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DEPENDENCE OF LIMIT OF RESOLUTION ON BACKGROUND INTENSITY, DETECTING INSTRUMENT AND STAGE OF RESOLUTION IN X-RAY SPECTRA.

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ABSTRACT. Ditchburn has suggested that a combination of stage of resolution desired and the detecting instrument is characterized by C, the ratio of the minimum to maximum of resultant intensity pattern of two spectral lines, for optimum resolution. If k is the ratio of background intensity to the intensity of two-X-ray spectral lines of half width b, the limit of resolution is given by

$$\Delta\lambda_r = b\left[\frac{\{5k(1-C)-6C+8\}+\sqrt{\{3k(1-C)-2C+8\}^2-32C}}{2\{C-k(1-C)\}}\right]^{1/2}$$

A table giving the values of $b!$ $b/\Delta\lambda_r$ for various values of k and C has been constructed.

INTRODUCTION

Sodha (1954) has discussed the effect of background intensity on the resolving power of optical instruments. He has also discussed the case when the instrumental width is negligible, the intensity distribution being governed by Doppler effect. His discussion is based on the Rayleigh criterion for resolution of spectral lines.

Ditchburn (1930) has emphasized the fact that the resolving power also depends on the detecting instrument and the stage of resolution desired. A given combination of detecting instrument and stage of resolution is characterized by the value C of I_{min}/I_{max} for optimum resolution of two spectral lines, where I_{min} and I_{max} denote the minimum and maximum of the resultant intensity pattern. Ditchburn has given the following values of C for three main stages of resolution when microphotometer is used as a detecting instrument.

Stage of resolution	C
(i) Detection of inhomogeneity in radiation	0.98
(ii) Partial resolution (approximate measurement of wavelength separation)	0.8
(iii) Complete measurement (measurement of wave length separation and relative intensities)	0.4

Sharma and Sodha (1954) and Mitra (1954) have discussed the dependence of resolving power of prism, grating, reflecting echelon, Fabry-Perot etalon and

Lummer Gehrcke plate C. Sodha (1954*a*) has also discussed the case when the lines have an intensity dis- tribution, governed by Doppler effect.

In this paper the author has discussed the effect of background intensity, detecting instrument and the stage of resolution desired on the limit of resolution of two X-ray spectral lines. An explicit expression for the limit of resolution has been obtained in terms of half-width, C and k the ratio of background intensity to the intensity of the spectral line.

INTENSITY CONSIDERATIONS

The intensity distribution of an X-ray spectral line is best represented by Hoyt (1932)

$$I_1 = \frac{I_0}{1+\{(\lambda-\lambda_0)/b\}^2}$$

where b is the half-width of the line. The intensity distribution of another line of equal intensity separated by $\Delta\lambda$ is

$$I_2 = \frac{I_0}{1+\{(\lambda-\lambda_0-\Delta\lambda)/b\}^2}$$

If the background intensity is kI_0 the resultant intensity pattern of the two lines is given by

$$I/I_0 = \frac{I_1+I_2}{I_0} = \frac{1}{1+X^2} + \frac{1}{1+(X-a)^2} + k \qquad \ldots \quad (1)$$

Neglecting shrinkage effect the intensity maxima ($x \approx o$ or a) and minimum ($x = \frac{a}{2}$) are given by

$$I_{max}/I_0 = k+1+\frac{1}{1+a^2} \qquad \ldots \quad (2)$$

$$I_{min}/I_0 = k+\frac{2}{(1+a^2/4)} \qquad \ldots \quad (3)$$

LIMIT OF RESOLUTION

We have

$$I_{min} = C \times I_{max} \qquad \ldots \quad (4)$$

which gives

$$a = \left[\frac{\{5k(1-C)-6C+8\}+\sqrt{\{3k(1-C)-2C+8\}^2-32C}}{2\{C-k(1-C)\}}\right]^{\frac{1}{2}} \quad \cdots \quad (5)$$

$$\Delta\lambda_r = b\left[\frac{\{5k(1-C)-6C+8\}+\sqrt{\{3k(1-C)-2C+8\}^2-32C}}{2\{C-k(1-C)\}}\right]^{\frac{1}{2}} \quad \cdots \quad (6)$$

where $\Delta\lambda_r$ is the limit of resolution, meaning the smallest wavelength difference, resolvable. The resolving power is given by

$$R = \frac{\lambda}{\Delta\lambda_r} = \frac{\lambda}{b}\left\{\frac{\{5k(1-C)-6C+8\}+\sqrt{\{3k(1-C)-2C+8\}^2-32C}}{2\{C-k(1-C)\}}\right\}^{-\frac{1}{2}} \quad \cdots \quad (7)$$

Table 1 gives the value of Rb/λ for various values of k and C; where $0 < k < C/(1 - C$.

Figures 1 and 2 illustrate the variation of Rb/λ with k and C.

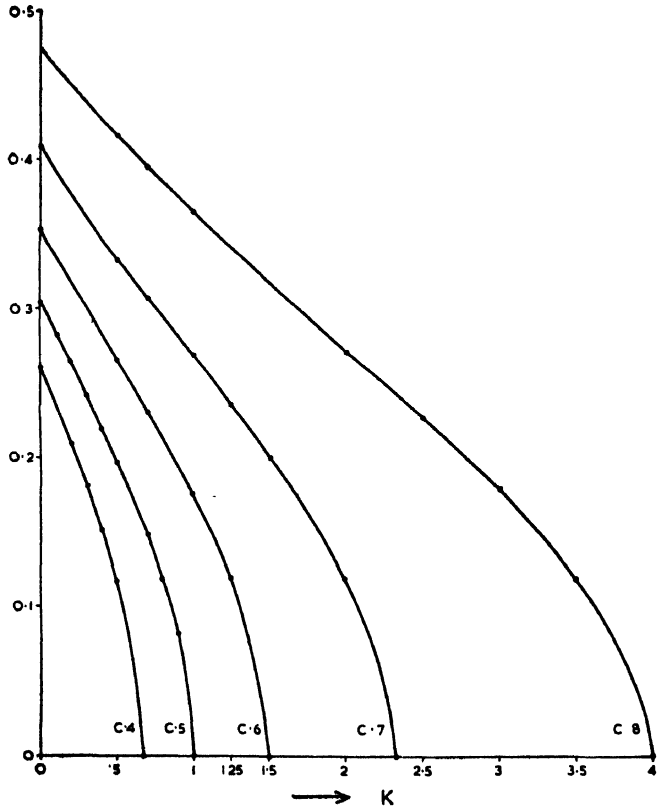

Fig. 1. Variatien of Rb/λ with k. $(0.4 \leqslant C \leqslant 0.8)$

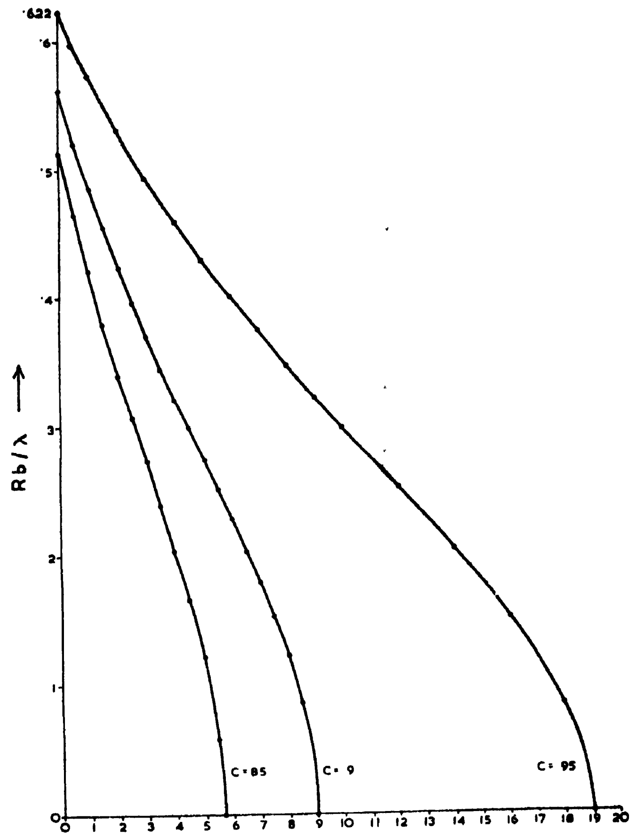

Fig. 2. Variation of Rb/λ with k. $(0.85 \leqslant C \leqslant 0.95)$.

TABLE I

Value of Rb/λ

k \ C	.4	.5	0.6	0.7	0.8	0.85	0.9	0.95
0	.260	.305	.353	.408	.474	.514	.562	.622
.1	.235	.284	—	—	—	—	—	—
.2	.209	.265	—	—	—	—	—	—
.3	.181	.241	—	—	—	—	—	—
.4	.152	.219	—	—	—	—	—	—
.5	.118	.197	.265	.331	.416	.464	.522	.596
.66	0	—	—	—	—	—	—	—
.7	—	.148	.222	.301	—	—	—	—

TABLE I (*contd.*)

.8	—	.119	—	—	—	—	—	—
.9	—	.083	—	—	—	—	—	—
1	—	0	.175	.268	.364	.42	.485	.572
1.25	—	—	.12	.235	—	—	—	—
1.5	—	—	0	.201	.317	.38	.454	—
2	—	—	—	.12	.27	.34	.424	.530
2.5	—	—	—	—	.226	.307	.396	—
3	—	—	—	—	.178	.273	.37	.493
3.5	—	—	—	—	.122	.238	.345	—
4	—	—	—	—	0	.203	.321	.459
4.5	—	—	—	—	—	.166	.300	—
5						.122	.274	.429
5.5						.059	.251	—
6							.228	.400
6.5							.204	—
7							.179	.375
7.5							.153	—
8							.123	.348
8.5							.086	—
9							0	.323
10								.300
12								.253
14								.205
16								.153
18								.085
19								0

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REFERENCES

Ditchburn, 1930, *Proc. Roy. Irish. Acad.*, 39, 58.
Hoyt, 1930, *Phys. Rev.*, **40**, 477.
Mitra, S. S. 1954, *Ind. J. Phys.*, **28**, 543.
Sharma and Sodha, M. S., 1954, *Ind. Phys.*, **28**, 437.
Sodha, M. S., 1954, *Ind J. Phys.*, **28**, 141.
Sodha, M. S. 1954*a*, Ph.D. Thesis, Allahabad University.