ON WIDE BAND-PASS EFFECT IN CRYSTALS ASSOCIATED WITH NEGATIVE IMPEDANCE ELEMENTS AND DEVELOPMENT OF WIDE-BAND LOW-LOSS CRYSTAL BAND-PASS FILTERS

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ABSTRACT. The band-pass crystal filters regarded to be the best from band-width point of view are (1) four-terminal lattice filter (consisting of crystals, condensers and coils) and (2) four-terminal resistance compensated filter. The former gives a maximum band width of 5 to 6 Kc/s at the expense of high attenuation in the transmission band. The latter can give a band-width of 10 Kc/s or so with lesser attenuation and is only suitable for use in unbalanced circuits such as for coupling unbalanced tube systems.

The present paper relates to "wide-band band-pass effect" obtained from combination of crystal and stabilized negative impedance elements in two-terminal networks and four-terminal balanced sections whereby new means have been provided to design and construct wide-band band-pass filters and couplings giving band-widths up to 10 Megacycles per second or more with lower attenuation in the transmission band. The best sharpness of cut-off obtainable in these filters is of the same order as in the two-section four-terminal lattice filter consisting of crystals, condensers and coils.

The mechanism of the band-pass action and the application of these wide-band band-pass filters and couplings to multi-channel radio-telephone and television systems are discussed.

1. PIEZO-ELECTRIC CRYSTALS AS ELEMENTS IN WAVE FILTERS

Electric wave-filters must transmit without distortion waves with frequencies lying between two limits say, f_1 and f_2 c. p. s., and attenuate sufficiently all waves with frequencies lying outside the above limits

The "percentage band-width" is given by $(f_2-f_1)/f_m$, and the "percentage separation-ranges" on the two sides are given by $({}_1f_\infty-f_1)/f_1$ and $({}_2f_\infty-f_2)/f_2$ where f_m = mean frequency of the band, and ${}_1f_\infty$ and ${}_2f_\infty$ are frequencies of infinite attenuation adjacent to f_1 and f_2 respectively.

At lower frequencies, a filter made up of coils and condensers can seperate frequencies well because the "percentage band-width" and the "percentage separation-range" are relatively larger. At high frequencies, the percentage band-width obtained from the same band width is much smaller and the insertion loss for a filter made up of coils and condensers cannot be made to increase faster with frequency than a certain percentage rate due to low 'Q' (*i.e.* reactance/resistance) values of coils and condensers and therefore a sudden frequency discrimination cannot be realised between transmission and attenuation bands.

If elements such as piezo-electric crystal resonators which have large 'Q' values are employed, filters having small percentage band-widths as well as attenuating in small percentage separation-ranges can be constructed.

It is only during last few years that filter sections containing crystal elements have been realized in the pratical field for use in high-frequency systems. Low-pass and high-pass crystal filter sections so evolved have been found to meet the requirements of the field satisfactorily, but the band-pass crystal filter sections are known to present certain limitations which have prevented their use in *wide-band* high-frequency systems.

2. LIMITATIONS OF EXISTING CRYSTAL BAND-PASS FILTERS

The simplest types of band-pass filters use crystal elements and condensers in ladder sections. The crystal elements of high Q value have been used with the best condensers having Q of the order of 10,000. The effect of capacitance connected in series with or parallel to a crystal is in the narrowing of the band-width. The limitations in the ladder type of sections are (1) very small transmission band-width and (2) the position of the attenuation peak frequencies.

By using lattice sections (consisting of crystal elements and condensers only), it is possible (1) to obtain a band-width roughly twice that of the ladder type and (2) to adjust the positions of attenuation peak frequencies with respect to the transmission band. The use of more than four crystals in any network configuration, employing crystal elements and condensers only, does not give larger band-width but contributes to higher attenuation in the transmision band on the other hand.

To obtain a filter section for a wider band-width at the same time maintaining advantages of sharply resonant crystals, lattice section using crystals, condensers and inductances has been developed[1,6]. The comparatively lower Q value of coils used in such a section gives large attenuations in the transmission band. A still wider band-width with somewhat lesser attenuation can be obtained from resistance compensated lattice filters[1] using similar elements.

Comparative figures regarding band-width, percentage band-width, percentage separation-ranges and attenuation in transmission band of some existing types[1] of crystal band-pass filters are given in Table I.

TABLE I

No.	Type	$f_2 \sim f_1$ (Kc/s)	$(f_2 \sim f_1)/f_m$	$({}_2f_m \sim f_2)/f_m$	Attenuation db
	Ladder section (crystals and condensers)	150·12 – 149·86 $=$ 0·26	·0017	·0014	1—2
2	Two Lattice sections (c condensers and coils)	65 3 – 62·7 $=$ 2·6	·04	·03	3·5
3	Do.	499·75 – 496·85 $=$ 2·9	·0058	·02	17·0 – 17·5
1	Resistance compe lattice section	470 – 460 $=$ 10	·021	·01	3·5

It will be seen from the Table I that two lattice sections formed of crystals, condensers and coils and connected in tandem suffer from two serious discrepancies :—(1) the maximum band-width obtained is still much narrower for most requirements of communication systems, (2) even for securing that band-width the attenuation in the transmission band becomes large and (3) the attenuation increases as sections are designed for higher and higher frequency range.

The limitations stated above have been found to be removed in the new types of band-pass filters developed for high frequency radio systems as a direct consequence of the "wide band-pass effect" discussed in the following sections. The best sharpness of cut-off obtained in the new type of four-terminal crystal filter is of the same order as the sharpness of cut-off of two-section lattice filter made up of crystals, coils and condensers.

3. BAND-PASS EFFECT IN PIEZO-ELECTRIC CRYSTAL CONNECTED IN SERIES WITH NEGATIVE IMPEDANCE ELEMENT

Quartz crystal (C T) resonator cut in such a way as to have only one frequency of vibration and mounted between electrodes has been used althroughout. The stabilized negative impedance element (N) consists of a screen-grid tube operated under secondary emission condition with condenser (C) and coil (L_2) to block the direct and high frequency currents respectively (Fig. 1). The stability and other properties of the negative impedance element are discussed [2,3] elsewhere and are therefore omitted here.

The attenuation or gain in db caused by the two terminal network consisting of the crystal, the blocking condenser and the negative impedance element has been measured by taking the input and the output voltage across the same impedance (AB and A′B′ respectively) by means of a screened Cambridge thermionic valve-voltmeter.

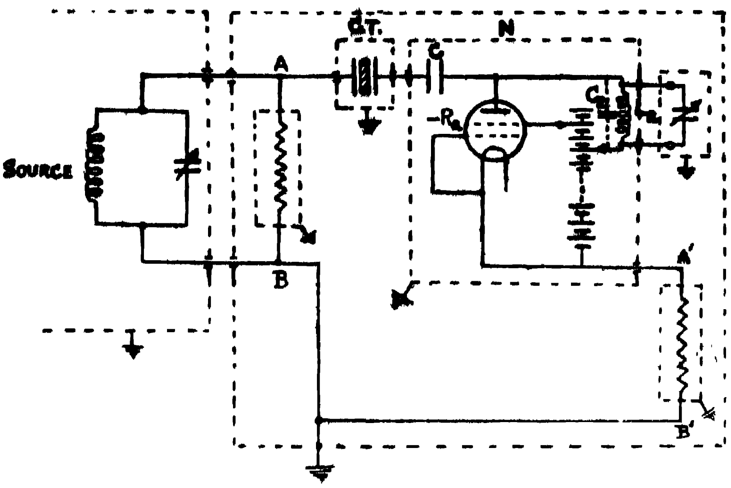

FIGURE 1

CASE I.—NEGATIVE IMPEDANCE ELEMENT DETUNED TO THE CRYSTAL FREQUENCY

Since the negative element (N) (omitting the blocking condenser) is equivalent to the circuit consisting of C_2, L_2, and $-R_a$ connected in parallel where C_2= the total of the anode—filament capacitance of the tube and the self capacitance of L_2 and $-R_a$ = the negative resistance (at the frequency concerned), it is possible

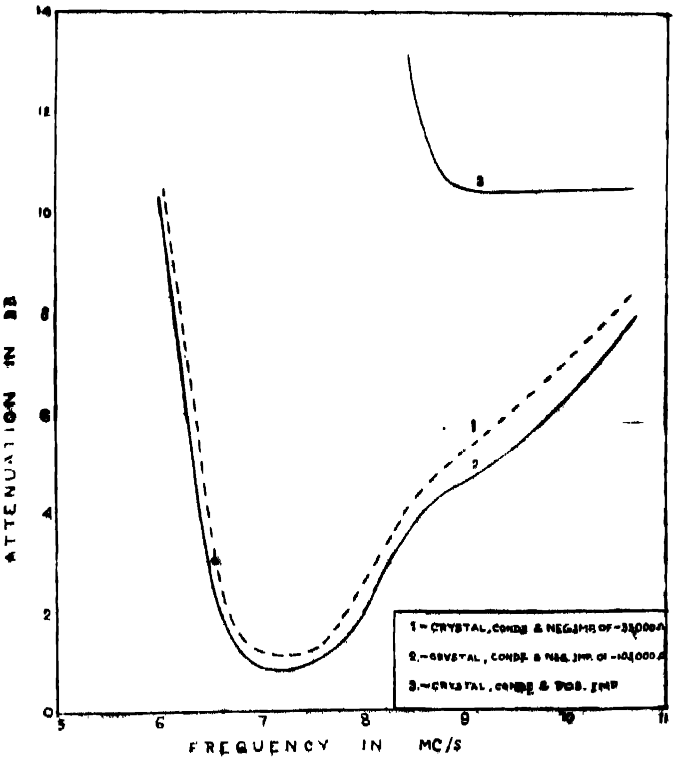

FIGURE 2

either to tune the L_2 C_2 circuit to the crystal frequency by an external screened condenser connected in parallel to L_2 or to leave it as its natural frequency (*i e.*, detuned to crystal frequency).

Fig. 2 shows the attenuation characteristics of the crystal in series with the negative impedance element (including blocking condenser) detuned to the crystal frequency. The attenuation characteristic of the crystal, condenser (of same value as the blocking condenser) and positive impedance of 50,000 ohms connected in series is plotted on the same sheet for comparison.

It will be seen that the crystal connected in series with condenser and positive impedance gives the characteristic of a high-pass filter of cut-of frequency

TABLE II

No.	$-{}_0R_0$ in series (Ohms)	f_2-f_1 (Mc/S)	$(f_2-f_1)/f_m$	$({}_2f_\infty-f_2)/f_2$	$({}_1f_\infty-f_1)/f_1$	Attenuation in *db*.
1	−33,000	7·9−6·7 =1·2	0.17	0·37	0·30	1·1-2·0
	-105,000	8·0−6·6 =1·4	0·20	0·41	0·30	0·8-2·0

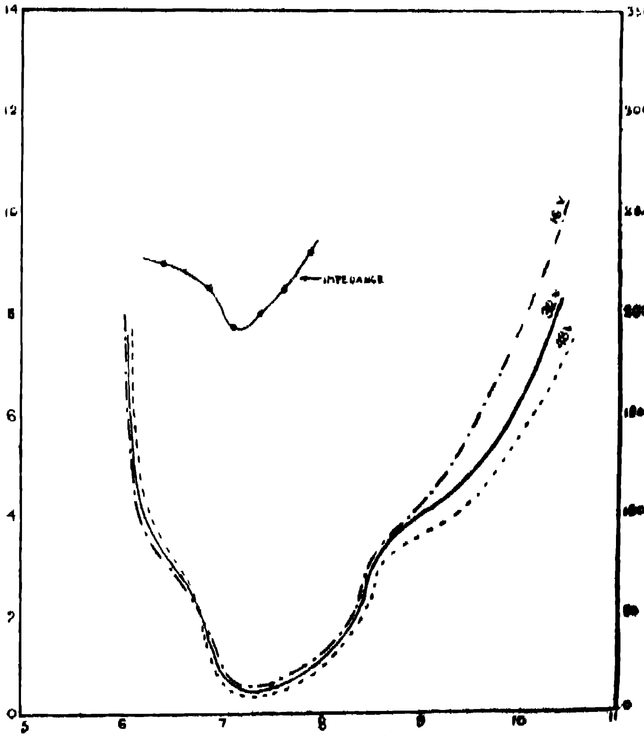

FIGURE 3

8·9 Mc/S whereas the crystal connected in series with negative impedance element (including blocking condenser) gives the characteristic of a wide-band band-pass filter, in which the attenuation in the transmission band is about 9 *db* less than that of the former. Table II shows the properties of band-pass filters so formed under detuned condition. $-{}_0R_n$ refers to value obtained from static characteristic.

Since capacitance in series or shunt with the crystal reduces the band-width, measurements have been repeated on the crystal in series with the negative impedance element without the blocking condenser. Further, as the omission of the condenser places across the crystal a D. C. voltage equal to the anode voltage, measurements have been repeated with different anode voltages.

Fig. 3 shows that the effect of omitting the condenser is to increase the band-width by about 14% and that of decreasing the anode voltage is to increase the sharpness of cut-off on one side. Measured variation of impedance of the two-terminal network (without blocking condenser) is also shown in Fig. 3.

CASE II. NEGATIVE IMPEDANCE ELEMENT TUNED TO THE CRYSTAL FREQUENCY

Fig. 4 shows the attenuation and impedance characteristics of the network with negative impedance element (without the blocking condenser) tuned to the crystal frequency. Table III shows the properties of band-pass filters so formed under tuned condition.

TABLE III

No.	$-{}_0R_n$ in series (Ohms)	$f_2 - f_1$ Mc/S	$(f_2 - f_1)/f_m$	$({}_2f_a - f_2)/f_2$	$({}_1f_a - f_1)/f_1$	Gain in *db*
1	−18,000	7·15-6·4 =0·75	0·110	0·25	0·14	4·5-5·3
2	−33,000	7·2-6·4 =0·80	0·117	0·25	0·14	4·7-5·4
3	−105,000	7·0-6·5 =0·50	0·074	0·21	0.14	5·5-5·9

The effect of tuning the element is therefore (*a*) to reduce the band-width, (*b*) to make the cut-off's sharper and (*c*) to decrease the attenuation in the transmission band.

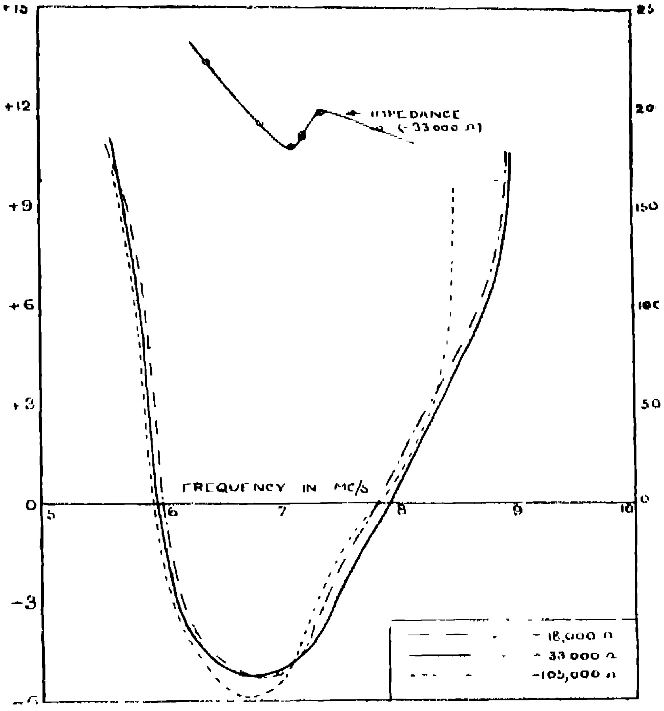

FIGURE 4

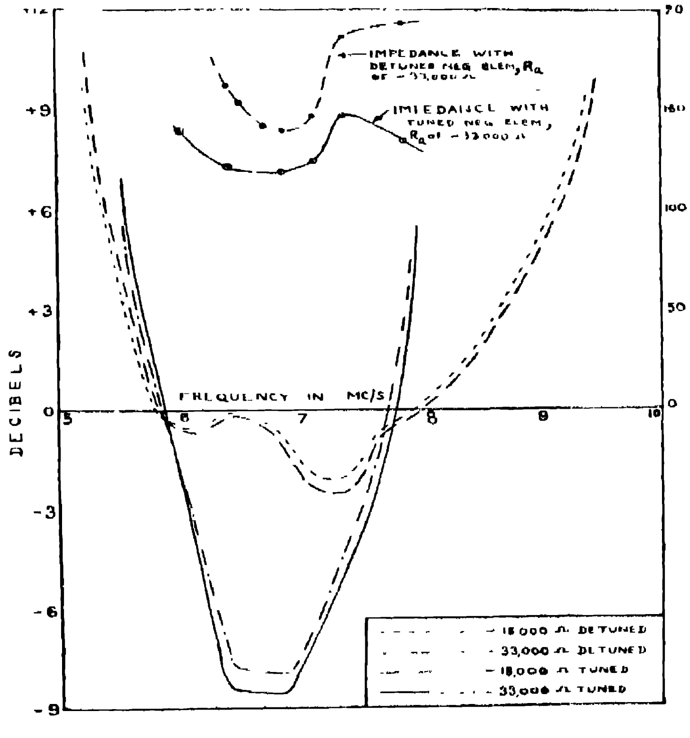

FIGURE 5

4. BAND-PASS EFFECT IN PIEZO-ELECTRIC CRYSTAL CONNECTED IN PARALLEL TO THE NEGATIVE IMPEDANCE ELEMENT

The stabilized negative impedance element (including the blocking condenser) is connected in parallel to the crystal. The blocking condenser cannot be omitted in this case as its absence may short-circuit the anode battery.

Fig. 5 shows the attenuation and impedance characteristics of the network under detuned and tuned conditions. Table IV shows the properties of the band-pass filters so formed under both conditions.

TABLE IV

No		$-_0R_n$ in parallel (Ohms)	$f_2 \sim f_1$ Mc/S	$(f_2 \sim f_1)/f_m$	$(_2f_r - f_2)/f_2$	$(_1f_r \sim f_1)/f_1$	Gain in *db*
(*a*) Detuned Case	1	−18,000	7·8-5·9 1·9	·28	·32	·13	15-2·1
	2	−33,000	7·8-5·9 − 1·9	·28	·32	·13	0·3-2·5
(*b*) Tuned Case	1	−18,000	6·9−6·4 −05	·076	·107	·14	7·8−7·95
	2	−33,000	6·85−6·4 0·45	·060	·107	·14	8·4−8·6

On comparing the 'parallel arrangement' under detuned condition with the 'series arrangement' under the same condition, it can therefore be said that the former gives (*a*) a larger band-width, (*b*) better sharpness of cut-off on one side, and (*c*) gain in the transmission band instead of attenuation. The effect of tuning the element in the parallel arrangement is similar to that of the 'series arrangement.'

5. BAND-PASS EFFECT IN FOUR-TERMINAL LATTICE SECTIONS CONTAINING PIEZO-ELECTRIC CRYSTAL AND STABILIZED NEGATIVE IMPEDANCE ELEMENT IN SERIES OR PARALLEL IN ONE OR BOTH ARMS

Four-terminal lattice sections consisting of two-terminal networks of the types discussed (in sections 3 and 4) in series and lattice arms have been grouped as follows :—

Class I.—In which piezo-electric crystal and negative impedance element connected in series or parallel are in one of the arms and series or parallel resonant circuit tuned to the crystal frequency or the negative impedance element is in the remaining arm (Fig. 6*a*).

Class II.—In which piezo-electric crystal and negative impedance element connected in series or parallel are in one of the arms and a similar arrangement is in the remaining arm, the frequency of the crystals in series and lattice arms being the same or different (Fig. 6b).

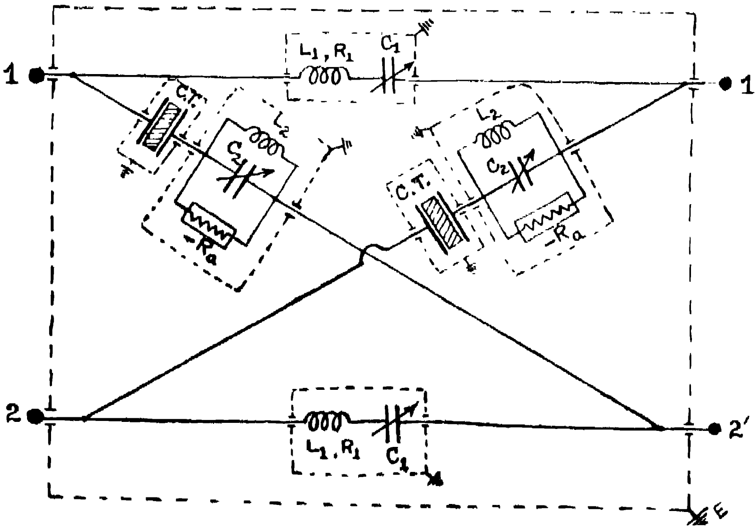

FIGURE 6 (*a*)

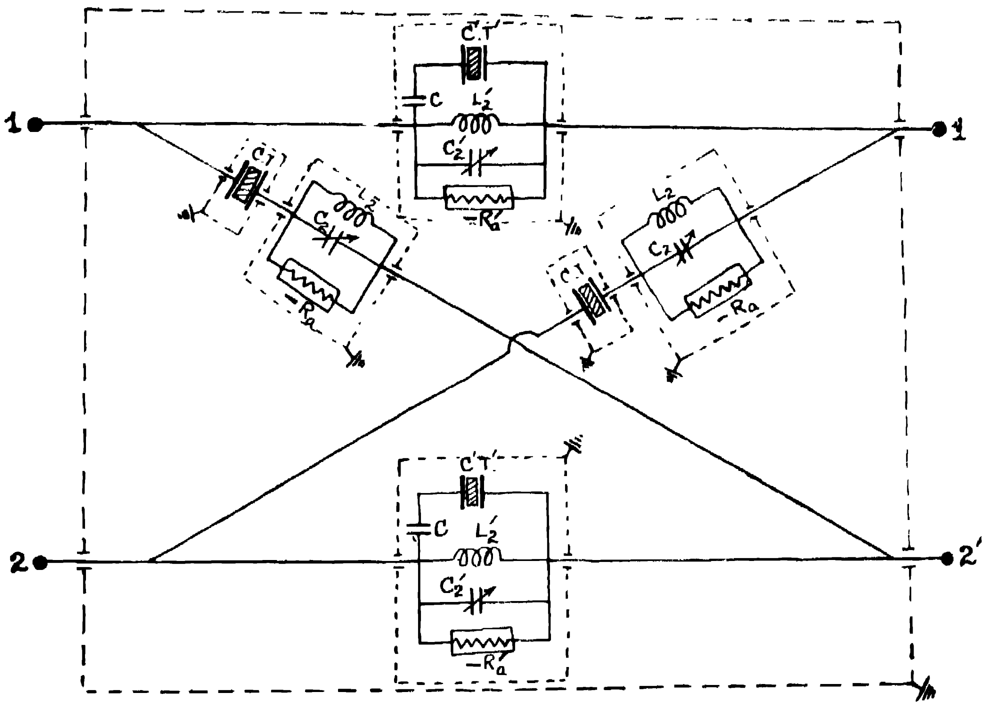

FIGURE 6 (*b*)

In sections of Class I, the negative impedance element can either be tuned to the crystal frequency or the frequency of the resonant circuit or remain detuned, and in sections of Class II the negative impedance elements can be tuned to frequencies of crystals in the same or other arms or remain detuned.

The present section relates to studies on the sections of Class I only, as little quantitative work could be done on the sections of Class II due to unavailability of desired components, etc., at the present war conditions. It is proposed to publish the studies on the sections of Class II at a later date.

The sections of Class I which have been discussed here consist of :—

(1) Series tuned circuits in the series arms, and the crystal connected in series with the negative impedance element in the lattice arms ; and

(2) Parallel tuned circuits in the series arms, and the crystal connected in parallel to the negative impedance element in the lattice arms.

Fig. 7 shows the attenuation and characteristic impedance measured at different frequencies under different conditions. In the figure, " I " stands for Class I ; (1) and (2) stand for particular sections of Class I referred to above ; and D, T and ST stand for ' detuned,' ' tuned to crystal ' and ' tuned to resonant circuit,' respectively.

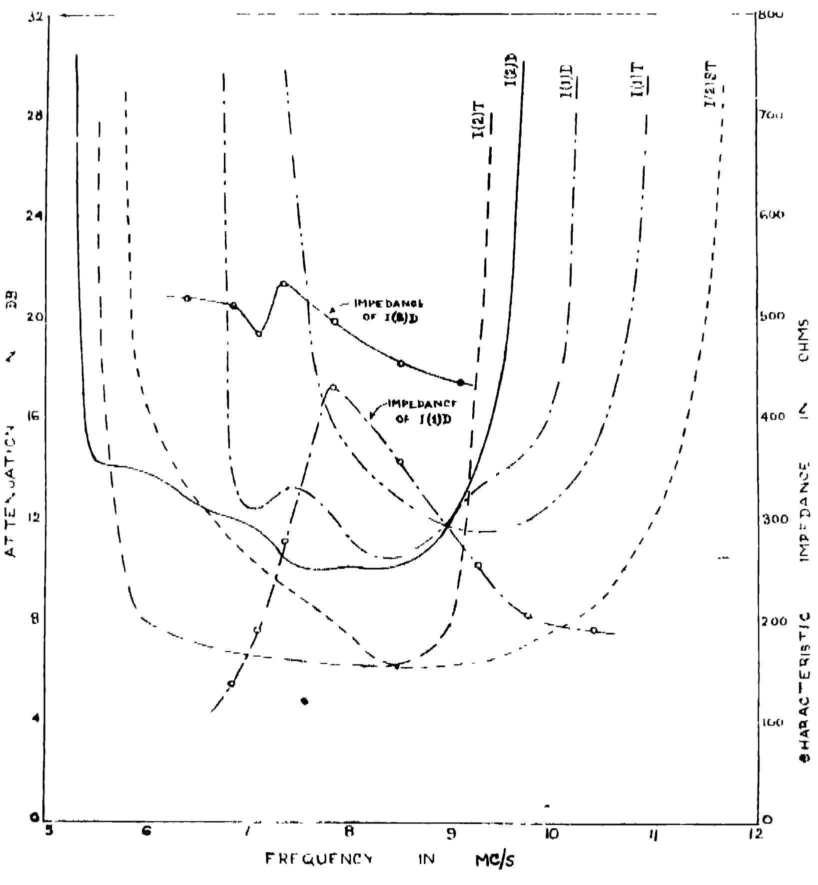

FIGURE 7

Table V shows the properties of the wide-band band-pass lattice filters of Class I so formed. The value of negative resistance (static) has been −33,000 ohms in all cases.

TABLE V

No	Class and No.	$f_2 \sim f_1$ Mc/S	$(f_2 \sim f_1)/f_m$	$({}_2f_\infty \sim f_2)/f_2$	$({}_1f_\infty - f_1)/f_1$	Attenuation in *db*
(*a*) Detuned Case 1	I – (1)	9·7-7·0=2·7	0·34	·051	·052	10·4-14·4
2	I – (2)	9·0-5·5=3·5	0·50	·083	·045	12·0-14·2
(*b*) Tuned Case (negative element tuned to crystal) 3	I – (1)	10·1-8·5=1·6	0·17	·045	·075	11·5-13·0
4	I – (2)	8·9-6·1=2·8	0·38	·055	·070	6·2-7·6
(*c*) Tuned Case (negative element tuned to resonant circuit) 5	I – (2)	10·5-7·0=3·5	0·41	·068	·09	6·1-10·0

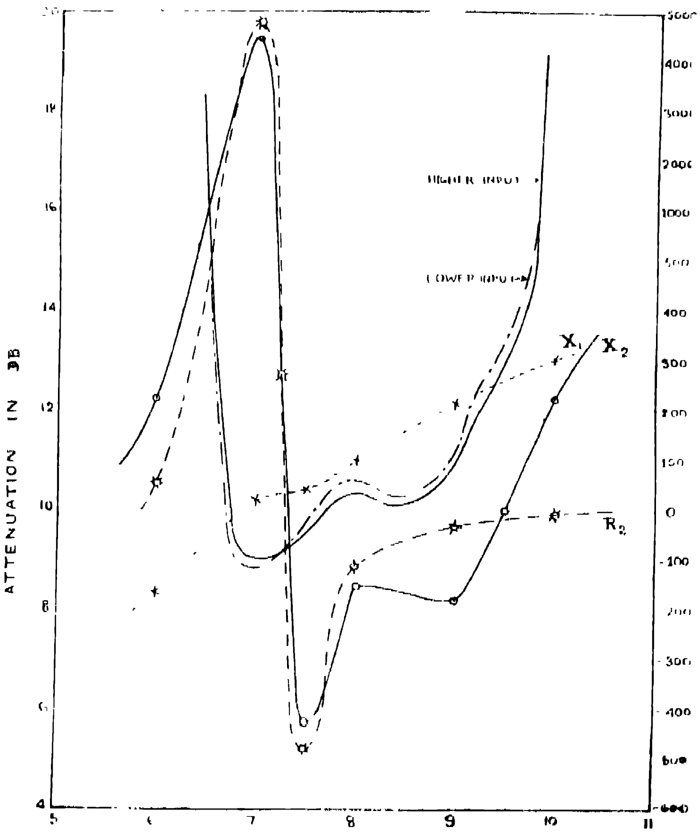

FIGURE 8

To test the linear performance of Class I types of filters discussed above, attenuation-frequency characteristics were taken with different inputs varying over a range of 60 decibels. Fig. 8 shows the curves corresponding to limiting values of input 0.1 and 100 volts for Class I—(1) filter. It will be seen that attenuation is fairly independent of the input for the range of variation.

6 MECHANISM OF THE BAND-PASS ACTION

Two-terminal Network

If the equivalent electric circuit of the crystal mounted between electrodes be a parallel circuit with L_0 and C_0 in series in one arm shunted by C' in another arm, the crystal impedance ${}_cZ_2$ is given by

$$ {}_cZ_2 = \frac{j\left(\omega L_0 - \dfrac{1}{\omega C_0}\right)}{1-\omega^2 L_0 C' + \dfrac{C'}{C_0}} \qquad \ldots \quad (1)$$

R_0 being neglected.

The impedance of negative elements consisting of L_2, C_2 and $-R_a$ in parallel is given by

$$ {}_NZ_2 = \frac{j\omega L_2 R_a}{R_a(1-\omega^2 L_2 C_2) - j\omega L_2} \qquad \ldots \quad (2)$$

If the two impedances are connected, say, in series, the total impedance

$$ Z_2 = \frac{-\dfrac{j}{\omega C_0}(1-\omega^2 L_0 C_0)}{1-\omega^2 L_0 C' + C'/C_0} + \frac{j\omega L_2 R_a}{R_a(1-\omega^2 L_2 C_2) - j\omega L_2} \qquad \ldots \quad (3)$$

The variation of total impedance $|Z_2|$ with frequency is shown in Fig. 3. The mechanism of action lies in the nature of variation of $|Z_2|$ which is minimum at a frequency at or about the middle of the transmission band, increases slightly from this value on both sides over a large frequency range and subsequently increases rapidly on both sides. The variation of $|{}_NZ_2|$ with frequency is largely responsible for giving such a variation of $|Z_2|$. Similar consideration applies when two impedances are connected in parallel.

Four-terminal Lattice Section

If the series arms consist of series resonance circuit (inductance $= L_1$, capacitance $= C_1$ and resistance $= R_1$) and the lattice arms consist of crystal connected in series with the negative impedance element, the circuit constants being the same as given in above paragraphs, then

$$\text{Reactance in series arm} = jX_1 = j\left(\omega L_1 - \frac{1}{\omega C_1}\right) \quad \ldots \quad (4)$$

Reactance in lattice arm

$$= jX_2 = j\left[\frac{\omega L_0 - 1/\omega C_0}{1 - \omega^2 L_0 C' + C'/C_0} + \frac{\omega L_2 R_a^2 (1 - \omega^2 L_2 C_2)}{R_a^2 (1 - \omega^2 L_2 C_2)^2 + \omega^2 L_2^2}\right] \quad (5)$$

The mechanism of "wide band-pass" action in a four-terminal lattice section of the types discussed is that the reactances in series and lattice arms remain of opposite sign over a *very wide frequency band*—a condition which has been brought about by the negative impedence element. It must be noted however that the resistance components of the impedances in series and lattice arms modify the width of the pass-band as illustrated below.

The resistance component in series arm is R_1 and that in lattice arm is

$$R_2 = -\frac{\omega^2 L_2^2 R_a}{R_a^2 (1 - \omega^2 L_2 C_2)^2 + \omega^2 L_2^2} \quad . \quad (6)$$

The attenuation caused by the positive resistance in series arms will be partially or completely reduced by the gain caused by the negative resistance in lattice arms. The wide band filters are therefore low-loss ones as well.

In Fig. 8 are shown the reactances in series and lattice arms of Class I—1) filter when the negative element is detuned to crystal as well as the resistance component R_2 of the lattice arm (R_1 being negligible). It will be seen that although the reactances remain of opposite signs between 6 and 9.5 Mc/s the resistance component of the lattice arm at 7.3 Mc/s is positive and of large value and at 9.5 Mc/s is negative but of small value. The pass-band from both these considerations is 7.3-9.5 Mc/s, whereas from actual attenuation measurements it is 7.0-9.7 Mc/s.

7. PROPOSED APPLICATION

The wide-band crystal band-pass filters discussed above appear to be well suited for use in wide-band H. F. Systems (*e.g.*, television and multi-channel radio telephony) as (*a*) band-pass couplings between stages of the transmitter and (*b*) band-pass filters in transmitting and receiving equipments.

Multi-channel radio telephone systems on short and ultra-short waves are at present generally designed for two and nine channels respectively. For two-channel system, the total band-width (including both side-bands) is 9 to 10 Kc/s therefore requires no special consideration. For nine-channel system as used by the British Post Office,[7] the carrier frequency is 76 Mc/s and the modulating band arising from nine channels is about 130 Kc/s wide (*i.e.*, 153-283 Kc/s).

Total band-width with two side bands is about 260 Kc/s. Attention has been directed to the design of several units (*e.g.*, modulation transformer, I. F. band-pass filter, I. F. negative feed-back amplifier, etc.) at transmitting and receiving terminals for as effective transmission of this wide-band as possible.

In a modern television transmitter as used by the French Post Office,[4,5] the carrier frequency is 46 Mc/s and the total band-width with two side-bands is 6 Mc/s. Penultimate and final R. F. power stages are designed as "inverted amplifiers" to work without balancing condensers and secure the essential condition for the wide band-width. Further, the penultimate stage is coupled to the final stage through the water cooled resistances for maintaining the load on the former stage fairly constant over the wide band of modulating frequencies.

Performance in the above cases, though satisfactory, is open to improvement in many ways. Most of the difficulties of wide band transmission could be obviated with economy and stability obtained by suitably using the wide-band crystal filters (specially of four terminal type) as band-pass coupling and band-pass filter in equipments. Take, for instance, the transmitting terminal equipment of a 100-channel radio telephone system employing series modulation at the penultimate stage. Fig. 9 shows the schematic diagram. The modulating band which consists of single side-bands of 100 carrier channels with spacings will be 0.5-1.1 Mc/s and the station carrier frequency is 30 Mc/s. Wide-band lattice band-pass filters 0.5-1.1 Mc/s and 28.9-31.1 Mc/s will be necessary at points shown by F_1 and F_2, respectively. Similar filters will be necessary at the receiving terminal.

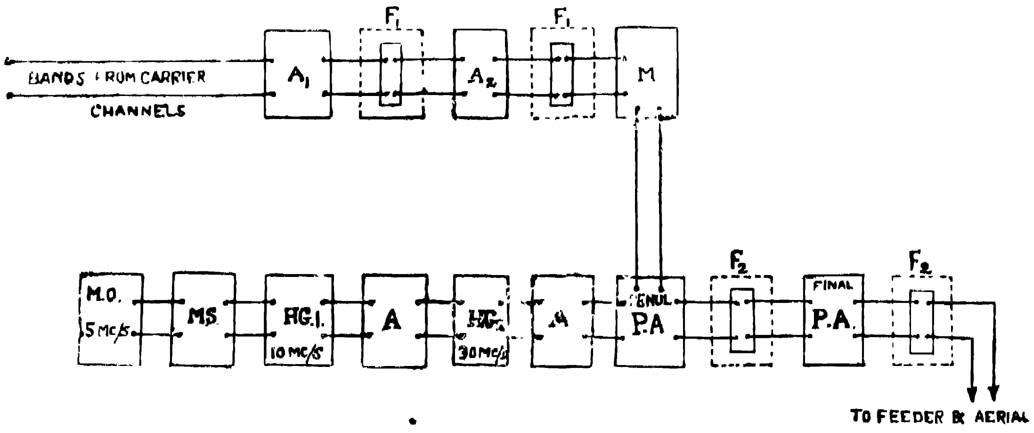

FIGURE 9

A_1 and A_2 = Amplifiers; M = Modulator, M.O. = Crystal controlled Master Oscillator;
M.S. = Master Separator and Amplifier; $H.G_1$ and $H.G_2$ = Hermonic Generators:
A = R.F. Amplifier; P.A. = R.F. Power Amplifiers.

8. CONCLUSION

The following conclusions have been arrived at :—

(1) A piezo-electric crystal connected in series with stabilized negative impedance element, detuned or tuned to the crystal frequency, acts as a two-terminal wide-band band-pass filter of low attenuation or gain in the transmission band.

Effect of tuning the element is (*a*) to reduce the band-width ; (*b*) to make the cut-off's sharper and (*c*) to decrease the attenuation in the transmission band.

(2) A piezo-electric crystal connected in parallel to the stabilized negative impedance element, detuned or tuned to the crystal frequency, acts as a two-terminal wide-band band-pass filter giving larger band-width, lesser attenuation in the transmission band and better sharpness of cut off (on one side) than the series arrangement referred to above under (1).

Effect of tuning the element is the same as mentioned above under (1).

(3) A lattice section, in which piezo-electric crystal connected in series with or parallel to stabilized negative impedance element detuned or tuned either to crystal frequency or to frequency of resonant circuit in the other arm is in one of the arms and series or parallel resonant circuit is in the remaining arm, acts as a four-terminal ultra wide-band band-pass filter having attenuation lesser than and sharpness of cut-off almost same as the two lattice sections made up of crystals, condensers and coils.

Effect of tuning the element to frequency of resonant circuit in the other arm is to make the band-width same as or greater than that of detuned case.

(4) For two-terminal filters, the impedance is in general minimum at a frequency within the transmission band and gradually increases on both sides of it.

For four-terminal filters, the characteristic impedance is maximum at a frequency near about the mid-frequency of the transmission band.

(5) The attenuation in the transmission band decreases in general with (*a*) the increase of negative resistance value, and (*b*) the tuning of the negative impedance element.

(6) The variation of attenuation or gain with frequency in the transmission band is in general less when the negative element is tuned than when it is detuned.

(7) The attenuation or gain of four-terminal filters is more or less independent of input at least upto a variation of 60 db. in the input.

9. ACKNOWLEDGEMENT

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