THE EFFECT OF FLUID MOTION ON HEAT TRANSMISSION

PART III, SOLIDS OF DIFFERENT SHAPES AND SIZES

By D. G. KAPADNIS*

NATIONAL PHYSICAL LABORATORY OF INDIA, NEW DELHI

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ABSTRACT. A detailed experimental study of the effect of a unidirectional stream of air on the rate of heat dissipation from solids of different shapes and sizes by forced convection has been made in a limited range of Reynolds numbers from 10^3 to 10^5. The experimentally derived values of the constants B and n in the Nusselts equation (employed for air)

$$\left(\frac{1}{A\Delta\theta}\,\frac{dQ}{dT}\,\frac{D}{K}\right)=B\left(\frac{VD}{\xi}\right)^{?}$$

(the symbols have their usual meaning) are respectively 0.505 and 0.516 for horizontal cylinders with ellipsoidal nosepiece, 0.033 and 0.783 for the same with plane blunt nosepiece; 0.78 and 0.517 for spheres; 0.12 and 0.654 for rectangular parallelopipeds placed in face-on position; and 0.24 and 0.576 for the same placed in edge-on position. Finally the experimental data have been compared with those of other investigators.

INTRODUCTION

The interchange of heat between a hot solid and a moving stream of relatively cool fluid or vice versa is a subject of considerable technical importance, and also not devoid of scientific interest. A considerable amount of work has been done on the cooling of solids at rest in moving fluids. Compared to the abundant experimental work on heat transfer in tubes and ducts relatively little has been done concerning the flow of the fluid parallel or perpendicular to a heating or a cooling surface when the fluid is not bounded by the walls of a tube or a channel. Moreover, our knowledge of the complex mechanism of heat dissipation by convection is as yet, especially from the theoretical standpoint, incomplete. More experimental evidence is necessary for the satisfactory understanding of the thermodynamics of heat-interchange between the solid and the surrounding stream of fluid under different ambient conditions. The experiments of heat dissipation from vertical cylinders (Kapadnis, 1953) were, therefore, extended to solids of various shapes and sizes with a view to throwing some light on this problem of convection in these cases.

EXPERIMENTAL ARRANGEMENT

The experimental arrangement used by the present author (1953) in studying the heat dissipation from vertical cylindrical vessels of different sizes filled with

* Now at Kamerlingh Onnes Cryogenic Laboratory of The State University of Leiden, The Netherlands.

hot water and placed in a current of air under different ambient conditions was completely modified in these investigations. A vessel A (figure 1) containing hot water was placed at a distance of about 90 cm from an electric fan F and a stream of air proceeding from the fan was directed on this vessel after allowing it to pass through a wire grid G situated at a distance of about 18 cm from the fan. The purpose of the grid was to change the divergent stream of air issuing from the fan into uniform parallel stream giving natural conditions resembling a wind tunnel

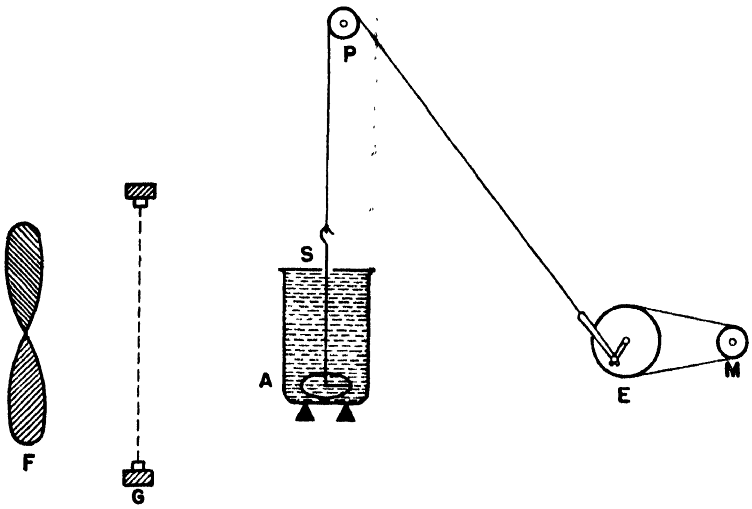


Fig. 1.

of infinite dimensions. The top of the vessel was closed by a lid having a hole in it. Through this hole a stirrer S was kept working up and down in the vessel by connecting it to a string passing over a pulley P. The other end of the string was connected to an eccentric arrangement E attached to slow-speed electric motor M, thus ensuring a vertical motion of the stirrer in the vessel. A uniform temperature was thus maintained throughout the whole mass of water at any instant. The usual precautions were taken to minimize heat losses due to conduction. radiation and evaporation. Separate experiments in still air were also performed in order to calculate the losses due to natural or free convection. All subsequent observations were corrected for losses due to radiation and natural convection. Under these circumstances the heat dissipation, when the vessel was subjected to air stream, was due to forced convection alone.

The fan was made by attaching four steel blades to the axle of the three-phase motor, the speed of which could be regulated by a rotor rheostat and by varying the voltage of the alternator supplying the current. A good range of wind velocity was thus obtained. The values of air velocity used by the author in the previous investigation were preferred in this work for the sake of convenience in calculations. The velocity of the air currents produced by the fan was measured by means of a cyclometer pattern type four-cup anemometer supplied by the Indian Meteorological Centre. This anemometer was arranged to indicate the run of the wind which had passed in any desired interval, by a simple system of gears terminating in a counting mechanism. This instrument measures the linear velocity but fails to measure the eddying motion of air produced near an obstacle in the turbulent region, therefore, the values of air velocity were checked up by means of a silvered Kata thermometer measuring the air movement by the cooling effect of the wind. Both of these instruments were previously calibrated.

Four copper-constantan thermocouples, having a potentiometer as an auxiliary measuring device, were used as temperature-measuring device in these experiments. The potentiometer circuit is shown in figure 2. Only one cold junction b common to all the four hot junctions a_1, a_2, a_3 and a_4 of the thermocou-

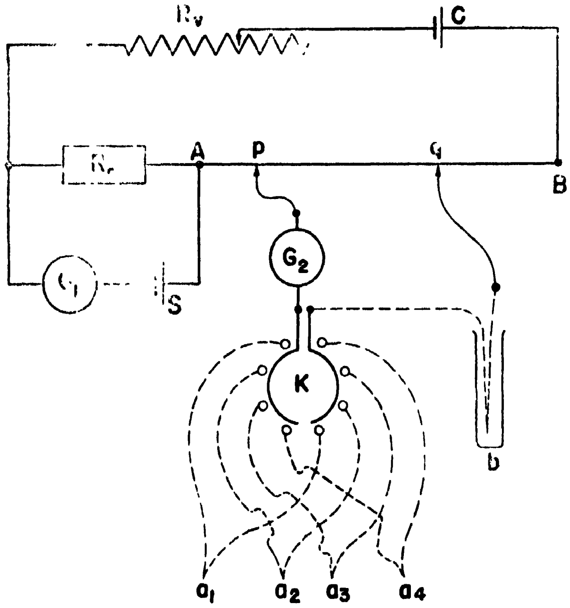


Fig. 2

ples was connected to each of them one by one through a multiple switch K. An accumulator C caused the current to pass round the circuit through the variable resistance R_v, the fixed standard resistance R_s and the potentiometer wire AB. The current was standardized by adjusting the variable resistance R_r until the voltage drop across R_s was just equal to the electromotive force of the standard cell S, as indicated by a zero reading of the galvanometer G_1. The potentiometer tappings p and q were then adjusted until the electromotive force of the thermocouple was balanced by the voltage drop along the potentiometer wire between them, indicated by a zero reading of the galvanometer G_2. The thermocouples were calibrated with a standard thermometer and a graph of temperature difference against the length of the segment of the potentiometer wire was plotted directly and this plot was used in subsequent expriments. The hot junctions of the thermocouples were soldered to the outer surface of the vessel for the measurement of its surface temperature. The method used by Kapadnis and Gogate (1952) was followed in calculating the rate of heat dissipation. The cylindrical vessels used were all circular. The diameters of the cylindrical and spherical vessels were taken as characteristic linear dimension in subsequent calculations. But in the case of rectangular vessels the diameter of the circular cylindrical vessel of equal exposed surface was used as the characteristic dimension. The density, viscosity and conductivity of the sourrounding stream of air were taken at the mean film temperature.

A detailed study of convective heat transmission from vessels filled with warm water to the surrounding air stream was made with vessels of three different shapes used in five ways—horizontal cylinders in two positions, spheres and rectangular parallelopipeds placed in face-on as well as edge-on position. The velocity of the air current was varied from 80 cm per second to 1095 cm per second. The experiments were repeated for vessels of four different sizes of each shape, a typical set of observations being recorded in Table I.

TABLE I

Shape of the vessel	Characteristic dimension of the vessel in cm	Air velocity in cm/sec	Rate of heat transmission in cal/cm2/sec/°C $\times 10^{4}$	$\frac{VD}{\xi}$ $\times 10^{3}$	$\frac{1}{A\Delta\theta}\frac{dQ}{dT}\frac{D}{K}$
Horizontal cylinder		80	3.06	2.6	27.7
		242	5.79	7.9	52.3
		405	7.70	13.2	69.5
	5.2	563	8.88	18.4	80.3
		721	9.82	23.6	88 7
		882	11.48	28.8	103.7
		967	11.71	31.6	105.9
		1095	12.22	35.7	110.4
		80	2.27	5.4	42.1
		242	3.94	16.3	73.3
		405	5.27	27.2	97.9
	10.7	563	6.42	37.8	119.3
		721	7.09	48.5	131.8
		882	7.72	59.2	143.5
		967	8.32	64.9	154.7
		1095	8.98	73.5	167.0
		80	1.92	7.5	50.0
		242	3.45	22 8	90.0
		405	4.27	38 1	113.9
	15.0	563	5.61	53.0	146.3
		721	5.96	68.0	155.2
		882	6.75	83.1	175.8
		967	7.16	91.0	186.7
		1095	7.49	103.0	195.2

TABLE I (*contd.*)

Shape of the vessel	Characteristic dimension of the vessel in cm	Air velocity in cm/sec	Rate of heat transmission in cal/cm²/sec/°C $\times 10^{-4}$	$\frac{VD}{\xi}$ $\times 10^3$	$\frac{1}{A\Delta\theta}\frac{dQ}{dT}\frac{D}{K}$
		80	1.64	11.0	62.0
		242	2.80	33.1	106.0
		405	3.65	55.4	138.2
	21.8	563	4.34	77.0	164.4
		721	4.99	98.9	188.9
		882	5.60	120.7	212.3
		967	6.10	132.3	231.1
		1095	6.72	149.8	254.6
Sphere		80	5.74	2.4	47.9
		242	9.98	7.3	83.2
		405	13.15	12.2	109.6
	4.8	563	14.42	17.0	120.2
		721	16.18	21.7	134.9
		882	17.34	26.6	144.5
		967	19.91	29.1	166.0
		1095	21.33	33.0	177.8
		80	3.07	5.2	55.0
		242	6.27	15.6	112.2
		405	8.46	26.2	151.4
	10.3	563	10.17	36.4	182.0
		721	11.95	46.6	213.8
		882	13.10	57.0	234.4
		967	13.41	62.5	239.5
		1095	15.04	70.8	269.2
		80	2.71	7.9	74.1
		242	5.27	24.0	144.5
		405	6.63	40.2	182.0
	15.8	563	7.79	55.9	213.8
		721	9.58	71.5	263.0
		882	10.27	87.5	281.8
		967	10.75	95.9	295.1
		1095	11.00	108.6	302.0
		80	2.52	10.2	89.1
		242	4.37	31.0	154.9
		405	6.17	51.9	218.8
	20.4	563	6.62	72.1	234.4
		721	8.33	92.3	295.1
		882	8.52	112.9	302.0
		967	9.78	123.8	346.7
		1095	10.97	140.3	389.0
Rectangular parallelopiped (Face-on position)		80	2.51	2.7	23.6
		242	4.63	8.2	43.5
		405	6.44	13.7	60.4
	5.4	563	8.53	19.1	63.5
		721	8.95	24.4	84.0
		882	10.99	29.9	103.0
		967	12.27	32.8	115.1
		1095	13.34	37.1	125.0
		80	1.96	5.1	34.8
		242	3.75	15.5	66.5
		405	5.12	25.9	90.8

TABLE I (*contd.*)

Shape of the vessel	Characteristic dimension of the vessel in cm	Air velocity in cm/sec	Rate of heat transmission in cal/cm2/sec/°C $\times 10^{-4}$	$\frac{VD}{\nu}$ $\times 10^{3}$	$\frac{1}{A\Delta\theta}\frac{dQ}{dT}\frac{D}{K}$
	10.2	563	6.52	36.1	115.6
		721	7.36	46.2	130.6
		882	8.36	56.5	148.3
		967	9.23	61.9	163.7
		1095	10.69	70.1	189.7
	15.2	80	1.80	7.6	41.2
		242	3.02	23.1	79.8
		405	4.27	38.7	112.7
		563	5.66	53.7	149.6
		721	6.78	68.8	179.1
		882	7.73	84.2	204.2
		967	8.36	92.3	220.8
		1095	9.38	104.5	247.7
	20.1	80	1.56	12.7	62.8
		242	2.97	30.5	103.8
		405	3.92	51.1	136.8
		563	5.09	71.0	177.8
		721	5.81	91.0	202.8
		882	6.89	111.3	240.4
		967	7.40	122.3	258.2
		1095	8.55	138.2	298.5
Rectangular parallelopiped (Edge-on position)	5.4	80	2.51	2.7	23.5
		242	4.60	8.2	43.2
		405	5.90	13.7	55.3
		563	7.46	19.1	70.0
		721	9.00	24.4	84.3
		882	9.38	29.9	87.9
		967	10.47	32.8	98.2
		1095	11.07	37.1	103.8
	10.2	80	1.88	5.1	33.4
		242	3.34	15.5	59.3
		405	4.75	25.9	84.3
		563	5.70	36.1	101.2
		721	6.05	46.2	107.4
		882	7.26	56.5	128.8
		967	8.04	61.9	142.6
		1095	8.09	70.1	143.5
	15.2	80	1.59	7.6	38.9
		242	2.98	23.1	78.7
		405	3.79	38.7	100.2
		563	5.04	53.7	133.0
		721	5.25	68.8	138.7
		882	6.32	84.2	167.1
		967	6.82	92.3	180.3
		1095	7.46	104.5	197.2
	20.1	80	1.47	12.7	55.3
		242	2.55	30.5	89.1
		405	3.61	51.1	125.9
		563	3.97	71.0	138.7
		721	4.66	91.0	162.6
		882	5.50	111.3	191.9
		967	6.01	122.3	209.9
		1095	6.62	138.2	231.2

RESULTS AND DISCUSSION

The method of dimensional analysis gives for forced convection the following simplified equation in the case of gases:

$$\left(\frac{1}{A\Delta\theta}\frac{dQ}{dT}\frac{D}{K}\right)=B\left(\frac{VD}{\xi}\right)^n \qquad \dots (1)$$

where $\frac{dQ}{dT}$ is the rate of heat transmission;

A the area of the solid exposed;

D the characteristic dimension of the solid;

$\Delta\theta$ the excess of the surface temperature of the solid over that of the surrounding air stream;

V the velocity of the air stream;

K the heat conductivity of air;

ξ the ratio of viscosity of air to its density;

and B and n are constants to be determined from the experimental data.

A logarithmic plot of the quantities in the parentheses of the above equation against each other should give a straight line having a slope equal to n and an intercept on the axis equal to B. Figures 3 to 6 represent such logarithmic plots in all the cases of vessels tried for different air velocities; the experimental data for differently shaped vessels are plotted in different figures, thus each figure corresponds to a particular shape of the vessel. All these curves satisfy equation (1), the constants B and n having different values for differently shaped vessels. The values of these constants revealed by the various curves are recorded in Table II. It is seen from all these figures that the experimental data for vessels of different shapes and sizes exposed to various streams of air current lie reasonably close to the straight lines (thick continuous lines in the figures), the slopes and intercepts on the axes of which give the values of n and B respectively. It is evident that the values of B and n change with the shape of the vessel, but they remain practically constant when the size of the vessel or the velocity of the air stream is changed. The logarithmic plot of the experimental data for cylinders given by various investigators slightly concaves upwards, suggesting a variation of B and n with the Reynolds number —mainly with the fluid velocity and the characteristic dimension of the solid. A very slight variation of B and n in some of the present investigations which were carried out in a limited range of Reynolds number (from 2.5×10^3 to 1.5×10^5) is insignificant. The experimental data of the present author for this limited range are, therefore, represented almost within the limits of experimental error by straight lines.

Equation (1) gives (Kapadnis, 1953)

$$\Delta \log \left(\frac{1}{A\Delta\theta} \frac{dQ}{dT} \right) = n\Delta \log V \tag{2}$$

$$\Delta \log \left(\frac{1}{A\Delta\theta} \frac{dQ}{dT} \right) = (n-1)\Delta \log D \quad \ldots \tag{3}$$

Equation (2) represents the effect of changing the velocity of the fluid stream alone on the rate of heat transmission, all other factors remaining unchanged;

TABLE II

Constants of Nusselts equation for forced convection from solids of various shapes (according to different investigators)

Shape of the solid	Observer	Reynolds number From	Reynolds number To	B	n
Vertical cylinders	Kapadnis	2000	40000	0.56	0.517
		40000	130000	0.185	0.62
Horizontal cylinders	Jakob and Dow	40000	100000	0.590	0.5
		100000	1500000	0.028	0.8
	Kapadnis	2600	150000	0.505	0.516
		22000	150000	0.033	0.783
Spheres	Williams (correlated data)	20	150000	0.33	0.60
	Nottage and Boelter	1000	100000	0.70	0.52
	Kapadnis	2400	140000	0.78	0.517
Rectangular parallelopiped (Face-on position)	Hilpert	5000	100000	0.092	0.675
	Reiher	2500	8000	0.160	0.699
	Kapadnis	2700	140000	0.12	0.654
Rectangular parallelopiped (Edge-on position)	Hilpert	5000	100000	0.222	0.588
	Reihar	2500	7500	0.261	0.624
	Kapadnis	2700	140000	0.24	0.576

while equation (3) gives the effect of changing the characteristic dimension of the vessel alone on the rate of heat dissipation. The experimental values of the slope n found in all the cases of differently shaped vessels are less than unity. Therefore, equation (2) expressed qualitatively for all these cases means that the rate of heat dissipation does not increase quite so fast as the air velocity. The values of n substituted in equation (3) give negative slopes, each less than unity. Therefore, as the characteristic dimension of the vessel is reduced the rate of convective heat transfer increases, of course, not quite so fast. The experimental data for these two cases when represented graphically (rate of heat dissipation against air velocity) give a family of curves for vessels of different sizes but of the same shape. The general nature of the various families of curves corresponding to various shapes of the vessels is similar to that obtained for vertical cylinders in the previous investigations (figure 2, Kapadnis, 1953). In all these cases as the air velocity increases the curves rise steadily with a continuous decrease in slope, the curves for vessels bigger in size lie below those for the smaller ones, showing a good agreement with equations (2) and (3). The two equations can also be used as an approximate check to the observations of the new set in which either the velocity of the air stream or the characteristic dimension of the vessel is changed. For example, when the velocity of the air stream increases from 450 to 882 cm per second—2.178 fold—the expected increase in the rate of heat dissipation as per equation (2) is 1.663 fold for a specific case of a rectangular vessel of 10.2 cm diameter. The experimentally observed and calculated values for the rate of heat loss per unit area per unit excess of temperature are 8.356×10^{-4} and 8.513×10^{-4} units respectively, the latter being about 1.9 percent higher than the former. Similarly, when there is a twofold increase in the characteristic dimension of the rectangular vessel for a constant velocity of 721 cm per second, the decrease in the heat dissipation should according to equation (3), be 1.265 fold; the calculated and observed values for the rate of heat loss per unit area per unit excess of temperature in this case are 5.821×10^{-4} and 5.808×10^{-4} units, the former being less than one percent higher than the latter.

The horizontal cylinders used were ellipsoidal at one end. The observations were taken by keeping the ellipsoidal edge facing the stream of air. The experimental results can be satisfactorily represented by equation (1) with $n = 0.516$ and $B = 0.505$, the logarithmic plot of which has been shown by a thick continuous line in figure 3. The dotted line has been drawn as per Pohlhausen's (1921) theoretical equation (employed for air) which has been confirmed by the experiments of Jakob and Dow (1946) except for a slight difference in the value of B. The slight deviation of the experimental results of the present author from Pohlhausen's equation might be due to less homogeneous air stream used. A few observations (not recorded in the table), were also taken by keeping the plane blunt edge of the cylinder facing the air stream. Because of the setting of turbulence higher values were obtained in this case. The values arrived at have been represented in figure 3 by broken line giving $n = 0.783$ and $B = 0.033$ for Reynolds numbers from 2×10^4 to 1.6×10^5. The experiments of Jakob and Dow (1946) led to $n = 0.80$ and $B = 0.028$, but for Reynolds numbers higher than 10^5.

The experimental data for spheres satisfies equation (1) with $n = 0.517$ and $B = 0.78$. The data have been represented by thick continuous line in figure 4. The values obtained are a bit higher than those arrived at for the same limited range of Reynolds number by Nottage and Boelter (1940). Their values shown in the figure by dotted line give $n = 0.52$ and $B = 0.70$. The broken line represents the data correlated by Williams (1942) in the range of Reynolds number from 20 to 1.5×10^5. The values obtained by the present author for Reynolds numbers higher than 10^4 are in close agreement with the data correlated by Williams.

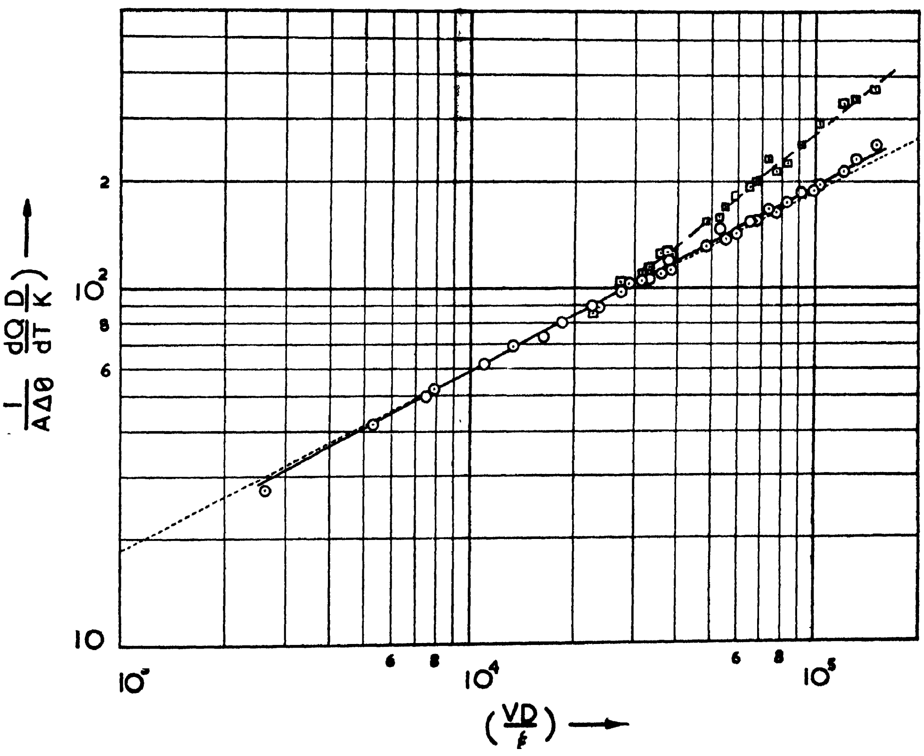
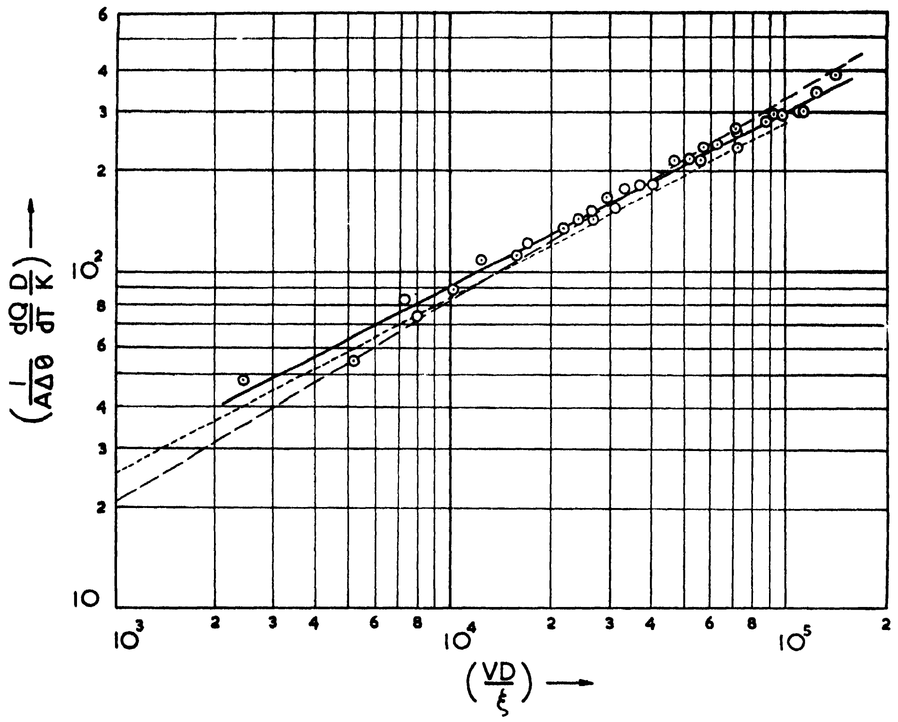


Fig. 3

The thick continuous lines in figures 5 and 6 represent the experimental data for rectangular vessels with face-on and edge-on positions respectively. The data lead to $n = 0.654$ and 0.576 and $B = 0.12$ and 0.24 respectively. The values are in agreement with those of Hilpert (1933), shown by broken lines in both the figures. The slight deviation might be due to the use of less homogeneous stream of air by the present author. Reiher's (1925) data are represented by dotted lines in both the cases. The values obtained by Reiher are much higher in comparison with those of Hilpert and of the present author, but they are limited to a small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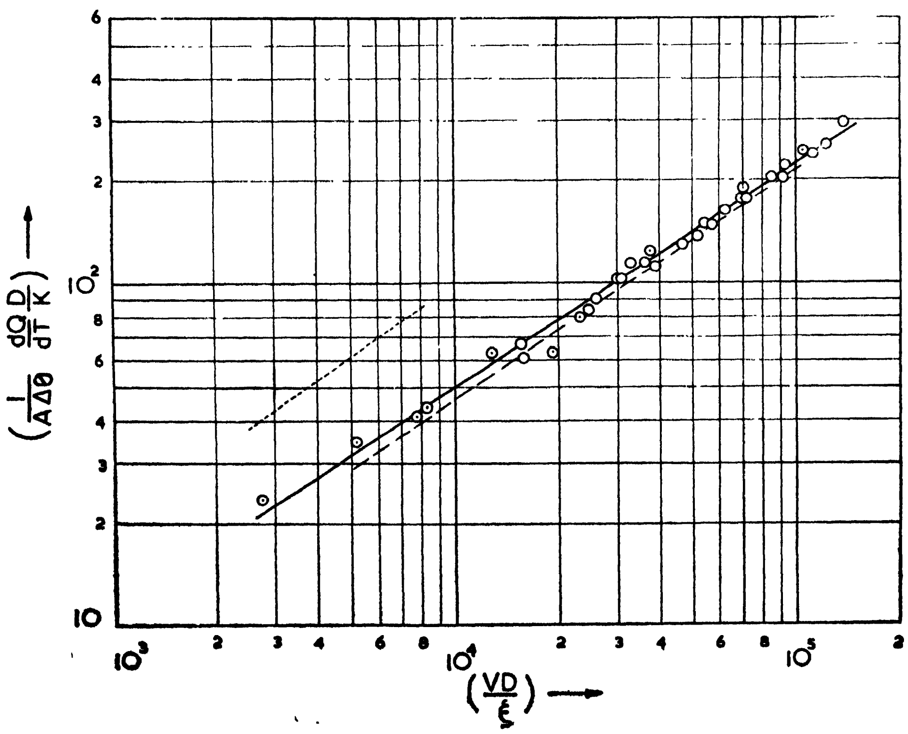
$\left(\frac{1}{A\Delta\theta}\frac{dQ}{dT}\frac{D}{K}\right)$
$\left(\frac{VD}{\xi}\right)$

Fig. 4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$\left(\frac{1}{A\Delta\theta}\frac{dQ}{dT}\frac{D}{K}\right)$
$\left(\frac{VD}{\xi}\right)$

Fig. 5

range of Reynolds number which was not so exhaustively studied in the present investigations.

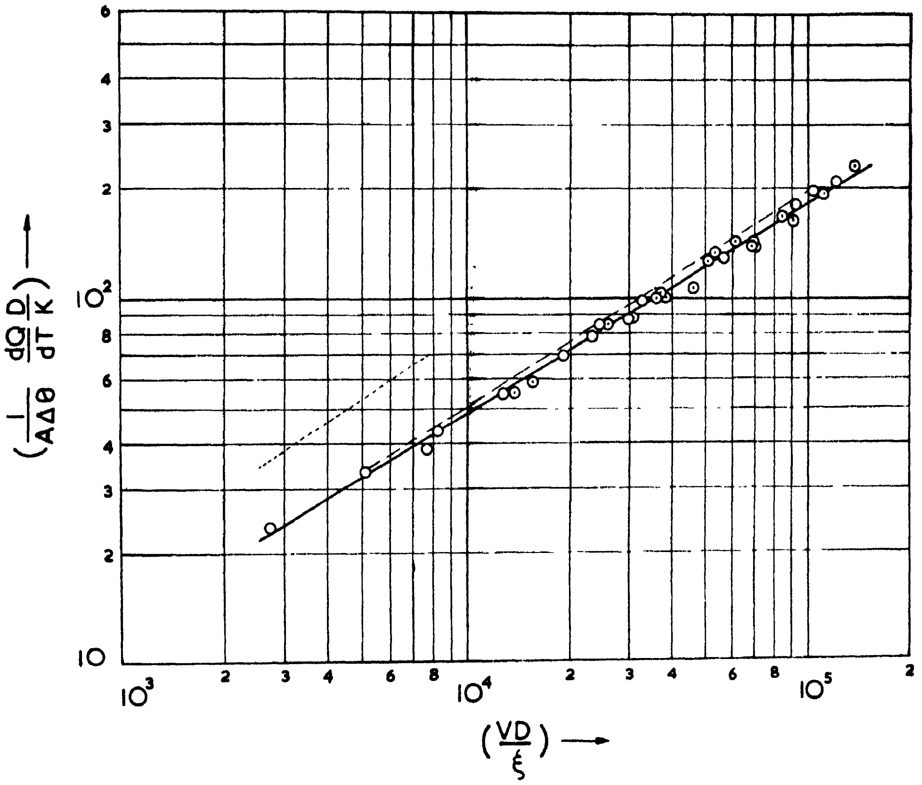

Fig. 6

A careful study of the author's (1953) experimental data on vertical cylinders shows a bend indicating a change in the character of the flow at Reynolds number 4×10^4 which escaped notice of the author at that time. The new values for n and B for Reynolds numbers higher than 4×10^4 are, therefore, 0.62 and 0.185 respectively, while the previously published values of n and B hold good for Reynolds numbers from 10^3 to 4×10^4. As the observations taken for Reynolds numbers less than 5×10^3 were very few in those investigations it is difficult to attribute the deviations of the values calculated from those observations to a less pronounced bend somewhere there. These newly calculated results have been also included in Table II.

For the sake of comparison at a glance the data, according to different investigators, discussed in the preceding paragraphs have been collected in Table II.

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