22

TRUE RELATIVE INTENSITY DISTRIBUTION FROM MICROPHOTOGRAMS

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ABSTRACT Harrison's test of self-consistency has been applied to rotating wire gauze screen type calibration method described earlier (*Nanda*, , 1945), for use in spectrophotometry. This device adheres to the test in at least the range of wavelengths 3,000 to 5,000 Å within the sensitiveness of the barrier-layer photo-cell microphotometer. Earlier calibration of various screens from area consideration is found to be correct from comparison with the self consistent inverse square method.

The effect of finite width of the spectrograph and microphotometer slits is considered in detail. In the study of sharp lines, the dispersion of the spectrograph must be such that the half width covers a greater length on the plate than that covered by the monochromatic image of the slit. It is shown that the modified contours on account of finite slit width have less curvature than the true ones, though the areas under the two are equal.

Among the methods of measuring the intensity distribution in microphotometer slit image, it is seen that differentiation of the pattern of a straight edge is most convenient and may also be advantageously used to determine the purity of an image formed by any microscope objective.

The microphotometer deflections depend on the intensity of the blackening on the photograph and the characteristic sensitiveness of the light-measuring device being employed in the microphotometer. As the response of the photographic plate as well as the light-measuring device is not always linear with the intensity of light incident on each, the deflections are not simply proportional to the original intensities which were photographed. Thus even if effect of finite width of slits have been allowed for, the deflections have to be reduced to true relative intensity magnitudes by some calibration device. The usual procedure is to get on the same photographic plate some exposures with proportionally reduced intensities; and by measuring these for the same setting (tracer light intensity, galvanometer sensitiveness, record to plate ratio) of the microphotometer, the deflections for known ratios of intensities can be determined. The set of calibration curves with λ are independent of slit widths, provided the original distribution is uniform or as in practice shows a slow variation warranting the neglect of such effects. From these curves the intensity relative to maximum, corresponding to any observed deflection at any wavelength can be easily determined.

A large number of devices are in vogue for this calibration of photographic plates. A brief description of some of these is given in a paper by Ditchburn (1927). It has been shown by the author (1945) in a recent paper that the simple wire screen type device has the largest sensitiveness and can be considered

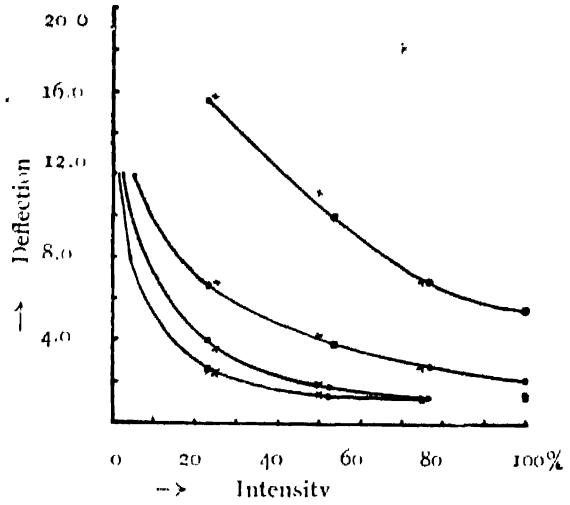
best for absorption spectrophotometry, for the reasons discussed there. Sawyer and Vincent (1943) have pointed out that any of these calibration devices should be used only after ascertaining their self-consistency. The method of doing this according to Harrison (1920) is to see if the common portion of any two calibration curves obtained by the device for two different initial intensities can be made to coincide by the same proportional sliding of one curve along the intensity axis. Grossman (1943) has found that the inverse square calibration is self-consistent and can serve as a check for any device.

It is attempted to show that the wire gauze screens calibrated from area considerations, give practically the same reduction of intensity as obtained by the calculated inverse square illumination. The inverse square check could be applied only in the visible or the near ultraviolet ($\lambda = 3{,}000$ to $\lambda = 5{,}000$ Å), as the ultraviolet source (the long water-cooled hydrogen discharge tube used in previous work) could neither be regarded as a point source nor could it be conveniently moved about to give inverse square intensity. A ground-glass screen was illuminated by a point-o-lite lamp. The ground glass was focussed on the slit by a suitable lens placed in the middle. Between the lens and the ground glass the short wooden tube capable of rotation in ball bearings was placed in which different wire gauze screens could be fixed. By keeping the distance between the lamp and the ground glass fixed, certain exposures were given with the same time of exposure and different wire screens (labelled 76.5%, 53 5%, 23.0%, etc.); later the wooden tube was kept empty while exposures for the same time were interspersed with the previous ones on the same plate (Ilford process) for different distances of the source from the ground glass. The ratios of successive distance to the first being $\sqrt{1.33} : 1$, $\sqrt{2} : 1$, $\sqrt{3} : 1$, $\sqrt{4} : 1$, exposures of true relative intensities 75%, 50%, 33%, 25%, were obtained. The ground glass is used so that light cone of the same width enters the spectrograph slit whatever the distance of the source, thus not altering the geometry of the spectrograph optics

After developing the plate carefully, sets of microphotometer deflections were observed for different exposures at the same wave lengths. To guard against the effect of photographic grain and any irregularities along the breadth of exposed strips, the microphotometer slit image was kept at its maximum length, *viz.*, 3 mm. The calibration curves were obtained from the two methods under comparison, at different points on the wavelength scale. The curves were found to coincide practically (Fig. 1). In view of no special precautions being taken to eliminate any variations of the light source or to ensure that there is no slight displacement regarding λ for points on the same curve, the maximum variation of 5% (which occurs only in small intensities) in the two sets of curves can be taken to be not much outside the experimental error. Taking the mean values from all the curves the labelled values may be altered to become 76·9%, 56·0% 23·4%.

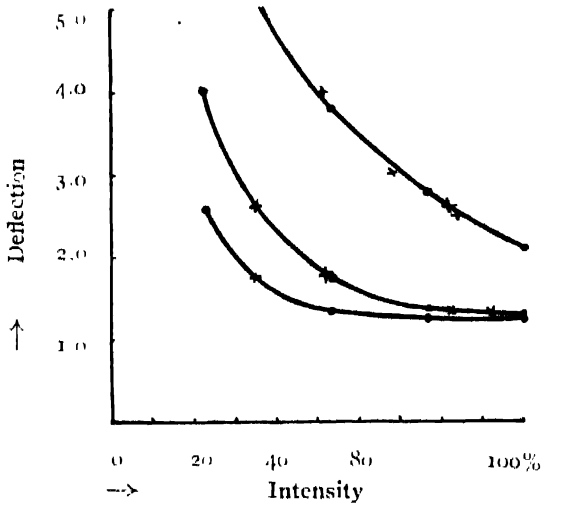
As regards self consistency any two curves for the calibration device, of the type obtained in figure 1, were tested and it was found that their common

portions could (making a proportional shift all along them) slide into each other (fig. 2). The divergences in the slow portions are due to poor accuracy in this region.



Comparison of wire gauze (o) and inverse square (×) calibration methods Different curves are for different maximum intensities.

FIG. 1



Self consistency of the wire gauze calibration method in the visible region— points marked (×) are obtained by transposition from other curves.

FIG, 2

The microphotogram can be directly converted into intensities using the calibration curves only if there were no modification of the contour, *e.g.*, by the finite width of the spectrograph slit and the microphotometer slit image. Changes brought about by these effects are prominent only for distributions

that are quite close as in the case of resonance lines or lines in absorption. Let us first consider the distortion on account of the spectrograph slit.

Following the notation of Van Cittert, in the coherent case to which the method of slit irradiation in actual practice corresponds, the slit image for monochromatic light of frequency ν gives an intensity distribution.

$$(J_{\phi d})_{\nu_0} \propto [Si(\psi_0 - \phi)d + Si(\psi_0 + \phi)d]^2 \qquad \ldots \quad (1)$$

where the function $Si(\alpha) = \int_0^{\alpha} (\sin \alpha/\alpha) d\alpha$ and $\psi_0 = \frac{2\pi\nu_0}{c} \psi_0'$ where ψ_0' is the half angular width of the slit viewed from the centre of the collimator lens. Similarly ϕ' is the angular distance of any point Q from the point P_0, where the monochromatic light reaches the plate after dispersion in the same spectrograph with infinitesimal slit-width, when viewed from the centre of the camera lens of the spectrograph, d is the aperture of the prism, etc., causing dispersion.

Let the incident light be of a continuous distribution $T/T\nu$ or $T_{\phi,\nu}$ on the plate if photographed through an infinitesimal slit. If as a special case $T_{\phi,\nu} = T$, a constant, in an ideal case any point P will simply have an intensity ordinate T_0 while in actual practice the point P will lie on all parts of curves analogous to $J_{\phi,d}$ with maxima at all points on the photographic plate. Therefore, the resulting intensity at P is just proportional to the integral of the curve $J_{\phi,d}$ with ϕ. The constant of proportionality in $J_{\phi,d}$ can be known if the resulting constant intensity is to be interpreted as T_0. Thus

$$T_0 = T_0 \int_{-\infty}^{+\infty} J_{\phi,d} \; d\phi, \text{ or } \int_{-\infty}^{+\infty} J_{\phi,d} \; d\phi = 1$$

For the distribution given by (1) the constant is $1/\pi'$. Now let $T_{\phi,\nu}$ vary with ν or in the same meaning with ϕ. To find the resultant intensity at P corresponding to ν, we have to write

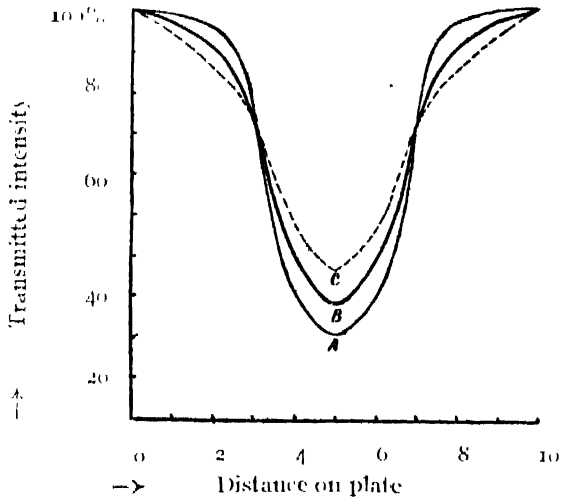
$$T_{j\nu} = \int_{-\infty}^{+\infty} T_{\phi,\nu} \; J_{\phi,d} \; d\phi \qquad \ldots \quad (2)$$

For integration, ψ_0 occurring in J can be taken as a constant for the narrow lines under discussion. The above integral can give results anywhere near the distribution T only when distribution J is much closer than T. If the original distribution does not extend beyond J of the monochromatic image of the slit, the photographed distribution will only be giving something near the monochromatic image distribution for all T, however the maximum amplitude shall depend on total intensity under the T-distribution.

It is clear that this modification of T-contour is only due to the resolution of the spectrograph, the least resolvable limit further levels out the original distribution, as in simple words the limiting resolvable distance is the size of slit image even if an infinitesimal slit is illuminated by monochromatic light.

Another effect occurs due to a limiting resolvable distance of the photographic emulsion, this levels out $T_{j,\nu}$ still further.

For usual spectrograph slit—width, the observed intensity, neglecting other effects, will be practically similar to the true one excepting for very narrow lines whose half width in terms of ψ becomes of the order of H_c, the half width for the monochromatic slit image ($H_c = .63\pi$ for optimum results for coherent arrangement). It is thus clear that for studying any line of a given order of sharpness there is a minimum resolution of the spectrograph which must always be maintained. When this consideration has been kept in view, the area of the observed curve which is generally needed for line intensity will be exactly equal to the true one. It will be clear from the accompanying treatment regarding effect of finite width of microphotometer slit image, that the width of a line that is uneffected by these effects is between the points of inflexion rather than between those of half intensity amplitude.

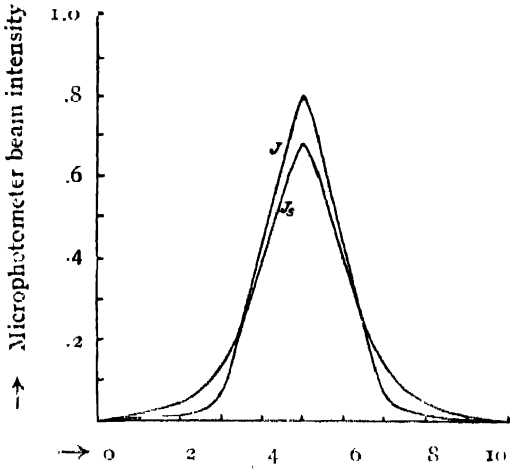


A—T_ν is the actual intensity distribution.
B—$T(j\nu)$ on account of spectrograph slit width.
C—$T(rj\nu)$ on account of finite resolution of the spectrograph and photographic plate as well.
The curves cut at the points of inflexion, Areas under all the curves are equal.

FIG. 3 (*a*)

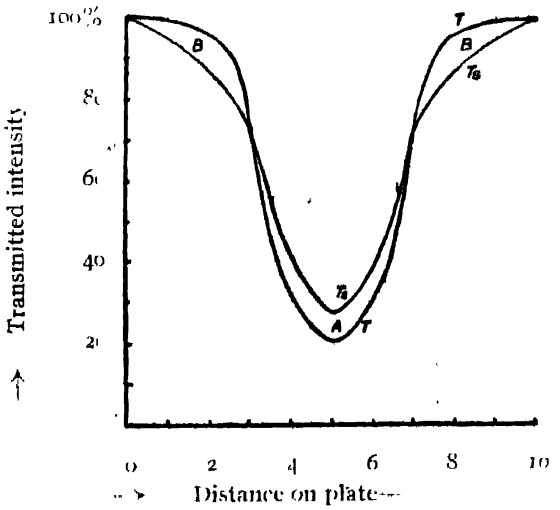
Before proceeding with the microphotometer slit effects it will be instructive to study how successive distributions arise, (Figs. 3). $T_{j\nu}$ in figure (3*a*) is the modified curve obtained by the changes brought about by finite spectrograph slit in the true curve T^ν (e.g., of an absorption line). $T_{j,\nu}$ will be further levelled out slightly on account of limiting resolving power of the prism (r_1) and the photographic emulsion (r_2) to give T (r, j, ν). The T-distribution will now give an F-distribution of the photographic blackening; for simplicity

F has been considered exactly similar to T. This F or T,-distribution gives the J-distribution of Fig (3*b*) standing for microphotometer beam intensities at different points; J_s^{ν} includes the effect of microphotometer slit width while J does not, for the same T-distribution. The J-distribution will further give the recorded curve D of deflections of the microphotometer, and again for simplicity



Js—on account of modification of J by the width of the microphotometer slit image. J curves are always narrower than T curves (opposite for emission lines). This necessitates use of calibration device even in the determination of width of spectral lines.

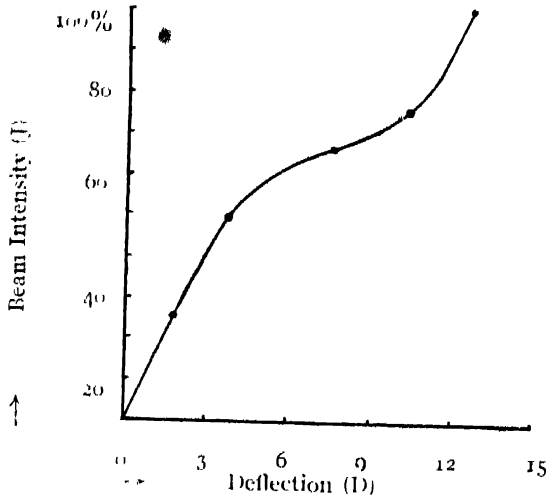
FIG. 3 (*b*)



Ts-curve is obtained if Js-curve is directly used without first correcting it to get the J-curve.
Area A is less than the areas B. (opposite for emission lines).

FIG. 3 (*c*)

J and D can be regarded as exactly similar, otherwise these too can be interchanged by an experimental conversion curve (Fig. 4).



Sensitiveness curves of the light-measuring device in the microphotometer, obtained by inverse square illumination Deflection can be converted into relative beam intensities from such a curve

FIG. 4

The observed calibration curves obtained even from a practically uniform spectral distribution, and expressed as a relation between J and T can be applied to any observed contour, only if all disturbing effects were absent. To get true T-curve, the $J(s, r, j, \nu,)$ curve is corrected first to give $J(r, j, \nu)$ and by the application of the calibration curve, $T(r, j, \nu)$ is obtained, which when corrected for the sundry effects gives $T\nu$. Ordinarily when corrections are known to be small and not considered we get $T(s,...\nu)$ distribution of figure $3(c)$ and not the T-distribution.

It can be shown that area under the complete curve $T\nu$ is equal to that under $T_{j,\nu}$, and also the same rule holds for the complete curves J. While areas under $T(s,...)$, T or $D(s,...)$, D are only approximately so. Langesth and Walles (1934) have calculated graphically the difference on T curves to be capable of being represented empirically by the formula $\Delta = 16A_0 p^2$ where A_0 is the maximum absorption amplitude (here) and p is the ratio of the effective slit-width to line half-width. The Δ in case of absorption lines is opposite in sign to that considered by Langesth and Walles, and so area under the T_s curve will be slightly larger than the true one. The area under T_s, T is practically the same and this fact has been used by Minkowski (1926) in his determination of radiation damping from total absorption rather than from integrated absorption coefficient, in case of sodium D lines. The area under $k\nu$ (the absorption coefficient) curve is not safe to work with especially if $T(s,...)$ falls to less than 20% in the centre as small spurious changes here give large error in k curve. The spurious effects described by Korff (1931), apart from those

of slit widths, are due to defective focussing the creep of the image on the emulsion at high exposures, and general scattering of light inside the spectrograph.

To take up the emergence of J_s in place of J, let x or x' stand for the distance along the plate, then at every point $P(x')$, the transparency of the plate is given by Jx' corresponding to the blackening Fx' at that point. When the point P comes exactly at the centre of the intensity pattern $S(x-x')$ of the slit image, the beam intensity (or deflection) $J_{s,x'}$ is said to correspond to $P(x')$. Evidently $J_{s,x'}$ is the convolution integral of the contour J_x and the pattern $S(x-x')$. Thus we have

$$J_{s,x'} = \int_{x'-\beta}^{x'+\beta} J(x)\, S(x-x')\, dx \qquad \ldots (3)$$

where 2β is the effective width of the slit image, *i.e.*, $S=0$ for $|x-x'| > \beta$. In the ideal case of an infinitesimally narrow slit, the S function for correct interpretation of results is the δ function *i.e.*, unity at $x=x'$ and zero at all other points. In the finite case, for constant $J(x)=C$, $J_{s,x'} = C\int_{x'-\beta}^{x'+\beta} S(x-x')\,dx$

and for regarding it as actually the same constant, $\int_{x'-\beta}^{x'+\beta} S(x-x') = 1 \qquad \ldots (4)$

which gives the correct value of the constant in the S function.

Repeated integration of R.H.S. of equation (3) above by parts gives

$$J_{s,x'} = \Big| J(x)D^{-1}S(x-x') - J'(x)D^{-2}S(x-x') + J''(x)D^{-3}S(x-x')\ldots \Big|_{x'-\beta}^{x'+\beta}$$

where negative powers of the operator D stand for the order of repeated integration. Since the function S is much more close than $J(x)$, which is important and as alternate D's have negative and positive symmetry, we can write down

$$J_{s,x'} = \Big[J(x'+\beta) + J(x'-\beta) \Big] \Big| D^{-1}S(x-x') \Big|_0^{x'+\beta} - \Big[J'(x'+\beta) - J'(x'-\beta) \Big] \times$$

$$\Big| D^{-2}S(x-x') \Big|_0^{x'+\beta} + \Big[J''(x'+\beta) + J''(x'-\beta) \Big] \Big| D^{-3}S(x-x') \Big|_0^{x'+\beta} - \qquad \ldots (5)$$

$$= \Big[J(x') + \frac{\beta^2}{2} J''(x') \Big] 2D_1 - \beta J''(x')\, 2D_2 + J''(x')\, 2D_3 \qquad \ldots (6)$$

The meaning of symbols D_1, D_2, etc., is obvious. For any particular S distribution these integrals can be worked out for small β. Two special cases may be considered :—

(*i*) $S(x-x')$ = a constant for 2β about x' and zero everywhere outside this range. From consideration of (4), $S(x-x') = 1/2\beta$ and the result is

$$J_{s,x'} = J(x') + \frac{\beta^2}{4} J''(x') + \ldots\ldots \qquad \ldots (7)$$

Effects like those of limited resolving power of the prism or the grating in the spectrograph and that of the photographic emulsion, already referred to, can be treated like this case in the first approximation.

(ii) When S has got some Gaussian or such symmetrical distribution within the range 2β of x it may be put as $\sqrt{\frac{a}{\pi}}\, e^{-a(x-x')^2}$ and outside this range as zero. Thus

$$J_{s,x'} = J(x') + \frac{\beta^2}{2} J''(x') \left\{ 1 - \sqrt{a} + \frac{a}{2} \right\} + \ldots\ldots \qquad \ldots (8)$$

Practically similar results are obtained if $S(x-x')$ were any other expression like $Si^2[a(x-x')]$ or $Si_2[b(x-x')]$ where Si has the usual meaning and Si_2 stands for the form $\int_0^{\alpha} \frac{\sin^2 \alpha}{\alpha^2}\, d\alpha$. Such cases arise whenever slit effects or in other words limited resolving power on account of finite slit widths is considered.

From (7) and (8) it is obvious that for the points of inflexion where $J''(x') = 0$ we have $J_{s,x'} = J(x')$. Thus as pointed out earlier the distance between the points of inflexion is independent of slit effects. It is also obvious that neglecting higher order terms than the second, the complete areas (or the areas in the symmetrical halves) $\int J_{s,x'} dx'$ and $\int J(x') dx'$ are equal. Thus the area A in fig. 3 is equal to the areas B. However similar areas are not equal on the blackening or on the deflection curves when these are not proportional to the illuminations. It is also clear from equation (7) as to how the curve $J_{s,x'}$ always faces the concave side of Jx'. For usual graphical calculations in connection with the slit effects, *viz.*, those of Langesth and Walles (*loc. cit.*), only the first term in (5) is kept in view and

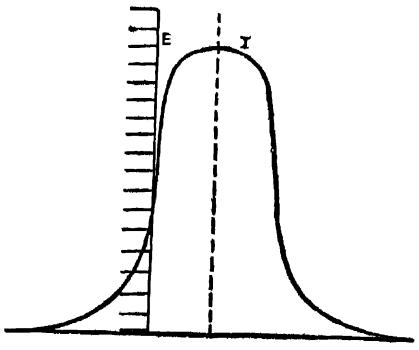
$$J_{s\,x'} = \tfrac{1}{2} \left[J(x'+\beta) + J(x'-\beta) \right] \qquad \ldots (9)$$

It must be clearly borne in mind that when a particular line has a distribution closer than that of the photometer slit image the observed distribution will in no way be near the real one but it will tend to correspond to the S distribution. Only in this case the maximum amplitude will be proportional to the area under the original curve. Thus for any photometer slit width $b = 2\beta$, line contours only of larger half width than this can be studied. Therefore, b must be small. But one does not gain much by reducing b to less than the value required to give a resolution equal to that of the photographic emulsion. The least distance that can be resolved by a photographic fine grain process emulsion is 12μ according to Mees (1931), and the slit distribution with a given purity of a projected slit width about 10μ can resolve distances up to 10μ according to Plaskett (1934). So b should at least be kept of the order 10μ.

For determination of the purity of the microscope objectives as well as the theoretical resolution of the microphotometer, the determination of effective

width of the slit image and the intensity distribution inside, is of interest. The simple procedure followed by Sandström (1933) of regarding the difference between the breadth of a microphotogram of a fine scale division at the foot and at the top as four times the effective breadth of the slit is evidently incorrect. Such a method can be applicable only if the intensity distribution inside the slit image is uniform falling suddenly to zero at the extremities. Even then the difference in breadth is only twice the width of the slit image and not four times. Also the microphotogram must be first corrected for the non-linear response of the barrier layer photo cell employed in the Siegbahn design.

Plaskett has obtained the distribution from the microphotogram of transmission across a narrow opening as a cut on an Abbe test plate. However this is an unnecessarily laborious method. The simple method of a straight edge mentioned also by him may be considered in the following way. In Fig. 5 the intensity distribution (I) inside the slit image is represented as being



A straight edge traversing the slit image.

FIG. 5

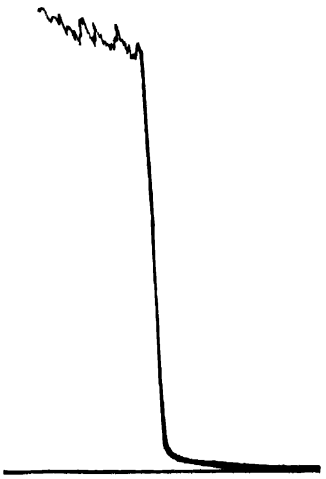
progressively unscreened by an extended obstacle with a straight edge (E). For any variable point P of plate falling on the x' centre of the slit image when the straight edge is at x, the beam intensity is given by

$$J(x') = \int_{x}^{x'+\beta} S(x-x')dx \qquad \ldots \quad (10)$$

Differentiating (10) we get $\frac{dJ(x')}{dx'} = S(x-x') - S(\beta) = S(x-x')$

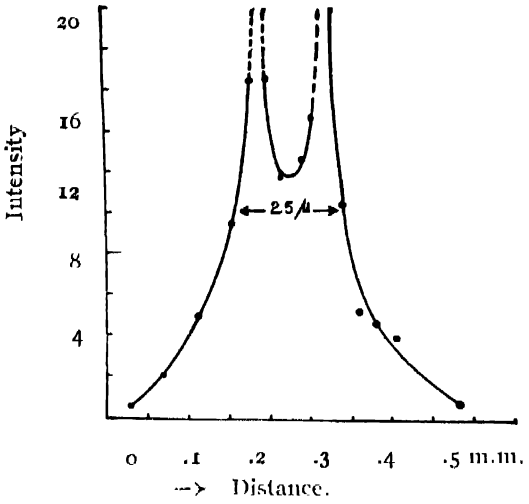
since $S(\beta)=0$. Thus we have simply to get a microphotogram when a straight edge traverses the microphotometer slit and then differentiate, at every point, the J curve computed from the deflection curve. Fig. 6*a* gives such a microphotogram in case of Siegbahn instrument of this laboratory. Fig. 6*b* gives intensity distribution derived from it. From the record to plate ratio the half width is found to be 25μ in this case. Slight structure clearly shown by the curve (also visible through a high power microscope) and the large width must

be ascribed to the causes as have been given in connection with methods of slit irradiation by Stockbarger and Burns (1931), namely, coherent illumination of



Microphotogram when a straight edge traverses the slit image

FIG. 6 (a)



Intensity distribution inside the image of a microphotometer slit
Doubling up is due to incorrect setting.

FIG. 6 (b)

primary slits of about double the size necessary to give single maxima.* The double structure was actually removed by readjustment on simply reducing the primary slit width inside the illumination unit of the microphotometer. The half width, at present is about 6μ.

CONCLUSION

(1) In absorption spectrophotometry the necessary calibration of individual photographic plates must be carried out by a self-consistent device. The rotating wire gauze screens have been found to give a self-consistent curve within the sensitiveness of the photo-cell arrangement. Inverse square comparison of the transmissions of different screens known from area consideration gave satisfactory results inside the wavelength region studied (process plates and point-o-lite in glass source).

(2) For the given resolution of a spectrograph, lines of certain minimum width alone can be studied. Otherwise the spurious effects cause the calculated transmission in the centre to be very much different from the actual which may lead to totally erroneous k_ν's. The effects of limited resolving power of the dispersing agent in the spectrograph and that of the photographic emulsion can be distinguished from the effects of finite size of the slit images for the spectrograph and the microphotometer. The calibration curves obtained especially from a source of uniform spectral distribution are free from all levelling errors described above. However, the observed contours must be corrected for all the

errors, before the use of the observed calibration curves can be made to give original intensities.

(3) All these errors individually tend to level out the contours, *i.e.*, depressing the convexities and lifting the concavities of the curves, the points of inflexion remaining undisturbed. Nevertheless, the complete areas of the true and the disturbed curves in each step are equal.

(4) Microphtogram of a straight edge gives an easy and accurate method of knowing the intensity distribution inside the slit pattern in a microphotometer assembly. This method may also be applied with advanatage in testing the purity of the image given by a microscope objective.

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