MEASUREMENT OF ACOUSTICAL IMPEDANCES *

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ABSTRACT. The present paper describes determination of *acoustical impedances* by measuring the electrical impedances of a speaker under different air loads. It is a continuation of the previous work. With a proper temperature control, very reliable results are obtained even for small acoustical impedances. Measurements of resistance, reactance and phase in the case of a reflecting surface of Treetax, Felt or simply a Helmholtz Resonator have been made at different frequencies. The variations are plotted graphically. Acoustical resistance of a Helmholtz Resonator, reactance and phase at different frequencies show fair agreement with the theory.

This paper is a continuation of previous work which has already been published.[1] The method is indirect but quite capable of measuring small acoustical impedances with repeatable results. The only defect is that it takes some time to complete one set of observations, otherwise quite consistent results are obtained with a little care and watchfulness. We have extended the method to the measurement of acoustical impedances of materials obtainable in small quantities and to Helmholtz resonators. Some results have been obtained in the case of artificial materials composed of a number of vaccine tubes. Since the experimental procedure is described in detail in the paper referred to above, we shall first discuss the theoretical background for the evaluation of the acoustical resistance and reactance and then the results.

Theory :—The electrical impedance Z_{ET} of a moving-coil receiver is given by

$$Z_{ET} = Z_{EW} + \frac{k^2}{Z_m + a^2 Z_1}$$

where Z_{EW} is the electrical impedance of the windings of the coil in the absence of vibration of the diaphragm. Z_m is the mechanical impedance of the vibrating system and Z_1 the air-load impedance. a is the area of the piston.

Writing $Z_m = R_p + jX_p$,

$$Z_{ET} = Z_{EW} + \frac{k^2}{R_p + jX_p + a^2 Z_1}. \quad \ldots \quad (1)$$

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From measurements at $X_{EL} = 0$

$R_{EL} = 3.2$ ohms

$l = 19.7$ cms.

Then $$R = \left(\frac{k^2}{R_{EL}} - R_p\right)\frac{1}{aZ_0} = \frac{.91854}{3.2} - .1701 = .11694$$

and $$X = -\frac{X_p}{aZ_0} = -.6056.$$

Putting these values in equation (9) we get

$$A = .5756 \log \frac{(1+.117)^2 + .6056^2}{(1-.117)^2 + .6056^2} = .08545.$$

From the tables of hyperbolic functions

$$\tanh A = .08524.$$

Putting this value of tanh A in equation (8) we get

$$\tan C = \frac{.11694 - .08524}{-.6056 \times .08524} = -.6142.$$

$$\therefore \quad C = -31^\circ.55.$$

But $$C = B + kl$$

or $$-31.55 = B + \frac{360}{30} \times 19.7$$

or $$\tan B = -\tan 87.95 = -28.0.$$

Radius of the piston tube $= 2.8$ cms.

$$\therefore \quad a = \pi \times 2.8^2.$$

Now substituting all these values in equations (10) and (11) we get

$$Z_r = \frac{42}{\pi \times 2.8^2} \cdot \frac{.08545(1+28^2)}{1 + .08545^2 \times 28^2} = 1.703,$$

similarly $$Z_x = -7.074.$$

From equation (12)

$$kB = +535.9 - 360 = 175^\circ.9$$

and $$\alpha = \text{Antilog}\left(\frac{-2 \times .08524}{2.303}\right) = .9286.$$

In this way readings were taken for Treetax and Felt at different frequencies and Z_r, Z_x, α and phase change calculated. Tables I and II give the results.

TABLE I

Treetax

λ	Z_r	Z_i	α	$k\beta$
28.4	12.74	− 9.55	.843	172.6
29.5	16.85	− 2.49	.820	178.3
30.0	17.03	− 7.07	.928	175.9
31.4	16.92	−29.60	.859	175.0
33.2	14.16	−10.31	.855	173.4
35.0	18.43	− 3.41	.836	178.1
36.0	24.13	− .67	.868	179.7

TABLE II

Felt

λ	Z_r	Z_i	α	$k\beta$
28.4	1.99	− 8.76	.922	159.0
29.5	3.34	− 8.77	.905	160.6
30.0	1.76	− 7.83	.916	156.8
31.0	1.24	− 6.64	.916	157.5
31.4	1.44	− 7.73	.927	155.9
33.2	1.68	− 8.71	.932	158.6
35.0	3.39	− 11.82	.928	164.8
36.0	1.79	− 9.54	.939	160.4

It can be seen that reflection coefficient varies with wavelength in both cases. It has maxima and minima at different wavelengths; for example, in the case of Treetax at $\lambda = 30$, the coefficient of reflection approaches a maxima, and then as the wavelength increases α falls down. Beyond $\lambda = 35$ it again starts increasing. Resistance also has maxima and minima as the wavelength of sound varies, but the change is more marked in the case of reactance. Figure I shows the results of Treetax graphically. It can be seen that there are marked minima and maxima in all cases and the curve of phase follows the resistance curve very closely. The phase goes on increasing with increasing wavelength, showing thereby that at lower frequency phase tends to 180°. It means that the material behaves like a perfect reflector at low frequency as far as phase is concerned.

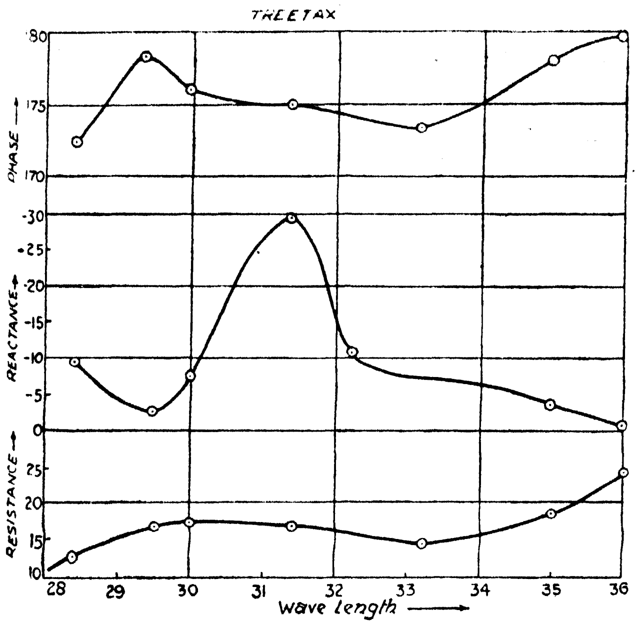

FIGURE 1

By comparing the two tables it can be seen that the reflection coefficients are smaller in the case of Treetax than Felt, but the changes are more pronounced in Treetax. Treetax is harder to touch than Felt, but it has greater resistance to sound waves than Felt. Mass per unit area is also greater in the case of Treetax. One more physical constant, namely, porosity has to be considered in discussing the question of impedance. The author proposes to find out the relation between porosity and resistance. Probably then it would be possible to correlate the coefficient of reflection with the various constants of the material.

Resonators :—Another type of acoustical system of which the acoustical impedance was determined is Helmholtz Resonator. A hole was bored at the centre of a piston disc and to this hole was attached a variable cylindrical chamber, so that the volume of the resonator could be altered. In the present set of experiments the holes were practically of the same size, the volume being adjusted for different frequencies. Below is given a typical calculation for the case of a resonator at $\lambda = 33.5$ cms.

The radius of the opening of the resonator = .27 cm.

Thickness of its neck = .36 cm.

$$k^2 = 1.2732\ aZ_0$$

$$R_p = .2122\ aZ_0$$

$$X_p = 1.1343\ aZ_0$$

$$R_{EL} = .9 \text{ ohm.}$$

$$l = 10.95 \text{ cms.}$$

Now $$R=\frac{1}{aZ_0}\left(\frac{k^2}{R_{EL}}-R_p\right)=\frac{1.2732}{.9}-.2122=1.20247$$

$$X=-\frac{X_p}{aZ_0}=-1.1343.$$

Putting these values in equation (9)

$$A=.5756\log\frac{(1+1.2025)^2+1.1343^2}{(1-1.2025)^2+1.1343^2}=.3839.$$

From tables of hyperbolic functions

$$\tanh A=.36522.$$

Putting this value of tanh A in equation (8)

$$\tan C=\frac{1.20247-.36522}{-1.1343\times .3652}=-2.021$$

$$C=-63^\circ.7.$$

But $$C=B+kl$$

$$\therefore\quad -63.7=B+\frac{360}{33.5}\times 10.95$$

$$\therefore\quad B=-181.4$$

$$\tan B=-.0244.$$

Substituting these values of tanh A and $-\tan B$ and $a=\pi\times 2.8^2$ we get

$$Z_r=\frac{42}{\pi\times 2.8^2}\cdot\frac{.3652(1+.0244^2)}{1+.3652^2\times .0244^2}=.6227$$

$$Z_x=\frac{42}{\pi\times 2.8^2}\cdot\frac{(-.0244)(1-.3652^2)}{1+.3652^2\times .0244^2}=-.03604$$

$$-\tan\phi=\frac{Z_x}{Z_r}=-\frac{.03604}{.6227}=-.0079$$

$$\therefore\quad \phi=-3^\circ.3$$

$$\alpha=\text{Antilog}\left\{-\frac{2\times .3652}{2.303}\right\}=.4818.$$

In the case of resonators it may be noted that the terminal impedance Z_2 gives the impedance of a resonator since the impedance of the remaining piston is infinite. Hence $Z_2=Z_r+jZ_x$. The phase in this case is calculated from $\tan\phi=Z_x/Z_r$ where ϕ is the phase inside the resonator. Table III gives the result for five different resonators of equal orifice but different volumes.

TABLE III—*Resonators*

I

λ	28.4	28.5	29.2	31.0	34.0	36.0
Z_r	1.493	.958	1.188	1.328	1.392	1.276
Z_x	1.097	.439	− 2.288	− 5.797	−11.24	−18.66
α	.365	.325	.641	.890	.963	.988
φ	36.3°	25°.0	−63.0	−77°.1	−83.°0	−86.°1

II

λ	28.2	29.5	30.0	31.0	32.0	34.0	36.0
Z_r	1.331	1.331	1.054	1.105	1.405	1.178	1.178
Z_x	4.166	1.672	.631	− .815	− 2.27	− 9.53	− 9.53
α	.812	.508	.357	.378	.602	.959	.959
φ	72°.3	51°.4	30°.9	−36°.4	−58°.2	−76°.3	−83°

III

λ	28.2	29.5	30.0	31.0	31.4	32.0	33.0	34.0
Z_r	1.226	1.332	1.318	.925	1.144	1.253	1.267	1.455
Z_x	5.546	2.805	2.117	1.148	.819	− 1.273	− 2.339	− 4.566
α	.887	.724	.591	.498	.369	.397	.634	.824
φ	77°.5	64°.6	58°.1	51°.1	35°.6	−38°.8	−61°.6	−72°.4

IV

λ	28.2	31.0	32.0	33.0	33.5	34.0	36.0
Z_r	1.311	1.247	1.117	.869	.623	1.017	1.409
Z_x	8.578	4.317	2.231	1.125	− .036	− 1.254	− 4.281
α	.944	.831	.646	.513	.482	.493	.812
φ	81°.3	73°.9	63°.4	52°.4	− 3°.3	−51°.0	−71°.8

V

λ	32.0	33.0	34.0	34.5	35.0	36
Z_r	1.417	1.447	1.144	.856	.714	1.042
Z_r	4.946	4.146	2.333	1.433	.632	− 1.61
α	.848	.798	.656	.576	.513	.556
φ	74°.0	70°.7	63°.9	59°.1	41°.5	−57°.1

The increase of resistance beyond the resonance point is a general effect observed in all cases. At resonance frequency the resistance is minimum. According to the current theory of Helmholtz Resonator the impedance of a resonator is given by

$$Z = R_r + R_f + j\left(\frac{\omega}{k} - \frac{c^2}{\omega v}\right)$$

$$\therefore \quad Z_r = R_r + R_f$$

where R_r represents the radiation resistance given by

$$R_r = \frac{\rho c k^2}{2\pi}$$

and R_f represents the resistance due to friction and is given by

$$R_f = \frac{l\rho}{\pi r^3}\sqrt{2v\omega}.$$

Here r is the radius of the neck of the resonator.
l, the length of the neck
ρ, the density of air
v, the kinematic viscosity of air
$k = 2\pi/\lambda$, where λ is the wavelength of sound
$\omega = 2\pi f$, f being the frequency of sound.

In the above case

$r = .27$ cm. $l = .36$ cm. $\lambda = 33.5$ cms. $f = 1037$ cycles/sec.

$$R_r = \frac{\rho \omega k}{2\pi} = \frac{2\pi f \rho}{\lambda} = \frac{2\pi \times .0013 \times 1037}{33.5} = .2528$$

$$R_f = \frac{l\rho}{\pi r^3}\sqrt{2v\omega} = \frac{.36 \times .0013}{\pi \times .27^3}\sqrt{\frac{4 \times .132 \times \pi \times 1037}{33.5}} = .3137$$

$$Z_r = R_r + R_f = .5665.$$

From the results it can be seen that the expected resistance at resonance is also of the same order. The increase of resistance beyond resonance is a general effect observed in all cases and cannot be attributed to experimental errors. Sivian[2] has pointed out that the internal resistance term R_i depends upon the magnitude of the particle velocity in the orifice, and it increases considerably with the particle velocity. Since the velocity amplitude is maximum at resonance frequency, it seems that the increase of the total resistance is due to 'end correction.' It can also be seen that the resistance does not go on increasing, but starts decreasing after reaching a certain limit. It is clear from the tables that the resistance term does not exceed 1.5 ohms in any case.

The reflection coefficient is also minimum at resonance, showing thereby that the absorption of sound is greatest at resonance. Theoretically also it can be seen that the value of α can never be zero but is minimum at resonance,

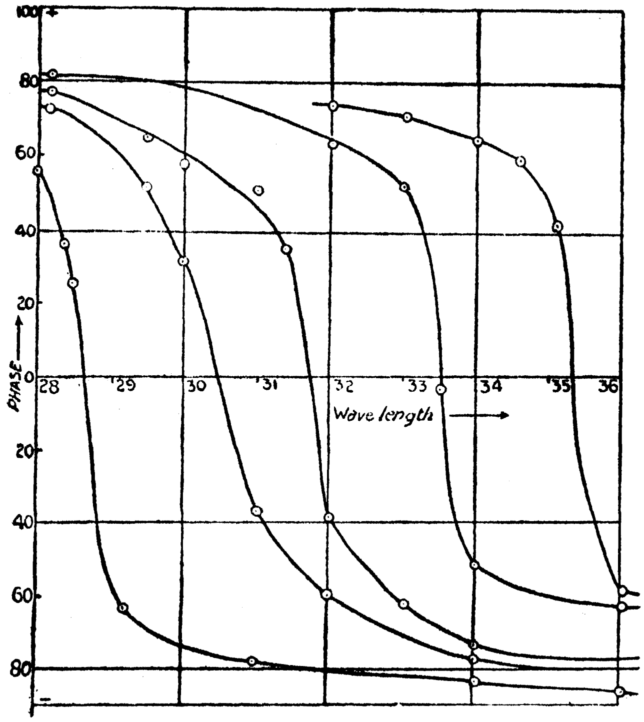

FIGURE 2

Figure 2 shows the variation of phase with wavelength. It can be observed that at resonance the phase becomes zero and, beyond it, it is negative. On both sides of resonance the phase does not go beyond 90° in any case. The change of phase is between +90° and −90° in every case.

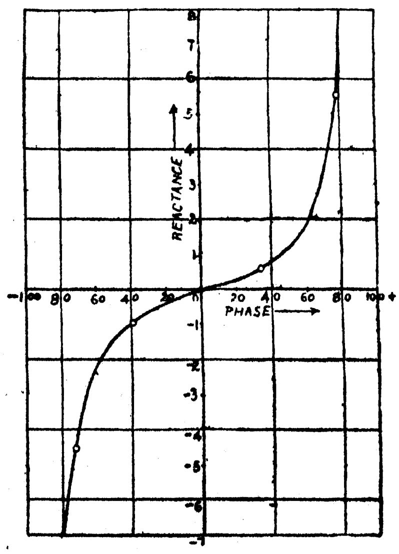

FIGURE 3

Figure 3 shows the relation between phase and reactance. It is found that the phase changes with reactance irrespective of the volume of the resonator. The reactance also becomes zero at resonance and then becomes negative which is entirely in agreement with theory. The curves for different resonators superimpose each other in this case.

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REFERENCES

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[2] Sivian, *Jour. Acoustical Soc. America*, V, 46.