50

THEORY OF SPHERICAL SYMMETRY METHOD FOR MEASUREMENT OF THERMAL NEUTRON ABSORPTION

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ABSTRACT. A new method has been developed for measuring thermal neutron absorption cross-section. The method is a variant of the beam attenuation technique. It is an absolute method requiring no standard neutron absorber for calibration. In this method the spherical symmetry of the experimental arrangement has been exploited to balance out the effect of scattering. Detailed examination has been made of the circumstances under which this balancing takes place. Effects of non-radial neutrons, scattering and absorption in the moderator, variation of detector efficiency for scattered neutrons, multiple scattering and absorption processes, thermalisation of scattered epithermal neutrons and non-attainment of thermal equilibrium in the moderator have been studied. Possible extension of this method to other energies of neutrons as well as to other types of radiations, have been discussed.

1. INTRODUCTION

Technological as well as theoretical importance of the interactions of thermal neutrons with matter has resulted in the accumulation of voluminous data on thermal neutron cross-sections. Nevertheless a study of the methods involved immediately reveals the fact that whereas the precise measurement of σ_t, the total interaction cross-section for thermal neutrons, is of comparatively little experimental difficulty, determination of absorption or scattering cross-sections separately, is rendered difficult by numerous sources of error. The principal methods which have been employed hitherto for the measurement of σ_a, the thermal neutron absorption cross-section, are the following ;

(a) The beam attenuation technique. This method is applicable only to very good absorbers in which the effect of scattering is negligible (Havens and Rainwater, 1946; Wu *etal* 1947 etc.)

(b) Activation method. This method is applicable to absorbers in which neutron absorption leads to the formation of beta-active nuclei. This method is a relative one requiring a standard absorber to calibrate the neutron beam used for activation. (Houtermans, 1941; Maurer and Ramm, 1942; Seren *et al*, 1947 etc.)

(c) Methods based on local reduction of neutron density in a solution or mixture, due to the presence of the absorber in it. This is also a relative method, requiring a standard absorber for calibration (Lapointe and Rasetti, 1940; Coltman and Goldhaber, 1946).

(d) Reactor methods, based on the diminution of power level in a nuclear reactor due to the presence of the absorber. In both, 'the danger coefficient' and the 'oscillation method', the two principal methods which fall under this category, the depression of neutron flux is required to be calibrated by a standard absorber, which is usually boron. (Anderson *et al*, 1947; Weinburg and Schweinler, 1948; Harris *et al*, 1950; Pomerance and Hoover, 1948; Pomerance, 1951; Raievski and Yvon, 1950 etc.)

(e) Methods based on the free decay of neutron flux in a moderator solution containing the absorber. This is an absolute method applicable only to absorbers obeying $1/v$ law of neutron absorption. So far this method has been applied to boron containing compounds only (Scott *et al*, 1954; Dardel and Sjostrand, 1954).

It will be observed that most of the above methods are relative ones and their accuracy thus depends entirely on the accuracy with which the thermal neutron absorption cross-section of standard boron absorber is known. The experimental value of this important nucleonic constant has changed from time to time with the improvement of the measuring technique and in Table I we have collected some of the values used for thermal neutron capture cross-section of boron to show this trend. We must note in this connection that the word 'thermal' in connection with neutrons refers to the Maxwellian distribution of neutron velocities corresponding to a temperature of 300°K. If the neutron velocity distribution is different, then corrections for this deviation must be included in discussing the cross-section value. We have corrected pre-war values quoted in Table I in this manner. Details of this correction will be discussed in a subsequent section.

Although the latest determination of the capture cross-section of boron has removed greatly the difficulty in fixing the capture value of standard absorber, it is obviously desirable to develop an absolute method for determining the thermal neutron absorption cross-section applicable to absorbers of not too large cross-section. The present method was developed with the idea of meeting this requirement.

TABLE I

Thermal neutron capture cross-section of natural boron

Author.	Thermal neutron cross-section in barns (at the standard neutron velocity of 2.2×10^5 cm/sec., wherever possible)
Pre-war value quoted by Lapointe and Rasetti (1940)	Varies from 500-700 barns when uncorrected; average value of 600 barns were used by the authors; corresponding corrected value is 684 barns
Ross and Story (1949)	710 ± 21
Neutron cross-section Advisory Group AECU (1952)	751.3 (3990 barns for B^{10} isotope which is 18.8% abundant)
Argonne Lab. Standard Hammermesh *et al* (1953)	755 ± 5
Brookhaven Lab. Standard Carter *et al* (1953)	749 ± 4
Harwell standard. Egelstaff (1953)	782 ± 5
Scott *et al* (1954)	744 ± 20
Dardel and Sjostrand (1954)	764 ± 3

II METHOD

The method we have employed is a variant of the beam attenuation technique*. The fundamental relation between the transmission factor ψ and the parameters involved in the passage of a neutron beam through an absorber can always be expressed in the form

$$\psi = I/I_0 = \exp.[-K\,N\rho\,S/M], \quad \ldots (1)$$

where I_0 and I are the neutron intensities recorded in the detector before and after the introduction of the absorber respectively, N is Avogadro's number, while ρ, M and S are the density, molecular weight and thickness of the absorber along

* (Reported to AEC India 1948; Bose Inst. Annual Report, 1949).

neutron path respectively. The interpretation of the constant K depends essentially on the geometry of the arrangement used. For a 'good' geometry transmission experiment in which all the scattered neutrons are excluded from the detector, the value of K is obviously equal to σ_t the total cross-section, provided the thickness of the absorber is such as to render the effect of multiple scattering negligible. As the geometry of the experiment is made 'poorer" more of the scattered neutrons strike the detector and the value of K gets smaller than σ_t. In the extreme case when the absorber is a spherical shell of small thickness surrounding the source and there is no limiting diaphragm between the source and the detector, as many neutrons are scattered into the detector (for instance neutron marked 'α' in figure 1) as are scattered out of the direct beam (neutron marked

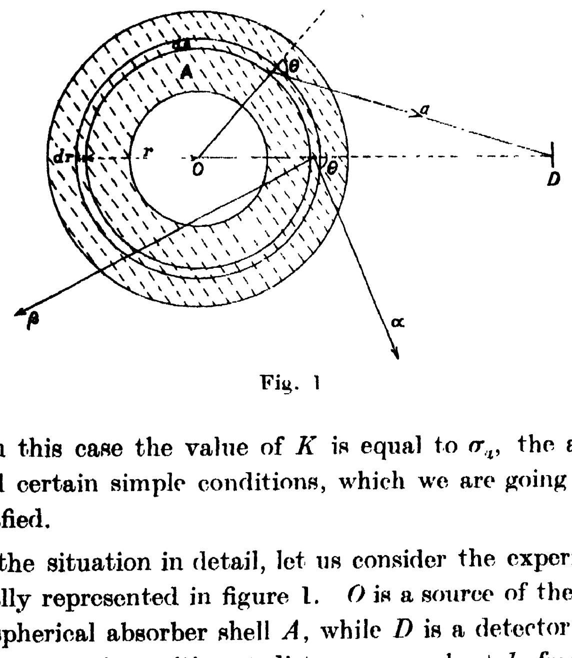


Fig. 1

α in figure 1); in this case the value of K is equal to σ_a, the absorption cross-section, provided certain simple conditions, which we are going to consider presently, are satisfied.

To analyse the situation in detail, let us consider the experimental arrangement schematically represented in figure 1. O is a source of thermal situated at the centre of a spherical absorber shell A, while D is a detector of thermal neutrons. Let the neutron intensities at distances r and $r+dr$ from the source be n and $n+dn$ respectively.

Obviously $$-dn = dn_a + dn_s \qquad \ldots \quad (1a)$$

where dn_a is the number of neutrons absorbed in the infinitesimal shell dA as shown in figure, while dn_s represents the drop in neutron intensity through scattering in dA. The expression for dn_a is $Nn\,\sigma_a\rho\; dS/M$, where dS is the average pathlength of neutrons through dA. Now, the neutron intensity n at r is composed of both primary as well as scattered neutrons; the path followed by the latter is not radial, in general, even if the primary beam happens to be radial. Hence

dS in the expression for dn_a is not, in general, equal to dr. However, if the source emits radial neutrons and if the absorber thickness is so small that the number of scattered neutrons is small enough to allow us to neglect the obliquity of their path in considering their contribution to n, we can write $dS = dr$ in the above expression for dn_a. Hence with these assumptions

$$dn_a = Nn\,\sigma_a\,\rho\,dr/M. \quad \dots \quad (1b)$$

While considering the expression for dn_s we note that the neutrons which are scattered in the forward direction (i.e. angle of scattering is not greater than $\pi/2$), e.g. the neutron marked α in figure 1, are available for absorption by nuclei outside the elementary shell and hence they do not cause any drop in the neutron intensity for transmission through dA. Neutrons which are scattered by more than $\pi/2$, e.g. the neutron marked β in the figure, will cross dA at points such as C and will be available for absorption, provided we can neglect the drop in neutron intensity through absorption between the points of scattering and the points of re-entrance. As before, we have neglected the effect of obliquity of the path of scattered neutrons with respect to their unscattered path. Under these assumptions, we can therefore set dn_s to zero and hence (1a) becomes

$$\frac{dn}{n} = \frac{-N\sigma_a\rho}{M}\,dr,$$

leading to

$$n = n_0 \exp.[-\sigma_a\,N\rho S/M] \quad \dots \quad (1c)$$

If the efficiency of the detector is independent of the direction of incidence of the neutron actuating it, so that scattered neutrons are detected with the same efficiency as the unscattered ones, the measured transmission will be equal to n/n_0 and hence we will get finally

$$= \frac{I}{I_0} = \frac{n}{n_0} = \exp.-[\sigma_a N\rho S/M] \quad \dots \quad (2)$$

which is the same as Eqn. (1) with K replaced by σ_a.

Collecting the assumptions made in deriving the above formula, we note that the following conditions must hold good, if the effect of scattering is to be balanced out by spherical geometry of the apparatus.

(1) The neutron flux is radial so that S in equation (2) is the radial thickness of the absorber.

(2) The thickness of the absorber is so small that the effect of the obliquity of the path of scattered neutrons compared to that of the unscattered ones is

negligible; we note that this condition in less stringent than that of neglecting multiple processes altogether.

(3) There is no absorber or scatterer between the source and the absorber whose cross-section is under investigation.

(4) Efficiency of the detector is independent of the angle of incidence of the neutron striking it.

(5) We have also tacitly assumed in the above that the absorber does not generate fresh thermal neutrons through slowing down by scattering of epithermal neutrons.

Deviations from the above assumptions occur in practice and in the subsequent sections we will examine them in detail.

III. NON-RADIAL FLUX OF NEUTRONS

To produce a spherically symmetrical thermal neutron beam in the laboratory using natural sources, the obvious and straight forward way is to surround a Ra-Be source with a spherical moderator of sufficient thickness to thermalise the fast neutrons issuing out of the source. It is well-known, however, that the neutrons emerging from the moderator surface are not radial and therefore the assumption (1) stated in the previous section is violated. In calculating the modified transmission factor, we will have to take into account the variation of the neutron path length S with its inclination with the radius, together with the angular distribution of the neutrons emerging from the moderator. A reference to figure 2 at once shows that the expression for S is given by

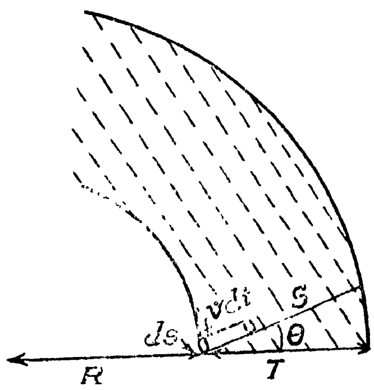


Fig. 2

$$S = S(\mu) = [R^2\mu^2 + T(T+2R)]^{\frac{1}{2}} - \mu R, \qquad \dots \quad (3)$$

where $\mu = \cos\theta$, R is the inner radius of the absorber which is assumed to be placed directly on the moderator and T is the radial thickness of the absorber.

The problem of determining the angular distribution of neutrons at the moderator surface is more complicated. If the dimensions of the moderator surface is large compared to the mean free path for scattering of thermal neutrons in it, we can apply the results which have been derived for semi-infinite plane moderators. Using certain simple assumptions Fermi deduced a simple angular distribution law for the neutron intensity (Fermi, 1936; Bethe, 1937). If we normalise to unit neutron density, the density distribution function is given by

$$\phi(\mu) = (1+\sqrt{3}\mu)/(1+\sqrt{3}/2), \quad \dots \quad (4)$$

where $\phi(\mu)d\mu$ is the neutron density between directions defined by μ and $\mu+d\mu$. The problem of neutron distribution from the surface of a semi-infinite moderator, which neither absorbs nor multiplies the neutrons, is completely analogous to the Milne problem in the astrophysics. The problem has been solved by Weiner and Hopf (Weiner and Hopf, 1931; Hopf, 1934). A modified derivation which is suitable for numerical calculations has been given by Plackzek and Seidel (Plackzek and Seidel, 1947; Plackzek, 1947). The new distribution function is given by

$$\phi(\mu) = \frac{1}{2(1+\mu)^{\frac{1}{2}}} \exp.\left[\frac{1}{\pi}\int_0^{\pi/2} \frac{x\tan^{-1}(\mu\tan x)}{1-x\cot x}\, dx\right] \quad (5)$$

Tables of numerical values of $\phi(\mu)$ have also been given by Plackzek, which shows that Fermi's function is accurate within a fraction of one per cent. More exact but complicated solutions of the problem at hand has been derived, but the error caused by using (5) in our calculations being of the order of a tenth of a per cent, we have refrained from using them. Experimentally the. distribution function has been verified by Hoffman and Livingston and more recently by Jonker and Blok (Hoffman and Livingston, 1938; Jonker and Blok, 1949).

To find out the numerical distribution function from the density distribution function, let us consider an elementary area dS of the moderator surface (figure 2). For simplicity let us assume that all neutrons travel with the same velocity v. Neutrons which are emitted at an angle θ in an interval of time dt will be contained in a cylinder with a base dS and slant height $v\,dt$. Hence the number of neutrons within the angles defined by μ and $\mu+d\mu$ is $\phi(\mu).\mu.vdt,\ d\mu ds$, showing that the numerical distribution function is proportional to $\mu\phi(\mu)$.

Hence the expression (2) for transmission is modified to

$$\psi = \int_0^1 \mu\phi(\mu)\exp.\left[-S(\mu)N\rho\sigma a/M\right]d\mu \Big/ \int_0^1 \mu\phi(\mu)d\mu \quad \dots \quad (6)$$

The above expression cannot be directly integrated and solved for σ and one has to take recourse to geometrical or algebraic methods. Using the geometrical constants of the apparatus used, one can tabulate corresponding values of ψ and $f = N\rho\sigma_a/M$. From this set of values either a graph of ψ *vs* f may be drawn to get f and hence σ_a from any measured value of ψ, or the usual interpolation formula may be applied for the same purpose. Alternatively, the universal transmission curves derived in the following paragraphs may be used.

Let us express all distances in terms of λ, the absorption mean free path of thermal meutrons in the absorber, where

$$\lambda = \frac{1}{f} = \frac{M}{N\rho\sigma_a} \tag{7}$$

The equation (3) now becomes

$$s(\mu) = \frac{S(\mu)}{\lambda} = [\mu^2 a^2 + t(t+2a)]^{\frac{1}{2}} - a\mu$$

where $a = R/\lambda$ and $t = T/\lambda$. Therefore,

$$s(\mu) = t[\{\mu^2 r^2 + (1+2r)\}^{\frac{1}{2}} - r\mu] = t.\alpha(r, \mu), \tag{8}$$

where $r = a/t$ is the ratio of inner radius of the absorber shell to the thickness of the absorber used and $\alpha(r, \mu)$ is the function

$$\alpha(r, \mu) = [\mu^2 r^2 + (1+2r)]^{\frac{1}{2}} - r\mu \qquad \ldots \quad (9)$$

The function $\alpha(r, \mu)$ is given for various values of r and μ in Table II and figure. 3.

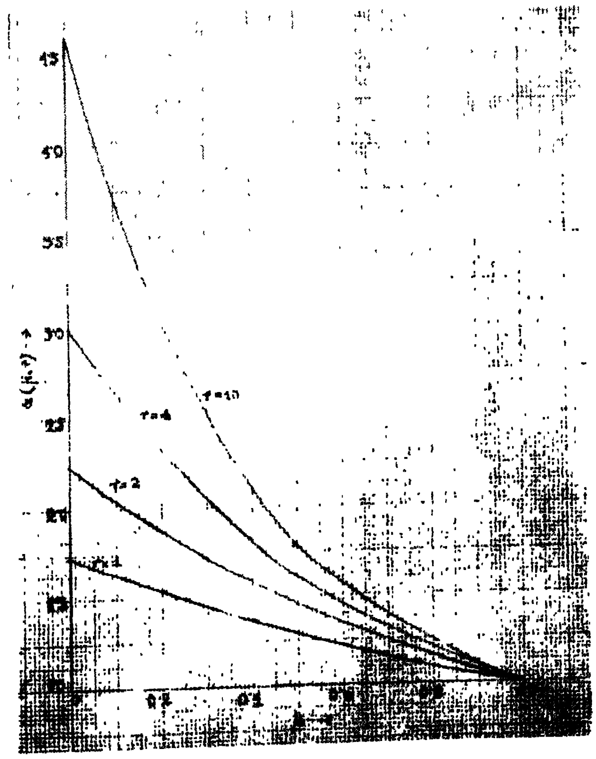


Fig. 3 $\alpha(r,\mu)$ as function of r and μ

TABLE II

Function $\alpha(r, \mu)$ for various values of r and μ

μ \ r	1	2	3	4	5	6	7	8	9	10
0	1.7320	2.2361	2.6457	3.0000	3.3166	3.6056	3.8730	4.1231	4.3589	4.5826
0.1	1.6349	2.0450	2.3627	2.6265	2.8541	3.0551	3.2357	3.4000	3.5508	3.6904
0.2	1.5436	1.8716	2.1129	2.3048	2.4641	2.6000	2.7182	2.8227	2.9159	3.0000
0.3	1.4578	1.7152	1.8946	2.0311	2.1401	2.2299	2.3057	2.3707	2.4273	2.4772
0.4	1.3776	1.5749	1.7052	1.8000	1.8730	1.9313	1.9791	2.0192	2.0533	2.0828
0.5	1.3028	1.4495	1.5414	1.6056	1.6533	1.6904	1.7202	1.7446	1.7650	1.7823
0.6	1.2330	1.3377	1.4000	1.4419	1.4721	1.4951	1.5131	1.5277	1.5397	1.5498
0.7	1.1682	1.2382	1.2779	1.3036	1.3219	1.3353	1.3458	1.3541	1.3583	1.3666
0.8	1.1079	1.1495	1.1721	1.1863	1.1962	1.2033	1.2088	1.2131	1.2166	1 2195
0.9	1.0519	1.0705	1.0802	1.0861	1.0902	1.0981	1.0953	1.0970	1.0984	1.0995
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

The expression (6) for transmission is now modified to

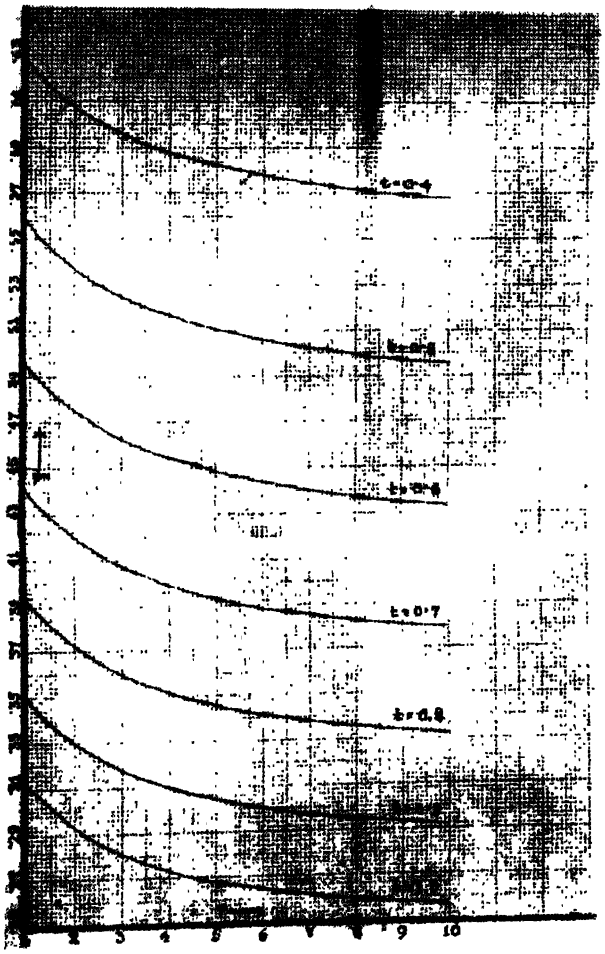
$$\psi = \int^{1} \mu\phi(\mu) \exp.[-s(\mu)]d\mu \Big/ \int^{1} \mu\phi(\mu)d\mu$$

$$= \int_{0}^{1} \mu\phi(\mu) \exp.[-t.\alpha(r, \mu)]d\mu \Big/ \int_{0}^{1} \mu\phi(\mu)d\mu \qquad \ldots \quad (10)$$

This is a function of t and r. This functional relation is shown in Table III and figure. 4, the latter being the universal transmission curves applicable to different geometrical dimensions of the apparatus used. These curves may be utilised as follows : From the dimensions of the apparatus we first determine the ratio $r = R/T$. Figure 4 is then used to determine the set of corresponding values of ψ and t, with the help of which the transmission curve (ψ *vs.* t) pertaining to the apparatus is drawn. Alternatively Table III may be used for the same purpose employing usual algebraic methods. In either case, the value of t corresponding to any experimentally obtained value of ψ is determined. From the value of t thus obtained and the value of T is cms. measured directly, the value of σ_a is calculated by using the relation

$$\sigma_a = \frac{M}{N\rho\lambda} = \frac{Mt}{N\rho T} \qquad \cdots \quad (11)$$

In Tables II and III and figures 3 and 4 we have covered the range $r = 1$ to 10 and $t = 0$ to 1.0; we can extend them to include values other than those considered here, in the manner indicated in the above paragraphs.



Fig, 4(a). $\psi(r,t)$ as function of r and t

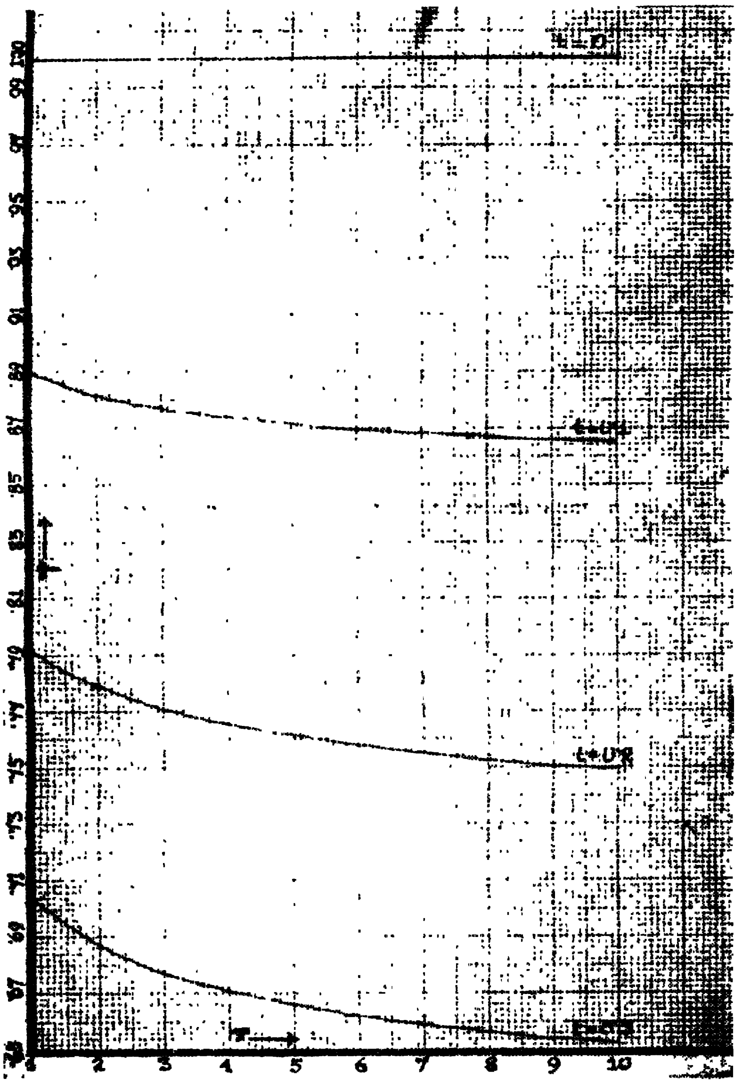


Fig. 4(*b*)

TABLE III

Transmission function $\psi(r, t)$ for various values of r and t

$t \setminus r$	1.	2	3	4	5	6	7	8	9	10
0.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.1	0.8891	0.8814	0.8777	0.8746	0.8721	0.8702	0.8687	0.8671	0.8663	0.8653
0.2	0.7914	0.7787	0.7710	0.7657	0.7616	0.7585	0.7560	0.7539	0.7522	0.7506
0.3	0.7040	0.6877	0.6778	0.6710	0.6660	0.6622	0.6590	0.6564	0.6544	0.6525
0.4	0.6266	0.6076	0.5963	0.5887	0.5831	0.5788	0.5753	0.5725	0.5703	0.5682
0.5	0.5578	0.5371	0.5250	0.5169	0.5111	0.5066	0.5030	0.5001	0.4978	0.4958
0.6	0.4966	0.4750	0.4625	0.4543	0.4484	0.4439	0.4404	0.4371	0.4353	0.4332
0.7	0.4422	0.4202	0.4078	0.3996	0.3938	0.3894	0.3860	0.3833	0.3811	0.3792
0.8	0.3939	0.3720	0.3597	0.3518	0.3462	0.3420	0.3387	0.3361	0.3341	0.3322
0.9	0.3509	0.3294	0.3175	0.3099	0.3046	0.3006	0.2976	0.2951	0.2932	0.2915
1.0	0.3127	0.2918	0.2804	0.2732	0.2682	0.2645	0.2617	0.2594	0.2577	0.2561

The error in the above table is within 0.2%, which is enough for the purpose at hand. For the sake of comparison we have given in Table IV, the values of ψ for radial distribution of neutrons. They are obviously equal to $\psi(0, t)$ in our previous notation.

TABLE IV

Transmission ψ, in absence of angular distribution of neutrons

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
ψ	1.0000	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066	0.3679

IV. MULTIPLE PROCESSES

When the absorber thickness reaches a value not small compared to the scattering mean free path λ_s of the neutrons, the error caused by the absorption of scattered neutrons cannot be neglected. Major contribution to this error comes from singly scattered neutrons; the effect of multiple scattered neutrons will be felt only when the absorber thickness becomes larger than λ_s. In this discussion we will confine our attention to first order corrections alone, as this is sufficient to cover most of the cases which will occur in practice. Referring to sec. II condition (2) for the validity of the transmission equation, we find that multiple processes affect our result through the alternation in the path-length of scattered neutrons from the path they would have followed in absence of scattering. Since on an average the length of the scattered path is greater than the corresponding undeviated path, more of the neutrons will be absorbed, as a result of which the apparent value of the absorption cross-section σ'_a will be greater than its true value. A rough estimate of the error caused may be obtained in the following manner. To find the order of increase in the neutron path due to scattering we note that if we pair off the neutrons scattered in the opposite directions, the total change in the path length remains of the same order as we shift the point of scattering from the inner to the outer surface of the absorber. At thickness $T/2$ of the absorber, the neutron pair scattered at angles $0°$ and $180°$ suffer a total increment of path length by T. Hence as a rough estimate we can suppose that the path-length increase through scattering is $T/2$ per neutron. Let ψ' and ψ be transmission in presence and in absence of scattering respectively. Hence the number of primary neutrons absorbed is $(1-\psi)$. Number of primary neutrons scattered is then obviously $\frac{\sigma_s}{\sigma_a}(1-\psi)$ where σ_s is the scattering cross-section of the absorber. Of these scattered neutrons nearly $\frac{1}{2}(1-\psi)$ fraction will be absorbed due to increase in path length alone. Hence neglecting second and higher order scattering

$$1-\psi' = 1-\psi+\tfrac{1}{2}\frac{\sigma_s}{\sigma_a}(1-\psi)^2$$

showing that the fractional error p in estimating absorption is of the order of

$$p = \tfrac{1}{2}\frac{\sigma_s}{\sigma_a}(1-\psi) \simeq \tfrac{1}{2}\frac{\sigma_s}{\sigma_a}(1-\psi')$$

If the value of $\frac{\sigma_s}{\sigma_a}$ is approximately known, then the above relation may be used to estimate the order of error involved.

A semi-empirical approach to the problem may be made as follows. The increase in absorption due to scattering is a function of absorber thickness T. This increased absorption may be imagined as due to a virtual source at the centre, the strength of the virtual source being a monotonically increasing function of T; we can write this source-strength as $n_0 f(T)$, as it is proportional to the real source-strength n_0 for obvious reasons. Hence the measured transmission is related to the corrected transmission by an equation of the form

$$n_0(1-\gamma') = n_0[1+f(T)](1-\psi')$$

or,

$$\psi' = \psi+f(T)\psi-f(T) \qquad \ldots \quad (13)$$

Since $\psi \leqslant 1$, ψ' is always greater than ψ, $f(T)$ being essentially positive. Now the expression for ψ when all neutrons are radial is

$$\psi = \exp.\left[-T\sigma_a N\rho/M\right] \qquad \ldots \quad (14a)$$

When the neutrons follow an angular distribution law, we assume that when T is varied keeping the moderator radius constant we can express the above equation in the form

$$\psi = \exp.\left[-\beta(T)\sigma_a N\rho/M\right] \qquad \ldots \quad (14b)$$

Similarly expressing ψ' in the form

$$\psi' = \exp.\left[-\beta(T)\sigma'_a N\rho/M\right] \qquad \ldots \quad (14c)$$

where σ_a' is the apparent absorption cross-section measured without heed to the scattering. Comparing (14a), (14b) and (14c) and remembering that by Plaekzek and Seidel distribution law most of the neutrons are emitted from the moderator surface making small angles with the radius, it is evident that for not too large value of T, we can expand $\beta(T)$ in the form

$$\beta(T) = T(A_0 + B_0 T + C_0 T^2 + \ldots\ldots). \qquad (15a)$$

When the absorber thickness T is very small, elementary considerations show that the strength-function $f(T)$ is also very small showing that $f(T) \rightarrow 0$ as $T \rightarrow 0$. For moderate values of T we can therefore write

$$f(T) = T(A_1 + B_1 T + C_1 T^2 + \ldots\ldots) \qquad \ldots \quad (15b)$$

Now, on substituting (14b) and (14c) in (13) and taking ln of both sides we have

$$\sigma'_a = \sigma_a + \frac{M}{N\rho} \cdot \frac{1}{\beta(T)} ln\left[1 + f(T)\left(1 - \frac{1}{\psi}\right)\right]$$

$$= \sigma_a + \frac{M}{N\rho} \cdot \frac{1}{\beta(T)}\left[f(T)\left(1 - \frac{1}{\psi}\right) + \text{higher terms}\right]$$

using (15*a*), (15b) and expansion of $\frac{1}{\psi} = \exp.\left[T \cdot \sigma_a \frac{N\rho}{M}\right]$, we have

$$\sigma_a' = \sigma_a + \frac{M}{N\rho}\,\frac{(A_1 + B_1 T + C_1 T^2 + \ldots\ldots)}{A_0 + B_0 T + C_0 T^2 + \ldots}\left[T\frac{\sigma_a N\rho}{M} + \text{higher terms}\right]$$

$$= \sigma_a(1 + AT + BT^2 + \ldots\ldots) \qquad \ldots \quad (16)$$

When a series of measurements of σ'_a is available for several values of T, we can fit them into the above relation to find the true value of the absorption cross-section σ .

V. ABSORPTION AND SCATTERING IN MODERATOR

If the moderator used in slowing down the neutrons has an appreciable absorption cross-section, a fraction of the scattered neutrons which encounter the moderator in their scattered path will be absorbed. This effect should be extremely small in all normal moderators, for which σ_a is only a small fraction of σ_t. On the other hand, most of the neutrons incident on the moderator will suffer multiple scattering within it. We can, in fact, assume that these neutrons, irrespective of their previous history, will emerge out from the surface of the moderator obeying the angular distribution law discussed in Sec. II.

A crude estimate of the correction factor, similar to Eqn. (12) above, may be derived easily if T is small compared to R, so that most of the neutrons scattered by more than $\pm\pi/2$ will strike the moderator. We neglect the increase in path between the point of scattering in the absorber and the point of incidence on the moderator; we assume however that the average change in path length for neutrons scattered through angles less that $\pm\pi/2$ is of the order of $T/2$. With these assumptions we can easily show that the presence of the moderator modifies the Eqn. (12) to

$$p \simeq \tfrac{1}{2}\left\{\frac{\sigma_s}{\sigma_a}\,\tfrac{1}{2}(1-\psi')+\frac{\sigma_s}{\sigma_a}\;(1-\psi')\right\} = \frac{3}{4}\,\frac{\sigma_s}{\sigma_a}\;(1-\psi') \qquad \ldots \quad (17)$$

Like Eqn. (12), the above equation is one useful only for indicating order of the error invoved and no other significance should be attached to it.

The nett effect of the presence of the moderator will be an increase in the neutron absorption, the increase being proportional to the number of primary neutrons scattered by the absorber. As discussed in the previous section, we can associate this absorption to imaginary sources at the centre and hence the form of the empirical correction formula as given by Eqn. (16) retains its validity and may be applied to derive the true value of σ_a.

VI. DETECTOR EFFICIENCY FOR SCATTERED NEUTRONS

Efficiency of a detector depends, in general, on the length of the path of the radiation through the active volume of the detector. Since the scattered neutrons follow a path different from the unscattered ones, efficiency and hence the number of neutrons recorded will vary with the magnitude of scattering by the absorber. The calculation of the relative efficiency of a detector depends, in general, on the type of the detector, the mode of its use as well as on its shape and size. In this discussion we will discuss a foil type neutron detector, the beta-activity induced on the exposed surface being taken as a measure of the neutron flux. We will also suppose that it is a circular disc of diameter d, with its axis passing through the centre of the absorber and moderator spheres. When a flux of neutrons is incident on the foil at an angle θ to the normal, the number of active nuclei produced between depths x and $x+dx$ is proportional to dx. sec θ exp. $(-\mu x \sec\theta)$, where μ is the absorption coefficient per cm. of the material of the detector for thermal neutrons. When the foil is presented for counting the activity produced, the recorded intensity will be approximately given by

$$\int \exp.\ (-\mu x \sec\theta) \sec\theta \exp.\ (-\mu' x)\ dx,$$

where μ' is the "absorption coefficient" for the beta rays emitted by the foil. In all practical cases $\mu' \gg \mu$. Hence the recorded activity and hence the efficiency of the detector is proportional to sec θ. The maximum value of θ (θ_m say) when the centre of the spheres is at a distance D is obtained by solving the equation (figure 5)

$$d = 2[D \tan\theta_m - (R+T) \cos\theta_m \tag{18}$$

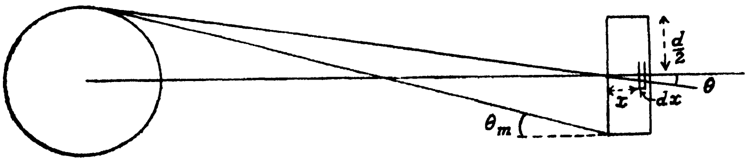


Fig. 5

We note that of the σ_s/σ_a $(1-\psi)$ scattered neutrons, about half will be rescattered by the moderator in a manner similar to the primary neutrons. Hence they do not cause any change in the efficiency of the detector. The remaining half will be scattered through a mean angle of the order of $\frac{1}{2}\theta_m$. Hence the measured transmission will be

$$\psi' = \psi + \tfrac{1}{2} \frac{\sigma_s}{\sigma_a} \left(\sec \frac{\theta_m}{2} \right) - 1\ (1-\psi)$$

showing that the fractional error in estimating the absorption is of the order of

$$p' = -\tfrac{1}{2} \frac{\sigma_s}{\sigma_a} \frac{1-\psi}{\psi} \left(\sec \frac{\theta_m}{2} - 1 \right) \quad \ldots \quad (19)$$

due to alteration in efficiency of the detector.

VII. SCATTERING OF EPITHERMAL NEUTRONS

A fast neutron source surrounded by a moderator of finite dimensions always emits epithermal neutrons in addition to the flux of thermal neutrons. In presence of the absorber, some of these neutrons will be thermalised by scattering, especially when these scattered neutrons encounter the moderator in their path. These neutrons can not be distinguished from the primary thermal neutrons and thus the dector will register an apparent increase in the value of ψ which will lower the measured absorption cross-section. The number of thermalised epithermal neutrons is a monotonic function of the absorber thickness T and tends to zero as T is made vanishingly small. Hence we can apply the semi-empirical equation (16) to correct for this effect as well.

VIII. ERROR DUE TO NON-ATTAINMENT OF THERMAL EQUILIBRIUM

In the moderating material surrounding the source, neutrons lose their energy through elastic and inelastic collisions till their energy is comparable to the energy of thermal agitation of the atoms of the slowing-down medium. We have assumed hitherto that the velocity of the neutrons eventually attains a Maxwellian distribution characteristic of the temperature T_0 of the moderator. This is, however, strictly true only in a moderator of infinite dimensions which scatter the neutrons but do not absorb them. The actual velocity distribution from a finite, moderator, will therefore, show deviation from the Maxwellian distribution. Experiments have shown that we can approximate the distribution closely by a Maxwellian distribution corresponding to a temperature T', different from T_0, over which is superimposed a pronounced tail of relatively fast neutrons extending far into the epithermal region. The contribution of epithermal neutrons can be experimentally determined by the usual cadmium difference technique. On the other hand, the estimation of the temperature T' characterising the velocity distribution cannot be determined in a simple manner. When strong sources are available one can employ the velocity selector techniques to determine the actual distribution. When the moderator is in room-temperature experiments of Manley *et al* (1946) and of Rainwater and Havens (1946) may be interpreted as showing that when a paraffin moderator of linear dimensions about 10 ± 5 cms. is used T' is given by $390° \pm 10°K$, if D—D neutrons are used. For Ra-Be source, the dimensions of the moderator are to be increased to take into account the higher initial energy of neutrons. Hence for spheres of paraffin of diameters lying between 15 and 25 cms., we expect the above value of T' to remain valid.

It is usual to define thermal neutrons as neutrons with Maxwellian velocity distribution corresponding to a temperature of 300°K. For low absorbers, therefore, the cross-section $(\sigma_a)T'$, measured for neutrons at temperature T' has to corrected by the following relation to get σ_a, the true thermal neutron absorption cross-section.

$$\sigma_a = (\sigma_a)T' \left(\frac{T'}{T}\right)^{\frac{1}{2}} \qquad \ldots \quad (20)$$

Inserting the value of T' stated above for paraffin spheres we have

$$\sigma_a = (1.1402 \pm 0.0145)\ (\sigma_a)T' \qquad \ldots \quad (20a)$$

We have used this correction factor in Table I.

For absorbers not obeying $1/v$ law partial compensation for non-attainment of thermal equilibrium may be obtained by lowering the moderator temperature. When strong sources are available a better plan is to use large amount of graphite moderator, which has a low absorption cross-section, as is done in atomic reactors for getting thermal neutron flux.

IX. SMALL DEVIATIONS FROM SPHERICAL SYMMETRY

If the absorber and the moderator spheres are not concentric, then the absorber thickness on one side will be greater than the opposite side. If, however, the centres are separated by a distance small compared to the absorber thickness, and detector is made to record the neutron intensity at different directions with respect to the spheres keeping its mean distance from the centres constant, then the increase in the number of scattered neutrons from one side will almost compensate the decrease in their number from the other side. The compensation in the value of the mean recorded intensity will not be exact but it will obviously be of second order of smallness. The degree of attainment of spherical symmetry can be obtained by noting the relation between the intensity I and the distance D between the detector and the mean centre of the source and the absorber. If the geometry of the arrangement is exactly spherical, for a point detector ID^2 should be constant. For a detector of finite size I/Ω should be constant, where Ω is the solid angle subtended by the detector at the centre of the spheres.

Small local variations of the density of the absorber, and other small deviations from the spherical symmetry are likewise smoothed out and their effects rendered insignificant, if measurements are taken in different directions as indicated above.

CONCLUSION

The above considerations show that the spherical symmetry method is realisable practically and it rests on firm theoretical foundations. Experimental details of the arrangement developed in our laboratory will be communicated in a separate paper.

The method can obviously be extended to other neutron energies provided a spherically symmetric source of such neutrons as well as a detector which responds uniformly to neutrons of different energies are available. The method can also be extended to study the absorption processes of other radiations, isolated from their scattering effects provided suitable sources and detectors are available. An important extension of the process is possible in the field of gamma rays, where account must be taken for the degeneracy and hence the variation of the detector efficiency through Compton effect. Experiment along this line is under way in the laboratory.

The authors wish to claim equal share in the publication of this work.

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