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SOUND-ABSORPTION CHARACTERISTICS OF INDIAN MATERIALS: PART II

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ABSTRACT. This paper is a continuation of a previous one by the authors [Chatterjee and Dutt (1944)] and presents a theoretical investigation of the sound-absorption characteristics of rigid porous materials based on the treatment given by Lord Rayleigh. The nature of variation of sound-absorption coefficient with angles of incidence as well as frequencies of sound wave incident on rigid porous surfaces of various characteristics has been studied. The condition of total absorption has also been incorporated. The absorption coefficient of a sample of jute suface has been calculated theoretically from the dimensions and properties of the sample and a comparison has been made with the results obtained experimentally.

INTRODUCTION

Sound waves striking on a plane surface forming the boundary between two different media are generally divided into reflected and transmitted portions, the amplitudes of which depend on the relative physical properties of the two media. If the boundary presents an unbroken solid or liquid surface, the amplitude of the reflected beam will predominate and only a minor portion of the energy will be found in the transmitted portion. If the surface is the boundary between air and a porous material a portion of the incident sound wave would penetrate into the pores of the second medium. Assuming the pores to be capillary channels there will be degradation of sound energy due to the viscous action of the air inside the pores. Thus in the case of porous bodies the amplitude of the reflected beam will be smaller due to the process of absorption.

The equation showing the relation between the absorption coefficient at any angle of incidence for a sound of definite frequency on a porous surface with pores of definite radius and a definite ratio between unperforated and perforated area and the kinematic viscosity of gas in the pores and ratio of sp. heats of the gas has been shown in Paper I to be as follows :

$$a_\theta = \frac{SM\{(2M^2+1) \pm \sqrt{4M^4+1}\}\cos\theta}{\{(2M^2+2M\cos\theta+1) \pm \sqrt{4M^4+1}\}^2} \qquad (1)$$

Variation of Absorption with Angle of Incidence

A graphical plot showing the relation between a_θ, θ and M is given in Fig. 1. The factor M involves not only the characteristics of the surface but also the effect of different frequency of the sound wave. As the angle increases from normal incidence value ($\theta=0°$), the absorption increases slowly, at first, and then more rapidly, it passes through a maximum value of unity and finally drops down rapidly to zero at the grazing incidence ($\theta=90°$).

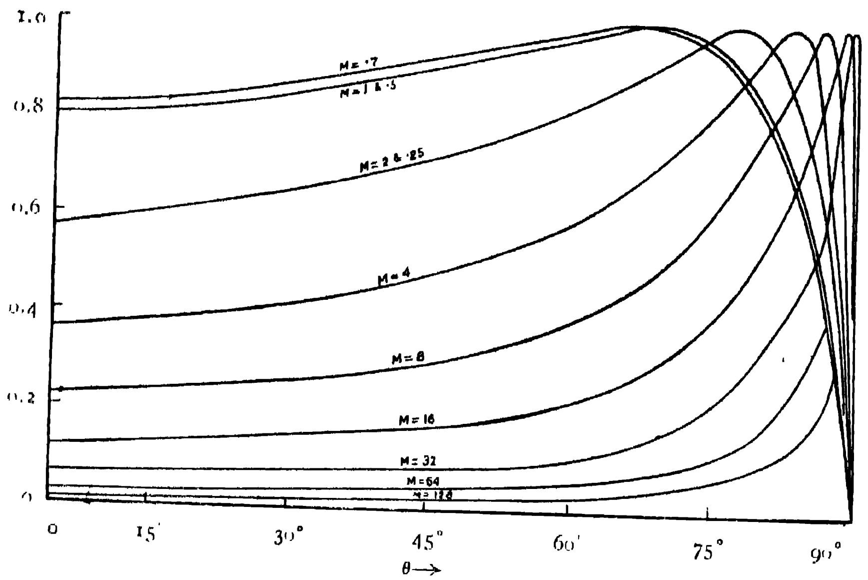

FIG. 1

The Case of Total Absorption

From Fig. 1, it is noted that for a particular value of M there is always some angle of incidence θ at which the absorption is complete. This optimum value of θ is found as follows :

Since $$a_\theta = 1 - \left\{ \frac{(1+g)\cos\theta - x}{(1+g)\cos\theta + x} \right\}^2 \quad \ldots \quad \text{[Paper I, } loc. cit.\text{]}$$

a_θ will be unity when the terms inside the bracket reduce to zero.

i.e. $$(1+g)\cos\theta - x = 0$$

or $$\cos\theta = x/1+g.$$

Substituting the value of x ... [Paper I, *loc. cit.*]

$$\cos\theta = \frac{(2M^2+1) \pm \sqrt{4M^4+1}}{2M} \quad \ldots \quad (2)$$

From (2) one gets $$M = \frac{(\cos^2\theta+1) \pm \sqrt{(\cos^2\theta+1)^2 - 8\cos^2\theta}}{4\cos\theta} \quad \ldots \quad (3)$$

For any value of θ, it is seen that corresponding to the positive and the negative signs preceding the term under the root, M will have two values at which the absorption would be unity.

When the term under the root sign becomes zero, M shall have only one value given by

$$M = \frac{\cos^2\theta + 1}{4\cos\theta}$$

and the value of the angle θ for which there is only one value of M at which absorption is unity is given as follows :

$(\cos^2\theta+1)^2 - 8\cos^2\theta = 0$; or $\cos\theta = 0.414$; or $\theta = 65°31'$.

For the angle of incidence $65°31'$, the numerical value of M at which unity absorption takes place is given by

$$M=\frac{\cos^2\theta+1}{4\cos^2\theta}=1/\sqrt{2}=0.707.$$

Fig. 2 shows the variation of the values of $(\cos^2\theta+1)^2-8\cos^2\theta$ with θ.

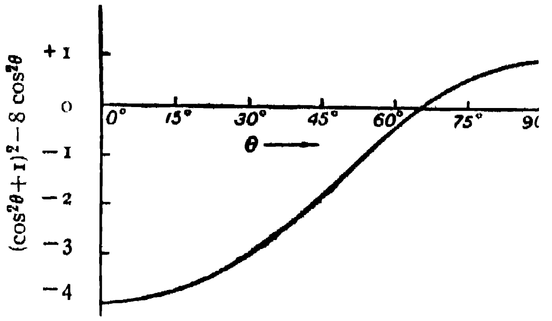

FIG. 2

It is noted from the curve that only for angles of incidence greater than $65°31'$ the term under the root sign in equation (3) is positive, and consequently there will be two real values of M at which the absorption will attain unity. For angles of incidence θ less than $65°31'$ the term $(\cos^2\theta+1)^2-8\cos^2\theta$ becomes negative and consequently M becomes a complex term, but from the definition, M is real and condition of unity absorption has been deduced by taking M to be real. So for any angle θ less than $65°31'$ absorption can never attain unity value, but it attains a peak value lower than unity for a particular value of M which can be determined by differentiating the expression (1) indicating the relation between a_θ and M and equating the result to zero.

Differentiating equation (1)

$$\frac{da_\theta}{dM}=\Big[\{(2M^2+2M\cos\theta+1)\pm\sqrt{4M^4+1}\}^2\Big[8\cos\theta\{(2M^2+1)\pm\sqrt{4M^4+1}\}+8M\cos\theta\Big\{4M\pm\frac{16M^3}{2\sqrt{4M^4+1}}\Big\}\Big]-$$

$$8M\cos\theta\{2M^2+1)\pm\sqrt{4M^4+1}\}\times 2\{(2M^2+2M\cos\theta+1)\pm\sqrt{4M^4+1}\}$$

$$\Big\{4M+2\cos\theta\pm\frac{16M^3}{2\sqrt{4M^4+1}}\Big\}\Big]\Big/$$

$$\{(2M^2+2M\cos\theta+1)\pm\sqrt{4M^4+1}\}^4.$$

This reduces to

$$\frac{da_\theta}{dM}=8\cos\theta\Big[4M^3\cos\theta+1\pm2\sqrt{4M^4+1}\pm(4M^4+1)\pm\frac{16M^5\cos\theta}{\sqrt{4M^4+1}}-$$

$$4M^4-2M\cos\theta\mp2M\cos\theta\sqrt{4M^4+1}\mp\frac{16M^6}{\sqrt{4M^4+1}}\mp\frac{8M^4}{\sqrt{4M^4+1}}\Big]\Big/$$

$$\{(2M^2+2M\cos\theta+1)\pm\sqrt{4M^4+1}\}^3.$$

Putting $\frac{da_\theta}{dM}=0$ one finds

$$8\cos\theta\left[4M^3\cos\theta+1\pm2\sqrt{4M^4+1}\pm(4M^4+1)\pm\frac{16M^5\cos\theta}{\sqrt{4M^4+1}}-\right.$$

$$\left.4M^4-2M\cos\theta\mp2M\cos\theta\sqrt{4M^4+1}\mp\frac{16M^6}{\sqrt{4M^4+1}}\mp\frac{8M^4}{\sqrt{4M^4+1}}\right]=0$$

or

$$(2M^2-1)\left\{2M\cos\theta-(2M^2+1)\mp\frac{2(4M^4+2M^2+1)}{\sqrt{4M^4+1}}\mp(2M^2+1)\pm\frac{2M\cos\theta\,(2M^2+1)}{\sqrt{4M^4+1}}\right\}=0$$

or

$$(2M^2-1)=0.$$

$$\therefore \quad M=\frac{1}{\sqrt{2}}=0.707.$$

Thus for angles of incidence lower than $65°31'$ absorption coefficient attains a peak value for a value of $M=1/\sqrt{2}$ at which absorption just attains unity at the critical angle of incidence of $65°31'$.

Variation of Absorption with the value of M

In order to examine the nature of absorption for different values of M, a graph has been plotted to indicate the relation of absorption and M. Here one finds a family of curves for different angles of incidence. Since the values of M extend over a very large range it was thought fit to plot the values of M on a logarithim scale. This is shown in Fig. 3.

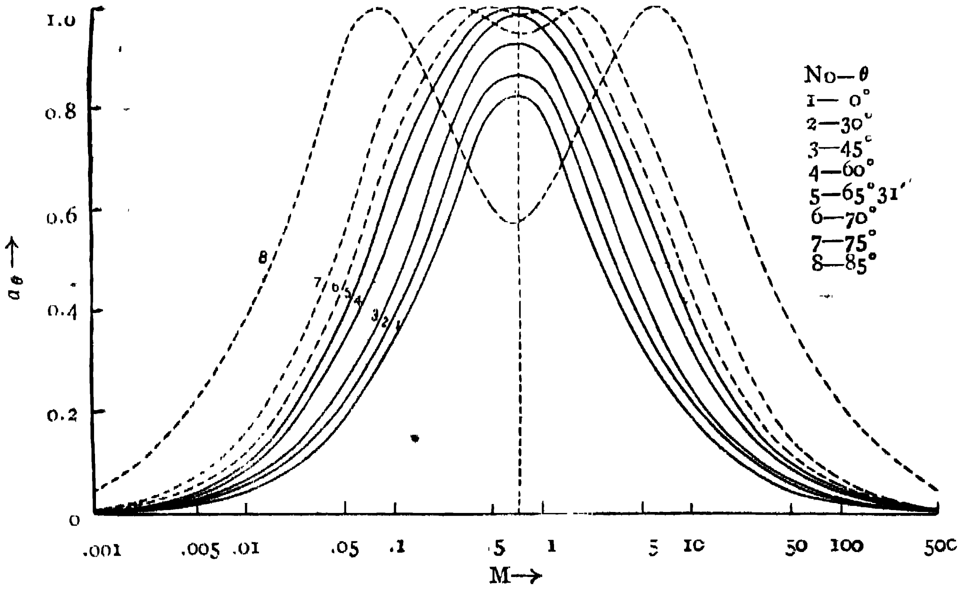

FIG. 3

The Fig. 3 shows that the absorption factor α_θ is related to the value of M, such that for the same angle of incidence there are two values of M at which equal values of absorption is attained and these two values are symmetrical with respect to the value of M at which peak absorption is found.

Taking $M_1 = M_0/n$ and $M_2 = nM_0$ and substituting the value of M_0 as $1/\sqrt{2}$ one finds for M_1

$$\alpha_\theta = \frac{\frac{8M_0}{n}\left\{\left(2\frac{M_0^2}{n^2}+1\right) \pm \sqrt{\frac{4M_0^4}{n^4}+1}\right\}\cos\theta}{\left\{\left(2\frac{M_0^2}{n^2}+2\frac{M_0}{n}\cos\theta+1\right) \pm \sqrt{\frac{4M_0^4}{n^4}+1}\right\}^2}$$

and for M_2

$$\alpha_\theta = \frac{8nM_0\{(2n^2M_0^2+1) \pm \sqrt{4n^4M_0^4+1}\}\cos\theta}{\{(2n^2M_0^2+2nM_0\cos\theta+1) \pm \sqrt{4n^4M_0^4+1}\}^2}.$$

Both these expressions reduce to

$$\frac{4\sqrt{2}\,n\{(n^2+1) \pm \sqrt{n^4+1}\}\cos\theta}{\{(n^2+\sqrt{2}\,n\cos\theta+1) \mp \sqrt{n^4+1}\}^2}.$$

The peak value of absorption is found to be at M_0 for angles of incidence less than that of the critical value of θ, *i.e.*, $65°31'$.

For angles higher than that of the critical value of θ the peak values are attained for two other values of M. In these cases, a depression of absorption is shown at M_0 value. As the angle of incidence increases the peaks are separated further apart and the depression is more marked. At the grazing incidence the entire range of value for M reduces to zero.

It may be pointed out in this connection that when the dissipation is negligible one would get a simplified expression for total absorption as has been shown by Rayleigh (1920).

In this case taking amplitude of reflection to be B one gets

$$\frac{B-1}{B+1} = \frac{\sigma}{\sigma+\sigma'} \quad \frac{1}{\cos\theta}$$

and then the condition of unity absorption, *i.e.*,

$$B = 0$$

gives

$$\cos\theta = \frac{\sigma}{\sigma+\sigma'} = \frac{1}{1+g}.$$

A graphical plot of $\alpha_\theta = 1 - B^2$ for different values of θ and g is given in Fig. 4.

This shows that even for normal incidence one would get total absorption which is not realised in practice. Paris (1927) from the consideration of acoustic impedance has shown that under no condition one would get maximum absorption at normal incidence.

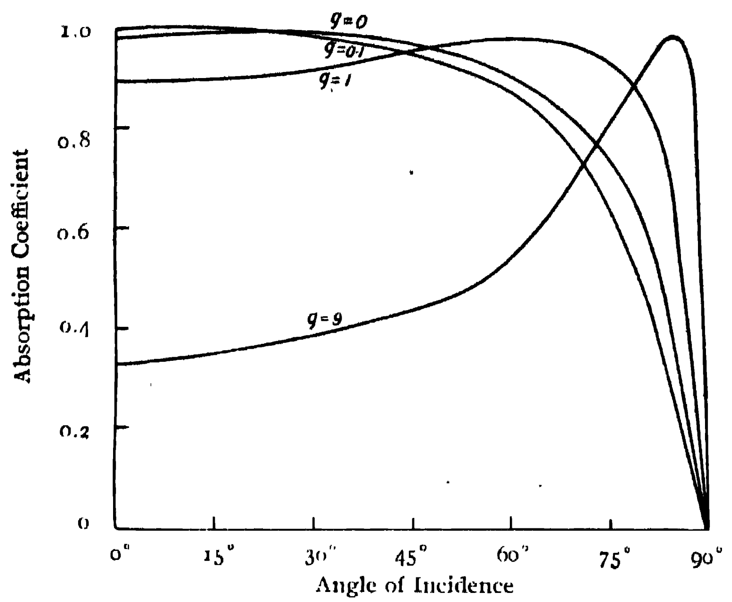

FIG. 4

Variation of α_θ with Frequency of Sound Wave and Physical Properties of the Sample

The general equation (1) does not give a direct relation between α_θ and frequency of the sound wave f. A direct relation between α_θ and f, can be obtained as follows :

We have $$M=\frac{2(1+g)\sqrt{\nu\gamma}}{r\sqrt{2\pi f}}=\frac{S}{\sqrt{f}}, \text{ say,}$$

where S is the factor depending on the surface condition of the sample.

Therefore $$S=\frac{2(1+g)\sqrt{\nu\gamma}}{r\sqrt{2\pi}}.$$

Substituting the values of ν and γ for air we have

$$S=0{\cdot}36\frac{1+g}{r}.$$

Thus S depends only on the nature of the surface of the sample that is on porosity and radius of the pores, and is constant for a particular sample. Let S be termed the 'Surface factor' of the sample.

Substituting $\frac{S}{\sqrt{f}}$ for M in equation (1) we have

$$\alpha_\theta=\frac{\frac{SS}{\sqrt{f}}\left\{\left(\frac{2S^2}{f}+1\right)\pm\sqrt{\frac{4S^4}{f^2}+1}\right\}\cos\theta}{\left\{\left(\frac{2S^2}{f}+\frac{2S}{\sqrt{f}}\cos\theta+1\right)\pm\sqrt{\frac{4S^4}{f^2}+1}\right\}^2}$$

Simplifying

$$a_\theta = \frac{8S\sqrt{f}\{(2S^2+f)\pm\sqrt{4S^4+f^2}\}\cos\theta}{\{(2S^2+2S\sqrt{f}\cos\theta+f)\pm\sqrt{4S^4+f^2}\}^2} \tag{4}$$

For normal incidence θ equals 0°, this reduces to

$$a_0 = \frac{4S\sqrt{f}}{2S^2+2S\sqrt{f}+f}$$

Eq. (4) gives a direct relation between absorption and surface condition of the sample and frequency and angle of incidence of the sound wave.

The Nature of S

The minimum value of the "Surface factor" S, which a substance can have, occurs when the value of g of the substance is minimum and r, the radius of the pores, is maximum. Let us assume the minimum value of g to be 0.1, *i.e.*, when about 90% of the surface is perforated. (As the Rayleigh's equation was deduced with the assumption that the pores were moderately small, less than 0·025 cm. diameter, to allow complete dissipation of sound energy into heat through viscous action in channels into which the sound waves penetrate), one can accept the maximum value of r to be about 0.012 cm. The minimum value of surface factor S comes near about 33.

When the pores are very small (say $r=0.001$ cm.) and the area of the unperforated surface is comparatively larger than that of the perforated surface (say $g=10$), so that 90% of the surface is unperforated, S has fairly large value near about 4000.

In practice porous surface can be obtained having surface factor varying from 33 to values more than 4000.

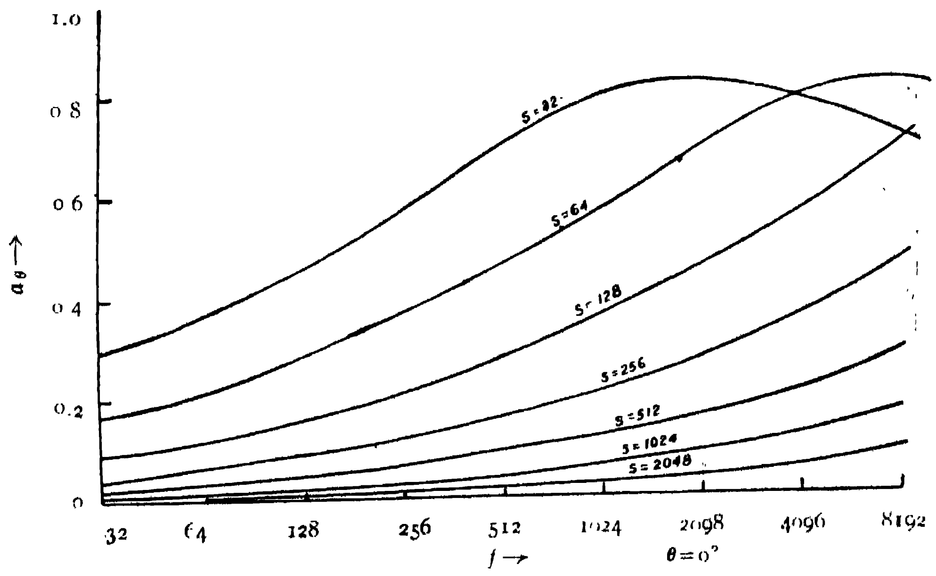

FIG. 5(a).

Fig. 5 (a, b, c, d and e) show the variation of a_θ with f, for substances of different values of S at various angles of incidence. It is seen that for all angles of incidence, substances having moderate values of S have a region of maximum absorption within the audio frequency band. As the value of S gradually increases the region of maximum absorption gradually shifts towards higher frequency till it is beyond the audio frequency band. Thus substances having high value of S show no selective absorption within audio frequency

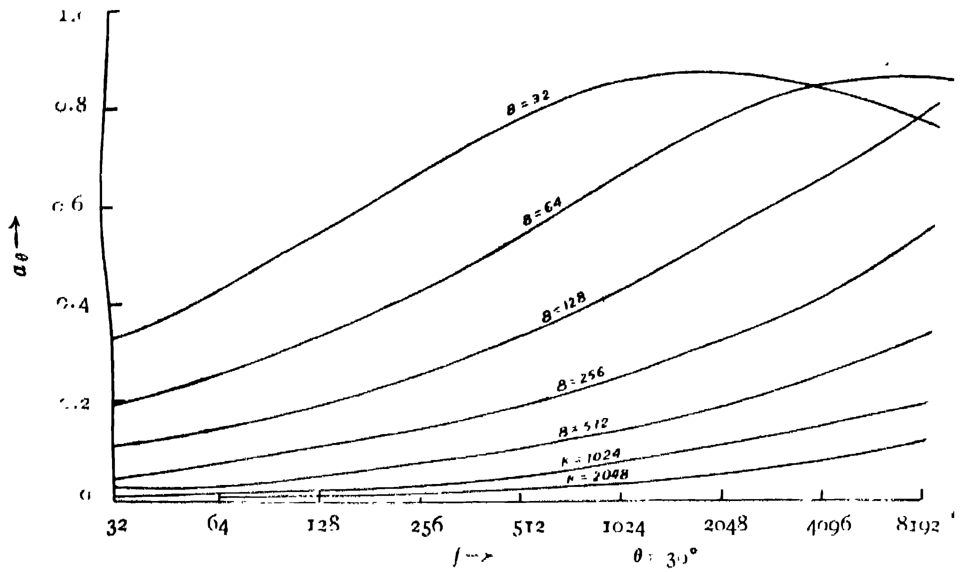

FIG. 5(b)

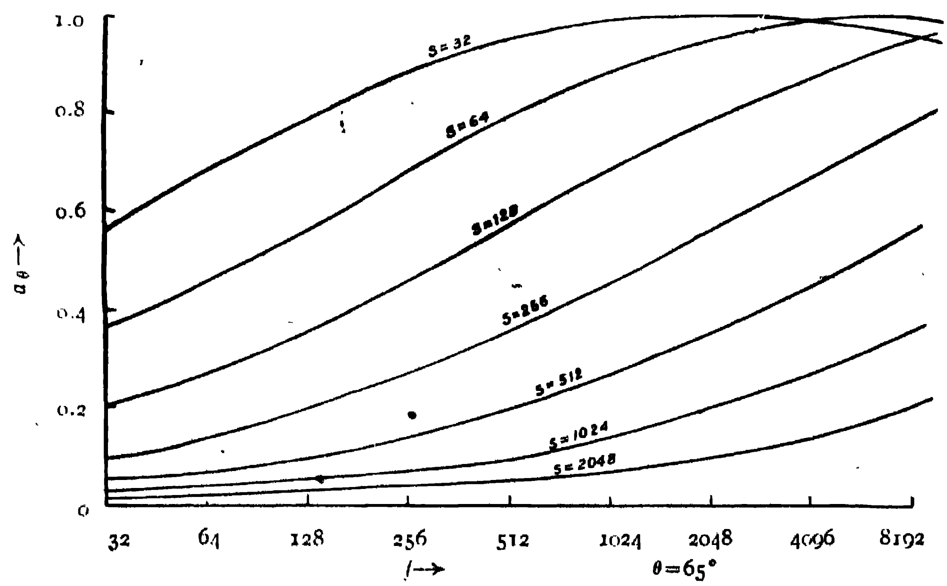

FIG. 5(c)

band and their absorption simply goes on increasing with frequency. For a particular value of S, the frequency at which maximum absorption takes place is constant for all angles of incidence less than 65°31′ and for angles greater than 65°31′ minima takes place at that frequency.

It is seen from the curves that in general the lower the value of S, the greater is its absorbing power. In other words the sound absorption coefficient is higher when the diameter of pores lies within a certain prescribed limit and the proportion of the perforated area of the surface is higher.

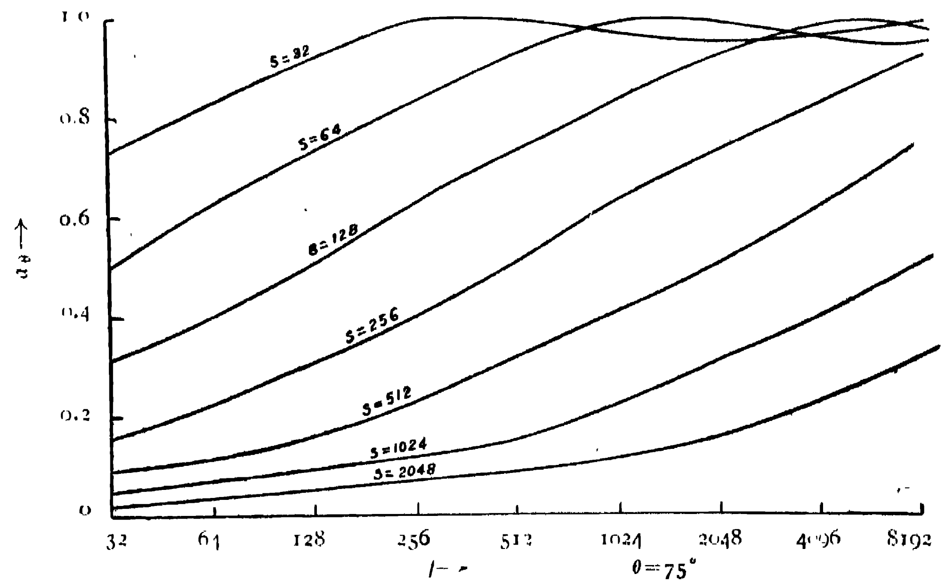

FIG. 5(d)

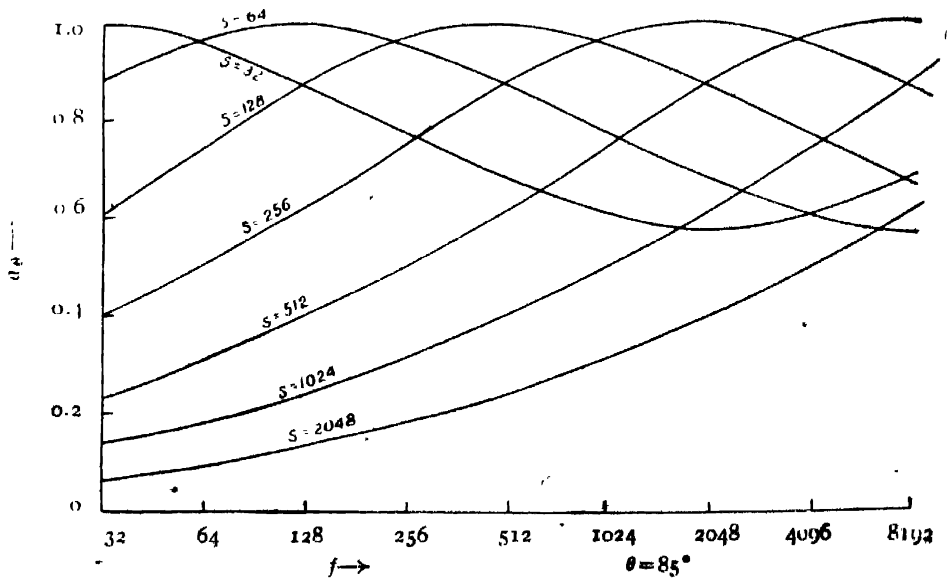

FIG. 5(e)

EXPERIMENTAL VERIFICATION

The absorption coefficients of some porous materials at different angles of incidence and frequencies of sound wave have been experimentally determined (See Paper I).

The variation of absorption coefficient with θ for one of these materials (Jute surface) at different frequencies is shown in Fig. 6. The nature of the curve agrees with the curves theoretically deduced as shown in Fig. 1.

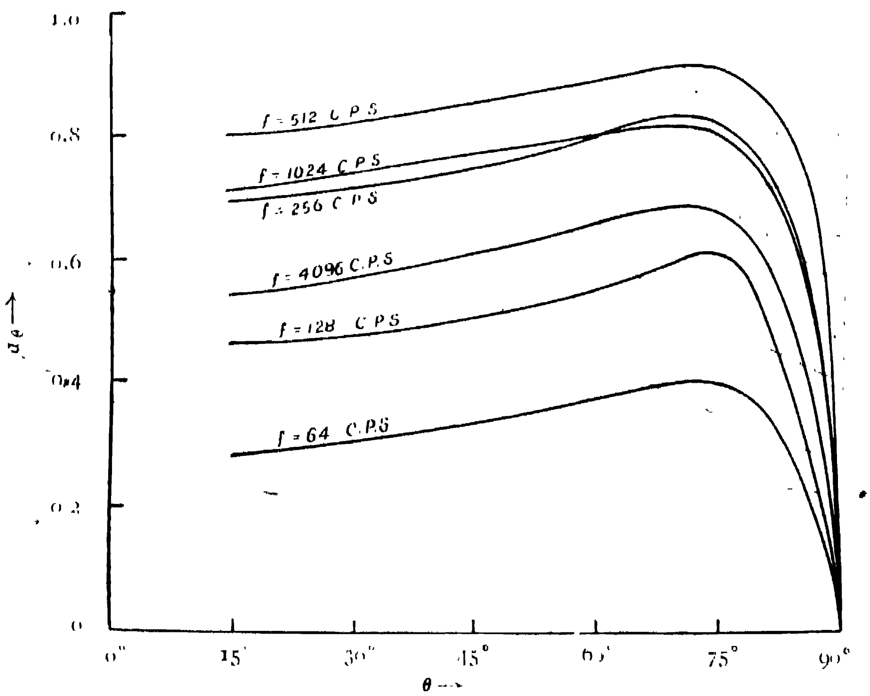

Fig. 6

In order to determine the absorption coefficient of any sample from the theoretical formula, a knowledge of the surface factor S of the sample is essential. S involves the determination of 'r' the radius of the pores and 'g' the ratio of the unperforated to the perforated surface of the sample.

The values of 'r' and 'g' of the sample formed by jute fibres were found from a microscopical study of the surface; 'r' was measured directly by means of the miscroscope and the value of 'g' was calculated from a measurement of the diameter of the fibres and the number of fibres per unit area. As the values of 'r' and 'g' are slightly different at various portions of the surface a mean of a large number of readings was taken.

The average values of r and g for the jute sample were found to be approximately ·01 cm. and ·1 respectively so that the surface factor of the jute sample came out to be about 40. In order to compare the deductions of the theoretical formula with the observational data the relation of the absorption coefficient of jute sample at an angle of incidence of 30° for various frequencies has been considered. On plotting the data in Fig. 7 one finds that for lower

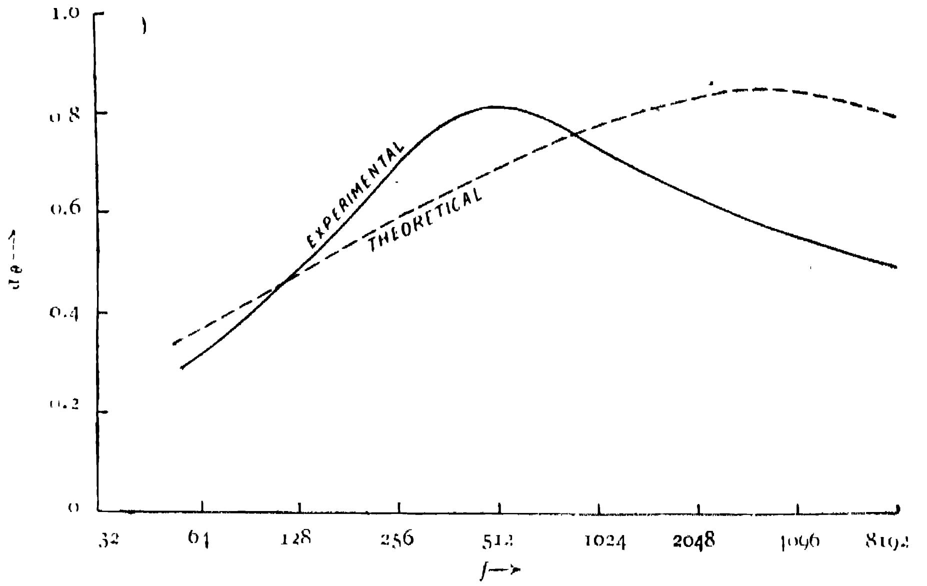

Fig. 7

values of frequencies there is a very fair degree of agreement between the theoretical value and the experimental results. But for the higher frequencies it is noted that the experimental curve of absorption is lower than the theoretical one. Further one finds that the maximum value of 0.88 is attained at 3000 c.p.s. according to the theoretical calculations. The experimental result, however, shows the maximum at a much lower frequency, namely 512 c.p.s., and the peak value of absorption is 0·83.

Such a divergence for higher frequencies requires an explanation. It may be remarked in this connexion that our theoretical formula was deduced from the consideration namely that the unperforated portion of the surface consists of rigid elements indicating that energy incident on this portion is definitely reflected at the solid boundary. But if these boundaries are of a resilient nature it is quite possible that some portion of energy is transmitted at this boundary before they are reflected. For low frequencies this time element does not play a significant role, but for higher frequencies the yielding nature of the fibre has a tendency to absorb the incident energy and the absorption will be affected by the reflected portion occurring after a definite interval. In that case the experimentally determined value of absorption will be lower for higher frequencies and the peak value of absorption will attain for the value of frequency at which the time lag factor is minimum. So absorption coefficient attains a reduced value at a lower frequency.

This aspect of the fibre regarding its yielding nature requires a closer study.

Here one may also expect certain variation between the phases of the incident and reflected waves due to yielding nature of the fibre.

ACKNOWLEDGMENT

The authors desire to express their best thanks to Prof. P. N. Ghosh, Head of the Department of Applied Physics, for his kind interest and valuable discussions throughout the progress of the work and to Mr. A. K. Sengupta for some helpful suggestions.

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