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STUDY OF THE BAND-PASS EFFECT BY CATHODE-RAY OSCILLOGRAPH

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ABSTRACT. The paper relates to study of the important phenomenon of "band-pass effect" by cathode-ray oscillograph method developed by the author utilizing Wood's phase angle measurement.

Measurements were carried out both on low-pass and high-pass filter sections terminated in stabilised negative impedance; and gain or attenuation, phase-shift angle and sign of phase-shift angle of the resulting band pass arrangement were obtained by oscillograph method.

The method not only enabled the phase-shift angle at different frequencies to be obtained with sufficient accuracy and comparative ease, but the variation of the sign of phase-shift angle as the frequency altered could be visually observed. Comparison between this method and a well-known non-oscillographic method has been made.

The method developed here is applicable to determination of gain or attenuation, phase-shift angle and sign of phase-shift angle of all types of wave-filter and network.

1. INTRODUCTION

It was experimentally discovered by the author (1938) about six years ago that both low-pass and high-pass *symmetrical* filter sections of any type terminated in stabilized negative impedances of magnitude more or less equal to their maximum characteristic impedance in the transmission band become "converted" to *unsymmetrical* band-pass filters and in addition, amplification was obtained over the transmitted band. Thus the two effects, namely the "band-pass effect" and "amplification effect," were simultaneously obtained. The band-pass effect was, however, more important than the latter effect. Use of negative impedance of magnitude greater and less than the maximum characteristic impedance in the transmission band for termination purpose had the effect of adjusting gain in the transmission band, transmission band-width and sharpness of cut-off of the resulting band-pass arrangement.

It has been found (Chakravarty, 1941) that the cause of the band-pass effect lies in (*a*) the nature of variation of the network gain or loss of the 'equivalent' network (formed from the original filter section and the negative impedance termination) with frequency and (*b*) the nature of variation of the reflection gain or loss between characteristic impedance of the original filter section and negative impedance of the termination with frequency. It has further been found that the former variation contributes little to the band-pass effect whereas the latter variation contributes mainly to it. The nature of variation of reflection gain or loss with frequency which intensifies the "band-pass effect" is again largely due to variation of negative impedance of the termination with frequency.

The present paper arose out of the desire to examine the "band-pass effect" by an *entirely different* method—by observing patterns on the screen of cathode-ray oscillograph and measuring various lengths and angles thereon. It has been possible by the oscillographic method developed to determine attenuation or gain, angle of phase-shift and sign of the angle of phase-shift at various frequencies for the resulting band-pass arrangement. Both low-pass and high-pass symmetrical filter sections have been considered. Advantages and disadvantages of this oscillographic method over other methods have been fully discussed.

2 CATHODE-RAY OSCILLOGRAPH METHOD

The diagram of connections for study of the band-pass effect by a cathode-ray oscillograph of Standard Telephones and Cables Co.'s make is shown in Figs. 1(*a*) and 1(*b*). Fig. 1(*a*) shows the beat-frequency audio-oscillator (model 70 D, manufactured by Clough Brengle Co.) feeding into a low-pass filter section terminated in a stabilized negative impedance *as well as* the connections from the input and output terminals of the filter section to the plates of the oscillograph

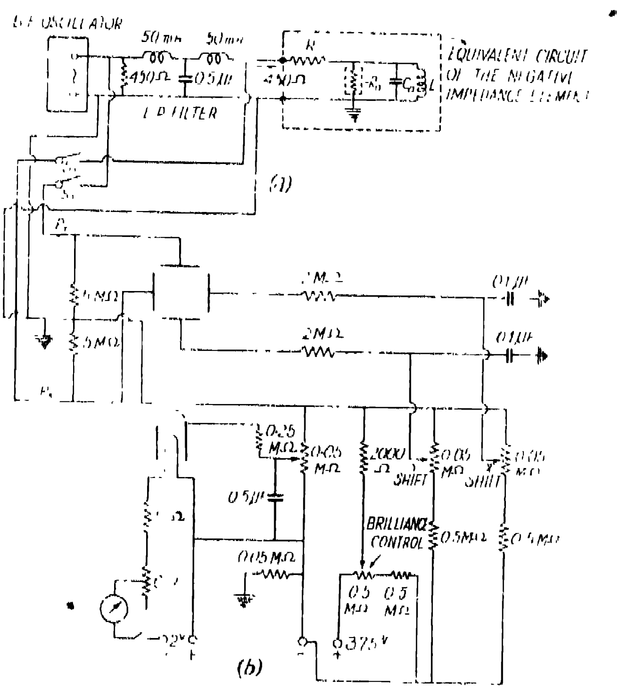

FIG. 1

The plate-to-plate capacitance (for each pair of plates of the cathode-ray oscillograph) is 10 to 12 $\mu\mu$ F and its impedance at 1 Kc/s is 15 8 to 13.2

megohms respectively shunting the input and output impedances of the filter. The anode-filament capacitance of the AC/SG tube together with stray capacitances in parallel and choke inductance both shunting the negative resistance are about 50 $\mu\mu$F and 40 H respectively and their impedance at 1 Kc/s is therefore 3.2 and 0.25 megohms respectively. A high resistance has to be connected across each pair of plates for steadying or stabilizing purpose and has to be of such value that the positive and negative impedances connected across input and output terminals of the filter section respectively are not appreciably altered. A resistance of 5 megohms has been used. Switches S_1 and S_2 enable one to examine only either input or output as desired.

Fig. 1(*b*) shows the circuit of the oscillograph with various controls, designed for the experiment.

Case I. When V_1=effective input voltage (to vertical deflector plates) and V_2=effective output voltage (to horizontal deflector plates), and V_1 and V_2 are of the same phase (*i.e.*, there is no phase-shift in transmission through the filter section).

At first only V_1 is applied across the vertical deflector plates and length of trace measured in cms. Then only V_2 is applied across the horizontal deflector plates and length of trace obtained in cms. Subsequently) the two voltages with no phase-shift between them applied simultaneously across their respective plates will give the resultant—a straight line of length $\sqrt{l_1^2+l_2^2}$ diagonally across the screen at an angle α (to the horizontal) given by

$\alpha = \tan^{-1} \frac{l_1}{\ }$.

Measurement of the lengths of traces l_1 and l_2 (in cms.) corresponding to $2V_1$ and $2V_2$ respectively or of the angle α will give the attenuation or gain directly. If the length of trace in vertical or horizontal direction is directly proportional to voltage at all frequencies and the constant of proportionality same for both directions at all frequencies, attenuation or gain can be obtained in decibels directly from the ratio of lengths. If not, voltages corresponding to lengths have to be obtained from calibration curves.

Case II. When V_1 and V_2 differ in phase.

In this case also, V_1 and V_2 are first applied separately to the respective pairs of plates of the cathode-ray oscillograph and lengths of traces are measured in cms. to obtain attenuation or gain in decibels as $20 \log_{10} l_1/l_2$.

Further, when both V_1 and V_2 are applied simultaneously, an elliptical trace whose dimensions depend upon the phase angle of V_1 and V_2 and their relative values is obtained.

The phase angle is obtained directly from lengths of traces due to V_1 and V_2 on screen, (*i.e.*, $l_1/2$ and $l_2/2$ respectively) as well as from the major and minor axes of the ellipse. If ϕ is the phase-shift between the voltages, then $$\phi = \sin^{-1}\frac{a.b}{4V_1V_2} \text{ (Wood)}^{*3} = \sin^{-1}\frac{a.b}{l_1 l_2},$$

where a = major axis of the ellipse, and b = minor axis of the ellipse.

When frequencies higher than 0.4 to 0.5 Mc/s are involved in wave-filters, direct application of V_1 and V_2 to the respective pairs of plates is not desirable. V_1 and V_2 are then applied to two exactly similar super-heterodyne amplifiers (consisting of high-frequency amplifier, mixer and intermediate-frequency amplifier stages) with the same local oscillator giving same gain and phase-shift and the corresponding I.F. voltages V'_1 and V'_2 are applied to the respective pairs or plates of the oscillograph. It is necessary in this case to have linear relation between r.f. input voltage to super-heterodyne system and i.f. output voltage as well as between i.f. output voltage and deflection in cms. on the screen of the oscillograph.

At high frequencies, direct application of V_1 and V_2 to the plates causes instability and loss by radiation and does not give linear relation between voltage applied and deflection on the screen.

At very high frequencies, in addition, the difference in distance between the deflector plates introduces an error in the phase angle measurement due to "Hollmann Phase Effect" (Hollman, 1933). The time of flight of the electron being d/V (where d = axial interplate distance and V = velocity of the beam), the phase angle is $\phi \pm \frac{\omega d}{V}$.

The method evolved here is convenient for finding out gain or attenuation, phase-shift angle, sign of phase-shift angle, etc., at various frequencies for any network or filter section. The advantages and disadvantages of this method are discussed in section 5.

3. LOW-PASS FILTER TERMINATED IN STABILISED NEGATIVE IMPEDANCE ELEMENT

A low-pass H-type filter section (total series inductance = 100 mH and total shunt capacitance = 0.5 μF) of nominal characteristic impedance 450 ohms and cut-off frequency 1500 c. p. s. was terminated by a negative impedance of magnitude nearly same as the nominal characteristic impedance obtained from AC/SG screen-grid tube under secondary emission condition.

Table I gives the results of observation on the traces on the screen of cathode-ray oscillograph. Negative sign in column under 'gain or attenuation' (Table I) signifies gain.

It will be seen from Table I that for first five frequencies (marked with asterisks) the pattern on combining voltage has been observed to be a straight line signifying that there is no phase-shift introduced in transmission at those frequencies. The length of the straight line is almost equal to $\sqrt{l_1^2+l_2^2}$ (where l_1 and l_2 are in cms.).

* V_1 and V_2 in the equation stand for corresponding lengths of traces due to them.

TABLE I

Frequency in c.p.s.	l_1 in cms. (corresponding to $2V_1$)	l_2 in cms (corresponding to $2V_2$)	Gain or attenuation db	Nature of pattern on combining voltages	Axes for elliptical pattern		Phase shift ϕ
					a cms.	b cms.	
*50	3.50	4.20	−1.58	Straight line 5.4 cms. at 40°			0°
*80	5.40	5.90	−.748	7.9 at ,,			0°
*100	5.30	6.00	−.984	8.0 ,, ,,			0°
*200	4.90	5.00	−.172	7.0 ,, ,,			0°
*300	3.70	4.90	−2.412	6.2 ,, ,,			0°
400	3.70	5.10	−2.75	Ellipse with major axis inclined to horizontal at 40°	6.00	0.70	12° 18′
500	3.50	5.10	−3.288	,, ,,	6.00	0.90	17°
600	3.50	5.00	−3.106	,, ,,	5.70	0.95	18°
700	3.50	4.90	−2.922	,, 50°	5.40	1.10	20° 24″
800	3.30	4.40	−2.542	,, ,,	4.60	1.30	24° 42′
900	2.90	3.00	−2.542	,, ,,	4.00	1.35	28° 15′
1000	2.80	3.60	−2.008	,, 60°	3.70	1.35	28′ 30′
2000	2.00	1.30	3.742	,, 140°	1.80	1.10	40° 44′
3000	2.00	1.10	5.102	,, 150°	1.80	1.10	65°
4000	2.60	0.90	12.766	,, 155°	2.50	0.60	75° 24′
5000	2.00	0.60	12.766	,, 160°	2.00	0.50	65° 15′
6000	2.70	0.40	16.178	,, 165°	2.50	0.30	44°
8000	2.70	0.10	30.158	,, 175°	2.50	0.20	90°

Fig. 2 shows the attenuation or gain and phase-shift with frequency of the band-pass arrangement. The cut-off frequencies are 500 to 1000 c. p. s. The phase-shift ϕ varies linearly with frequency over the transmission band of the band-pass arrangement.

As the frequency is altered from 50 c. p. s. to 8000 c. p. s., the pattern on the screen of cathode-ray oscillograph undergoes remarkable variation. Up to 300 c. p. s., the pattern appears to the eye to be a straight line which may be a much elongated ellipse in which the minor axis has been negligibly small. At 400 c. p. s. and near about the ellipse is an elongated one with major axis at 40° to the horizontal. As frequency increases up to 1000 c. p. s. and near about, the ellipse broadens out gradually with major axis making larger and larger angle with the horizontal. Subsequently, after 2000 c. p. s. the broadening decreases, the ellipse getting thinner and thinner and the major axis makes angles larger than 90° with the horizontal so as to rotate itself to the second quadrant.

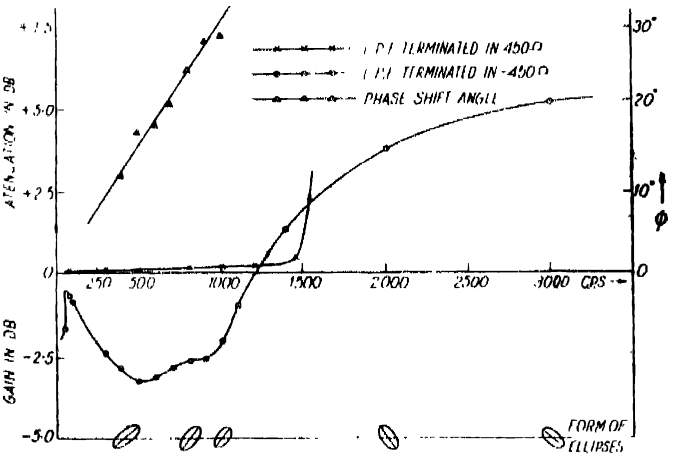

FIG. 2

As the phase angle ϕ between the voltages increases, the ellipse becomes broader ; and as ϕ decreases it becomes narrower. The rotation of ellipse will depend upon the sign of the phase-angle with variation of frequency (*i.e.*, the phase-angle may be ' lagging ' at some frequencies and ' leading ' at others).

It will be seen that within the transmission band (500-1000 c. p. s.) ϕ increases linearly with frequency but remains same with regard to sign (*i.e.*, ellipse is not rotated).

4. HIGH-PASS FILTER TERMINATED IN STABILISED NEGATIVE IMPEDANCE ELEMENT

A high-pass H-type filter section (total series capacitance = 0·075 μF and total shunt inductance = 50 mH) of nominal characteristic impedance 814 ohms and cut-off frequency about 1330 c. p. s. was terminated by a negative impedance of 814 ohms obtained in a similar manner as before.

Table II gives the results of observation on the traces on the screen of cathode ray oscillograph. Negative sign in column under gain or attenuation (Table II) signifies gain.

It will be seen from Table II that for four frequencies (marked with asterisk) it is likely that the pattern is much elongated ellipse with negligibly small minor axis appearing to be a straight line since the length of the line is $<\sqrt{l_1^2+l_2^2}$.

Fig. 3 shows the attenuation or gain and phase shift with frequency of the band-pass arrangement. The cut-off frequencies are 2150 and 4000 c. p. s. Since the sign of ϕ in the transmission band is negative (as explained below), ϕ plotted on negative scale will show a linear variation with frequency over the transmission band.

TABLE II

Frequency in c.p.s.	l_1 in cms corre-sponding to $2V_1$	l_2 in cms corre-sponding to $2V_2$	Gain or attenua-tion db	Nature of pattern on combining voltages	Axes for elliptical patterns		Phase-shift ϕ
					a cms.	b cms.	
*500	3.55	0.60	14.9	St. line 3.1 cms at 10° to horizontal			0°
*600	3.15	0.65	13.7	Do.			0°
700	3.15	0.70	13.1	Ellipse with major axis inclined to horizontal at 15°	3.00	.35	26° 42′
800	3.15	0.85	10.95	Do.	3.00	.45	30° 18′
900	2.95	1.00	9.4	at 20°	3.00	.60	37° 48′
1000	2.90	1.20	7.7	at 22°	3.00	.80	44°
1100	2.80	1.30	6.7	,, 25°	2.80	.90	44°
1200	2.70	1.45	5.6	,, 25°	2.85	1.00	46° 48′
1300	2.50	1.70	3.5	,, 30°	2.80	1.20	54° 1′
1400	2.42	1.75	2.9	,, 30°	2.70	1.42	65°
1500	2.30	2.00	1.2	,, 35°	2.05	1.60	67°
1800	1.90	2.43	2.1	,, 60°	2.42	2.00	90°
2000	1.75	2.60	−3.4	,, 80°	2.60	1.85	90°
2400	1.65	2.85	−4.8	,, 105°	2.90	1.60	81° 6′
2800	1.65	2.90	−4.9	,, 120°	3.10	1.25	54°
3600	1.70	2.95	−4.8	,, 130°	3.30	1.10	46° 12′
4000	1.80	2.00	−4.13	,, 130°	3.35	1.00	40° 18′
4200	1.90	2.65	−3.1	,, 130°	3.30	.90	36° 15′
5500	2.02	2.11	−.40	,, 132°	3.30	.50	21° 15′
6000	2.00	2.05	−.20	,, 135°	3.15	.33	14° 30′
*7000	1.50	1.50	0	St. line 1.95 cms at 135°	—	—	0°
*7500	1.60	1.50	·55	St. line 2.0 cms at 135°	-	—	0°

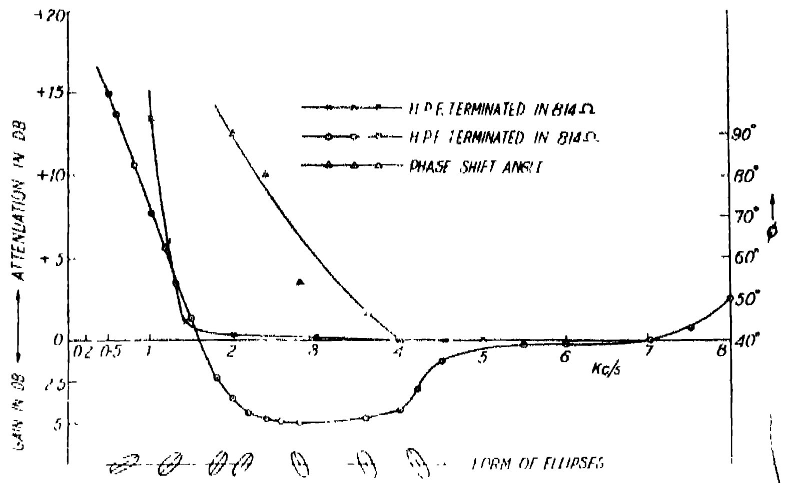

FIG. 3

Here also as the frequency is altered the pattern on the screen of cathode ray oscillograph undergoes remarkable variation. φ increases with frequency and has the same sign up to 2000 c. p. s. and near about. Thereafter φ decreases and also changes in sign. φ, however, has the same sign within the transmission band.

5. DISCUSSION ON MEASUREMENTS BY CATHODE-RAY OSCILLOGRAPHIC METHOD

The oscillographic method employed here utilizes the method of phase-angle determination evolved by Wood (1932), and is capable of giving gain or attenuation as well as phase-angle and its sign with sufficient accuracy both over low and high frequency ranges.

Another cathode ray oscillograph method which might have proved suitable for the purpose was developed by F. de la C. Chard (1938), for measuring gain or attenuation as well as phase angle. It suffers from several disadvantages as follows:—

(1) It requires two pairs of magnetic deflection coils with a number of accessories; (2) the mutual inductance between deflection coils causes inaccuracy specially at higher frequencies; (3) there is also liability of loss by radiation from coils at high frequencies; (4) to find out whether current in coil B say leads or lags that in coil A (that is, to find out the sign of phase-angle) the direction of rotation of the cathode-ray beam has to be observed by viewing the tube screen through a stroboscopic disc (requiring an induction motor rotating a single-holed disc at 1470 r.p.m.); and (5) further the cathode-ray tube should not be of the type which gives a long-sustained afterglow.

The non-oscillographic method developed by Messrs. Hinton, Rendall and White (1920), for measurement of phase-shift angle is capable of giving both gain or attenuation as well as phase-shift angle but requires a special type of transformer, two exactly similar linear amplifiers, a third linear amplifier for combined voltages and a calibrated thermionic voltmeter. For higher frequencies good balancing and great deal of care in circuit design are necessary. If only gain or attenuation is desired but not the phase-shift angle, measurement requires only a calibrated thermionic voltmeter.

Table III shows the results of observation on the same high-pass filter (total series capacitance $=0.25$ μF and total shunt inductance $=150$ mH; nominal characteristic impedance $=770$ ohms and cut-off frequency $=400$ c. p. s.) terminated by -700 ohms (negative impedance) by oscillographic method employed here as well as non-oscillographic method of Messrs. Hinton, Rendall and White. The negative sign in Table III signifies gain.

TABLE III

f in c. p. s.	Oscillographic method		Non-oscillographic method	
	Gain or attenuation db	Phase-shift in degrees	Gain or attenuation in db	Phase-shift in degree
[illegible]	1[illegible].0	0	15.0	0°
400	7.5	38°	8.3	30°
600	2.0	41° 30′	4.0	1[illegible]°
800	−3.5	14°	2.8	43° 30′
900	−5.7	46° 48′	−5.1	46°
1000	0.3	75° 21′	7.4	76° 30′
1200	−0.7	65°	−7.5	66°
1400	−0.7	50°	7.3	60°
1600	−0.5	50° 18′	−7.3	55° 30′
2000	−2.7	46° 48′	2.1	46°
3000	1.6	41°	1.0	43°
4000	−1.5	30°	−0.8	28° 30′
5000	−1.1	20° 24′	−0.5	19°
10,000	−0.8	0°	0	0°
12,000	−0.5	0°	0	0°
15,000	0	0°	0.5	0°

It will be seen from Table III that there is some discrepancy between gain or attenuation measurements by the two methods and that the non-oscillographic method gives larger gain and better sharpness of cut-off. Further a closer agreement between phase-shift angles measured by the two different methods will be observed.

By comparing the two methods it will be seen that in oscillographic method both gain and phase-shift can be determined from *four* observations per frequency whereas in the method developed by Messrs. Hinton, Rendall and White (1929) both gain and phase-shift can be obtained from *three* observations per frequency but a larger number of equipments (some of special features) as stated above as well as skilful manipulation are necessary. The oscillographic method gives one a visual picture as to how the ellipse undergoes alteration in shape (*i.e.*, broadens out or narrows down) as phase-shift angle increases or decreases with frequency and as to how the ellipse rotates as the sign of ϕ changes with variation of frequency. Thus the oscillographic method has *distinct* advantage over the other method at least regarding the examination of the phase-shift problem.

The disadvantages of the oscillographic method are as follows:—

1. Measurement of lengths of traces on the cathode-ray oscillograph screen is liable to inaccuracy. By means of two adjusters the thickness of lines on the screen can be reduced to less than 0.6 mm. and the whole length can be brought to equally good brilliancy. As the ellipse becomes more and more elongated, measurement of minor axis is liable to introduce great error. Measurement of voltages by thermionic voltmeter for gain or attenuation and phase determinations can give better accuracy.

2. It will be seen that if frequency band involved is large, direct determination of attenuation or gain and phase-angle from lengths measured on the screen of cathode-ray oscillograph is liable to error for reasons discussed below. The wide band difficulty is involved in case of high-pass filter terminated in stabilised negative impedance.

The length of the trace (vertical or horizontal) may not be directly proportional to the a.c. voltage impressed across the plates for all voltages over the frequency range involved and the constant of proportionality may not be the same for traces in vertical and horizontal directions.

An experiment has been carried out to test the relation between length of trace and voltage impressed across the plates for the S. T. C. Co.'s cathode-ray oscillograph (Type 4050A—filament current =0.9A, anode voltage =350V, and shield voltage =25V with respect to filament) used at 1000 and 8000 c. p. s. Table IV shows the results of the experiment.

TABLE IV

Frequency 1000 c. p. s.				Frequency 8000 c. p. s.			
Vertical		Horizontal		Vertical		Horizontal	
Impressed volts	Deflection cms.	Impressed volts	Deflection cms.	Impressed volts	Deflection cms.	Impressed volts	Deflection cms.
3.1	1.0	3.3	1.0	2.9	1.0	3.6	1.0
6.6	2.0	7.1	2.0	6.4	2.0	7.4	2.0
9.9	3.0	10.6	3.0	9.8	3.0	11.2	3.0
13.2	4.0	14.0	4.0	13.3	4.0	14.8	4.0
16.4	5.0	17.4	5.0	17.0	5.0	18.7	5.0

It will be seen from Table IV that the variation is more or less linear in all cases but slight discrepancies occur in the variation for two directions as well as at different frequencies.

6. ACKNOWLEDGMENTS

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