NUCLEAR STRUCTURE OF LIGHT ATOMS

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ABSTRACT. It has been suggested in the following paper that the nucleus may be regarded as a kind of crystalline structure rather than an ensemble of protons and neutrons, as a static or at any rate a quasi-static system, rather than dynamical. The principles and methods of Quantum Mechanics have not, therefore, been applied. Born's unitary theory gives a field which differs only slightly from the Coulomb field, which may be supposed to be playing an important role in the nuclear structure. On this hypothesis, it has been shown that the ratio between two fundamental distances in the assumed structure for α-particles lies between two narrow limits. Calculations from mass-defects give the distance between elementary particles of the nucleus as of order 10^{-14} cms., the figure generally accepted. The structure of the isotopes of hydrogen, helium and lithium has also been considered. Since the identity of the different elementary particles of the nucleus of the same isotope is denied, the difficulty about the continuous energy-spectrum of β-radiation seems to admit of a simple solution.

Modern views on the nuclear structure regard the ultimate constituents as neutrons and protons which are supposed to form an ensemble obeying the Bose-Einstein or Fermi Dirac statistics according as the number of the constituent particles are even or odd. The interaction forces, according to Majorana [1] are of the Exchange type, which seeks to explain the high binding energy of the helium nucleus as compared with the deuteron. It is held that this hypothesis clears, to a certain extent, the difficulties which were experienced about the spin. It is an odd multiple of $\frac{1}{2}h'$, if the number of particles is odd, and even, if the number of particle is even. This simple rule encountered its first exception in nitrogen on the older electron-proton hypothesis. But the newer neutron-proton hypothesis overcomes this difficulty. There remains again the standing difficulty of the continuous energy spectrum of the β-radiation, which has been explained by Fermi on the assumption of a new elementary particle, the neutrino which though incapable of observation, is considered necessary if the Principle of Conservation of Energy is to be saved.

Among the objections to the scheme may be put forward the following considerations. The two systems of statistics are based on entirely different assumptions about the behaviour of the component particles. The Bose-Einstein statistics presuppose an entire absence of interaction so that any number of particles can occupy the same cell of the generalised position and momentum space, while in the Fermi-Dirac system, the interaction is so strong that not more than one member can occupy one such cell. It is difficult to see why the addition of a single particle should make such a great difference in the behaviour of the constituent particles.

Regarding Majorana exchange forces, Tamm and Ivanenko [2] have calculated that on the assumption of Fermi's expression, the distance between each pair of protons and neutrons must be of order, 10^{-17} or less, before they can be effective. This is certainly very small compared with the accepted figures for the radii of the nuclei or of the ultimate particles.

Thirdly, by the replacement of electrons and protons as ultimate particles neutrons and protons does not really solve the difficulty about the spin momentum, but merely puts it back one place. It has been found that neutrons, protons and electrons all possess spin momentum of $\frac{1}{2}h'$, so the same difficulty re-appears in the case of the neutrons, which are regarded as built up of protons and electrons in some way or other. The principle of conservation of angular momentum cannot be salvaged if only algebraic addition is allowed.

The main idea of the present paper is to consider the nucleus as a sort of crystalline structure in which the complex nuclei are built out of the simpler materials. The ultimate particles are assumed to be protons and electrons.

In the first place, this meets the difficulty about the spin of the neutron. The idea of a crystalline structure implies vector addition of spin, and it is only by vector addition that $\frac{1}{2}h'$ added to $\frac{1}{2}h'$ can give $\frac{1}{2}h'$. Born [3] has formulated a unitary field theory in which matter has been sought to be blended with the electro-magnetic field. It is a consequence of that theory that the Potential Function due to a charge e at distance r is expressed as $\phi\ (r) = \frac{e}{r_0}\ f\left(\frac{r}{r_0}\right)$ where $f(x) = \int_x^{\infty} \frac{dx}{\sqrt{1+x^4}}$ where $x = r/r_0$, r_0 being a standard length. If we tabulate the values of $f(x)$ for different values of x we find that the law does not differ materially from the Coulomb Law when $r/r_0 > 1$.

TABLE I.

x	$f(x)$	$xf(x)$
1	0·927	·927
1·11	0·854	·948
1·25	0·792	·965
1·43	0·685	·980
1·67	0·592	·989
2	0·499	·994
2·50	0·399	·998
3·33	0·300	·999
5	0·200	1
10	0·1	1

It may be mentioned here that Born's value for r_0, "radius of the electron" is $2{\cdot}28 \times 10^{-13}$ cm. which probably is higher than what would be acceptable in the light of modern experimental facts.

For simplicity, in the following pages, the law of forces has been taken as that of Coulomb which plays the most important role. Besides, there would be other forces at work, *e.g.*, those due to magnetic moments, whose nature is but imperfectly known. But from the known energies of β-particles, which range up to 12×10^6 electron Volts, it seems reasonable to assume that these forces have a very low potential compared to the Coulomb potential. For, it is generally accepted that the linear dimensions of the complex nuclei are of order 10^{-12} cms. The distances between contiguous particles must therefore be of order 10^{-14} at most. At this distance, the Coulomb potential energy of an electron in the nucleus may be taken as of order $\lambda e^2/r$ where λ is an arithmetical constant of order 1, and r, the distance from the nearest particles, which is sufficient to account for the energy of order 23×10^{-6} ergs or 13×10^6 electron Volts.

Taking the simplest case of the deuteron, $_1H^2$, we may suppose it to be made up of one electron and two protons, one on each side, at distance a from the former. The mass defect is known to be $2{\cdot}25 \times 10^6$ ev $= 3{\cdot}6 \times 10^{-6}$ ergs If therefore, we consider Coulomb Potentials only,

$$\frac{3}{2}\frac{e^2}{a} = 3{\cdot}6 \times 10^{-6} \text{ which gives } a = 9{\cdot}6 \times 10^{-14}.$$

Similarly $_2He^3$ may be regarded as being made up of one electron and 3 protons symmetrically arranged in the three corners of an equilateral triangle with the former at the centroid. If a be the distance of the electron from each of the protons, it is easily verified that the latter are prevented from flying away by the Coulomb forces. The mass defect of the nucleus is known to be $7{\cdot}2 \times 10^{-8}$ M. U. $= 106{\cdot}6 \times 10^{-7}$ ergs. The Coulomb potential energy is easily calculated to be $1{\cdot}27e^2/a$.

Equating the two, we get $a = 2{\cdot}9 \times 10^{-14}$ cms.

But the most interesting case is that of the helium nucleus, $_2He^4$, which may be supposed to be made of 2 electrons and 4 protons placed symmetrically on the three axes of a rectangular Cartesian set. Each of the protons is at a distance a from the origin, while each of the electrons is at a distance d from the origin. We shall first show that considered as a statical system, the particles have no tendency to fly away.

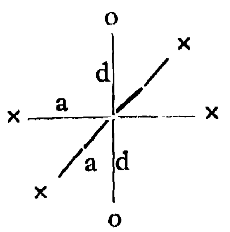

FIGURE 1.

Let us calculate the force on one of the electrons represented by a circle directed towards the origin. This equals

$$e^2\left\{\frac{4d}{(a^2+d^2)^{3/2}} - \frac{1}{4d^2}\right\}.$$

In order that this may be positive

$$16\, d^3 > (a^2 + d^2)^{3/2}$$

or, $$2^8\, d^6 > (a^2 + d^2)^3$$

or, $$4^3 \sqrt{4}\, d^2 > a^2 + d^2$$

or, $$5{\cdot}4\, d^2 > a^2$$

or, $$d^2 > {\cdot}18 a^2$$

or, $$d > {\cdot}4a.$$

For a proton the force directed towards the origin is

$$e^2 \left\{ \frac{2a}{(a^2 + d^2)^{3/2}} - \frac{1}{\sqrt{2}\, a^2} - \frac{1}{4a^2} \right\}.$$

In order that this may be positive

$$\frac{2a^3}{(a^2 + d^2)^{3/2}} > \frac{1}{4} + \frac{1}{\sqrt{2}}$$

or $$\frac{a^3}{(a^2 + d^2)^{3/2}} > {\cdot}48$$

or $$a^2 > {\cdot}61\ (a^2 + d^2)$$

or $${\cdot}39\, a^2 > {\cdot}61\, d^2$$

or $$d^2 < \frac{{\cdot}39}{{\cdot}61}\, a^2$$

$$< {\cdot}63\, a^2$$

or $$d < {\cdot}8a.$$

Therefore, to prevent disintegration

$${\cdot}4a < d < {\cdot}8a.$$

The Potential Energy of the system is

$$e^2 \left\{ \frac{1}{2d} - \frac{8}{(a^2 + d^2)^{1/2}} + \frac{2}{2a} + \frac{4}{\sqrt{2a}} \right\}.$$

Taking $d = \cdot 5a$, this

$$= e^2\left\{\frac{2+2\sqrt{2}}{a} - \frac{16}{\sqrt{5a}}\right\} = -\,2\cdot30\, e^2/a.$$

Since the mass-defect of an α-particle is $42\cdot3\times10^{-6}$ ergs, we have

$$2\cdot3\times\frac{e^2}{a} = 42\cdot3\times10^{-6}$$

$$\therefore\qquad a = \frac{e^2\times\cdot23\times10^{6}}{423} = 1\cdot2\times10^{-14}\ \text{cms.}$$

This gives approximately the accepted dimensions.

The question of spin has then to be considered. It must be pointed out here, that the generally accepted fact that the spin momentum of the elementary particles protons, electrons, positrons, neutrons (and even neutrinos) is $\frac{1}{2}h'$ leads apparentiy to insuperable difficulties. The term spin in Quantum Mechanics has not, of course, the same definite sense as it possesses in classical Mechanics. But if it has the magnitude given by its "eigenvalues," easy calculations show that the velocity at the periphery of the particle exceeds the velocity of light.

If this difficulty be brushed aside, the electrons and the protons in the above model must be supposed to have antiparallel spins, symmetrical with respect to the origin, to make the resultant spin of the α-particle zero.

Li has two isotopes $_3Li^7$ and $_3Li^6$ which occur in the proportion of 94:6. The following model may be suggested for the former, the crosses standing for the protons and the circles for the electrons. This indicates that $_3Li^7$ will break up easily into two α-particles if an additional proton be supplied—an experimental fact. In the absence of any knowledge of the magnetic forces, it is futile to attempt any definite theory about the direction of the spin, but obviously an odd multiple of the unit $\frac{1}{2}$ h′ (1 or 3) is indicated by the presence of the three protons in a straight line while the spins of the pairs of protons and electrons may be supposed antiparallel and therefore neutralise one another.

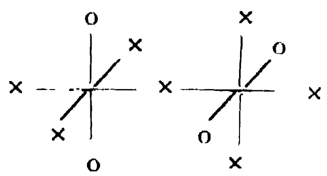

FIGURE 2.

For $_3Li^6$, the following model is suggested. The presence of two protons and an electron near the centre suggests the possibility of disintegration by shedding a neutron or a deuteron. β-disintegration has been a great stumbling block in the path of nuclear theories. Fermi's hypothesis of neutrinos invests them with residuary properties and places them at the same time, beyond possibilities of experimental verification—at least with our present resources. This is hardly satisfactary. It is obvious that some such hypothesis is necessary if the principle of conservation of energy is to be retained, provided the principle of identity of the different nuclei and the different elementary

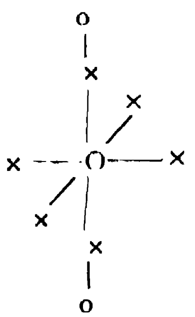

FIGURE 3.

particles is assumed. But if it be assumed that the internal structure is crystalline, the electrons in the different positions will be at different energy levels. It is conceivable that two nuclei of the same isotope have different internal energy levels. The energy spectrum of β-radiation is then capable of a simple explanation.

REFERENCES.

[1] Majorana, *Zs. f. Phys.*, 82, 137 (1933).

[2] Ivanenko, *Nature*, 133, 981 (1934).
Tamm, *Ibid*, *Proc. R.S.*, 143, 432 (1934).

[3] Born and Infeld, *Proc. R.S.*, 144, 425 (1934).