ON THE ANOMALOUS MAGNETIC MOMENTS OF THE NUCLEONS

S. P. MISRA

MATHEMATICS DEPARTMENT, RAVENSHAW COLLEGE, CUTTACK

AND

B. B. DEO

PHYSICAL LABORATORY, RAVENSHAW COLLEGE, CUTTACK

(*Received for publication, October* 19, 1955).

ABSTRACT. We have here calculated the anomalous magnetic moments of the nucleons with the help of a fourth order meson equation given by Bhabha and Thirring. The electromagnetic current density of this meson field has been evaluated for our purpose. The results are in much better agreement with the experiments than what we get in conventional meson theories.

1. INTRODUCTION

Several attempts have been made to get a correct value of the anomalous magnetic moments of the nucleons making use of conventional meson theories (Case, 1949 ; Slotnick and Heitler, 1949 ; Borowitz and Kohn 1949 ; Goto, 1954)) ; Except for partial qualitative success, the disagreements of the results obtained by them with the experiments are too conspicuous. Even the ratio of the anomalous neutron and proton moments is almost eight times the experimental value, although this ratio is independent of the rather uncertain coupling constants. The treatment of Sachs (1952) with the help of a definite model is more or less made to agree with experiments; but the arbitrariness of this model is an essential defect of this approach. Such failures suggest that an altogether different meson theory may help us in this direction. We have here chosen a fourth order meson equation given by Bhabha (1950) and Thirring (1950) for the consideration of the same problem. It is seen here that for our calculations no infinite renormalisation is necessary and that the values thus obtained agree with the experiments to a much greater extent.

2. FIELD EQUATIONS AND MESON CURRENT DENSITIES

The fourth order meson field equation is

$$(\square^2-\kappa^2)^2\phi_i = 0 \qquad \dots \quad (1)$$

The invariant lagrangian density $L(x)$ which, with the general field equation

$$\frac{\partial L}{\partial \phi_i}-\frac{\partial}{\partial x_\mu}\frac{\partial L}{\partial \frac{\partial \phi_i}{\partial x_\mu}}+\frac{\partial^2}{\partial x_\mu \partial x_\nu}\frac{\partial L}{\partial \frac{\partial^2 \phi_i}{\partial x_\mu \partial x_\nu}}=0 \quad \ldots (2)$$

gives us the field equation (1) which can be written as

$$L=-\left(-\frac{1}{2\kappa^2}\frac{\partial^2 \phi_i}{\partial x_\mu}\frac{\partial^2 \phi^*_i}{\partial x_\tau{}^2}+\frac{\partial \phi_i}{\partial x_\mu}\frac{\partial \phi^*_i}{\partial x_\mu}-\tfrac{1}{2}\kappa^2\phi_i\phi_i^*\right) \quad \ldots (3)$$

Here we have adopted the summation convention for repeated indices, and $x=(\vec{x}, ct)$ and $x_\mu x_\mu = \vec{x}^2 - c^2t^2$.

For our purpose it is necessary to deduce a current density for the above field. The lagrangian here contains the second order derivatives of the field operators. But proceeding according to Wentzel (1949), the gauge invariance of the first kind enables us to write the current density s_μ as

$$=-i\epsilon\left\{\frac{\partial L}{\partial \frac{\partial \phi_i}{\partial x_\mu}}\phi_i-\frac{\partial}{\partial x_\nu}\frac{\partial L}{\partial \frac{\partial^2 \phi_i}{\partial x_\mu \partial x_\nu}}\phi_i+\frac{\partial L}{\partial \frac{\partial^2 \phi_i}{\partial x_\mu \partial x_\nu}}\frac{\partial \phi_i}{\partial x_\nu}\right.$$

$$\left.-\text{the complex conjugate expression}\right\} \quad (4)$$

where ϵ is a constant related to the charge. A direct evaluation of $\frac{\partial s_\mu}{\partial x_\mu}$ with the subsequent application of the field equations(2) gives us

$$\frac{\partial s_i}{\partial x_i}=-i\epsilon\left\{\frac{\partial L}{\partial \phi_i}\phi_i+\frac{\partial L}{\partial \frac{\partial \phi_i}{\partial x_\mu}}\frac{\partial \phi_i}{\partial x_\mu}+\frac{\partial L}{\partial \frac{\partial^2 \phi_i}{\partial x_\mu \partial x_\nu}}\frac{\partial^2 \phi_i}{\partial x_\mu \partial x_i}-c.c.\right\}$$

But the above quantity is the coefficient of α under an infinitesimal gauge transformation $\phi_i \to \phi_i \exp(i\alpha)$, $\phi_i^* \to \phi_i^* \exp(-i\alpha)$ and thus must vanish. This was at the basis of the choice of the current density (4). Thus for (3) we obtain

$$s_\mu = i\epsilon\left\{\left(\frac{\partial \phi_i^*}{\partial x_\mu}\phi_i-\frac{\partial \phi_i}{\partial x_\mu}\phi_i^*\right)-\frac{1}{2\kappa^2}\left(\frac{\partial \Box^2 \phi_i^*}{\partial x_\mu}\phi_i-\frac{\partial \Box^2 \phi_i}{\partial x_\mu}\phi_i^*\right)\right.$$

$$\left.+\frac{1}{2\kappa^2}\left(\Box^2\phi_i^*\frac{\partial \phi_i}{\partial x_\mu}-\Box^2\phi_i\frac{\partial \phi_i^*}{\partial x_\mu}\right)\right\} \quad (5)$$

However, when there is an external electromagnetic field with the four vector potential A_μ, then the lagrangian density (3) must be changed to

$$L(x)=-\left\{-\frac{1}{2\kappa^2}\left(\frac{\partial}{\partial x_\mu}-\frac{ie}{\hbar c}A_\mu\right)^2\phi_i\left(\frac{\partial}{\partial x_\nu}+\frac{ie}{\hbar c}A_\mu\right)^2\phi_i^*\right.$$

$$\left.+\left(\frac{\partial}{\partial x_\mu}-\frac{ie}{\hbar c}A_\mu\right)\phi_i\left(\frac{\partial}{\partial x_\mu}+\frac{ie}{\hbar c}A_\mu\right)\phi_i^*-\tfrac{1}{2}\kappa^2\phi_i\phi_i^*\right\}, \quad (6)$$

and equation (1) is to be changed similarly.

Thus by (4) the current density s_μ becomes

$$s_\mu = \frac{ie}{\hbar c}\left\{\left(\frac{\partial \phi_i^*}{\partial x_\mu}\phi_i - \frac{\partial \phi_i}{\partial x_\mu}\phi_i^*\right) - \frac{1}{2\kappa^2}\left(\frac{\partial \Box^2\phi_i^*}{\partial x_\mu}\phi_i - \frac{\partial \Box^2\phi_i}{\partial x_\mu}\phi_i^* - \Box^2\phi_i^*\frac{\partial \phi_i}{\partial x_\mu}\right.\right.$$

$$\left.\left.+\Box^2\phi_i\frac{\partial \phi_i^*}{\partial x_\mu}\right)\right\} - \frac{2e^2}{(\hbar c)^2}A_\mu\left\{\phi_i^*\phi_i + \frac{1}{2^2}\left(\Box^2\phi_i^*\phi_i + \Box^2\phi_i\phi_i^*\right)\right\}$$

$$-\frac{ie^3}{(\hbar c)^3}A_\mu A_\nu\frac{1}{\kappa^2}\left(\frac{\partial \phi_i^*}{\partial x_\nu}\phi_i - \frac{\partial \phi_i}{\partial x_\nu}\phi_i^*\right) + \frac{2e^4}{(\hbar c)^4}\frac{1}{\kappa^2}A_\mu A_\nu^2. \quad \ldots (7)$$

Here we have made the substitution $\epsilon = e/\hbar c$ and have arranged the terms in the ascending powers of e.

The continuity equation follows from the gauge invariance of the first kind of the lagrangian density (6). The theory is also invariant for gauge transformations of the second kind (Wentzel, 1949, p. 68), this gauge invariance being necessary since $F_{\mu\nu} = \frac{\partial A_\nu}{\partial x_\mu} - \frac{\partial A_\mu}{\partial x_\nu}$ determines the electromagnetic field.

In our calculations we shall keep only the terms involving the first power of e.

3. GENERAL THEORY

We can take the Tomonaga equation as

$$i\hbar c\frac{\delta \Psi(\sigma)}{\delta \sigma(x)} = H(x)\Psi(\sigma)$$

Here $H(x) = H_i + H_1^{ext} + H_2^{ext}$

$$H_i(x) = if_\mu\bar{\psi}(x)\gamma_5\tau_\mu\psi(x)\phi_\mu(x) \quad \ldots (8)$$

$$H_1^{ext} = j_\mu A_\mu = -ie\bar{\psi}\frac{1-\tau_3}{2}\gamma_\mu\psi A_\mu \quad \ldots (9)$$

$$H_2^{ext} = s_\mu A_\mu$$

$$= -(e/\hbar c)A_\mu\left(\phi_1\frac{\partial \phi_2}{\partial x_\mu} - \phi_2\frac{\partial \phi_1}{\partial x_\mu}\right)$$

$$+e/\hbar c\, A_\mu(1/2\kappa^2)\left(\phi_1\frac{\partial \Box^2\phi_2}{\partial x_\mu} - \phi_2\frac{\partial \Box^2\phi_1}{\partial x_\mu} - \frac{\partial \phi_1}{\partial x_\mu}\Box^2\phi_2 + \frac{\partial \phi_2}{\partial x_\mu}\Box^2\phi_1\right) \ldots (10)$$

In the above τ_1, τ_2, τ_3 are the isotopic spin matrices, τ_4 is unit matrix in isotopic spin space, ψ is the nucleon wave function denoting the proton and neutron states for $\tau_3 = \mp 1$ respectively. ϕ_μ, $\mu = 1, 2, 3, 4$ are real fields, pseudoscalar in space-time with the corresponding complex fields describing the charge mesons given as $\phi = \frac{1}{\sqrt{2}}(\phi_1 - i\phi_2)$ and $\phi^* = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$ which has been applied in deducing (10). f_μ, $\mu = 1, 2, 3, 4$ are the corresponding coupling constants for the above fields. As mentioned earlier in deducing (10) we have neglected the higher powers of e. The anticommutation relations and the vacuum expectation values for the nucleon fields are written as

$$[\psi_\alpha(x),\ \psi_\beta(x')]_+ = -iS_{\alpha\beta}(x-x') \qquad \dots \ (11)$$

$$<P(\bar{\psi}_\alpha(x)\psi_\beta(x'))>_0 = \tfrac{1}{2}S_{F\beta\alpha}(x-x') \qquad \dots \ (11a)$$

where $$S_{F\beta\alpha}(x) = -\left(\gamma_\mu \frac{\partial}{\partial x_\mu} + \kappa_0\right)_{\beta\alpha} \Delta_F(x) \qquad \dots \ (11b)$$

$$\Delta_F(x) = -2i/(2\pi)^4 \int d^4k (k_\mu^2 + \kappa_0^2)^{-1} \exp\,(ik_\mu x_\mu). \qquad \dots \ (11c)$$

The integral in (11c) is to be understood with the usual convention of adding a small negative imaginary part to the mass of the nucleon.

The commutation relations and the vacuum expectation values of the meson field can be written as (Thirring, 1950).

$$[\phi_\mu(x),\ \phi_\nu(x')] = i\hbar c\ \delta_{\mu\nu} D(x-x') \qquad \dots \ (12)$$

$$<P(\phi_\mu(x)\phi_\nu(x'))>_o = \tfrac{1}{2}\hbar c\ \delta_{\mu\nu} D_F(x-x') \qquad \dots \ (12a)$$

where $$D_F(x) = \frac{-2i}{(2\pi)^4}\kappa^2 \int d^4k (k_\mu^2 + \kappa^2)^{-2} \exp\,(ik_\mu x_\mu). \qquad \dots \ (12b)$$

The convention of the (12b) integral is the similar to that of (11c). The κ^2 was introduced in the above integral to keep the dimensions of the propagation function remain unchanged; it could as well have been absorbed in the hamiltonian, as has been done by Thirring.

4. CALCULATIONS

Because of the presence of the virtual meson fields, the electromagnetic properties of the nucleons will be modified. In the second order for the meson field, this change is given by (Case, 1949)

$$H'_{eff}(x_o) = \tfrac{1}{2}(-i/\hbar c)^2 \int_{-\infty}^{\infty} d^4x_1 \int_{-\infty}^{\infty} d^4x_2\ P(H^{ext}(x_0)\ H_i(x_1) H_i(x_2))$$

which are split up into the terms

$$H_1(x_0) = \frac{-ief_\nu f_\sigma}{2\hbar^2 c^2} A_\mu(x_0) \int_{-\infty}^{\infty} d^4x_1 \int_{-\infty}^{\infty} d^4x_2\ P(\phi_\nu(x_1)\phi_\sigma(x_2))$$

$$\times P(\psi(x_0)\frac{1-\tau_3}{2}\gamma_\mu \bar{\psi}(x_1)\tau_\nu\gamma_5\psi(x_1)\bar{\psi}(x_2)\tau_\sigma\gamma_5\psi(x_2)) \qquad \dots \ (13)$$

$$H_2(x_0) = \frac{-ief_\nu f_\sigma}{2\hbar^3c^3} A_\mu(x_0) \int_{-\infty}^{\infty} d^4x_1 \int_{-\infty}^{\infty} d^4x_2\, P(\bar{\psi}(x_1)\tau_\nu\gamma_5\psi(x_1)\bar{\psi}(x_2)\tau_\sigma\gamma_5\psi(x_2))$$

$$\times\, P\Big((\phi_1(x_0)\frac{\partial\phi_2(x_0)}{\partial x_{0\mu}} - \phi_2(x_0)\frac{\partial\phi_1}{\partial x_{0\mu}}\Big)\,\phi_\nu(x_1)\phi_\sigma(x_2)$$

$$+\frac{ef_\nu f_\sigma}{2\hbar^3c^3} A_\mu(x_0)(1/2\kappa^2) \int_{-\infty}^{\infty} d^4x_1 \int_{-\infty}^{\infty} d^4x_2\; P(\bar{\psi}(x_1)\tau_\nu\gamma_5\psi(x_1)\bar{\psi}(x_2)\tau_\sigma\gamma_5\psi(x_2))$$

$$\times P\Big((\phi_1(x_0)\frac{\partial\,\Box_0^2\phi_2}{\partial x_{0\mu}} - \phi_2(x_0)\frac{\partial\,\Box_0^2\phi_1}{\partial x_{0\mu}} - \frac{\partial\phi_1}{\partial x_{0\mu}}\Box_0^2\phi_2 + \frac{\partial\phi_2}{\partial x_{0\mu}}\Box_0^2\phi_1\Big)\phi_\nu(x_1)\phi_\sigma(x_2) \quad \ldots \quad (14)$$

Equations (13) and part of (14) are identical with those of (Case, 1949) (equations (20) and (21)).

For our problem we now take the one-nucleon, zero-meson vacuum expectation values. As has been shown in the appendix, then we can write in terms of a single momentum variable

$$H_1(x_0) = -\frac{ieA_\mu(x_0)}{8\hbar c}\frac{i\kappa^2}{2\pi^4}\int d^4k\bar{\psi}(x_0)T\gamma_\nu\gamma_\mu\gamma_\lambda\psi(x_0)$$

$$\times k_\nu k_\lambda((k_\mu-P_\mu)^2+\kappa_0^2)^{-1}((k_\mu-P'_\mu)^2+\kappa_0^2)^{-1}(k_\mu^2+\kappa^2)^{-2}, \quad \ldots \quad (15)$$

and

$$H_2(x_0) = \frac{ie}{2\hbar c}f_1f_2\, A_\mu(x_0)\frac{\kappa^2}{2\pi^4}\int d^4k\, \bar{\psi}(x_0)\tau_3 i\,\gamma_\nu\psi(x_0)$$

$$\times\frac{(k_\nu-P'_\nu)(P_\mu-k_\mu)}{(k_\mu^2+\kappa_0^2)((k_\mu-P_\mu)^2+\kappa^2)^2((k_\mu-P'_\mu)^2+\kappa^2)^2}$$

$$+\frac{ef_1f_2}{2hc}\frac{\partial A_\mu(x_0)}{\partial x_{0\lambda}}\frac{\kappa^2}{2\pi^4}\int d^4k\bar{\psi}(x_0)\tau_3 i\gamma_\nu\psi(x_0)$$

$$\times\frac{(k_\nu-P_\nu)(P_\mu-k_\mu)(P_\lambda-k_\lambda)}{(k_\mu^2+\kappa_0^2)((k_\mu-P_\mu)^2+\kappa^2)^2(k_\mu-P'_\mu)^2+\kappa^2)^2} \quad \ldots \quad (16)$$

The description of the symbols appears in the appendix along with the calculations. Using the representations

$$1/(abc) = 2\int_0^1 dx\int_0^x dy(ay-b(x-y)-c(1-x))^{-3}$$

for the product denominators in (15) and (16), we get

$$H_1 = \frac{-ie}{8\hbar c} A_\mu(x_0) \frac{i\kappa^2}{2\pi^4} 6 \int d^4k \int_0^1 dx \int_0^x dy \quad \frac{(1-x)\bar{\psi}(x_0)T\gamma_\nu\gamma_\mu\gamma_\lambda\psi(x_0)k_\nu k_\lambda}{(k_\mu^2-2k_\mu(P'_\mu y+P_\mu(x-y))+\kappa^2(1-x))^4}$$

$$H_2 = \frac{ief_1f_2}{2\hbar c} A_\mu(x_0) \frac{\kappa^2}{2\pi^4} 6 \int d^4k \int_0^1 dx \int_0^x dy(1-x)\bar{\psi}(x_0)\tau_3 i\gamma_\nu\psi(x)$$

$$\times \frac{(k_\nu-P'_\nu)(P_\mu-k)}{(k_\mu^2-2k_\mu(P_\mu(x-y)+P'_\mu(1-x))+\kappa_0^2(1-2y)+\kappa^2(1-y))^4}$$

$$+ \frac{ef_1f_2}{2\hbar c}\frac{\partial A_\mu(x_0)}{\partial x_{0\lambda}}\frac{\kappa^2}{2\pi^4} 24 \int d^4k \int_0^1 dx \int_0^x dy (1-x)(x-y)\bar{\psi}(x_0)\tau_3 i\gamma_\nu\psi(x)$$

$$\times \frac{(k_\nu-P_\nu)(k_\mu-P_\mu)(k_\lambda-P_\lambda)}{(k_\mu^2-2k_\mu(P_\mu(x-y)+P'_\mu(1-x))+\kappa_0^2(1-2y)+\kappa^2(1-y))^5}$$

We find that the k-integration above is automatically convergent. Proceeding as in Feynman, (1949), we get.

$$H_1 = -\frac{ieA_\mu(x_0)}{8\hbar c}\cdot\frac{\kappa^2}{2\pi^4} \int_0^1 dx \int_0^x dy(1-x)\bar{\psi}(x_0)T(K_1)^{-2}$$

$$\times(-\gamma_\nu\gamma_\mu\gamma_\lambda(\Delta P_\nu y-P_\nu x)(\Delta P_\lambda y-P_\lambda x)+\gamma_\mu K_1)\psi(x_0),$$

where

$$K_1 = y(x-y)(\Delta P_\nu)^2-\phi(x) \qquad \ldots \quad (17)$$

$$\phi(x) = \kappa_0^2 x^2+\kappa^2(1-x) \qquad \ldots \quad (18)$$

and $\qquad \Delta P_\nu = P_\nu-P'_\nu. \qquad \ldots \quad (19)$

Similarly

$$H_2 = -\frac{ief_1f_2}{8\pi^2\hbar c} A \ (x_0)\kappa^2 \int_0^1 dx \int_0^x dy(1-x)\bar{\psi}(x_0)\tau_3\gamma_\nu\psi(x_0)$$

$$(K_2)^{-2}(2(x\Delta P_\nu-yP_\nu)((1-x)\Delta P_\mu-yP_\mu)-\delta_{\mu\nu}K_2)$$

$$-\frac{ef_1f_2}{8\pi^2\hbar c}\frac{\partial A_\mu(x_0)}{\partial x_{0\nu}}\kappa^2 \int_0^1 dx \int_0^x dy(1-x)(x-y)\bar{\psi}(x_0)\tau_3\cdot$$

$$[(K_2)^{-2}(\gamma_\nu(\Delta P_\mu(1-x)+P_\mu y)+\gamma_\mu(\Delta P_\nu(1-x)-P_\nu y))$$

$$\cdot(K_2)^{-3}\gamma_\lambda(P_\lambda y+\Delta P_\lambda(1-x))(P_\mu y+\Delta P_\mu(1-x))(P_\nu y+\Delta P_\nu(1-x))]\psi(x_0) \quad \ldots \quad (20)$$

where $K_2 = (1-x)(x-y)(\Delta P_\nu)^2+\phi(y)$.

In duducing (20) we have used

$$\int \frac{k_\nu k_\mu k_\lambda d^4k}{(k_\mu^2-2k_\mu Q_\mu-\Delta)^5} = \frac{\pi^2}{48i}\left[\frac{4Q_\nu Q_\lambda Q_\mu}{(Q_\mu^2+\Delta)^3}-\frac{Q_\nu\delta_{\mu\lambda}+Q_\mu\delta_{\nu\lambda}+Q_\lambda\delta_{\mu\nu}}{(Q_\mu^2+\Delta)^2}\right]$$

and the continuity equation $\dfrac{\partial A_\mu(x_0)}{\partial x_{0\mu}} = 0$

Equations (17) and (20) are simplified by repeated use of (A3) and the anticommutation rules of the γ-matrices. We also use the result

$$\bar{\psi}(x_0)(2iP_\mu)\psi(x_0) = \bar{\psi}(x_0)(\sigma_{\mu\nu}\Delta P_\nu-2\kappa_0\gamma_\mu+i\Delta P_\mu)\psi(x)$$

with

$$\sigma_{\mu\nu}=(-i/2)(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)$$

We also use the result

$$\int d^4x_0 \frac{\partial A_\mu(x_0)}{\partial x_{0\nu}} \bar{\psi}(x_0)Q\psi(x_0) = -i\Delta P_\nu \int d^4x_0\, A_\mu(x_0)\bar{\psi}(x_0)Q\,\psi(x_0) \qquad \ldots \quad (21)$$

(21) is employed to simplify the second term in (20). It gives as a particular case that terms of the type

$$A_\mu(x_0)\ (\text{constant})\ \bar{\psi}(x_0)\Delta P_\mu f((\Delta P_\nu)^2)\psi(x)$$

are effectively zero, as has been mentioned in Case (1949).

Now we write,

$$H_1 = \frac{ieA_\mu(x_0)}{16\pi^2\hbar c}\kappa^2 \int_0^1 dx \int_0^x dy(1-x)\bar{\psi}(x_0)T[(K_1)^{-2}\kappa_0 x^2\sigma_{\mu\nu}\Delta P_\nu$$
$$-\gamma_\mu(K_1)^{-2}(\kappa_0^2x^2+\phi(x))+\gamma_\mu(\phi(x))^{-2}(\kappa_0^2x^2+\phi(x))]$$

The last term above inside the square bracket has been added for renormalisation. It is to be noted that this renormalising term is finite, as has been mentioned in the introduction. The physical significance of this renormalisation may be realised when we see that the matrix element above vanishes when ΔP is zero, and that the correction due to renormalisation is independent of the momenta P and P'. We may also add that the concept of renormalisation has nothing to do with the divergencies as such, although necessarily the arguments are *more* consistent when the renormalising terms are finite, as is the case here (Kallen, 1953). Thus H_1 simplifies to

$$H_1 = \frac{ieA_\mu(x_0)}{16\pi^2\hbar c}\kappa^2 \int_0^1 dx \int_0^x dy(1-x)\bar{\psi}(x_0)T[(K_1)^{-2}\kappa_0 x^2\sigma_{\mu\nu}\Delta P_\nu$$
$$-(K_1\phi(x))^{-2}\gamma_\mu y(x-y)(\Delta P_\nu)^2(\kappa_0^2x^2+\phi(x))$$
$$(2\phi(x)+y(x-y)(\Delta P_\nu)^2)\psi(x_0). \qquad \ldots \quad (22)$$

For the simplification of H_2 we remember (21) and proceed in a similar way as before. Thus we get

$$H_2 = \frac{ief_1f_2}{8\pi^2\hbar c} A_\mu(x_0)\kappa^2 \int_0^1 dx \int_0^x dy(1-x)\bar{\psi}(x_0)\tau_3[(K_2)^{-2}\kappa_0 y^2\sigma_{\mu\nu}\Delta P_\nu$$
$$-(K_2\phi(y))^{-2}\gamma_\mu(1-x)(x-y)(\Delta P_\nu)^2(4\phi(y)\kappa_0 y^2-(\phi(y))^2-$$
$$-(1-x)(x-y)(\Delta P_\nu)^2(\phi(y)-2\kappa_0^2y^2)\psi(x) \qquad \ldots (23)$$

+terms that involve $(\Delta P\ )^2$ throughout, and thus by the subsequent section will not contribute anything to the magnetic moment.

5. MAGNETIC MOMENTS

The terms involving $\sigma_{\mu\nu}\Delta P_\nu$ above will contribute to the anomalous magnetic moments of the nucleons. For this purpose we neglect $(\Delta P_\nu)^2$ and write $A_\mu(x_0)$ $\bar{\psi}(x_0)\Delta P_\nu\Gamma\psi(x_0)$ in the form $+\ i\ \frac{\partial A_\mu}{\partial x_{0\nu}}\bar{\psi}(x_0)\Gamma\psi(x_0)$, the Γ above being an operator depending on the γ's and the τ's. This gives

$$H_1 = \frac{e\kappa^2}{16\pi^2\hbar c} I_1\tfrac{1}{2}F_{\mu\nu}(x_0)\bar{\psi}(x_0)T\ \sigma_{\mu\nu}\psi(x_0).$$

$$H_2 = \frac{ef_1f_2}{8\pi^2\hbar c} I_2\tfrac{1}{2}F_{\mu\nu}(x_0)\bar{\psi}(x_0)\tau_3\sigma_{\mu\nu}\psi(x_0)$$

where

$$I_1 = \int_0^1 dx \int_0^x dy(\phi(x))^{-2}((1-x)\kappa_0 x^2),$$

$$I_2 = \int_0^1 dx \int_0^x dy(\phi(y))^{-2}((1-x)\kappa_0 y^2).$$

This finally with $\vec{H} = (F_{23},F_{31},F_{12})$ giving the only nonzero kennzahlen of the field tensor $F_{\mu\nu}$ and with the usual spin matrix vector $\vec{\sigma}$ we get for a proton

$$H'_{eff} = \left(-\frac{e(f_3^2+f_4^2)}{16\pi^2\hbar c}\kappa^2 I_1 - \frac{ef^2}{8\pi^2\hbar c}\kappa^2 I_2\right)(-\psi^*(x_0)\vec{\sigma}\cdot\vec{H}\psi(x_0))$$

and for the neutron

$$H'_{eff} = \left(-\frac{ef^2}{8\pi^2\hbar c}\kappa^2 I_1 - \frac{ef^2}{8\pi^2\hbar c}\kappa^2 I_2\right)(-\psi^*(x_0)\vec{\sigma}.\vec{H}\psi(x_0)).$$

In the above $f_1 = f_2 = f$ has been taken. Confining our attention to the symetrical theory ($f_1 = f_2 = f_3 = f, f_4 = 0$) we get the respective anomalous magnetic moments as

$$\Delta\mu_P = \frac{ef^2\kappa^2}{8\pi^2\hbar c} (I_2 - \tfrac{1}{2}I_1)$$

and

$$\Delta\mu_N = -\frac{ef^2\kappa^2}{8\pi^2\hbar c} (I_2 + I_1)$$

The integrals I_1 and I_2 are elementary although lengthy. Evaluating them we have

$$I_1 = \frac{1}{\kappa_0^3} \Big(-3/2 + \tfrac{1}{2} \ln(1/\lambda) + \frac{\lambda^{\frac{1}{2}}}{(4-\lambda)^{3/2}} (18 - 13\lambda + 2\lambda^2) \cos^{-1}(\lambda^{\frac{1}{2}}/2)$$

$$+ \tfrac{1}{2}\lambda - \frac{3\lambda - \lambda^2}{4(4-\lambda)} - \lambda \ln (1/\lambda)\Big),$$

and

$$I_2 = \frac{1}{\kappa_0^3} \Big(1 - \tfrac{1}{2} \ln (1/\lambda) + \frac{2 - 12\lambda + 9\lambda^2}{\lambda^{\frac{1}{2}}(4-\lambda)^{3/2}} \cos^{-1}(\lambda^{\frac{1}{2}}/2) - \frac{2 - 4\lambda + \lambda^2}{4(4-\lambda)}$$

$$+ \tfrac{1}{2} \ln(1/\lambda) - 4\lambda\Big).$$

where $\lambda = \left(\frac{\kappa}{\kappa_0}\right)^2$. With $\lambda^{\frac{1}{2}} = 0.15$, we get

$$I_1 = \frac{1}{\kappa_0^3}\, 0.80 \text{ nearly},$$

and

$$I_2 = \frac{1}{\kappa_0^3}\, 1.20 \text{ nearly}.$$

Hence

$$\Delta\mu_P = (G^2/4\pi^2\hbar c)\; 0.80$$

and

$$\Delta\mu_N = -(G^2/4\pi^2\hbar c)\; 2.00$$

where the quantities are expressed in nuclear magnetons and we have substituted

$$\frac{f\kappa}{\kappa_0} = G.$$

Thus $|\Delta\mu_P/\Delta\mu_N| = 0.40$ nearly.

If we take $\Delta\mu_P = 1.79$, then $G^2/4\pi^2\hbar c = 7$ nearly, ... (24)

and if we take $\Delta\mu_N = -1.91$, then $G^2/4\pi^2\hbar c = 3$ nearly. ... (25)

DISCUSSIONS

The above results are in qualitative agreement with experiments as regards the signs and the relative magnitudes of the magnetic moments. The quantitative values, though not satisfactory, do not 'contradict violently the experimental results' (Goto, 1954) as in earlier theories. Our result 2.50 for the coupling-constant-independent ratio $|\Delta\mu_N/\Delta\mu_P|$ is a significant improvement over those of Case (1949), and Borowitz and Kohn (1949), which is 8 nearly, as compared to the experimental value 1.07. Even the relativistic cut-off method of Goto (1954), yields the value 3.1 nearly, which is slightly worse than ours.

Again, the calculated values of the coupling constants in (24) and (25) do not differ widely (previously they differed as much as 56 and 7), and although the values thus obtained are comperatively small, they are not small enough to make the second order calculations very reliable, and the differences still present may be attributed to this fact.

In our calculations here no infinite renormalisation was necessary, which is an encouraging feature of this theory. But it may be noted that higher order corrections to this theory will contain infinite renormalisations, since the meson self-energy remains unchanged. Thus the processes which involve this graph will have to be dealt with more or less in the usual manner of subtracting infinite quantities.

ACKNOWLEDGMENTS

The authors wish to express their sincere thanks to Dr. D. Basu for suggesting the problem and also for pointing out an error in the calculations.

APPENDIX

Evaluation of H_1:

After taking the vacuum expectation values, we can write $H_1 = H_1^a + H_1^b$ where

$$H_1^a = \frac{ief_\nu^2}{8\hbar c} A_\mu(x_0) \int_{-\infty}^{\infty} d^4x_1 \int_{-\infty}^{\infty} d^4x_2 \; D_F(x_1-x_2)\bar{\psi}(x_2)\gamma_5\tau_\nu\psi(x_2)$$
$$Sp(\tau_\nu\gamma_5 S_F(x_0-x_1)\gamma_\mu \frac{1-\tau_3}{2} S_F(x_1-x_0)) \qquad \ldots \quad \text{(A1)}$$

and

$$H_1^b = -\frac{ief_\nu^2}{8\hbar c} A_\mu(x_0) \int_{-\infty}^{\infty} d^4x_1 \int_{-\infty}^{\infty} d^4x_2 \bar{\psi}(x_1)\gamma_5\tau_\nu S_F(x_0-x_1)\gamma_\mu \frac{1-\tau_3}{2}$$
$$S_F(x_2-x_0)\tau_\nu\gamma_5\psi(x_2)D_F(x_1-x_2)$$

To evaluate H_1^a we note that the spur written down in (A1) vanishes, as has been shown by Case (1949) with the same quantities as in (A1), but with a different D_F function. Thus the arguments of Case for the vanishing of the H_1^a with the conventional D_F function continues to hold here as well.

Rearranging the isotopic spin matrices, H_1^b can be simplified to

$$H_1^b = -\frac{ie}{8\hbar c} A_\mu(x_0)\int_{-\infty}^{\infty} d^4x_1 \int_{-\infty}^{\infty} d^4x_2 \bar{\psi}(x_1) T\gamma_5 S_F(x_0-x_1)\gamma_\mu S_F(x_2-x_0)\gamma_5 \psi(x_2) D_F(x_1-x_2),$$

where

$$T = \tfrac{1}{2}(1-\tau_3)(f_3^2+f_4^2)+\tfrac{1}{2}(1+\tau_3)(f_1^2+f_2^2). \quad \ldots \quad (A2)$$

Also we take

$$\psi(x) = u_p \exp(iP_\mu x_\mu), \quad \bar{\psi}(x) = u'_p \exp(-iP'_\mu x_\mu)$$

such that $P_\mu^2 = P'^2_\mu = -\kappa_0^2$. We now represent the invariant functions by means of the integrals (11c) and (12b) and integrate with respect to x_1 and x_2, giving rise to δ-functions with the help of which we finally express

$$H_1^b = -\frac{ie}{8\hbar c} A_\mu(x_0) \frac{i\kappa^2}{2\pi^4}\int d^4k \bar{\psi}(x_0) T\gamma_5 \times \frac{(i\gamma_\nu(P'_\nu - k_\nu)-\kappa_0)\gamma_\mu(i\gamma_\lambda(P_\lambda - k_\lambda)-\kappa_0)\gamma_5\psi(x_0)}{((k_\mu-P_\mu)^2+\kappa_0^2)((k_\mu-P_\mu')^2+\kappa_0^2)(k_\mu^2+\kappa^2)^2}$$

With repeated applications of the results

$$(i\gamma_\mu P_\mu+\kappa_0)u_p = u_p'\ (i\gamma_\mu P'_\mu+\kappa_0) = 0 \quad \ldots \quad (A3)$$

and $\gamma_5^2 = 1$, the above expression for H_1^b can be seen to simplify to the expression (15) already written down.

Evaluation of H_2 :

We first write $H_2 = H'_2+H''_2$, where

$$H'_2 = -\frac{ef_\nu f_\sigma}{2\hbar^3c^3} A_\mu(x_0)\int_{-\infty}^{\infty} d^4x_1 \int_{-\infty}^{\infty} d^4x_2 P(\bar{\psi}(x_1)\tau_\nu\gamma_5\psi(x_1)\bar{\psi}(x_2)\tau_\sigma\gamma_5\psi(x_2)) \times P\left(\left(\phi_1(x_0)\frac{\partial\phi_2}{\partial x_{0\mu}} - \phi_2(x_0)\frac{\partial\phi_1}{\partial x_{0\mu}}\right)\phi_\nu(x_1)\phi_\sigma(x_2)\right)$$

$$H_2'' = \frac{ef_\nu f_\mu}{2\hbar^3c^3} A_\mu(x_0)\frac{1}{2\kappa^2}\int_{-\infty}^{\infty} d^4x_1 \int_{-\infty}^{\infty} d^4x_2\, P(\bar{\psi}(x_1)\tau_\nu\gamma_5\psi(x_1)\bar{\psi}(x_2)\,\tau_\sigma\gamma_5\psi(x_2))$$

$$\times P((\phi_1(x_0)\frac{\partial\Box_0^2\phi_2}{\partial x_{0\mu}} - \phi_2(x_0)\frac{\partial\Box_0^2\phi_1'}{\partial x_{0\mu}} - \frac{\partial\phi_1}{\partial x_{0\mu}}\Box_0^2\phi_2 + \frac{\partial\phi_2}{\partial x_{0\mu}}\Box_0^2\phi)(\phi_\nu(x_1)\phi_\sigma(x_2)))$$

The calculation of H_2' is similar to that of Case (1949), and proceeding as before, the final result can be written as

$$H'_2 = + \frac{ief_1f_2}{2\hbar c} A_\mu(x_0) \frac{\kappa^4}{2\pi^4} \int d^4k \, \bar{\psi}(x_0)\tau_3 i\gamma_\nu\psi(x_0)$$

$$\times \frac{(k_\nu - P_\nu')(P_\mu - k_\mu)}{(k^2_\mu+\kappa_0^2)((k_\mu - P_\mu)^2+\kappa^2)((k_\mu - P'_\mu)^2+\kappa^2)^2}$$

To simplify H_2'' we note that

$$< P(\phi_\nu(x_1)\phi_\sigma(x_2)(\phi_1(x_0)\frac{\partial\Box_0^2\phi_2}{\partial x_{0\mu}} - \phi_2(x_0)\frac{\partial\Box_0^2\phi_1}{\partial x_{0\mu}} - \frac{\partial\phi_1}{\partial x_{0\mu}}\Box_0^2\phi_2(x_0) + \frac{\partial\phi_2}{\partial x_{0\mu}}\Box_0^2\phi_1)) >_0$$

$$= (\hbar^2c^2/4)(\delta_{1\nu}\delta_{2\sigma} - \delta_{1\sigma}\delta_{2\nu})\,(D_F(x_0-x_1)\frac{\partial\Box_0^2 D_F(x_0-x_2)}{\partial x_{0\mu}} - D_F(x_0-x_2)\frac{\partial\Box_0^2 D_F(x_0-x_1)}{\partial x_{0\mu}}$$

$$- \frac{\partial D_F(x_0-x_1)}{\partial x_{0\mu}}\Box_0^2 D_F(x_0-x_2) + \frac{\partial D_F(x_0-x_2)}{\partial x_{0\mu}}\Box_0^2 D_F(x_0-x_1))$$

This gives

$$H''_2 = - \frac{ef_1f_2}{8\hbar c} A_\mu(x_0) \frac{1}{2\kappa^2} \int_{\infty}^{\infty} d^4x_1 \int_{-\infty}^{\infty} d^4x_2\bar{\psi}(x_1)(\tau_1\gamma_5 S_F(x_2-x_1)\tau_2\gamma_5$$

$$-\tau_2\gamma_5 S_F(x_2-x_1)\tau_1\gamma_5)\psi(x_2)$$

$$\times \Big[D_F(x_0-x_1)\frac{\partial\Box_0^2 D_F(x_0-x_2)}{\partial x_{0\mu}} - D_F(x_0-x_2)\frac{\partial\Box_0^2 D_F(x_0-x_1)}{\partial x_{0\mu}}$$

$$- \frac{\partial D_F(x_0-x_1)}{\partial x_{0\mu}}\Box_0^2 D_F(x_0-x_2) + \frac{\partial D_F(x_0-x_2)}{\partial x_{0\mu}}\Box_0^2 D_F(x_0-x_1)\Big] \qquad \ldots \text{(A4)}$$

We now use $\tau_1\tau_2 = -\tau_2\tau_1 = i\tau_3$. Also in (A4), the x_0 integration is implicit. Carrying out partial integration with respect to x_0 and using the result $\frac{\partial A_\mu(x_0)}{\partial x_{0\mu}} = 0$, the second and third terms inside the square bracket can be seen to be respectively equal to the first and fourth terms. For examlpe,

$$- \int_{-\infty}^{\infty} dx_{0\mu}A_\mu(x_0)\frac{\partial D_F(x_0-x_1)}{\partial x_{0\mu}}\Box_0^2 D_F(x_0-x_2)$$

$$= \int_{-\infty}^{\infty} dx_{0\mu}\frac{\partial}{\partial x_{0\mu}}(A_\mu(x_0)\Box_0^2 D_F(x_0-x_2))D_F(x_0-x_1)$$

$$= \int_{-\infty}^{\infty} dx_{0\mu}\, A_\mu(x_0)\frac{\partial\Box_0^2 D_F(x_0-x_2)}{\partial x_{0\mu}} D_F(x_0-x_1).$$

But again applying partial integration two times in the different variables for the fourth term in (A4), we obtain this term as equal to

$$\int d^4x_0 \Box_0^2 \left(A_\mu(x_0) \frac{\partial D_F(x_0-x_2)}{\partial x_{0\mu}} \right) D_F(x_0-x_1)$$

$$= \int d^4x_0 \left[A_\mu(x_0) \frac{\partial \Box_0^2 D_F(x_0-x_2)}{\partial x_{0\mu}} + 2 \frac{\partial A_\mu(x_0)}{\partial x_{0\nu}} \frac{\partial^2 D_F(x_0-x_2)}{\partial x_{0\nu}\partial x_{0\mu}} \right] D_F(x_0-x_1)$$

where we have applied $\Box_0^2 A_\mu(x_0) = 0$.

Thus we get

$$H_2'' = -\frac{ief_1f_2}{2\hbar c}\frac{1}{\kappa^2}\int_{-\infty}^{\infty} d^4x_1 \int_{-\infty}^{\infty} d^4x_2 \bar{\psi}(x_1)\tau_3\gamma_5 S_F(x_2-x_1)\gamma_5\psi(x_2)$$

$$\times \left[A_\mu(x_0) D_F(x_0-x_1) \frac{\partial \Box_0^2 D_F(x_0-x_2)}{\partial x_{0\mu}} + \frac{\partial A_\mu(x_0)}{\partial x_{0\nu}} D_F(x_0-x_1) \frac{\partial^2 D_F(x_0-x_2)}{\partial x_{0\mu}\,\partial x_{0\nu}} \right].$$

We again apply the integrals (11c) and (12b) and thus finally obtain, as in case of H_1^b and H'_2,

$$H''_2 = \frac{ief_1f_2}{2\hbar c}\frac{\kappa^2}{2\pi^4}\int d^4k \bar{\psi}(x_0)\tau_3 i\gamma_\nu\psi(x_0)$$

$$\times \frac{A_\mu(x_0)(k_\nu-P'_\nu)(P_\mu-k_\mu)(P_\lambda-k_\lambda)^2 - i\frac{\partial A_\mu(x_0)}{\partial x_{0\lambda}}(k_\nu-P'_\nu)(P_\mu-k_\mu)(P_\lambda-k_\lambda)}{(k^2_\mu+\kappa_0^2)((k_\mu-P_\mu)^2+\kappa^2)^2((k_\mu-P'_\mu)^2+\kappa^2)^2}$$

Thus adding the values of H'_2 and H''_2 and simplifying, we get the value of H_2 as mentioned in formula (16) earlier.

REFERENCES

Bhabha H. J., 1950, *Phys. Rev.*, **77**, 665.
Borowitz S. and Kohn, W., 1949, *Phys. Rev.*, **76**, 818.
Case, K. M., 1949, *Phys. Rev.*, **76**, 1.
Feynman, R. P., 1949, *Phys. Rev.*, **76**, 784 (appendix).
Goto S., 1954, *Progress of Theor. Phys.*, **12**, 699.
Källen G., 1953, *Physica*, **19**, 850.
Sachs R. G., 1952, *Phys. Rev.*, **87**, 1100;
Slotnick, M. and Heitler W., 1949, *Phys. Rev.*, **75**, 1645;
Thirring, W., 1950, *Phil. Mag.* **41**, 653.
Wentzel G., 1949, Quantum Theory of Fields, Interscience Publishers. p. 68