MEASUREMENT OF THE ANGLE OF INCIDENCE AT THE GROUND OF DOWNCOMING SHORT-WAVES FROM THE IONOSPHERE

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Plates VI and VII

ABSTRACT. Measurement of downcoming angles for a number of European short-wave stations have been made. The apparatus consists of two commercial superheterodyne receivers with two parallel horizontal dipole aerials for picking up energy. Downcoming angle is inferred from the phase difference between the voltages on the two aerials.

Average values of the downcoming angles for the B.B.C. transmitters during the months of May, June, September and November, 1940, came out to be 16°; 20°.6; 15°.2 and 14°.2 respectively.

The average values obtained for the German stations for the months of May, June and November, 1940, came out to be 20°.8; 19° and 15°.

INTRODUCTION

A knowledge of the angle of incidence at the ground of short-waves propagated *via* the ionosphere is essential for the efficient design of both the receiving and the transmitting aerials, for point-to-point working in particular and broadcasting in general.

For most efficient operation the transmitting aerial, besides being orientated to radiate maximum energy along the great circle path joining the transmitter and the receiver, should also concentrate the energy so that the angle of elevation above the horizontal of the radiated beam is most favourable for reception at the receiving point. Energy radiated at other than the correct angle is wasted in so far as the receiving point is concerned. A consideration of the reciprocity theorem will show that the optimum angle of transmission will be the same as the downcoming angle measured at the receiving point. Likewise the receiving aerial should also be orientated in the correct horizontal direction and designed to receive maximum energy from the correct angle in the vertical plane. This will help to discriminate against atmospherics and other kinds of interfering signals, besides building a strong signal to overcome receiver noises, etc.

Determination of the downcoming angles is important to the physicist engaged in the study of the ionosphere. This information provides him a means of experimentally verifying his theories of electromagnetic wave propagation *via* the ionosphere. Determination of the height of the ionosphere can also be made from a knowledge of these angles of incidence at the ground.

The set-up described in this article is what is usually termed the "phase measurement method," and differs from conventional methods[1] in that it makes use of two ordinary commercial superheterodyne receivers (having R. F. sensiti vity control) adopted for the experiment with the following modifications:

(1) The internal oscillators of the two receivers are made inactive.

(2) A common external local oscillator is used as a heterodyning oscillator for either.

No great care is exercised to have exact phase symmetry between the two receivers. The individual tuning control adjusts the tuning of the R. F. stages and thus controls the phase. This control can be used for phase compensation. The local oscillator is used for tuning in the required signal.

Such an arrangement is relatively easy to set up and requires no elaborate components and instruments for making special receivers, exactly matched for phase and gain.

THEORY

The basic principle underlying the method is that when radio waves from a certain direction arrive at two receiving aerials separated by a known distance the phase angle ϕ between the induced voltages is :

$$\phi = \frac{2\pi}{\lambda} d \sin\theta \sin\Delta \quad \text{radians,}$$

where d = the spacing in metres between the two receiving aerials set up parallel to each other ;

λ = the wavelength in metres of the waves under investigation ;

θ = the angle of incidence (measured from the vertical) of the down-coming waves ; and

Δ = the angle between the Azimuth of the transmitting station and the direction of the receiving aerials.

The signals from the two receiving aerials are amplified independently after being heterodyned with a common heterodyne oscillator (to ensure that proper phase relationships are maintained) and are then applied to the plates of a cathode-ray oscillograph tube where they give rise, in general, to an elliptical trace. It is possible from this pattern to calculate the relative phase difference between the two E.M.F's applied to the C.R.O. plates. Knowing this value of phase difference and granting that no extra phase difference has been introduced by the associated circuits, it is a simple matter to calculate the angle of incidence to the ground of the downcoming waves. A mathematical note, showing that when two E.M F's with a phase difference are heterodyned with a common heterodyning oscillator, they retain their original phase difference, is given in Appendix I. Appendix II presents complete mathematical discussion for the interpretation of the ellipse.

EXPERIMENTAL SET-UP

An idea of the experimental set-up can be obtained from photographs Nos. 1 and 2 (Plates VI and VII) and the lay-out diagram shown in Fig. 1.

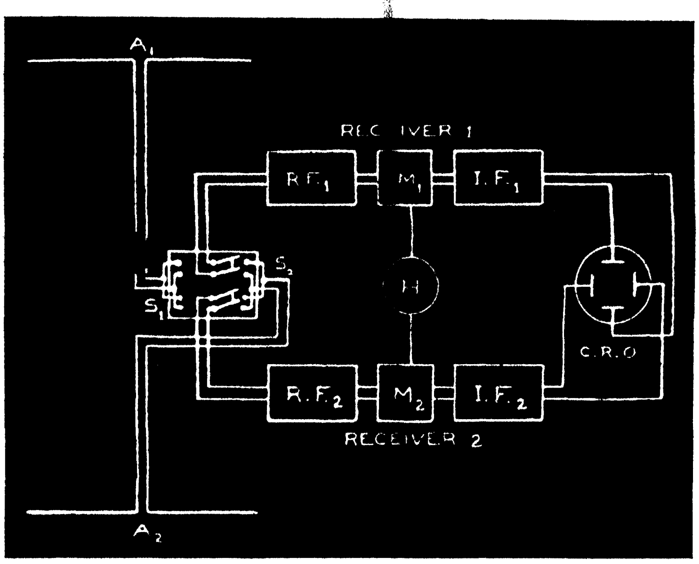

FIG. 1

A_1 and A_2 are 12 ft. high horizontal receiving dipole aerials separated by a distance '*d*' metres (approximately 1.2λ) and cut for approximately half wave operation on 19 metres band. Care is taken that the two receiving aerials are exactly symmetrical. The aerials are joined to the apparatus located in a nearby wooden hut by means of two equal lengths of twisted flex. S_1 and S_2 are D.P.D.T. knife switches (suitably shielded to avoid spurious pick-up). RF_1 and RF_2 are R.F. stages of the two commercial receivers, local oscillators of which have been made inactive. Their mixers stages M_1 and M_2 are fed in parallel from a common heterodyne oscillator H. In this arrangement the two receivers were Hammerlund Super-pro model SP-110S which have a sensitivity of 5 to 2 μv. for 601BS signal to noise ratio and the common heterodyne oscillator was the oscillator section of an R.C.A. receiver type 8T, its mixer stage having been disconnected. IF_1 and IF_2 are I.F. amplifiers working at the intermediate frequency (456 kc/s) in order to amplify the signals to a level high enough to give a suitable deflection on the cathode-ray oscillograph marked C.R.O.

PROCEDURE

Preliminary Experiments and Adjustment :—

The D.P.D.T. switches S_1 and S_2 are used to enable any one of the two receivers to be connected to any of the aerials separately or together.

With any one of the aerials connected to both the receivers in parallel, the common heterodyne oscillator H and the ganged R.F. stages of the receivers could be tuned to receive any desired transmission. The tuning and the R.F. gain-control of the receivers are then manipulated such that the trace on the C.R.O. screen becomes a line inclined at 45° to the horizontal and the vertical axes. Under these conditions the two receivers have been adjusted so that the phase retardation in one compared to that in the other receiver is a whole number of complete cycles or is zero. All that is then necessary is to put the two receivers on independent aerials and record the resultant ellipse appearing on the C.R.O. screen, either photographically or by making a tracing, though the latter will be only approximate unless the signals are very steady and no fading is present. For preliminary observations ordinary bromide photo-printing paper can be exposed directly on the C.R.O. screen when a contact print of the desired ellipse pattern will appear on the paper after development, or the pattern can be recorded by the help of an ordinary camera using rapid negative material. The ellipse pattern is then used for calculating the required phase difference. Under proper working conditions the ellipse will have its major axis inclined at 45° to the oscillograph axes and will maintain its axis and eccentricity even if the aerials are changed over by operating switches S_1 and S_2. Under such conditions the angle subtended by the minor axis at the ends of the major axis will give the desired phase angle between the two component e.m.f's which produced the ellipse (see Appendix II). Under improper conditions both the major axis at which the ellipse appears on the C.R.O. screen and its eccentricity may change on reversing the switches S_1 and S_2.

Such an improper condition when it exists is found to be mainly due to the coupled impedance thrown back by the tuned secondaries of the input transformers into their primaries being different in the two cases, with the result that when the receivers are put in parallel to work from the same aerial in order to be aligned for zero phase difference in the two limbs the impedances alter and become unsymmetrical as soon as the receivers are put on the two aerials independently. In the arrangement set up this effect was made negligible by connecting two very low resistances (about 1 ohm each) across the input of the two receivers. This helped to keep the input impedance in both the cases steady and resistive. The reduced gain of the receivers was compensated for by additional I.F. amplifiers IF_1 and IF_2.

For any measurement two successive photographs were taken by interchanging the aerials connected to the two receivers. The mean of two such measurements was taken as the final result. This procedure helped to further reduce the error introduced by the changes in the receiver input impedance.

BEHAVIOUR OF THE PATTERNS APPEARING ON THE C.R.O. SCREEN

Under actual working conditions it was observed that during the day signals from long distance broadcasting stations such as B.B.C., Berlin, Moscow, etc., rarely give ellipses which are steady and useful for downcoming angle measurements. The ellipses are often seen to rotate, change their size, shape and eccentricity in a random manner, the rapidity of rotation and the agitation of the ellipses being greatest on the magnetically disturbed days. Ellipse patterns were in general found to be steady in the nights and before dawn in the mornings, when the ionisation levels in the F and the E layers are relatively low. Long distance transmitters towards the east of Delhi were generally observed to show a tendency to give steadier ellipses soon after sunset, and those to the west of Delhi in the early morning up to sunrise. Ellipse patterns from short distance short-wave stations at Delhi, Bombay, Madras and Calcutta were seldom observed to be steady either in the mornings or in the evenings up to 11 P.M. The ellipse patterns from these stations were generally more highly agitated than those obtained from long distance short-wave stations, with the result that measurements on the Indian short wave stations were not always possible.

A study of the mathematical note (Appendix II) will show that this behaviour of the ellipse patterns is to be ascribed to simultaneous arrival of energy along more than one path, with the result that the resultant direction of arrival is changing with the fading of the signals and interference between the multiple rays arriving at the receiving aerials. An experimental verification of this theory to some extent was obtained by observing the ellipse patterns by using two special diamond aerials directed towards B.B.C. which are highly directional both in the horizontal and in the vertical planes. Since the angular spread from which energy can be received in this case was considerably limited as compared to the ordinary dipole aerials the ellipse patterns from B.B.C. transmitters were seen to be relatively steadier in the case of B.B.C. diamond aerials. The limited angular spread from which energy could be received in the case of B.B.C. diamond aerials helped to cut out some of the energy arriving in multiple paths with the result that effects due to interference became less acute.

RESULTS

The apparatus was ready for making angle measurements on the 20th May, 1940, when a direct exposure was made on the C.R.O. for obtaining an ellipse pattern for GSF signals at 10 P.M. The angle of arrival of GSF waves as obtained from this photograph came out to be 16° to the horizontal. This result has been more or less confirmed by later observations and measurements. A typical direct photograph is shown in Fig. 1 (Plate VI), from which it will be observed that the ellipse pattern is a dark patch on a white background. This is because of the fading of the signal which causes the pattern to vary from zero amplitude to the maximum marked by the contour of the dark ellipse.

It must be remarked here that, with such a crude method as the one used in the earlier period of experimentation, successful photographs as the one shown in Fig. 1 (Plate VI) are subject to a certain amount of practical skill and the conditions prevailing at the time the recordings were made. In case the signals are suffering from bad fading and the exposure is made during the time that the ellipse is going through its worst agitation, a very disappointing result may be obtained, which will be utterly useless from the point of view of any measurements. Subsequent measurements were therefore made by taking instantaneous recordings with the help of an ordinary camera. A typical set of such camera recordings is shown in Figs. 2, 3, 4, 5 and 6 (Plate VI).

Preliminary adjustment for no phase difference in each of these six cases was a straight line making an angle of 135° with the positive direction of X-axis. The phase difference as shown below the photograph is the angle subtended by the minor axis at the end of the major axis. Actual phase difference will, however, be given by the following consideration :—

If the ellipse lies in the 2nd and the 4th quadrant, the phase difference can be ϕ, $360 \pm \phi$, $2 \times 360 \pm \phi$, etc., where ϕ is less than 90°.

On the other hand, for the ellipse to lie in the 1st and 3rd quadrant the phase can be $180 \pm \phi$, $540 \pm \phi$, etc.

By looking at the photograph and knowing the value of the maximum possible phase difference $(2\pi d/\lambda)$, the most probable value of the additional figure can be easily picked out. Thus, having found out the actual phase difference, the value of θ can be calculated.

It may be further added that the major axis of the ellipse in Figure 1 was actually at an angle of 135° with the positive direction of X-axis. It has been rotated through an angle of 45°, thus making the major axis appear vertical.

At this stage it was considered advisable to devise methods for experimentally checking up the behaviour of the apparatus. One of the easiest and the most reliable mothods that lent itself for this purpose was the use of the ground wave signals from the Delhi short-wave transmitters at a time when the ionisation levels in the ionosphere were low enough not to return any energy incident vertically. Under such conditions, which were mostly confined to the night after 11 P.M. or the early morning before 8 A.M., in the months of June-July, 1940, ellipses were found to be sufficiently steady to permit of good photographic records or tracings to be made. As the tall Delhi masts supporting the short-wave transmitting aerials were visible at the Todapur Receiving Centre of A.I.R. (where these experiments were conducted), the angle between the direction of the receiving aerial and that of the Delhi short-wave transmitting aerial could be measured with a high degree of accuracy with the help of a theodolite. Knowing the wavelength of the Delhi transmission used to develop any ellipse patterns, and the fact that wave propagation is horizontal, together with the angle measured between the direction of the receiving aerials and the Delhi transmitting aerial, the expected phase difference could be calculated and then checked with the phase difference obtained from the ellipse pattern appearing on the C.R.O. screen.

A typical ellipse pattern obtained with the use of the Delhi transmissions in the 19 metre band is shown in Fig. 2 (Plate VI), from which the measured phase difference comes out to be 74°. Phase difference calculated from the geometry of the set-up was found to be $77^\circ.3$ which is considered to be a fairly close agreement.

It was found that the average value for the downcoming angle in the case of B.B.C. transmitters during the months of May, June, September and November, 1940, came out to be 16°, $20^\circ.6$, $15^\circ.2$, and $14^\circ.2$ respectively.

The average values obtained for the German stations for the months of May, June and November, 1940, came out to be $20^\circ.8$, 19° and 15°.

The results, as will be apparent, indicate a general lowering of the F_2 layer height during the winter evenings as compared to the summer evenings.

Variation in the measured values :—

During the course of these observations it was noted that the results varied by as much as $\pm 20\%$ even on the same evening. This variation has to be ascribed partly to the well-known fact that the rays from any particular station do not come as single rays but have a certain spread in the horizontal and the vertical planes, the angular spread being more in the latter case. Results are further affected by fading, which is considered to be the chief factor contributing to such a wide variation as $\pm 20\%$.

CONCLUSION

As the photographs were made by exposure of the film at the moment that the ellipse reached its maximum size they represent the main ray, and the direction inferred from that ellipse refers to the direction of the principal ray. This is particularly true in the case of the contact photographs in which measurements can only be made from the contour of the dark patch. Hence under conditions of measurement the subsidiary ray must have either faded out completely or so changed in polarization as to give no effect on the aerials. The direction of arrival of the subsidiary rays cannot be inferred from these experiments.

For finding out the direction of arrival of other rays than the principal, it will be necessary to have a series of photographs of the changing ellipse at small known intervals and to analyse them so as to find out what combination of rays can give the kind of change observed.

This method, however, on the face of it is very complex and may not give very definite or easily interpretted results.

If, however, the transmitter gives short-duration pulses, each subsidiary ray will arrive at an interval of time different from that of others and will give a separate ellipse of its own. There will then appear ellipses equal in number to the number of rays present. These ellipses will represent the directions of arrival and intensity of the rays by their eccentricity and size respectively. The pulse method of observation is therefore more promising for yielding useful results relating to the more complex propagation conditions in the ionosphere.

ACKNOWLEDGMENTS

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APPENDIX I

Let $A_1 \sin \omega t$ and $A_2 \sin (\omega t+\varphi)$ be the signals applied to the two mixers, respectively in the two receivers.

Suppose further that the signal introduced by the local oscillator is :

$$B \sin (\gamma t+\xi).$$

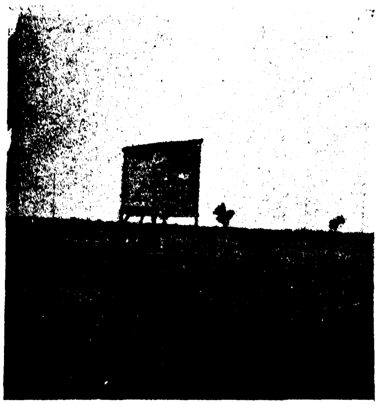
Now, the mixer characteristic may be taken of the general form :

$$i=a_0+a_1 e+a_2 e^2$$

where i = output current ;

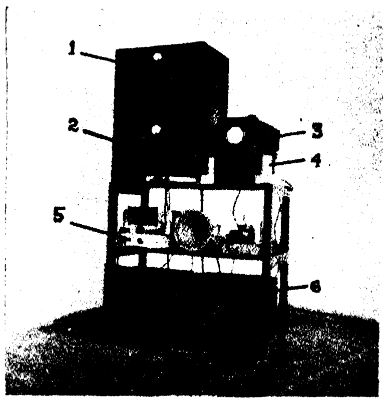
e = input voltage ;

and a_0, a_1 and a_2 are constants.



(1)

Aerial system and receiving hut
at
Toda-Pur.



(2)

1. Receiver No : 1
2. Receiver No : 2
3. Cathode Ray Oscillograph
4. Additional I.F. Amplifier
5. Common Hetrodyning Oscillator
6. Power supplies.

For mixer No. 1, the output current i_1 may therefore be written as :

$$i_1 = a_0 + a_1 \ (A_1 \sin \omega t + B \sin \overline{\gamma t + \xi})$$
$$+ a_2(A_1 \sin \omega t + B \sin \overline{\gamma t + \xi})^2.$$

This after simplification may be written as :

$$i_1 = a_0 + a_2/2(A_1^2 + B^2) + a_1 A_1 \sin \omega t + a_1 B \sin \overline{\gamma t + \xi}$$
$$- a_2 A_1/2 \cos 2\omega t - a_2 B/2 \cos 2(\gamma t + \xi)$$
$$+ a_2 A_1 B \cos \{(\gamma - \omega)t + \xi\} - a_2 A_1 B \cos \{(\gamma + \omega)t + \xi\}.$$

This expression shows that the output of the mixer contains (1) the fundamental frequencies ; (2) frequencies of twice the value of the fundamentals ; (3) sum and difference of the fundamentals. As the output of the mixer is tuned to the intermediate frequency which is equal to the difference of the fundamental frequencies then the only term of importance (and which is further amplified) is :

$$a_2 A_1 B \cos \{(\gamma - \omega)t + \xi\}. \qquad \ldots \quad (1)$$

Similarly it may be shown that the I.F. output of the 2nd mixer is :

$$a_2 A_2 B \cos \{(\gamma - \omega)t + \xi + \phi\}. \qquad \ldots \quad (2)$$

It will be seen from (1) and (2) that the phase difference of the intermediate frequencies produced is ϕ, which is the phase difference between the original R.F. signals.

APPENDIX II

Intrepretation of the Ellipse forms appearing on the CRO in the experiment of "Measurement of the Angles of Down-coming Rays."

SUMMARY

It is shown that for $E_h = E_r$, the ellipse will not precess but only change its ellipticity with changing values of the phase difference ϕ. If, however, E_r does not remain equal to E_h, the ellipse will rotate. It will rotate continuously for variable E_r/E_h, and changing ϕ. Further the ellipse may not precess in case of a single ray but will do so when rays more than one are present simultaneously.

(I)—Ellipse formation and its interpretation.

Let us confine ourselves, in this section, only to what happens in the CRO itself.

A sinusoidal voltage $E = E_h' \sin \omega t$ is applied to the plates, which produces deflection in the horizontal (X-axis) direction. The deflection produced is proportional to E. Hence at any time (t) displacement (x) of the beam from the centre is given by

$$x \propto E_h' \sin \omega t$$

or

$$x = kE_h' \sin \omega t = E_h \sin \omega t \qquad \ldots \quad (1)$$

where k is a constant of the CRO and its working voltages and $kE_h' \equiv E_h$.

A voltage of the same frequency ($\omega/2\pi$) is also applied to the vertically deflecting plates. This second voltage may have a different amplitude E_r' and phase ϕ; hence the vertical deflection

$$y = k'E_r' \sin(\omega t + \phi) = E_r \sin(\omega t + \phi) \qquad \ldots \quad (2)$$

where k' may be different from k of (1) and $k'E_r' \equiv E_r$.

Actual path traced by the spot will be determined by the combined action of (1) and (2).

Eliminating (ωt) from relations (1) and (2), we have

$$y = \frac{xE_r}{E_h} \cos\phi + E_r \sin\phi \sqrt{1 - x^2/E_h^2}$$

Transposing, squaring and simplifying, we get

$$\frac{x^2}{E_h^2 \sin^2\phi} + \frac{y^2}{E_r^2 \sin^2\phi} - 2\frac{xy\cos\phi}{E_r E_h \sin^2\phi} = 1. \qquad \ldots \quad (3)$$

Comparing (3) with the standard equation of the ellipse

$$ax^2 + by^2 + 2hxy = 1,$$

we find that the curve appearing on the CRO will be an ellipse. If ψ is the angle made by the direction of the major axis with the X axis, then

$$\tan 2\psi = \frac{2h}{a-b} \; *$$

which for equation (3) after simplification becomes

$$\tan 2\psi = 2\Big/\left(\frac{E_h}{E_r} - \frac{E_r}{E_h}\right) \times \cos\phi. \qquad \ldots \quad (4)$$

If α and β are the semi-major and the semi-minor axes respectively, then

$$\alpha/\beta + \beta/\alpha = \frac{a+b}{\sqrt{ab-h^2}} \; \dagger$$

$$= \left(\frac{E_h}{E_r} + \frac{E_r}{E_h}\right) \times \frac{1}{\sin\phi}. \qquad \ldots \quad (5)$$

Case I. Let $E_r = E_h$; *i.e.*, amplitudes of the horizontal and vertical deflections be equal. From equation (4) we get

$$\tan 2\psi = \infty \text{ or } \psi = 45^\circ.$$

And equation (5) gives

$$\alpha/\beta + \beta/\alpha = 2/\sin\phi.$$

* 'Conic Section' by C. Smith, p. 227

† 'Conic Section' by C. Smith, p. 240

Solving this quadratic equation for α/β, we get

$$\frac{\alpha}{\beta} = \tan \phi/2 \text{ or } \cot \phi/2$$

or ϕ is the angle subtended at the end of one axis by the other axis.

Thus the direction of the axes is fixed to be along 45° and 135° lines but the ellipticity depends upon the phase difference ϕ of the component oscillations.

If $\phi = 0$, the ellipse reduces to a st. line along the 45° axis [$x = y$ from (3)].

If $0 < \phi < 90^\circ$, the ellipse lies in 1st and 3rd quadrants and changes from a st. line to a circle.

For $\phi = 90^\circ$, the ellipse becomes a circle.

For $90^\circ < \phi < 180^\circ$, the major axis lies along the 135° axis and the ellipse collapses from a circle to a st. line [$x + y = 0$ from equation (3)].

To change the major axis from the 45° axis to 135° axis, and for the ellipse to have the same shape, ϕ should change to $180 \pm \phi$.

It will be observed that to deduce the value of ϕ from the shape of the ellipse ϕ is always the angle subtended by the axis of the ellipse along 135° at the end of the axis along 45°, provided the adjustment for $\phi = 0$ was a st. line along 45° (as assumed already).

Also, it is to be seen that the ellipse for $E_v = E_h$ does not rotate but passes through a circle to the other axis, direction of the axes being always confined to 45° and 135° lines.

Case II. E_v and E_h are unequal, E_v/E_h being supposed to be constant, then

$$\tan 2\psi = 2 \cos \phi \Big/ \left(\frac{E_h}{E_v} - \frac{E_v}{E_h} \right) \qquad \dots (4)$$

and

$$\alpha/\beta + \beta/\alpha = \left(\frac{E_h}{E_v} + \frac{E_v}{E_h} \right) \times \frac{1}{\sin \phi} \qquad \dots (5)$$

and the ellipse will rotate as well as change shape with changing values of ϕ.

Let $\phi = 0$,

then $\tan 2\psi = \dfrac{2E_v E_h}{E_h^2 - E_v^2}$ or $\tan \psi = E_v/E_h$.

and $\alpha/\beta + \beta/\alpha = \infty$ $\therefore \beta = 0$ and α finite,

showing that the ellipse is a st. line ; the line is inclined at an angle $\tan^{-1}(E_v/E_h)$ to the X-axis. As ϕ increases from 0 to 90°, $\cos \phi$ decreases from 1 to 0, ψ decreases from the value $\tan^{-1}(E_v/E_h)$ to 0, and the ratio of β/α increases to a maximum, *i.e.*, to E_v/E_h from 0. Thus with the phase difference ϕ increasing from 0 to 90° the ellipse rotates from the direction $\tan^{-1}(E_v/E_h)$ to the direction of X-axis, at the same time changing in shape from a st. line to ellipse with maximum β/α. As ϕ increases from 90° to 180°, $\cos\phi$ is negative and changes from 0 to -1. ψ will also be negative and change to $(-\tan^{-1}\overline{E_v/E_h})$ from 0. Thus for constant value of E_v/E_h the ellipse rocks about the X-axis through the limits $\pm\tan^{-1}(E_v/E_h)$ and at the same time changes shape so as to be st. line in the extreme positions and fattest ellipse (maximum β/α) in the position $\psi=0$.

Case III. E_h/E_v variable, but ϕ constant.

For E_h/E_v changing from ∞ to 1, $\tan 2\psi$ changes from 0 to ∞, or ψ changes from 0 to 45°.

For E_h/E_v changing from 1 to 0, ψ changes from 45° to 90°. If then ϕ changes by 180° and the change in E_h/E_v continues, the ellipse will undergo a complete rotation.

To recapitulate we have arrived at the following results :—

(1) If the ellipse has the directions of its axes fixed to 45°-135° axes, but it changes shape from a circle to a st. line along 45° or 135° axes, or *vice versa*, the amplitudes of the component oscillations must be equal but the relative phase changes and can be deduced by measuring the angle at the end of the axis along 45° subtended by the axis along 135°.

(2) If the direction of the major axis rocks about the X-axis, the ellipse becoming fattest when the major axis is coincident with X-axis and is a st. line at the maximum deviations from X-axis direction, then E_v/E_h is constant but ϕ is changing values.

If the ellipse rotates continuously, say, in the anti-clock wise direction, then E_h/E_v is constantly decreasing when the ellipse is in the 1st and 3rd quadrant; the phase difference ϕ may also be changing. Holding ϕ constant, however, E_h/E_v, changing from ∞ to 0, makes the ellipse to rotate from the X-axis to Y-axis, change in ϕ may help or retard the rotation according as the $\cos\phi$ happens to decrease or increase with E_h/E_v. From a rotating ellipse which also changes shape it is rather difficult to infer as to what is changing, E_h/E_v, ϕ or both. In all likelihood, both may be varying under a certain set of practical conditions. It is, however, possible to understand the changes from a study of a number of instantaneous forms succeeding each other under experimental conditions.

Interpretation of a particular form follows :—

Let OX and OY (Fig 1, Plate VII) be the coordinate axes along which deflections take place. Let the ellipse meet the axes of X in points X and X′ and Y in points Y and Y′. Draw a rectangle ABCD circumscribing the ellipse and having its sides parallel (Fig. 2) to the coordinate axes. Let it touch the ellipse in points A′, B′, C′ and D′. It cuts the axes of X and Y in points A″, B″, C″ and D″ as shown in Fig. 2.

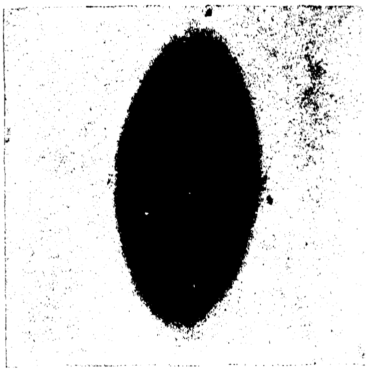


Fig : 1
GSF on 21.5.'40, at 21-30 I.S.T.
$\lambda = 19{\cdot}82$ M $d = 57{\cdot}6$ FT :
$\phi = 49{\cdot}2^\circ$ $\Delta = 90^\circ$
$\theta = 77^\circ$

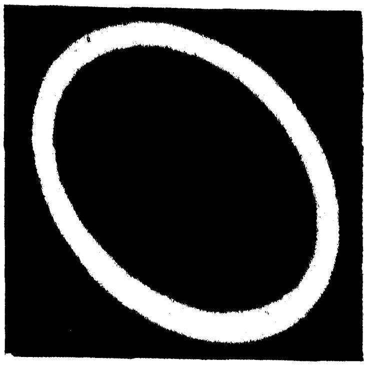


Fig : 2.
Delhi VUD3 on 16.11'40 at 23-00 I.S.T.
$\lambda = 19{\cdot}62$ M $d = 81{\cdot}1$ FT ·
experimental $\phi = 74^\circ$ Δ $9{\cdot}81^\circ$
calculated $\phi = 77{\cdot}3^\circ$

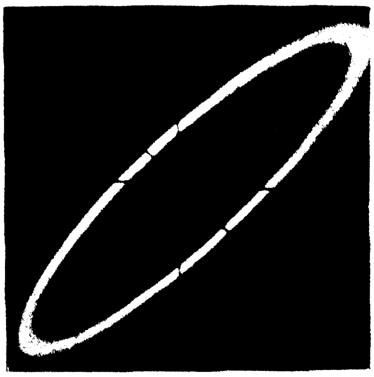


Fig : 3
GSG on 17.11.'40 at 19-00 I.S.T.
$\lambda = 16{\cdot}86$ M $d = 81{\cdot}1$ FT :
$\phi = 26{\cdot}8^\circ$ $\Delta = 90^\circ$
$\theta = 76{\cdot}5^\circ$

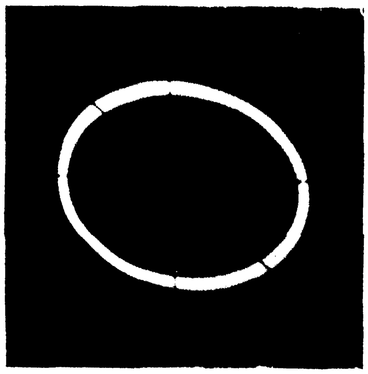


Fig : 4
GSF on 17.11.'40 at 21-30 I.S.T.
$\lambda = 19{\cdot}82$ M $d = 81{\cdot}1$ FT :
$\phi = 75{\cdot}3^\circ$ $\Delta = 90^\circ$
$\theta = 75{\cdot}8^\circ$

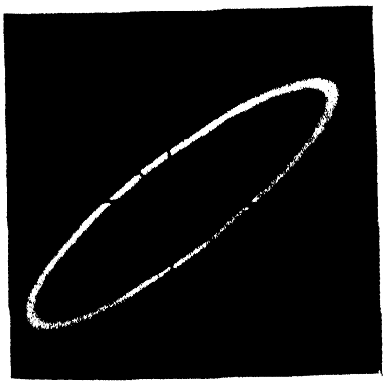


Fig 5
DJH on 17.11.'40 at 19-00 I.S.T.
$\lambda = 16{\cdot}81$ M $d = 81{\cdot}1$ FT :
$\phi = 27{\cdot}8^\circ$ $\Delta = 89{\cdot}4^\circ$
$\theta = 75{\cdot}4^\circ$

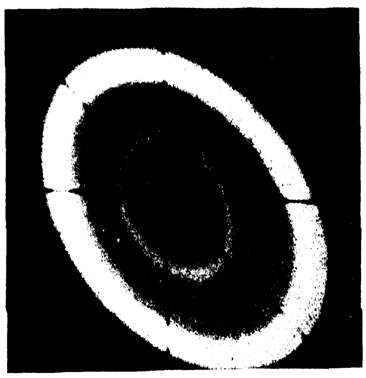


Fig : 6.
Moscow on 26.11'40 at 21-00 I.S.T.
$\lambda = 19{\cdot}47$ M $d = 81{\cdot}1$ FT :
$\phi = 70^\circ$ $\Delta = 80{\cdot}3^\circ$
$\theta = 72{\cdot}7^\circ$

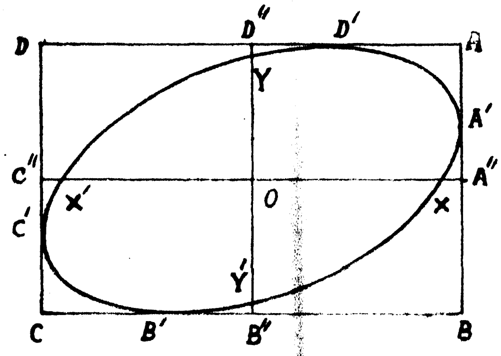


FIG. 2

(*a*) To determine E_v/E_h :

In equation (3) put $x=0$, then $OY=E_v \sin\phi$ ⎫
next $y=0$, then $OX=E_h \sin\phi$. ⎭ ... (6)

From (6) we have : $E_v/E_h = OY/OX$.

$$=\frac{\text{Intercept on Y-axis}}{\text{Intercept on X-axis}}$$

also $$E_v/E_h = OD''/OA''$$

$$=\frac{\text{Maximum deflection along Y-axis}}{\text{Maximum deflection along X-axis}}.$$

As we know, that maximum deflections (OD'' and OA'') are proportional to E_v and E_h.

(*b*) $$\frac{OY}{OD''}=\frac{E_v \sin\phi}{E_v}=\sin\phi$$

similarly $$OX/OA''=\sin\phi$$

thus ϕ can be inferred.

(*c*) In equation (3) put $x=E_h$: solving for y, we get $y=E_v \cos\phi$.

$$\frac{A'\,A''}{A''\,A}=\frac{\text{Ordinate of the pt. where } x \text{ is maximum}}{\text{Maximum ordinate of the ellipse}}$$

$$=\cos\phi$$

similarly $D'D''/D''A=\cos\phi$.

Knowing ϕ and E_v/E_h for a number of succeeding ellipses, we may be able to infer as to what changes and in what way.

II—Study of the case when a single incident ray is present.

Let A and B (Fig. 3) represent sections of the two parallel horizontal aerials through their middle points. A uniform wave-front of any kind of polarization is proceeding downwards, in the plane of the paper,

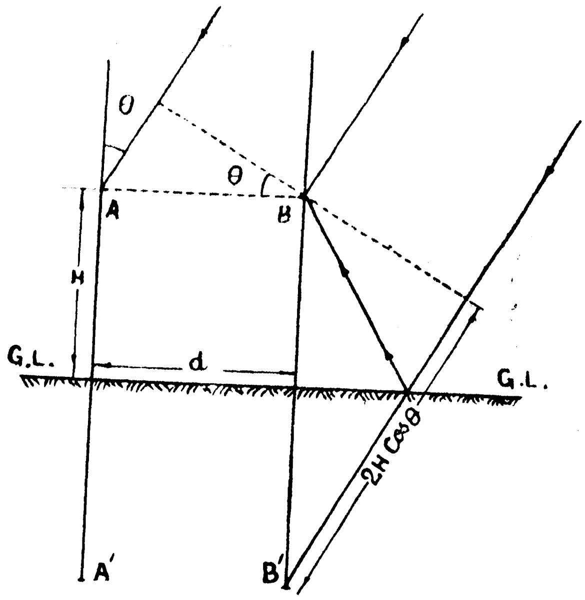


FIG. 3

the direction of propagation making an angle θ with the vertical. A part is incident on B, and induces a voltage E proportional to $E_H \sin \omega t$ where E_H is the horizontal component of the electric vector. The component of the vector perpendicular to E_H does not affect the aerial ; consequently the nature of polarization is immaterial so long as extraneous conductors and the vertical feeders do not play an important part in the pick-up. A change in polarization or rotation of the plane of polarization will only show up along with fading of the signal.

A second ray reaches B, after reflection from the ground. It has undergone the following changes :—

(1) The amplitude of the horizontal component has changed in the ratio 1 : A, where A is the coefficient of reflection.

(2) A sudden phase change ξ occurred on reflection. A and ξ depend on the nature of the reflecting soil.

(3) A phase retardation ($=4\pi$ H $\cos \theta/\lambda$) due to path difference has taken place. H is the height of the aerial above earth.

The effect of the reflected ray at B can therefore be represented to be proportional to :

$$A\, E_H \sin\left(\omega t + \frac{4\pi H \cos\theta}{\lambda} + \xi\right)$$

$$= AE_H \sin(\omega t + \chi)$$

where

$$\chi \equiv \frac{4\pi H \cos\theta}{\lambda} + \xi.$$

The resultant voltage induced in B is therefore proportional to

$$E_n \sin \omega t + AE_n \sin (\omega t + \chi)$$

$$= E_n \sqrt{1 + A^2 + 2A \cos \chi} \quad \sin\left(\omega t + \tan^{-1} \frac{A \sin \chi}{1 + A \cos \chi} \right). \quad \dots \quad (7)$$

Now consider the aerial A. It is similarly affected except that the influence takes place a little later than that on B.

Path difference between the direct ray reaching A and that reaching B $= AB \sin \theta = d \sin \theta$, where $AB = d$. Hence the directly incident ray induces a a voltage proportional to

$$E_n \sin\left(wt + \frac{2\pi d \sin \theta}{\lambda} \right).$$

Similarly, for the reflected ray, the induced voltage is proportional to

$$AE_n \sin\left(\omega t + \frac{2\pi d \sin \theta}{\lambda} + \chi \right).$$

The resultant is :

$$E_n \sqrt{1 + A^2 + 2A \cos \chi} \sin\left(\omega t + \frac{2\pi d \sin \theta}{\lambda} + \tan^{-1} \frac{A \sin \chi}{1 + A \cos \chi} \right). \quad \dots \quad (8)$$

Comparing (7) and (8), we see that, provided the nature of the ground is uniform in the vicinity of the apparatus and it is level, phase difference between the voltages induced in the two aerials depends only on the path inclination θ and the distance apart of the aerials. The amplitudes of voltages in both depend only on the horizontal component E_n and upon the nature of the ground. Thus the amplitude of the induced voltage in either of the aerials should be equal. Hence $E_h = E_v$ always, if the gains are once adjusted to be equal. The ellipse should be with its axes along 45°-135° axes and should change only its shape for changing ϕ but should not rotate. The phase difference $\phi = \frac{2\pi d \sin \theta}{\lambda}$ can be measured straight away by the angle subtended at the end of the major axis by the minor axis.

We can have a single ray at a particular moment by transmitting short duration pulses from the transmitter, so that each component ray reaches the receiver at a different moment from the others. A multiplicity of ellipses will then be obtained, each ellipse corresponding to a definite angle of arrival θ.

It may be noted that change in the nature of polarisation or a sudden phase discontinuity or a gradual phase or amplitude change in the single incident ray will not make the ellipse to change its shape as the phase difference in the voltages induced depend only on the path inclination.