A SELF CONSISTENT METHOD OF DETERMINING THE MASS OF MESOTRON

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ABSTRACT A new method is developed for finding the mass of mesotron existing in neutrons and protons. The method is self-consistent. The accuracy of the mass thus determined, which is 11[illegible] electron unit, depends on the accurate value of the binding energy of deuteron already known from the experiment of Chadwick. The interaction potential is taken in the form suggested by Yukawa. The mass determined by the present method is the same as the mass obtained by Kar (1942) from the theory of proton-proton scattering.

In the present paper we shall discuss a new method of determining the mass of mesotron. The method is perfectly rigorous and self-consistent. The accuracy with which the mass is determined depends on the knowledge of the accurate value of the binding energy of deuteron.

It is well known that the neutron-proton attractive force in a deuteron is of Yukawa type, derivable from a potential of the form

$$V = -\frac{A}{r} e^{-ar} \qquad \ldots \quad (1)$$

Again, because there is a special affinity between a neutron and a proton, the attractive force between them should be of *exchange type*. The particle which takes part in the exchange is evidently the mesotron. The experiments done so far prove definitely that the charge of this particle is e the electronic charge ; but the mass determined by several experimenters till now varies between 100⟶400 electron mass approximately. Our proposed self-consistent method gives the mass accurately inasmuch as the binding energy of deuteron is known with fairly great accuracy.

Now, it is already known (Bethe and Bacher, 1936-37) that in an α-particle the exchange force is completely saturated both with respect to the spatial and spin coordinates, whereas in a deuteron it is saturated with respect to the spatial coordinates only. Consequently the interaction potential between a neutron and a proton should be as given by Majorana. Thus referring the motion to the centre of mass of the interacting neutron and proton and assuming the potential to be spherically symmetrical and of Yukawa type, we have for the wave equation of the neutron or proton initially at $\mathbf{r}(r, \theta, \phi)$ and after exchange at $-\mathbf{r}(r, \pi-\theta, \pi+\phi)$

$$\frac{h^2}{4\pi^2 M} \Delta\chi(\mathbf{r}) + E\chi(\mathbf{r}) = -\frac{A}{r} e^{-ar} \cdot (-\mathbf{r}) \qquad \ldots \quad (2)$$

where M is the mass of the proton or neutron and E the binding energy being negative. Now, because

$$\chi(-\mathbf{r}) = R(r) P_l^m(\cos(\pi-\theta)) e^{im(\pi+\phi)}$$

$$= (-1)^l R(r) P_l^m(\cos\theta) e^{im\phi}$$

and
$$\chi(\mathbf{r}) = R(r) P_l^m(\cos\theta) e^{im\phi}$$

we at once get from (2)

$$\frac{h^2}{4\pi^2 M}\left\{\frac{d^2}{dr^2}(rR) - \frac{l(l+1)}{r^2}(rR)\right\} + E.(rR) = -(-1)^l \frac{A}{r} e^{-ar}(rR) \qquad \ldots \quad (3)$$

There are reasons to believe that no excited state of the deuteron exists which differs from the ground state with respect to the orbital motion. Thus $l=0$ and so (3) becomes

$$\frac{h^2}{4\pi^2 M} \cdot \frac{d^2}{dr^2}(rR) + E.(rR) = -\frac{A}{r} e^{-ar} \cdot (rR) \qquad \ldots \quad (3.1)$$

Even at the ground state the deuteron may have two varieties according as the spins of proton and neutron are parallel and antiparallel. In the former case the deuteron is in a triplet state having threefold degeneracy, while in the latter it is in a non-degenerate singlet state. However, if we assume Yukawa interaction of purely Majorana type, the binding energies should be same in both the cases.

Now, because a is very great, the Yukawa potential is appreciably changed for a very slight variation of r due to the exponential factor e^{-ar}, the factor A/r practically remaining constant. Thus to solve the wave equation (3.1) for deuteron, we may take A/r to be constant to a first approximation and equal to $A/\bar{r}$, where $\bar{r}$ is, strictly, the mean distance between neutron and proton. More rigorous solution of of (3.1) may, however, be obtained by using δ-function. Thus putting $V_0 = A/\bar{r}$, we have from (3·1) up to first approximation

$$\frac{h^2}{4\pi^2 M} \frac{d^2}{dr^2}(rR) + (E + V_0 e^{-ar})(rR) = 0 \qquad \ldots \quad (3.2)$$

If we put $x = e^{-ar}$, (3.2) reduces to

$$x\frac{d}{dx}\left\{x\frac{d}{dx}(rR)\right\} + \frac{4\pi^2 M}{a^2 h^2}(E + V_0 x)(rR) = 0 \qquad \ldots \quad (3.3)$$

Let us again put

$$z = \frac{4\pi M^{\frac{1}{2}} V_0^{\frac{1}{2}} x^{\frac{1}{2}}}{\ldots} \qquad \ldots \quad (3.4)$$

Hence (3.3) reduces to

$$\frac{d^2}{dz^2}(rR)+\frac{1}{z}\frac{d}{dz}(rR)+\left(1+\frac{16\pi^2 ME}{a^2h^2}\cdot\frac{1}{z^2}\right)(rR)=0 \qquad \ldots \quad (3.5)$$

which is the usual Bessel equation, when E is negative. Its well known solution is

$$rR=CJ_s\left(\frac{4\pi M^{\frac{1}{2}}V_0^{\frac{1}{2}}e^{-\frac{1}{2}ar}}{ah}\right) \qquad \ldots \quad (4)$$

where

$$s=\frac{4\pi(-EM)^{\frac{1}{2}}}{ah} \qquad \ldots \quad (4.1)$$

and C the averaging factor. Because R should be bounded at $r=0$, we have from (4)

$$J_s\left(\frac{4\pi M^{\frac{1}{2}}V_0^{\frac{1}{2}}}{ah}\right)=0 \qquad \ldots \quad (5)$$

From tables of Bessel Functions $J_s(z)$ one can easily find the value of $z=z_s$ (say) for which the function is zero for a given s. In the same way one finds easily that for a given s, Eq(5) is satisfied if

$$\frac{4\pi^{\frac{1}{2}}M^{\frac{1}{2}}V_0^{\frac{1}{2}}}{ah}=B_s \qquad \ldots \quad (5.1)$$

where the numerical value of B_s is obtained from tables of Bessel Functions (Watson.)

Now, from (3.2) we have for the wavestatistical value of the binding energy of deuteron

$$E=-\frac{h^2}{4\pi^2 M}\int_0^\infty\frac{d^2}{dr^2}(rR).\,rR.\,dr-V_0\int_0^\infty e^{-ar}.\,R^2r^2dr \qquad \ldots \quad (6)$$

where (rR) is given in (4). From the substitutions already made (*vide* (3.4)) we have

$$\frac{dz}{dr}=-\frac{a}{2}z$$

and therefore

$$\frac{d^2J_s}{dr^2}=\frac{a^2}{4}z\left(\frac{d^2J_s}{dz^2}.z+\frac{dJ_s}{dz}\right)$$

and also

$$e^{-ar}=\frac{a^2h^2}{16\pi^2MV_0}.z^2$$

Thus we easily get from (6), denoting differentiation with respect to the argument by dash

$$E=\frac{h^2c^2}{4\pi^2M}\cdot\frac{a}{2}\int_0^{B_s}J_s(zJ_s''+J_s')\,dz-\frac{h^2c^2}{4\pi^2M}\cdot\frac{a}{2}\int_0^{B_s}J_s^2zdz \qquad \ldots \quad (6.1)$$

where B_s is given in (5.1). The averaging factor C in (6.1) is determined in the usual way from the condition

$$C^2 \int_0^\infty J_s^2 dr = \frac{2C^2}{a} \int_0^{B_s} J_s^2(z) \frac{dz}{z} = 1 \qquad \ldots \quad (6.2)$$

Again, from the wellknown recurrence formulae of Bessel functions, *viz.*,

$$J_{s-1}(z) + J_{s+1}(z) = \frac{2s}{z} J_s(z)$$

$$J_s'(z) = \frac{s}{z} J_s(z) - J_{s+1}(z)$$

we have

$$J_s'(z) = \tfrac{1}{2}\{J_{s-1}(z) - J_{s+1}(z)\}$$

$$J_s''(z) = \tfrac{1}{4}\{J_{s-2}(z) - 2J_s(z) + J_{s+2}(z)\}$$

On using the above relations, (6.1) may be easily transformed to

$$E = -\frac{ah^2C^2}{8\pi^2M}\left\{\frac{1}{2}\int_0^{B_s} J_s^2(z)z\,dz - \int_0^{B_s} J_{s+1}(z)J_s(z)dz + s\int_0^{B_s} J_s^2(z)\frac{dz}{z}\right.$$

$$\left. + \frac{1}{4}\int_0^{B_s} J_{s+2}(z)J_s(z)z\,dz + \frac{1}{4}\int_0^{B_s} J_{s-2}(z)J_s(z)z\,dz\right\} \qquad \ldots \quad (6.3)$$

On substituting the experimental values of binding energy (E) *viz.*, 2.15 MV $= 3.421 \times 10^{-6}$ erg, we get sets of values of s and a satisfying (4.1). Of these only one set will obviously be correct. To find out these correct values we make use of the wavestatistical eq. (6.3) of binding energy. Now, for a given value of s, B_s is known from tables and so the integrals in (6.3) are evaluated by the graphical method. The averaging factor C may also be determined from the condition (6.2). Thus the binding energy is calculated from (6.3) for a given set of values of s and a satisfying (4.1). The binding energy so calculated is not in general the correct one. We, therefore, calculate a number of binding energies from (6.3) for different values of s and a satisfying (4.1) and draw a graph giving E for different values of s and a taken. This enables us to find the exact values of s and a for which the binding energy is just what is obtained experimentally. Having got the correct values of s and a by the above method which is evidently self-consistent, we get the mass of mesotron from the usual formula

$$m = \frac{ah}{2\pi c} \qquad \ldots \quad (7)$$

The accuracy of the mass so determined depends on the accuracy with which the binding energy of deuteron is known. The values of s and a calculated by the present method are 1.59 and $.2848 \times 10^{13}$ respectively. With the above value of a in (7) the mass of mesotron is found to be 110 e.u. This is the value obtained experimentally by Corson and Brode (1938).

It should be noted that from the theory of proton-proton scattering already developed by one of us, (Kar, 1942) the value of a has been obtained as

$.2866 \times 10^{13}$ and the corresponding mass calculated from (7) is 110.8 e.u. It is in very good agreement with the mass calculated by the present method. This strongly suggests that the origin of the short range attractive force must be same in both the cases. The attraction is due to meson field surrounding a neutron and a proton.

If, however, it is assumed that the proton-proton interaction is due to exchange and the attractive force is of Majorana type, then, it is clear, the particle taking part in the exchange cannot be mesotron having charge e. It should be 'neutretto' having the same mass but without the free charge e. As, however, in the formation of nucleus, the proton is not found to exhibit any affinity for proton, it appears to us that there is no exchange although there is attraction between the protons. Accordingly, the assumption regarding the existence of a new kind of exchange particle, *e.g.*, neutretto, is unnecessary.

It is thus evident that the attraction between a neutron and a proton or between two protons is due to the meson field. In the former case there is exchange while in the latter case there is none. Accordingly the interaction between protons cannot be of Majorana type. It should be that given by Wigner. But for $l=0$, the wave equation is same as (3.1). The next question that is to be settled is whether the short range meson field permanently exists in the protons and neutrons. As, however, the short range force is always found to be attractive and never repulsive, it appears likely that the field is of electrical origin and is temporarily developed during close interaction, due to polorisation of neutrons and protons and consequent creation of mesotrons within the particles.

In conclusion we may note that as α and B_s are already determined we readily evaluate V_0 *i.e.*, $A/\bar{r}$ from (5.1). The unknown parameter A of Yukawa potential cannot be determined by the present method. It is determined from neutron-proton scattering which we shall discuss on a different occasion. However, we may just mention at present that the value of $\sqrt{A}$ so determined is $\sim 6e$ which is just the value obtained from proton-proton scattering (Kar, 1942). On comparing the values of A and V_0 we find $\bar{r} \sim 3.118 \times 10^{-13}$ from which a rough estimate of the scattering cross-section may be made.

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