FRICTION BETWEEN A LIQUID SURFACE AND A SOLID NOT WETTED BY IT

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ABSTRACT. If the narrow uniform stem attached to an oscillating float is given a thin coating of paraffin wax, the oscillations are very quickly damped and the value of the logarithmic decrement increases as the amplitude decreases. The hypothesis is made that this is due to friction between the solid and the liquid, which is of the same nature as that between two solids in relative motion. Applying the analysis of vibrations damped by solid friction in addition to fluid friction to these oscillations, it has been possible to calculate the value F, of the limiting kinetic frictional force per cm. of the line of contact. The values obtained with three stems of diameters 0.246 cm., 0.296 cm. and 0.360 cm. have been found to be practically uniform: 15.36, 14.86 and 15.42 dynes per cm. respectively. It is also found that the value of this friction decreases at low velocities and that it is considerably reduced by contamination of the water surface. For rough surfaces it is found to be higher and, in addition, to show a rise in value as the velocity increases. For palmitic acid F is found to be 17.02 dynes per cm., and for stearic acid 27.90 dynes per cm.

These experiments confirm the hypothesis made by Adam and Jessop, in regard to the hysterisis of contact angle, that it may be the effect of a frictional force, F, operating along the solid surface with equal intensity as the liquid advances or recedes.

INTRODUCTION

While working on the oscillations of a float for verifying Stokes' theory of a sphere oscillating in a liquid,[1] it was found that the damping was very high sometimes and the logarithmic decrement, λ, several times greater than the normal value (*vide* Tables 1 and 2). Another interesting observation was that the value of λ increased as the value of the amplitude of oscillation became smaller. This is possible if the resistance offered to the motion of the float is independent of its velocity like friction between two solid surfaces in relative motion. This additional damping, on investigation, was found to be due to the contamination of the stem, for after repeated cleaning and washing of the stem, the damping was reduced to the normal value. It thus became evident that if the surface of the stem is contaminated, the surface of water offers frictional resistance to motion across it. Contaminating the stem in a known manner—giving it a coating of paraffin wax, for example,—it should be

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possible to study this frictional resistance between the surface of water and paraffin wax. The present paper is the result of investigations made in this direction.

This resistance to the motion of solid across the surface of water has a bearing on the angle of contact. It is found in the determination of the latter that the angle is much larger usually if the liquid is advancing than it is receding. This difference between the advancing and receding angles is called the 'hysterisis' of the contact angle, and so far the cause of this hysterisis is considered to be obscure.[2] Adam and Jessop[3] attempted to formulate the hysterisis as the effect of a frictional force, F, operating along the surface with equal intensity when advancing and receding motions were just prevented. It can be easily shown that if γ_{LA} is the surface tension of the liquid, θ_R and θ_A are the receding and advancing angles respectively, $2F=\gamma_{LA}(\cos\theta_R-\cos\theta_A)$. They, however, consider the description to be 'formal' for 'it is difficult to see how there can be a permanent frictional resistance to the motion of a liquid over a solid.' An alternative explanation adopted by most writers is that the work of adhesion between the liquid and the solid surface is actually different for a dry surface and for one that has been wetted even for a short time. The results of the experiments reported here go to show that the hypothesis of friction between the liquid surface and the solid is able to account for the large damping observed in the oscillation of the float in a satisfactory manner. The hypothesis of Adam and Jessop may therefore be taken to be confirmed by these experiments and their description considered to be real and not merely formal.

THE OSCILLATING FLOAT

The experiment was conducted on similar lines to the previous experiment[1] on the verification of Stokes' theory. The hollow spherical body of the float of diameter about 6″ has a glass stem of diameter about 2.5 mm. and length about 25 cm. attached to it radially. The sphere is suitably loaded with lead shot so as to make the sphere go under water with a portion of the stem above the surface. At the top of the stem is attached a small pan in which weights are placed to make the float sink to the desired point on the stem which is about its middle. A millimetre scale of length 10 cm. is attached to the stem below the pan and the turning points of the float as it oscillates up and down are read with the help of a reading telescope from a distance of $1\frac{1}{2}$ metres. The tank of water in which the float oscillates is oval in shape and is about $5' \times 3' \times 1\frac{1}{2}'$. It is kept in a closed room from which draughts of air are excluded. The float is initially kept displaced upwards from its equilibrium position and the wire attached to the top pan clamped between two vertical jaws. Releasing the clamp made the jaws go apart, when the float went down and oscillated. This method of starting was found to be as satisfactory as the previous one in which the float was tied up by an unspun silk fibre and started to oscillate by burning it. The method has an advantage in that the setting in the required position can be obtained more easily and speedily.

Cleaning the water surface : Before observations were made the water in the tank was made to overflow and the surface blown over from one side by an electric fan. This rendered the surface of water clean.

The experiment was first conducted with a clean stem without a coating and the damping observed. Paraffin wax was dissolved in benzene and the solution then applied to the stem from top to bottom by means of a small brush previously cleaned in benzene to form a very thin layer. The experiment was repeated after the coating dried.

Below are given typical sets of observations of consecutive turning points taken (1) with the clean stem and (2) with the stem coated with paraffin wax :—

Mean diameter of the spherical body of the float : 7.71 cm.

Mean diameter of the stem : 0.246 cm.

Period of oscillation : 51.6 sec.

TABLE 1

(*With clean stem*)

Turning Points	Extent of Swing A	$Log_{10}A$	$Log_{10}A_n - Log_{10}A_{n-1}$
13.38 cm.			
19.72	6.34 cm.	0.8021	
14.17	5.55	0.7443	0.0578
18.99	4.82	0.6830	0.0613
14.73	4.26	0.6294	0.0536
18.49	3.76	0.5752	0.0542
15.19	3.30	0.5185	0.0567
18.09	2.90	0.4624	0.0561
15.52	2.57	0.4099	0.0525
17.77	2.25	0.3522	0.0577

Mean of 4 similar observations : 0.0566 Mean Value : 0.0562

TABLE 2

(*With paraffin wax coating*)

Turning points	Extent of Swing A	$Log_{10}A$	$Log_{10}A_n - Log_{10}A_{n-1}$
12.62 cm.	6.04 cm.	0.7810	0.1485
18.66	4.29	0.6325	0.1885
14.37	2.78	0.4440	0.2708
17.15	1.49	0.1732	0.5821
15.66	0.39	$\bar{1}.5911$	0.9890
S 16.05	0.04	$\bar{2}.6021$	0.0000
16.01	0.04	$\bar{2}.6021$	0.0000
16.05			
...	...	...	...
...	...	...	...

It is seen from this table that not only is the logarithmic decrement much higher than that observed with the clean stem but it also increases gradually as the amplitude decreases. This goes on until the stage marked S is reached when the amplitude becomes very small and the oscillations are observed through the telescope to continue for a time with much less damping. The oscillations thus appear to be divided into two series : the large oscillations with considerable damping and the small oscillations with much less damping. Further, it is found that the period of the large oscillations is very nearly the same as that observed with the clean stem, 51·6 sec. in the above case, while the period of the small oscillations is much less, only about 18 sec. The cause for these small oscillations is not quite clear. They may be due to the circulation set up in the water by the oscillation of the float which does not stop by the time the oscillations proper of the float die out, which happens in a short interval of time in this case on account of friction. However, since this is a minute effect and its nature has not been understood, the small oscillations will be omitted from the present study and the considerable damping of the large oscillations taken up for investigation.

Starting with an extent of swing equal to 6·04 cm. the large oscillations come to an end after 5 semi-oscillations. This large damping together with the fact that the damping is proportionately greater at smaller amplitudes suggests that the resisting force responsible for this may be constant in magnitude, reversing its sign at the end of every semi-oscillation, as in the case of friction between two solid surfaces. This hypothesis may be tested by applying the results obtained for oscillations damped by solid friction to these large oscillations.

ANALYTICAL STUDY OF VIBRATIONS DAMPED BY SOLID FRICTION IN ADDITION TO FLUID FRICTION

The values of the logarithmic decrement λ of the float with a clean stem are found to be constant (*vide* Table I), which shows that the surrounding fluid offers a resistance which is proportional to its velocity. With the paraffin wax coating, resistance of the type of solid friction is brought into play in addition, so that the effect of both fluid friction and solid friction on the vibrating body has to be considered. Jenkins and Thomas[4] have given a spiral construction to represent this kind of motion, extending that given by Rowell[5] for the case of solid friction only. It will be assumed that the bulk of the liquid in which the float moves is at rest so that the relative velocity between the float and the liquid at a distance can be considered to be the velocity of the float $\dot{x}$ itself and the fluid friction to be $c\dot{x}$, where c is a constant. The equation of motion may be written to be

$$M\ddot{x} = \pm f - Rx - c\dot{x}, \qquad \dots \quad (1)$$

R being the restoring force per unit displacement from the position of equilibrium between the weight of the float and the upward thrust, and f the limiting kinetic frictional resistance. R is evidently equal to $a\rho g$ where a is the area of cross-section of the stem, ρ the density of the fluid, and g the acceleration due to gravity. M is the virtual[1] mass of the float.

Writing $p=\sqrt{\frac{R}{M}}$ and $n=\sqrt{p^2-\frac{c^2}{4M^2}}$, we find that when the fluid damping is not excessive so that $p^2>\frac{c^2}{4M^2}$ or $R>\frac{c^2}{4M^2}$, the solution is

$$x=\pm\frac{f}{R}+Ae^{-\frac{ct}{2M}}\cos(nt+\epsilon) \qquad \ldots (2)$$

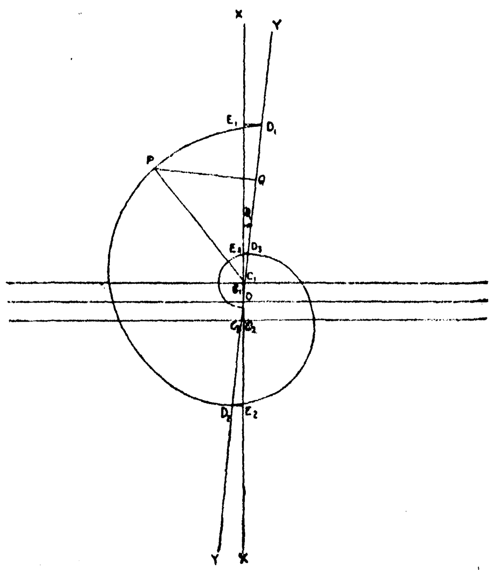
and $$\dot{x}=-Ae^{-\frac{ct}{2M}}\left\{n\sin(nt+\epsilon)+\frac{c}{2M}\cos(nt+\epsilon)\right\} \qquad \ldots (3)$$

$$=-pAe^{-\frac{ct}{2M}}\sin(nt+\epsilon+\phi) \qquad \ldots (4)$$

where $\cos\phi=\frac{n}{p}$ and $\sin\phi=\frac{c}{2Mp}$.

It should be noted that the sign of f is the same as $-\dot{x}$. Also f is a limiting value and equation (1) ceases to apply when $\dot{x}$ is zero. The constants A and ϵ in equation (2) depend upon the initial conditions.

The motion is represented diagramatically in the figure. Starting the



reckoning of time when the vibrating body is about to start from one extreme position, we have, if d_1 is the displacement at this instant,

$$d_1 = \frac{f}{R} + A_1 \cos \epsilon_1 \qquad \ldots \quad (5)$$

and since the velocity $\dot{x}$ is zero

$$o = -pA_1 e^{-\frac{ct}{2M}} \sin(\epsilon_1 + \phi) \qquad \ldots \quad (6)$$

from which it is seen that

$$\epsilon_1 = -\phi \qquad \ldots \quad (7)$$

$$\therefore \quad d_1 = \frac{f}{R} + A_1 \cos \phi. \qquad \ldots \quad (8)$$

If YY be drawn making an angle $-\phi$ with XX, a point c_1 taken on it so that $oc_1 \cos \phi = \frac{f}{R}$ and a point D_1 so that $c_1D_1 = A_1$, it is seen that the displacement d_1 is given by the vertical projection oE_1 of oD_1 upon XX. At any subsequent moment in the next half oscillation

$$x = \frac{f}{R} + A_1.e^{-\frac{ct}{2M}} \cos(nt - \phi) \qquad \ldots \quad (9)$$

so that the displacement at the time t may be derived from the vertical projection of the rotating arm $A_1e^{-\frac{ct}{2M}}$ which revolves at angular velocity n about the centre c_1, the extremity of the arm describing an equiangular spiral. And

$$-\frac{\dot{x}}{p} = A_1.e^{-\frac{ct}{2M}}.\sin nt. \qquad \ldots \quad (10)$$

Thus values of $\left(-\frac{\dot{x}}{p}\right)$ are given by the lengths of the perpendicular projectors such as PQ from the extremity of the rotating arm OP on to YY. When the spiral curve intersects YY, the velocity of the vibrating mass is zero, the tangent to the spiral curve is horizontal and the vertical projection of the point of intersection gives the extreme displacement of the vibrating body. Thus $\frac{f}{R}$ next reverses sign at D_2 when $\dot{x} = o$, that is, when $t = \frac{\pi}{n}$, so that for the next half oscillation, changing the constants to meet the altered conditions, we have

$$x = -\frac{f}{R} + A_2.e^{-\frac{ct}{2M}}.\cos(nt + \epsilon_2) \qquad \ldots \quad (11)$$

$$\dot{x} = -p.A_2.e^{-\frac{ct}{2M}}.\sin(nt+\epsilon_2+\phi). \quad \dots \quad (12)$$

Commencing anew at the point of discontinuity for this half oscillation again, we have $t=0$ at the instant represented by the point D_2, so that, since both sets of equations (9) and (10), and (11) and (12) must be satisfied for this point

$$\dot{x} = -p.A_2\sin(\epsilon_2+\phi) = 0 = \mp p.A_1 e^{-\frac{c\pi}{2Mn}} \sin\pi \quad \dots \quad (13)$$

or

$$\epsilon_2 = (\pi - \phi) \quad \dots \quad (14)$$

and

$$d_2 = x = -\frac{f}{R} + A_2\cos(\pi-\phi) = \frac{f}{R} + A_1 e^{-\frac{c\pi}{2Mn}}\cos(\pi-\phi) \quad \dots \quad (15)$$

or

$$A_2\cos(\pi-\phi) = \frac{2f}{R} + A_1.e^{-\frac{c\pi}{2Mn}}\cos(\pi-\phi)$$

i.e.

$$-A_2\cos\phi = \frac{2f}{R} - A_1 e^{-\frac{c\pi}{2Mn}}.\cos\phi$$

or

$$A_2 = -\frac{2f}{R\cos\phi} + A_1 e^{-\frac{c\pi}{2Mn}}. \quad \dots \quad (16)$$

If we take a point c_2 now on the other side of YY, so that, $c_2 o = \frac{f}{R\cos\phi}$, it is seen that $c_1 D_2 = A_1 e^{-\frac{c\pi}{2Mn}}$, and $c_1 c_2 = \frac{2f}{R\cos\phi}$, so that $A_2 = c_2 D_2$. Thus c_2 now represents the centre of the oscillation and $A_2 e^{-\frac{ct}{2M}}$ will be the rotating arm of the spiral for the next half oscillation. In this way the construction may be continued, the centres of the spiral shifting from c_1 to c_2 and *vice versa* until the spiral cuts the line YY at a point between c_1 and c_2, when the body will come to rest, the restoring force now acting being balanced by the force of friction. For this to happen, $A_n e^{-\frac{c\pi}{2Mn}}$ must be less than $\frac{2f}{R\cos\phi}$ or more strictly speaking $\frac{f+f_s}{R\cos\phi}$, where f_s is the limiting value of statical friction and A_n is the value of the rotating radius at the beginning of the last half oscillation.

The above investigation may now be extended to determine the positions corresponding to the centres of oscillation c_1 and c_2 from the turning points of the vibrating body. The value of the frictional force f can then be obtained. If $E_1, E_2, \dots$ are the positions of the turning points, $OE_1 = d_1$, $OE_2 = d_2, \dots$ Draw

$c_1 b_1$ and $c_2 b_2$ horizontal through c_1 and c_2 cutting the vertical through o in b_1 and b_2 respectively.

Then $$ob_1 = oc_1 \cos\phi = \frac{f}{R}. \quad \dots \quad (17)$$

Similarly $$ob_2 = oc_2 \cos\phi = \frac{f}{R} \quad \dots \quad (18)$$

so that $$b_1 b_2 = \frac{2f}{R}. \quad \dots \quad (19)$$

Again $$b_1 E_1 = c_1 D_1 \cos\phi = A_1 \cos\phi \quad \dots \quad (20)$$

and $$b_1 E_2 = C_1 D_2 \cos\phi. \quad \dots \quad (21)$$

But $$c_1 D_2 = A_1 e^{-\frac{c\pi}{2Mn}}. \quad \dots \quad (22)$$

$$\therefore \quad b_1 E_2 = A_1 \cos\phi \, e^{-\frac{c\pi}{2Mn}} = b_1 E_1 . e^{\frac{c\pi}{2Mn}} \text{ [from (10)]} \quad \dots \quad (23)$$

$$\therefore \quad E_1 E_2 = (b_1 E_1 + b_1 E_2) = b_1 E_1 (1 + e^{-\frac{c\pi}{2Mn}}). \quad \dots \quad (24)$$

If the same oscillating system can be studied without solid friction and if a_1, a_2, a_3,...... be the magnitudes of the successive swings from one turning point to the next obtained, then we have the case of damped simple harmonic motion for which it is known that

$$\frac{a_1}{a_2} = \frac{a_2}{a_3} = \frac{a_3}{a_4} = \quad \dots \quad \dots \quad \dots \quad = k = e^{\frac{c\pi}{2Mn}} = e^{\lambda} \quad \dots \quad (25)$$

where λ is the logarithmic decrement of the system. If λ be known, k can be calculated and

$$e^{-\frac{c\pi}{2Mn}} = \frac{1}{k} \quad \dots \quad (26)$$

so that substituting in (24), $E_1 E_2 = b_1 E_1 (1 + \frac{1}{k}) = b_1 E_1 \frac{(k+1)}{k}$

or $$b_1 E_1 = E_1 E_2 \cdot \frac{k}{(k+1)}. \quad \dots \quad (27)$$

When the turning points E_1 and E_2 are known, the value of $b_1 E_1$ can be calculated from the above equation and the position of b_1 determined. Similarly if E_2, E_3 are the turning points for the next half oscillation, it follows that

$$b_2 E_2 = E_2 E_3 \cdot \frac{k}{(k+1)} \quad \dots \quad (28)$$

and the position of b_2 can be determined. In this way the centre of oscillation for each half oscillation can be calculated, b_1 and b_2 being obtained from alternate half oscillations. When the value of $b_1 b_2$ is thus obtained, the value of the frictional force f is easily calculated from (19) : $\frac{2f}{R} = b_1 b_2$.

APPLICATION TO THE OSCILLATING FLOAT

The value of k corresponding to the value of $\lambda = 0.0566$ to base 10 as recorded in table 1 is 1.140. Applying the above method of obtaining b_1 and b_2 to the observations recorded in Table 2, we get the following results :—

TABLE 3

Turning Points	Extent of Swing E_1E_2 etc.	E_1C_1 etc.	C_1 $(E_1+E_1C_1)$	C_2 $(E_2-E_2b_2)$
12.62 cm.	6.04 cm.	3.22 cm.	15.84 cm	
18.66	4.29	2.29		16.37 cm.
14.37	2.78	1.48	15.85	
17.15	1.49	0.79		16.36
15.66				
16.05	Mean Values:	C_1	15.845 cm.	
		C_2	16.365 cm.	
		C_1C_2	0.52 cm.	

The last half oscillation of the series is not included in the above calculations as the small oscillations that commence then introduce some uncertainty in the position of the last turning point. If the observed turning point 16.05 is accepted, the corresponding value obtained for C_1 will be 15.87 cm. The results obtained are of great significance. They show that the damping of the large oscillations is not an irregular phenomenon but it is the consequence of a systematic frictional force operating on the vibrating body. The experiment is repeated, varying the initial position of start. The following mean values are obtained for C_1C_2 :

0.52 cm. ; 0.52 cm. ; 0.50 cm. ; 0.50 cm. ; 0.51 cm. ; 0.50 cm. ; 0.52 cm. ; 0.52 cm. ; 0.49 cm. ; 0.49 cm. ; 0.50 cm. ; 0.52 cm.

It should be noted that a millimetre scale is read through the telescope in the experiment so that the second decimal place recorded in the observations is an estimated value. The values of C_1C_2 may therefore be taken to be constant. The mean of all the above values is 0.51 cm.

Now if we put $\frac{f}{R}$ equal to h,

$$h=\frac{b_1 b_2}{2}$$

and

$$f=R.h.$$

But

$$R=a\rho g$$

$$=\pi r^2\rho g,$$

where r is the radius of cross-section of the stem.

$$\therefore \quad f=\pi r^2\rho g h \qquad \ldots \quad (29)$$

A simple assumption that we may make in regard to f is that it acts along the line of contact of the liquid surface and the stem and that it is proportional to the length of the line along which it acts. Then if F is the force per unit length

$$f=2\pi r.F$$

or

$$F=\frac{f}{2\pi r}$$

$$=\frac{r.\rho g h}{2} \quad [\text{from } (29)]. \qquad \ldots \quad (30)$$

(I) In the above experiment

$$r=0.123 \text{ cm.}$$

$$\rho=1 \text{ gm. per c.c.}$$

$$g=979 \text{ cm. per sec. per sec.}$$

and

$$h=\frac{0.51}{2}=0.255 \text{ cm.}$$

$$\therefore \quad F=\frac{0.123\times 1\times 979\times 0.255}{2}=15{\cdot}36 \text{ dynes/cm.}$$

$\therefore$ The limiting value of the kinetic frictional force per / cm. between paraffin wax and water is 15·36 dynes.

(II) The experiment is repeated with a second glass stem of mean diameter 0.296 cm. A typical set of observations is given below :

Mean value of logarithmic decrement to base 10 : 0.0532.

Corresponding value of k : 1.131 ; Period$=$43.2 sec.

TABLE 4

Turning Points	Extent of Swing	E_1C_1 etc.	C_1	C_2	C_1C_2
20.14 cm.	7.11 cm.	3.77 cm.		16.37 cm.	
18.03	5.47	2.90	15.93 cm.		0.44 cm.
18.50	4.06	2.15		16.35	0.42
14.44	2.84	1.50	15.94		0.41
17.28	1.77	0.94		16.34	0.40
15.51	0.89	0.47	15.98		0.36
				16.29	
16.40	0.20	0.11			0.31
16.20 } small					
16.23 } oscilla-					
16.20 } tions.					

Mean Value of C_1C_2 (of the first four) : 0.42 cm.

Mean values of b_1b_2 or $2h$ obtained by repetition of the experiment are : 0.42 cm. ; 0.41 cm. ; 0.41 cm. ; 0.41 cm. ; 0.41 cm. ; 0.41 cm. ; 0.40 cm. ; 0.41 cm.

Mean of these values : 0.41 cm.

$$\therefore \quad h = 0.205 \text{ cm.}$$

Substituting these values in (30), we get

$$F = \frac{0.148 \times 1 \times 979 \times 0.205}{2}$$

$$= 14.86 \text{ dynes per cm.}$$

(III) Values obtained for a third stem :

Mean diameter of the stem :	0.360 cm.
Logarithmic decrement to base 10 :	0.0472
Value of k :	1.115
Period of oscillation :	35.14 sec.

TABLE 5

Turning Points	Extent of Swing	E_1C_1 etc.	C_1	C_2	C_1C_2
17.40 cm.	5.10 cm.	2.69 cm.	14.71 cm.		0.36 cm.
12.30 cm.	3.88	2.05		14.35 cm.	0.33 cm.
16.18	2.85	1.50	14.68		0.33 cm.
13.33	1.94	1.02		14.35	0.34 cm.
15.27	1.10	0.58	14.69		
14.17					
14.59					
14.51					
14.58					

Mean Value : 0.34 cm.

Mean values of b_1b_2 or $2h$ obtained by repetition are :

0.34 cm. ; 0.34 cm. ; 0.36 cm. ; 0.36 cm. ; 0.35 cm. ; 0.35 cm.

Mean of these values : 0.35 cm.

$$\therefore \quad h = 0.175 \text{ cm.}$$

$$F = \frac{0.180 \times 1 \times 979 \times 0.175}{2}$$

$$= 15.42 \text{ dynes per cm.}$$

Thus the values of the limiting kinetic frictional force obtained with the three stems respectively are 15.36 dynes per cm.,

14.86 dynes per cm.

and 15.42 dynes per cm.

The fair agreement between these values shows that the assumption made that this frictional force is proportional to the length of the line of contact is justifiable.

Kinetic and Statical Friction:—From the theory of vibrations developed above it is to be expected that the float should come to rest at end of the nth semi-oscillation, when $A_n e^{-\frac{c\pi}{2Mn}}$ is less than $\frac{f+f_s}{R}$, f_s being the limiting statical friction. The float, on the other hand, is found to execute small oscillations. This variance is due to the short-coming of the theory, in that no account has been taken of the disturbance in the water of the tank which is produced by the oscillations. The centre of small oscillations, however, does not generally coincide with the point of normal equilibrium, o, which is the mid-point of C_1C_2. From Table 3, it is seen that the position of o is 16.11 cm., while the centre of small oscillations is 16.03 cm. Similarly in Table 4 the centre of small oscillations is 16.22, while o is 16.15. It is also found that the centre of small oscillations may lie on either side of o. Since the float ultimately comes to rest at this centre, it is seen that the float can remain in equilibrium at any point within a small range on either side of o, showing thereby that there is also a statical force of friction between paraffin wax and water. *It is an interesting observation that a floating body can have more than one position of equilibrium if the liquid does not wet the surface of the float.* It has been possible, by trial, to keep the float with the 0.246 cm. stem in equilibrium at different points within a range of 2 mm. A satisfactory method has yet to be devised by which the range corresponding to statical friction can be accurately determined.

Attention may also be drawn to another point which may be illustrated from Table 4. Values of C_1C_2 obtained from the first 5 half oscillations are fairly constant or may be considered to have a very slight gradual decrease ; but when we come to the 6th half oscillation, for which the extent of swing is 0.89 cm., there is a sudden and appreciable shift in the position of C_1 (15.98) towards o (16.15), showing thereby that the value of kinetic friction has become less. The time for this half oscillation is 21.6 sec. so that the average velocity during this motion is 0.89/21.6 cm./sec. or 0.41 mm. per sec. This appreciable reduction in the value of kinetic friction when the amplitude is reduced is a common observation in these investigations. This is another reason for omitting the last large half oscillation in the above calculations, from which the steady value of F is obtained.

Ablett's work :—These observations are in qualitative agreement with the results obtained by Ablett[6] for the receding and advancing contact angles between paraffin wax and water. From the relation $2F=\gamma_{LA}(\cos\theta_R-\cos\theta_A)$ it is possible to calculate the value of the frictional force (kinetic) from the values of θ_R and θ_A obtained by him, since γ_{LA}, the surface tension of water, is known. Ablett finds that the values of the advancing and receding angles remain practically constant for speeds of 0.44 mm. per sec. and above, while for lower speeds the difference between the two angles is less. His values are as follows :—

TABLE 6

Velocity mm./sec.	θ_A	θ_R	$\theta_A-\theta_R$
0.1273	109°−24′	99°−34′	9°−50′
0.1943	110 −31	98 −45	11 −46
0.1313	112 −8½	97 −5	15 −3½
	Mean	Values	
0.441 and above	113 −9	96 −20	16 −49

Assuming γ_{LA} to be 75 dynes per cm. the value of F corresponding to the mean values noted above is

$$2F=75\,(\cos 96^\circ.12'-\cos 113^\circ.9')$$

or

$$F=\frac{75\times 0.282}{2}=10.58 \text{ dynes/cm.}$$

the value of the kinetic frictional force is therefore 10.58 dynes per cm. This is decidedly less than the values calculated from the float experiments. This may be due to the difference in the nature of the surface of paraffin wax obtained on the stem from that in Ablett's experiment. There may be also another explanation for this difference. It is not stated in his paper whether the surface of water is cleaned before making the observations. Any small contamination present on the surface of water is found to affect the value of this friction considerably (as will be seen in the next section) and the lower value obtained by Ablett may also be due to a trace of contamination that might have been present if the surface was not cleaned.

CONTAMINATION OF THE WATER SURFACE

Before the study of the effect of contamination is taken up the upper edge of the tank is given a coating of paraffin wax since it is found that reliable results with thin films of fatty acids can be obtained under those conditions. The contamination is supplied by dissolving one drop of oleic acid in 6 c. c. of benzene and adding 4 drops of the solution to the surface of water in the tank. The vigorous spreading of the last benzene drop showed that the film was not complete, for it was found previously that when the film is complete there is no vigorous spreading but the drop collects in the form of a lens and evaporates slowly. Results obtained with this amount of contamination are given below. The experiment was conducted with the stem of diameter 0.246 cm.

TABLE 7

Turning Points	Extent of Swing	E_1C_1 etc.	C_1	C_2
12.23 cm.	7.02 cm.	5.75 cm.	15.98 cm.	
19.25	5.88	3.14		16.11 cm.
13.37	4.91	2.62	15.99	
18.28	4.08	2.18		16.10
14.20	3.39	1.81	16.00	
17.59	2.76	1.47		16.12
14.83	2.19	1.17	16.00	
17.02	1.70	0.91		16.11
15.32	1.25	0.67	15.99	
16.57	0.86	0.46		16.11
15.71	0.48	0.26	15.97	
16.19		Mean Values :	15.99	16.11
16.03				

$$\therefore \quad b_1 b_2 = 0.12 \text{ cm.}$$

and

$$h = 0.06 \text{ cm.}$$

$$\therefore \quad F = \frac{0.123 \times 1 \times 979 \times 0.06}{2}$$

$$= 3.61 \text{ dynes per cm.}$$

The value of the kinetic frictional force is reduced to about one-fourth by the contamination. This shows how even a small amount of contamination has a considerable effect on the frictional force between the surface of water and paraffin wax.

Palmitic acid

The experiments were also conducted with coatings of palmitic acid and stearic acid given to the stem (of diameter 0.296 cm.) in the same manner by dissolving in benzene. These have the property of spreading on water unlike paraffin wax, and conditions of the water surface as well as the stem gradually alter as the float oscillates up and down. Observations taken at the first start are given below:

TABLE 8

Turning Points	Extent of Swing	E_1C_1 etc.	C_1	C_2
13.99 cm.	5.03 cm.	2.67 cm.	16.66 cm.	
19.02	3.55	1.88		17.14 cm.
15.47	2.27	1.20	16.67	
17.74	1.16	0.61		17.13
16.58				
16.87				
16.82				
		Mean Values :	16.665	17.135

These values may be taken to be fairly constant. The mean value of b_1b_2 is 0.47 cm. so that F is equal to 17.02 dynes per cm. In subsequent oscillations the value of b_1b_2 is found to change gradually, the consecutive values in an experiment being 0.54 cm., 0.51 cm. and 0.48 cm. respectively.

Stearic acid

Observations of the first series of oscillations with a coating of stearic acid on the same stem are as follows:

TABLE 9

Turning Points	Extent of Swing	E_1C_1 etc.	C_1	C_2
11.30 cm.	7.98 cm.	4.23 cm	15.53 cm.	
19.28	5.59	2.97		16.31 cm.
13.69	3.50	1.86	15.55	
17.19	1.64	0.87		16.32
15.55				
15.81				
15.76				
		Mean values :	15.54	16.31

The mean value of b_1b_2 is 0.77 cm. from which F is calculated to be 27.90 dynes per cm. a value much higher than that for paraffin wax or palmitic acid.

ROUGH SURFACES

The experiments described above were worked on the day next to the one on which the coating was given. This time was sufficient for the coating to dry. The values obtained for kinetic friction were fairly uniform. But if the coated stem was allowed to stand exposed for some days, the nature of the surface was found to alter, it became more rough, and the friction increased. The values of b_1b_2 were, moreover, not uniform but showed an increase with increase in amplitude. Observations obtained with the stem of diameter 0.296 cm. (normal value of b_1b_2=0.41 cm.) a week after the first experiments were performed are given below :—

TABLE 10

Turning Points	Extent of Swing	E_1C_1 etc.	C_1	C_2	C_1C_2
11.60 cm.	6.37 cm.	3.38 cm.	14.98 cm.	—	—
17.97	4.58	2.43	—	15.54 cm.	0.56 cm.
13.39	3.12	1.66	15.05	—	0.49
16.51	1.81	0.96	—	15.55	0.50
14.70	0.77	0.41	15.11	—	0.44
15.47	—	—	—	—	—
15.35	—	—	—	—	—
15.40	—	—	—	—	—
....	—	—	—	—	—

These are the types obtained whenever the surface is rough. It is thus a common observation that for rough surfaces the kinetic friction is not only greater but shows also a rise as the amplitude (*i.e.*, the average velocity) increases.

FRICTION AND HYSTERISIS OF CONTACT ANGLE

The reasults of the experiments described here are in close agreement with the observations on the hysterisis of contact angle. The agreement with Ablett's work has already been mentioned. The reduction in the value of friction on adding contamination to the surface of water can be deduced from the following equations :

$$W_{SL} = \gamma_{LA}(1 + \cos\theta), \qquad \ldots \quad (31)$$

when W_{SL} is the work of adhesion between the solid and the liquid, and

$$2F = \gamma_{LA}(\cos\theta_R - \cos\theta_A). \qquad \ldots \quad (32)$$

From equation (31) it follows that when γ_{LA} is lowered on account of the contamination, the contact angle (the receding as well as the advancing) becomes less, since W_{SL} cannot be altered by the change. These changes and the reduction in the value of γ_{LA} may be taken to be responsible for lowering the value of the friction. In the case of rough surfaces also there is agreement. It is found that roughness of surfaces increases the angle of contact and also the hysterisis.[8] The float experiments give the additional information that kinetic friction increases with increase of velocity in these cases instead of attaining a steady value as in the case of smooth surfaces. From these considerations it follows that there is a close correspondence between hysterisis of contact angle and friction.

It becomes necessary therefore, in the light of these results, to reconsider the ideas entertained in regard to hysterisis. It appears doubtful whether the explanation "that the work of adhesion between the liquid and the solid surface is actually different for a dry surface and for one that has been previously wetted even for a short time"[9] can be accepted. In the float experiments the surface of the stem which gives the receding angle as the float rises up must be considered to be one which has been wetted previously, but as soon as the float reverses its motion and begins to sink, it will have to be considered as one which is dry so as to give the advancing angle an explanation that is not convincing. Preference will therefore have to be given to the alternative hypothesis of Adam which postulates the same frictional force to be operating in both the cases and which further is amply supported by the results of these experiments.

FRICTION AND WETTING

In conclusion, the connection between friction and wetting may be pointed out. When the stem is clean, it is wetted by water and a thin layer of water adheres to it as it comes out of the surface. In subsequent movements water outside moves over this layer and there is little friction. With the paraffin wax coating, on the other hand, there may be no possibility for such a layer as the stem is not wetted and frictional force comes into play each time the water moves over the surface of paraffin wax. The absence of the adhering liquid layer next to the surface of the solid may therefore be considered to be responsible for the friction observed when the liquid moves over the solid. Friction between a solid surface and a liquid is probably a general phenomenon. If such a view is taken, it should follow that even in the case of a solid that is wetted by a liquid there is a frictional force operating when the liquid advances over the dry surface of the solid for the first time. The low values obtained for the rise of water in a capillary tube when the walls are dry may be due to this cause. While there may be some doubt in regard to this view that friction between a solid and liquid is general, there can be no uncertainty however as to its being present between a solid and a liquid that does not wet it, as is found with paraffin wax, palmitic acid and stearic acid.

It may be noted in this connection that the place where the friction comes into play is not the whole area of contact between the liquid and the solid. The friction is mainly due to the relative motion between the solid surface and the layer of liquid next to it, but in the case of liquids it is commonly understood that there is no slip between the solid and the layer of liquid next to it so long as the liquid is in contact with the solid. There can be this relative motion only at the place where the surface of the liquid is advancing or receding or over the solid, so that it may be said that the friction is between the surface of the liquid and the solid. Reduction in the value of friction when the surface of water is contaminated (which affects the surface and not the body of liquid), and the result that the force of friction is proportional to the length of the line of contact lend support to this view. The title to the paper is given accordingly.

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