of the effective inductance of the oscillator (primary) coil which in turn causes a change in the emitted frequency of the oscillator. This change in the frequency of the oscillator is directly proportional to the change of reactance which is $\gamma^2 X_1$. The frequency-change is then given by

$$\Delta f \propto \frac{\omega^2 M^2}{R^2 + x^2} . X_1$$

Now it can be easily proved that for ωM = constant, the change in frequency is maximum, when $R^2 = X^2$ or when $R = \pm x$

i.e.

$$R = \left(\omega L - \frac{1}{\omega C'} \right)$$

or

$$R = \left(\frac{1}{\omega C''} - \omega L \right) \quad \ldots \quad (3)$$

Here $C' + C''$ are the values of the secondary capacity for the two alternative conditions for maximum change of frequency as given in (3). Combining now the two alternative conditions, we get

$$R = \frac{1}{2\omega} \left(\frac{1}{C''} - \frac{1}{C'} \right) \quad \ldots \quad (4)$$

3. GENERAL EXPERIMENTAL PROCEDURE IN WILLANS' METHOD

The change in the emitted frequency due to the change in the effective reactance of the oscillator circuit can be discerned as an alternation of the beat note in the telephones connected to an oscillating receiver. Let us consider three circuits (1), (2) & (3) as shown in fig. 1. Circuit (1) is the measuring

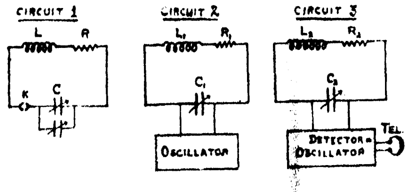

FIG. 1

circuit which comprises an inductance L, a resistance R, a suitable standardised variable condenser C and a switch K, circuit (2) is an oscillator circuit with an inductance L_1, a resistance R_1 and a suitable standardised condenser C_1. Circuit (3) is a detector-oscillator of the same type as the circuit (2) with an inductance L_2 a resistance R_2 and a variable condenser C_2. A pair of telephones is in the anode circuit of this detector oscillator. When the frequencies of the two oscillators are suitably adjusted, it is possible to hear the heterodyne beat-note in the telephones.

The experimental procedure is as follows: With the switch of circuit (1) open, the frequency of (3) is first adjusted to the desired value and the frequency of (2) is adjusted by varying C_1 till no beat-note is heard. With the circuit (1) closed, the condenser C varied till the beat-note again disappears; *i.e.* the frequency of (2) is back to its original value. This gives C_r the resonance value of C. Next displacing C by a small amount the beat-note is again heard due to a change in the frequency of (2). C_1 is now changed till the beat-note disappears. If the process is repeated for other positions of C, it will be found that the variation of C_1 required to balance the effect of variation of C is of the same form as the variation of frequency shown in fig. 2. Here the ordinate

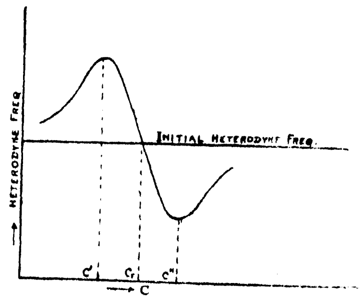

FIG. 2

represents the heterodyne frequency (which depends on the value of C_1) and abscissa, the value of C. Two peaks are observed on both sides of the C-value which corresponds to the original frequency and these peaks appear for two different values of C *viz.*, C' & C''. The H.F. resistance R of the measuring circuit (1) can then be obtained from equation (4).

The curve cuts across the horizontal line representing the capacity of C_1 corresponding to the initial frequency of (2), at a certain value of the capacity C which is the same as the resonance capacity C_r.

4. EXPERIMENTAL TECHNIQUE IN THE PRESENT INVESTIGATION

The complete arrangement of the apparatus is shown in fig. 3. The oscillators were of the Hartley type. A pair of telephones was inserted in the

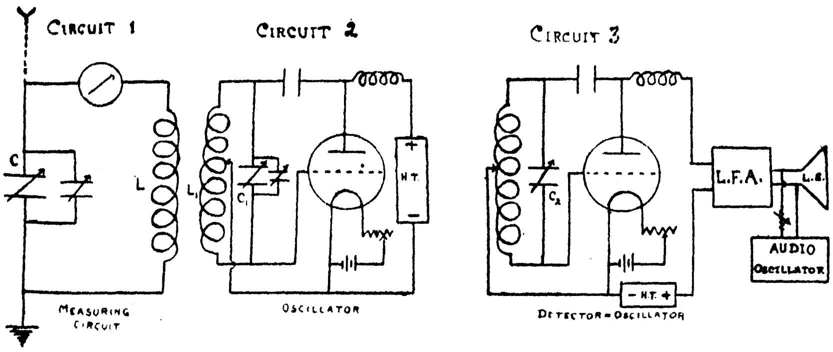

FIG. 3

anode circuit of the detector-oscillating valve. When the frequency of each was adjusted, it was possible to hear the heterodyne whistle. The L.F. e.m.f. developed across the telephone was then fed into a 3-valve amplifier of the conventional type. A loudspeaker was connected to the output of the amplifier. Through this loudspeaker was passed an audio-frequency current from the secondary circuit of an audio oscillator of a suitable constant frequency. A variable resistance was inserted to adjust the amplitude of this audio-frequency current so that the heterodyne whistle and the sound due to the audio-frequency current from the audio-oscillator could be made of the same order of intensity. As a result of the superposition of these two audio-frequency currents through the loudspeaker, beats could be heard. The tuning condenser C_1 or C_2 of either oscillator could be adjusted till no beats were heard. This double-beat technique increased the accuracy of the condenser settings to a considerable extent.

All the condensers were properly shielded and calibrated very accurately. A mercury-link was included in the measuring circuit. Usually more than one condensers fitted with slow-motion dials were used in parallel both for C and C_1. Long glass rods fixed to the condenser knobs were employed for turning the condensers.

5. MEASUREMENT OF THE ANTENNA RESISTANCE AND REACTANCE AND THE THEORY OF THE METHOD

First, the measuring circuit consisted of an inductance L and a standardised variable capacity C. The H.F. resistance R of this circuit was measured by the method of Willans. The resonance value of C was C_r. Next, the aerial under investigation was connected to one end of the inductance and the other end was put to the earth. Similar procedure was followed to measure the equivalent resistance R′ of the complex circuit. The resonance capacity of the variable condenser in this case was C'_r which was different from C_r.

Measurements of R and R′ were made successively and the coupling between the coil of the oscillator and the coil of the measuring circuit was kept fixed for each frequency.

The theory of the method is indicated below. If R_e be the effective antenna resistance, it can be shown that

$$R' = R_e + \frac{R}{\omega^2 LC'_r}, \quad \text{since } \omega RC'_r << (1 - \omega^2 LC'_r).$$

Therefore $$R_e = R' - \frac{R}{1-\omega^2 LC'_r} = R' - \frac{R}{1-\omega^2 LC_r\left(\frac{C'_r}{C_r}\right)}.$$

Since $\omega^2 LC_r = 1$, we can write

$$R_e = R' - R.\frac{C_r}{C_r - C'_r} \qquad \ldots (5)$$

The effective reactance X_e of the antenna can be obtained in the following manner. If X′ be the reactance of the complex circuit when the antenna is connected in the manner shown in fig. 3, the condition of resonance is given by $X_e + X' = 0$. Under the conditions of the experiment, it can be shown

$$X' \approx \frac{\omega L}{1-\omega^2 LC'_r} \approx \omega L.\frac{C_r}{C_r - C'_r},$$

so that the antenna reactance will be given by

$$X_e = -\omega L.\frac{C_r}{C_r - C'_r} \qquad \ldots (6)$$

The values of the antenna resistance and reactance were obtained with the help of (5) and (6).

6. EXPERIMENTAL RESULTS—VARIATION OF ANTENNA RESISTANCE AND REACTANCE WITH FREQUENCY

The results of measurements for the earthed vertical aerial, the inverted L-aerial and the T-aerial are given in figs. 4, 4(*a*), 5, 5(*a*) and 6, 6(*a*).

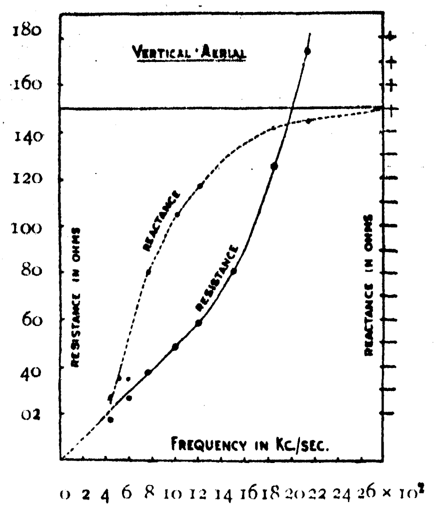

FIG. 4

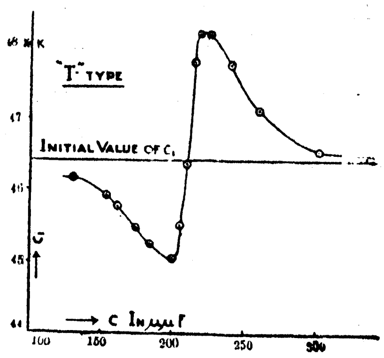

FIG. 5(a)

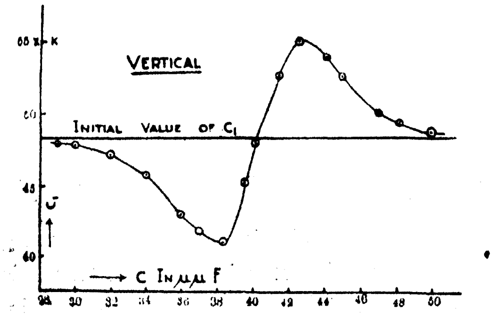

FIG. 4(a)

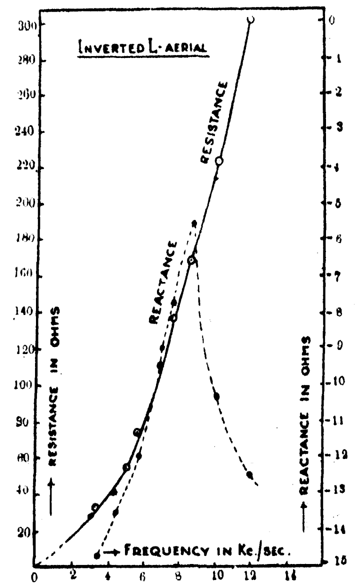

FIG. 6

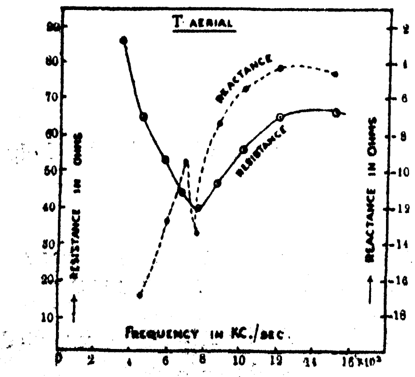

FIG. 5

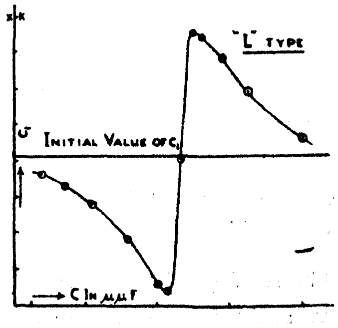

FIG. 6(a)

7. VARIATION OF ANTENNA RESISTANCE AND REACTANCE WITH THE LENGTH OF THE HORIZONTAL PART OF THE T-AERIAL AND THE L-AERIAL

The experiments on the variation of the antenna resistance and reactance with the horizontal top length were carried with freq : 845 kc/sec. ($\lambda = 355m$). The results of measurements with a "T"-aerial are given in table IV.

TABLE IV

Earthed T-aerial : Frequency : 845 kc/sec. ($\lambda = 355m$).

Top length (metres)	Without aerial				With aerial				R_e (ohms)	X_e (ohms)
	C' ($\mu\mu f$)	C'' ($\mu\mu f$)	C_r ($\mu\mu f$)	R (ohms)	C' ($\mu\mu f$)	C'' ($\mu\mu f$)	C'_r ($\mu\mu f$)	R' (ohms)		
11.0					292	350	319	54.2	41.8	-7.01×10^2
10.5					305	365	332	50.46	37.4	-7.36 ,,
10.0	579	600	589	5.7	312	370	339	46.9	33.5	-7.57 ,,
9.5					312	366	344	43.45	29.7	-7.72 ,,
8.5					328	382	354	39.8	25.5	-8.05 ,,

Similar experiments were performed with an L-aerial of different horizontal top-lengths for a definite frequency. The results of measurements are given in table V. These are illustrated in fig. 7.

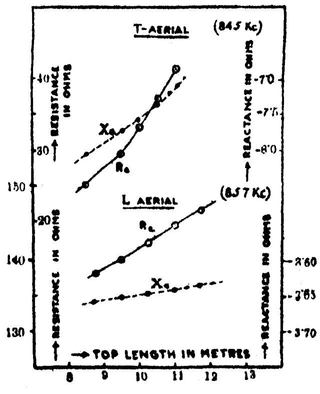

FIG 7

TABLE V

Earthed L-aerial (inverted). Frequency : 857 kc/sec. ($\lambda = 350m$).

Top length (metres)	Without aerial				With aerial				R_e (ohms)	X_e (ohms)
	C' ($\mu\mu f$)	C'' ($\mu\mu f$)	C_r ($\mu\mu f$)	R (ohms)	C' ($\mu\mu f$)	C'' ($\mu\mu f$)	C'_r ($\mu\mu f$)	R′ (ohms)		
11.75					123	150	137	150.2	146.6	-2.885×10^2
11.0					124	151	138	148.2	144.6	−2.888 ,,
10.25	778	788	783	3.0	125	152	139	145.9	142.25	−2.893 ,,
9.5					126	153	140	143.5	139.85	−2.897 ,,
8.75					127	154	141	141.7	138.0	−2.902 ,,

8. DISCUSSION OF THE EXPERIMENTAL RESULTS

In the case of the earthed vertical aerial and the earthed inverted L-aerial, the effective antenna resistance R_e was found to increase steadily with frequency as can be seen from figs. 4 & 6. The smallest wavelength in the investigation was however larger than the natural fundamental wavelength of the antenna.) In the case of the T-aerial, on the other hand, the resistance at first decreased steadily with the increase of frequency and attained a minimum value but subsequently it increased with further increase of frequency. The minimum value of the antenna resistance appeared at 750 kc/sec. The existence of such a minimum in the value of antenna resistance had also been previously observed by earlier investigators.

There had been attempts also to explain the nature of the variation of the antenna resistance with frequency. An approximate theory for the case of a perfectly conducting straight vertical wire connected to homogeneous earth with a horizontal surface, had been worked out by Eccles.[7] It had been shown that if R is the antenna resistance at any wavelength λ, then

$$\frac{R_0 - R}{R_0} = \frac{3\lambda_0(\lambda - \lambda_0)}{\lambda^2},$$

where λ_0 is the natural wavelength (fundamental) of the antenna and R_0 is the antenna resistance at that wavelength. The equation shows that if the resistance R be plotted against frequency, the curve is a parabola. It can be seen that the minimum resistance is $R_0/4$ and occurs at $\lambda = 2\lambda_0$ and that when $\lambda = 3\lambda_0$, the curve is straight and directed through the origin. The resistance is $R_0/3$ for $\lambda = 3\lambda_0$. This ideal curve was, of course, departed in many practical cases. According to Colebrook's[8] theory, the antenna resistance of a vertical or an L-aerial should vary much in the same way as in our experiments. The observed variation of the antenna resistance of a T-aerial resembled Eccle's theoretical curve to a certain extent.

Referring to the antenna reactance of a vertical earthed aerial, it can be seen from fig. 4 that its value was negative and that it increased steadily with frequency, tending to pass through the zero value. In the case of the earthed inverted L aerial, the antenna reactance (which was also negative) increased with frequency and in the region of 900 kc/sec., it suddenly began to decrease with the increase of frequency, as is evident from fig. 6. In the case of the T-aerial, the antenna reactance was found to fall suddenly in the frequency region of minimum antenna resistance; otherwise, *on the whole*, the antenna reactance showed an increase with the increase of frequency. (See fig. 5)

Regarding the variation of the antenna resistance and reactance with the horizontal top-length of the T-aerial or that of the inverted L-aerial for a definite frequency, the experimental results showed that both resistance and reactance increased steadily with the length of the horizontal part. (See fig. 7.)

9. SUMMARY

Experiments to study the variation of the total-loss-resistance and reactance of receiving aerials with frequency were carried out following Willans' double-beat method of measuring H.F. resistance. Three different types of aerials were employed *viz.* the vertical aerial, T-aerial and the inverted L-aerial each being grounded. It appears that in the case of the vertical aerial and the inverted L-aerial, the antenna resistance steadily increased with frequency. The range of frequencies did not however include the natural fundamental frequency of the aerial or its harmonics. In the case of the T-aerial a definite minimum for the antenna resistance appeared to exist within the range of frequencies employed in these measurements.

The antenna reactance was found, in general, to increase with the increase of frequency. In the case of the inverted L-aerial the rise was followed by a sudden decrease with the increase of frequency at a certain frequency. In the case of the T-aerial, there appeared a sudden diminution in the value of the antenna reactance in the region where the minimum antenna resistance was observed.

The experiments also showed that the antenna resistance and reactance of a T-aerial and an inverted L-aerial increased with the increase of the top-length of the horizontal part of each aerial.

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