EFFECT OF VARIATION OF THE MAGNITUDE AND PHASE ANGLE OF THE LOAD ON POWER OUTPUT IN COMMUNICATION CIRCUITS*

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ABSTRACT. In communication circuits a great deal of stress is laid on the need for exact matching of the load to the source. While this impedance matching principle is true as a general rule, under actual conditions, results indicate that over a wide range of values of impedance on either side of the source impedance, the loss due to mismatch is not appreciable. The paper derives conditions for the maximum transfer of power to load when both the source and load are complex impedances, and treating the source as a fixed one. Actually, it is found mathematically

(1) that when mismatch of impedance must exist, it is better to mismatch to a higher than a lower load impedance (in magnitude);

(2a) in the case of small phase angles of source, it is also derived that, where perfect matching is not possible, the load phase angle should be nearer zero, and

(2b) for higher phase angles of the source, matching becomes critical

INTRODUCTION

The subject of impedance matching in communication circuits has been dealt with in several text books and journals from a very general point of view, most often treating both the source and the load as pure resistances and thus ignoring the phase angle or power factor of both the source and the load. Also, often the source impedance is considered fixed and the load impedance variable. The object of this paper is to consider the effect on the power output when both the magnitude and phase angle of the load are treated as variable (one at a time), while the source itself is treated as a complex impedance. This paper also suggests choosing the magnitude of the impedance and phase angle of the load where perfect matching is not possible.

NOTATION

(1) Z_S = Impedance of the source
(2) Z_L = Impedance of the load
(3) ϕ_S = Phase angle of the source
(4) ϕ_L = Phase angle of the load
(5) E = E. M. F. of the source
(6) W_L = Power in the load

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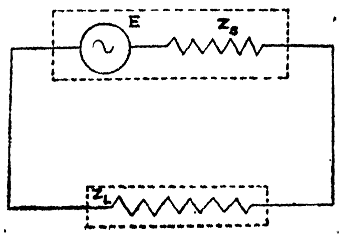
CONDITIONS FOR MAXIMUM TRANSFER OF POWER

Any communication network can be reduced by Thevenin's theorem to a source, generating an E. M.F.,E, a complex impedance of the source Z_s in series with the generator and a complex load Z_L.



(Fig. 1)

Let us consider the conditions under which maximum power is transferred to the load.

With the above notation and Fig. 1, it is proposed to derive an expression for the power W_L in the load.

The power in the load is given by :

$$W_L = \tfrac{1}{2} |I|^2 \times |Z_L| \times \cos \phi_L \qquad \dots \quad (1)$$

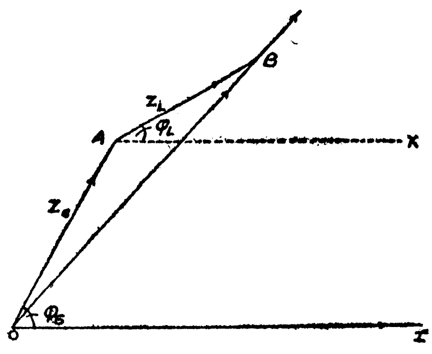
But,

$$|I| = \frac{|E|}{|Z_s + Z_L|} \qquad \dots \quad (2)$$

Therefore,

$$W_L = \tfrac{1}{2} |E|^2 \times \frac{|Z_L| \cos \phi}{|Z_L + Z_s|^2} \qquad \dots \quad (3)$$

(Signal Training Manual, 1936)



(Fig. 2)

From Fig. 2, $|Z_s + Z_L|^2 = |Z_s|^2 + |Z_L|^2 + 2|Z_L Z_s| \cos \overline{\phi_s - \phi_L}$ $\qquad \dots \quad (4)$

Substituting this in equation (3) above, we have :

$$W_L = \tfrac{1}{2} |E|^2 \frac{|Z_L| \cos \phi_L}{|Z_s|^2 + |Z_L|^2 + 2|Z_L Z_s| \cos(\phi_s - \phi_L)} \qquad \dots \quad (5)$$

Equation (5) gives the general expression for the power in the load. It is often not possible to vary one or both of the factors, magnitude and phase of the source, while both the magnitude and/or the angle of the load can be varied for obtaining maximum transfer of power from the source to the load ; *i.e*, $|Z_S|$ and ϕ_S are often fixed and $|Z_L|$ and ϕ_L are usually variable.

(*a*) *Variation of* $|Z_L|$, *keeping* ϕ_L *constant*

Glasgow (1936) says that this condition actually obtains in practice in the speech coil of a dynamic loudspeaker. First let us keep ϕ_L constant and see the effect on W_L by varying $|Z_L|$ only.

By differentiating the expression (5) with respect to $|Z_L|$ and equating to zero,

we get $$|Z_L| = |Z_S| \qquad \ldots \quad (6)$$

(Everitt, 1937)

(*b*) *Variation of* ϕ_L, *keeping* Z_L *constant and equal to* $|Z_S|$

After making $|Z_L| = |Z_S|$ let us vary ϕ_L to obtain maximum power in load.

Equation (5) becomes on substituting $|Z_S|$ for $|Z_L|$

$$W_L = \tfrac{1}{2} \, |E|^2 \frac{|Z| \cos\phi_L}{2\,|Z_S|^2 Z\,(1+\cos(\phi_S-\phi_L))} \qquad \ldots \quad (7)$$

On differentiating the above expression with respect to ϕ_L and equating to zero and solving, we get :

$\sin\phi_L = -\sin\phi_S$

or $\phi_L = -\phi_S$ or $(n\pi+\phi_S)$ where n is odd $\qquad \ldots \quad (8)$

Since the values of ϕ_L other than $(-\phi_S)$ result in an infinite value of W_L, the only possible solution is $\phi_L = -\phi_S$

Thus the two conditions for maximum power transfer to load are

$$\left.\begin{array}{ll} & (1)\ |Z_S| = |Z_L| \\ \text{and} & (2)\ \phi_S = -\phi_L \end{array}\right\} \qquad \ldots \quad (9)$$

(*c*) *Physical significance of the conditions for maximum transfer of power to the load*

The significance of equation (9) can be interpreted thus : If both the magnitude and angle of the load could be adjusted, then the load impedance should be conjugate of the generator or source impedance, *i.e.*, the resistive components should be equal, while their reactive components should also be equal in magnitude but opposite in sign—that is, if one is inductive, the other should be capacitative.

Symbolically, if $Z_S = R_1 + jx_1$;

then, Z_L should be ; $R_1 - jx_1$

Under these conditions, the impedances (Z_L and Z_S) are said to match and any deviation from these conditions is described as impedance mismatch.

EFFECT OF IMPEDANCE MISMATCH

In what follows, for convenience $|Z_L|$ and $|Z_S|$ are denoted merely by Z_L and Z_S.

From equation (9), values of Z_L and ϕ_L can be determined to obtain maximum power output in the load. It is often impossible to match the load to the source for all frequencies (e. g. speech coil of a loudspeaker to the plate of the output tube), since the load impedance is invariably a function of frequency in communication circuits.

It is therefore proposed to consider the effect of mismatch on the output (load) power.

(*a*) The effect of mismatch due to a difference between the load impedance Z_L and source impedance Z_S will be considered first, the phase angles being constant.

It has been derived that for maximum output Z_L should be equal to Z_S (*vide* equation 7). In this case, the power output is :

$$W_S = \tfrac{1}{2}|T|^2 \frac{\cos\phi_L}{2Z_S(1+\cos\overline{\phi_S-\phi_L})} \quad \ldots \quad (10)$$

When, however, $Z_L \neq Z_S$, we have the power output

$$W_L = \tfrac{1}{2}|E|^2 \frac{Z_L \cos\phi_L}{Z_L^2 + Z_S^2 + 2Z_LZ_S\cos\overline{\phi_S-\phi_L})} \quad \ldots \quad (11)$$

Dividing (11) by (10) we have

$$\frac{W_L}{W_S} = \frac{2\,Z_LZ_S\,(1+\cos\overline{\phi_S-\phi_L})}{Z_L^2+Z_S^2+2Z_LZ_S\cos\overline{\phi_S-\phi_L}} = \frac{2(1+\cos\overline{\phi_S-\phi_L})}{\dfrac{Z_L}{Z_S}+\dfrac{Z_S}{Z_L}+2\cos\overline{\phi_S-\phi_L}} \quad \ldots \quad (12)$$

The power loss with reference to power output at matched condition is :

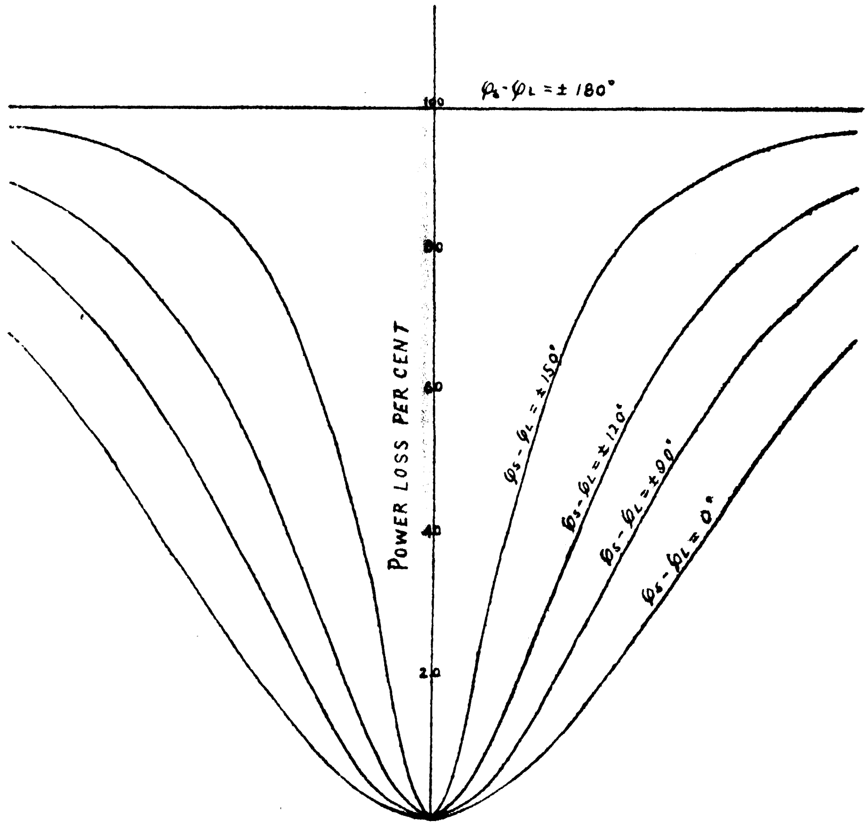
$$\frac{W_S-W_L}{W_S} = 1-\frac{W_L}{W_S} = \frac{\dfrac{Z_L}{Z_S}+\dfrac{Z_S}{Z_L}-2}{\dfrac{Z_L}{Z_S}+\dfrac{Z_S}{Z_L}+2\cos\overline{\phi_S-\phi_L}} \quad \ldots \quad (13)$$

From equation (13) the following conclusions can be drawn :—

(1) If ϕ_S and ϕ_L are constant, the percentage power loss due to an inequality between Z_L and Z_S is solely dependent on the ratio Z_L/Z_S and not on the individual values of Z_L and Z_S.

(2) As the expression contains only the difference between ϕ_L and ϕ_S and not their individual values the power loss percent will be the same for a given Z_S/Z_L although ϕ_S and ϕ_L may be given different sets of values, provided $(\phi_S-\phi_L)$ is constant.

(3) The power loss percent will be the same for a given Z_S/Z_L whether the sign of $(\phi_S-\phi_L)$ is positive or negative because, in either case, $\cos(\phi_S-\phi_L)$ is the same.



GRAPH 1

Z_S/Z_L Ratio

A graph (GRAPH 1) has been drawn showing the variation of power loss with the ratio Z_s/Z_L on a log-linear graph paper.

The conclusion (1) given above has been verified by calculating the power loss for various standard values of Z_S (*viz*., 37, 50, 74, 200, 500, 550 and 600 ohms being the standard characteristic impedances for audio and radio-frequency transmission lines) and plotting the curve for Z_S/Z_L versus power loss percent. It found that all the curves coincide for a given value of $(\phi_L - \phi_S)$. For the curve for is which the value of $(\phi_L - \phi_S)$ is assumed to be 90°, the values of ϕ_S and ϕ_L are $-45°$ and $+45°$ respectively, making the difference of ϕ_L and ϕ_S equal to 90° in both cases. This curve also coincides with the previous one, thereby showing that the curve is universal for a particular value of $(\phi_L - \phi_S)$

(*a*) *Inferences from and applications of Graph No.* 1

(*i*) Maximum power is transferred to the load when $Z_L = Z_S$ $(\phi_S - \phi_L)$ being **constant.**

(*ii*) The output power is zero when $Z_L = 0$ *i. e.*, no power is delivered when the source is short-circuited.

(*iii*) The curve is symmetrical about $Z_S/Z_L = 1$ and an interesting property can be derived from this curve.

If Z_1 and Z_2 are two values of load impedance on either side of Z_S such that $Z_1 > Z_S > Z_2$ and also $Z_L = \sqrt{Z_1 \; Z_2}$, then, the power loss for Z_S/Z_1 = power loss for Z_S/Z_2, even though $(Z_1 - Z_S) > (Z_S - Z_2)$.

Alternatively, if Z_1 and Z_2 are two load impedances on either side of Z_S such that $(Z_1 - Z_S) = (Z_S - Z_2)$, then the loss of power due to the mismatch between Z_1 and Z_S will be less than that due to the mismatch Z_2 and Z_S so that an error of a few ohms on the higher side has less effect than the same numerical difference on the lower side of the source impedance.

(*iv*) Curves have been drawn for different values of $(\phi_S - \phi_L)$. in this set of curves that for $(\phi_S - \phi_L) = 0$ is the same as that given by Beitman (1943). It is found that the smaller the value of this difference the flatter will be the curve. This shows that for a given value of $Z_S \;/Z_L$, the power output will be greater the smaller the numerical value of $(\phi_S - \phi_L)$;

(*v*) when $(\phi_S - \phi_L)$ is equal to $\pm 180°$, no pôwer is delivered to the load. The power loss is constant and equal to 100% for all values of Z_S/Z_L ; and

(*vi*) the curve for $(\phi_S - \phi_L)$ equal to zero, gives the minimum variation of percentage power loss for variation in the Z_S/Z_L ratio.

(*b*) The effect on power output due to mismatch of phase angles of the source and load will now be considered. the impedances of the source and load being as assumed to be matched $(Z_S = Z_L)$.

For maximum power output under the above conditions, $\phi_S = (-\phi_L)$.

Then the power output $(W_S) = \frac{1}{2} \mid E \mid^2 \dfrac{\cos(-\phi_S)}{2Z_S[1 + \cos(2\;\phi_S)]}$

$$= \tfrac{1}{2} \mid E \mid^2 \frac{1}{4Z_S \cos \phi_S} \qquad \dots \quad (14)$$

When, however $\phi_S \neq -\phi_L$,

the power output $$(W_L) = \tfrac{1}{2} \mid E \mid^2 \frac{\cos \phi_L}{2Z_S\,(1 + \cos \phi_S - \phi_L)} \qquad \dots \quad (15)$$

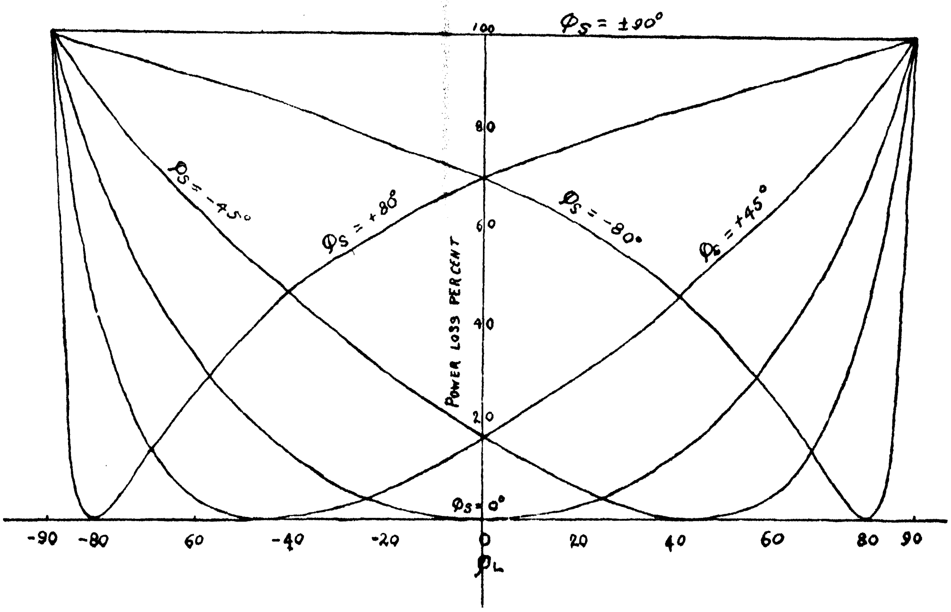
Dividing (15) by (14),

$$W_L/W_S = \frac{2 \cos \phi_L \cos \phi_S}{1 + \cos(\phi_S - \phi_L)}.$$

The power loss with reference to power output at matched condition is :

$$\frac{W_S - W_L}{W_S} = 1 - W_L/W_S = \frac{1 - \cos(\phi_S + \phi_L)}{1 + \cos(\phi_S - \phi_L)} \quad \dots \quad (16)$$

The power loss values have been calculated for different values of ϕ_L ranging from $-90°$ to $+90°$ for five particular values of ϕ_S namely $+45°, -45°, +80°, -80°$ and $0°$. The results have been plotted in graph 2.



GRAPH 2

The following inferences can be drawn from the graph :

(1) Maximum power is delivered when $\phi_L = -\phi_S$.

(2) Output is zero both when $\phi_L = +90°$ or $-90°$.

(3) At higher values of ϕ_S, the curves become steeper on either side of the matching point ($\phi_L = -\phi_S$). This means that the effect of mismatch increases as ϕ_S is increased and hence at the larger values of ϕ_S perfect matching becomes essential. It is therefore better to have ϕ_S as small as possible.

This same inference can be drawn from conclusion (*v*) para (*a*) above, where it is stated that for low power loss ($\phi_S - \phi_L$) should be as small as possible. The value of ($\phi_S - \phi_L$) when the angles are matched, is equal to $2\phi_S$ and if this is to be small, ϕ should be small. There can be most perfect matching with regard to impedances and phase angles in the case of pure resistances because, in that case, both the conditions, $\phi_S = -\phi_L$ and $\phi_S - \phi_L = 0$, are satisfied ($\because \phi_S = -\phi_L = 0$) ; and maximum output will be delivered to the load.

(4) For smaller values of ϕ_S (up to about 50°), if perfect matching is not possible, it is preferable to have ϕ_L some value between $(-\phi_S)$ and o.

(5) When ϕ_S is $\pm 90°$, the output is zero for all values of ϕ_L.

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