ANALYSIS OF G-M COUNTER IMPULSES BY THE METHOD OF DELAYED COINCIDENCES

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ABSTRACT. An interval analyzer circuit for studying the origin of spurious pulses in G-M counter tubes has been described. The method of delayed counts vs delay time has been employed in studying their time distribution. An expression for the probability that a real discharge creates an after discharge has been derived. The present investigations give :

(i) An exponential decrease of dead and recovery times with overvoltage,

(ii) The life of the petroleum ether filled counter and

(iii) The values of coefficient of secondary emission The coefficient of secondary emission seems to increase exponentially with overvoltage

It is concluded that the alteration of the optimum gas composition by deterioration with use is the main cause responsible for the high values of this coefficient. The possibility of negative ion formation by molecular dissociation is also suspected.

INTRODUCTION

The plateau of a Geiger-Müller counter is defined as the voltage range over which the counting rate at a constant intensity of irradiation is substantially independent of voltage No Geiger counter exhibits an ideally flat plateau characteristic for any considerable range about the threshold voltage. The increase in the counting rate with overvoltage is due to the increase in sensitivity but mainly to the occurrence of spurious counts. Since accurate measurements with Geiger counter require an essentially flat plateau, it was considered worthwhile to make a study of the spurious pulses.

Spurious counts result as a consequence of an earlier discharge and have been defined by Korff (1946) as those caused by any agency whatever other than the entity which it is desired to detect, or the normal contamination or cosmic ray background. When these spurious counts are few in number it is very difficult to distinguish them from genuine counts The only method by which spurious counts are recognised is by studying their time distribution.

Were there no spurious counts, we would expect the counter to deliver randomly distributed pulses in time, modified only by the absence of intervals shorter than the dead time of the counter. The spurious counts, if present, may arise from a number of causes [Wilkinson (1950), Spatz (1943), Sanborn and Brown (1948), Sanborn et al (1950)] In that case they will not be randomly distributed but their times of occurence will be related systematically to those of the pulses which caused them. Thus an excess of intervals of a particular range of the order of 10^{-4} sec shows the presence of spurious counts (Curran and Craggs, 1950). The delay of occurence of a spurious pulse from a genuine one depends on the mechanism responsible for its production.

The method of time-interval analysis has been employed by a number of workers [Medicus (1936); Driscoll et al (1940); Roberts (1941); Ward (1942-43); Curran and Rae (1947); Putman (1948); Willard and Montgomery (1950); Guimaraes and Sampaio (1949); Kupperian et al (1951); Picard and Rogozinski (1953)]. In the present study the principle is similar to that employed by Curran and Rae (1947), but the resolving time of the coincidence circuit has been rendered discrete by the introduction of two shaping circuits, one in each of the two channels. Kupperian et al (1951) employed a multistage delay line for a similar investigation. The delay line has the disadvantage of its bulk and presents unavoidable distortion. In the present investigation a univibrator was used since its delay could be varied over wide limits, (Elmore and Sands, 1949). The circuit is simpler than the previous ones, but is based on the same principle as that of Curran and Rae (1947). The values of dead time and recovery time agree in a general way with those reported by earlier workers for a counter of these dimensions.

The circuit is extremely useful in studying the origin of spurious pulses. An equation has been developed for calculating the value of the coefficient of secondary emission caused by the impact of positive ions on the cathode surface, and it is shown that the value of K increases exponentially with overvoltage. This would be expected since the increase in overvoltage increases the number of positive ions per impulse and the probability that these ositive ions will release secondary electrons at the walls becomes greater, (Stever, 1942). The results of the present paper show an exponential decrease of the dead and recovery times with overvoltage.

THE PULSE INTERVAL ANALYZER

The block diagram of the whole electronic circuitry is shown in figure 1.

The negative impulses from the counter are amplified by the pulse amplifier. These negative impulses from the amplifier feed simultaneously the shaping circuit I in the direct channel and the grid of the deley univibrator, which in its turn produces a square wave of variable duration. This gating pulse is differentiated into a negative and a positive part from its leading and trailing edges respectively; the positive part gives rise to a negative pulse at the plate of the inverter, which triggers the shaping circuit II in the delay channel. Thus the series of pulses from the shaping

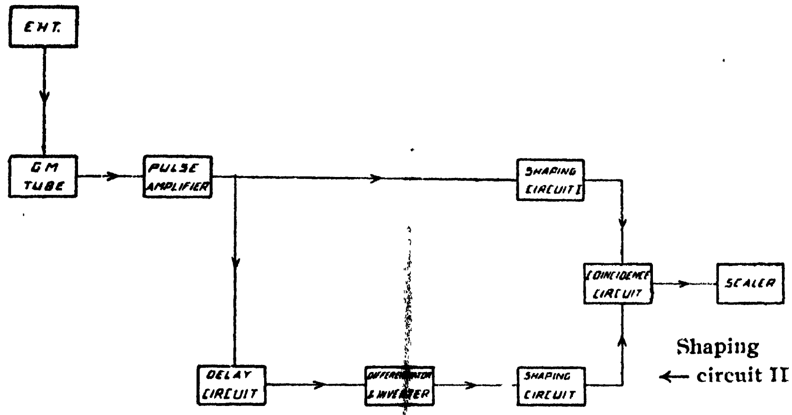

FIG 1. Block diagram of the circuit

circuit II has no pulses spaced more closely than the time occupied by the gating pulse, which obviously depends on the setting of the fine and coarse controls of the delay multivibrator. These two series of pulses, one direct and the other from the delay channel are fed to the diode coincidence circuit. By measuring the rate of coincidences between a direct count and the delayed count as a function of delay, the time correlations in the ionizing events in the tube were determined.

I. *Cathode Input Amplifier.* Cathode input amplifier employs two (7C7) tubes, the first functioning as a cathode follower (power amplifier) and the second as voltage amplifier, with cathode coupling in between. The chief function of the amplifier is to increase the sensitivity of the apparatus and consequently the precision of measurement. Its input sensitivity is 0.2 volts and it furnishes very sharp negative pulses, which are fed simultaneously to the delay univibrator and the shaping circuit I.

II. *Delay Circuit.* The delay circuit also consists of two 7C7 tubes, which are cross-coupled to each other by condensers. In the normal condition the first 7C7 is conducting whereas the transconductance of the second is zero, since its grid is at a voltage beyond cut off, i.e. −15v.

As soon as it receives a negative pulse from the trigger amplifier, the circuit goes to a quasistable state whose duration is governed by the discharging time of C_7 through R_{13} which can be varied conveniently from 50 microseconds to 800 microseconds.

The negative rectangular signals (figure 3) of present duration are taken from the plate of the second 7C7 tube. *Once the delay univibrator is flipped into operation, it is insensitive throughout the time occupied by the wave.*

III. *Differentiator and Inverter.* The negative rectangular signal from the delay circuit is differentiated into negative and positive parts from its leading and trailing edges respectively. The negative pulse does not trigger the selector, since its grid is at −15v far beyond cut-off, thus it is the positive part which gives a negative pulse at its plate. It is in this manner that a pulse whose delay is T_d is realized from the pulse from the counter.

Half of 6H6 has been used as coupling element, so that the operation of the next trigger circuit may not feed a signal back to the source.

IV. *Shaping Circuits I & II.* Two identical univibrators I and II are employed to put into form the direct and the delayed pulses respectively. The positive rectangular signals of duration t_1 and t_2 depend on the time constants R_7C_4 and $R_{21}C_{11}$. t_1 and t_2 can take all the values from 2 microseconds to 25 microseconds respectively, thus the resolving time of the succeeding coincidence circuit can be varied from 5 microseconds to 50 microseconds.

In order to render this time accurately determinate, it was thought necessary to incorporate these pulse shaping circuits both in the direct and delay channels ; and it is an advantage over the previous circuits.

V. *Coincidence Circuit.* Two diodes are used to make a circuit which produces an output pulse only if its two input terminals are simultaneously excited by input pulses. The schematic diagram of the whole assembly is shown in figure 2.

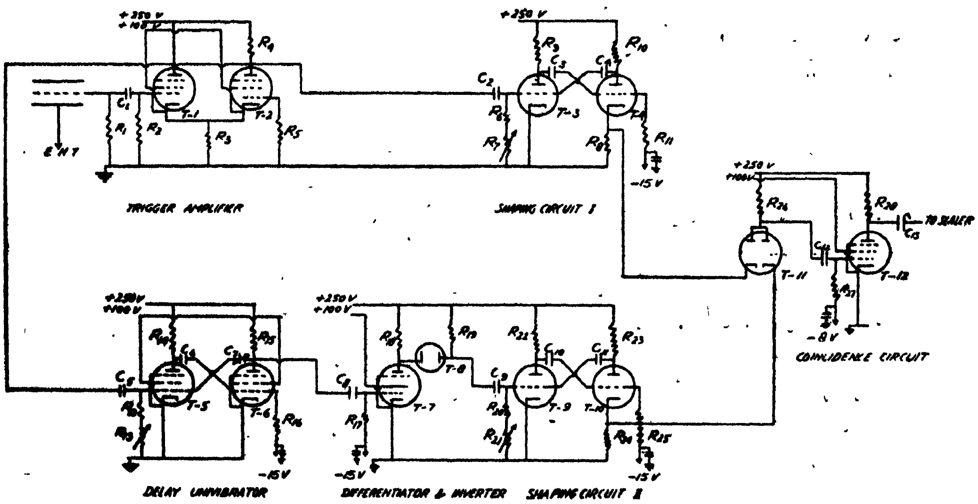

FIG. 2. Circuit of delayed coincidences. The values of components are listed below :

$R_1 = 1M\Omega$	$C_1 = C_4 = C_5 = C_8 = C_9 = C_{11}$	$T_1 = T-2 = T-5 = T-6$
$R_2 = R_4 = R_{14} = R_{18} = R_{16}$	$= 50pf$	$= T-7 = 7C7$
$= R_{26} = R_{27} = R_{28} = 100K$	$C_2 = 10pf$	$T-3 = T-4 = T-9 = T-10$
$R_5 = R_9 = R_{17} = R_{22} = R_{11}$	$C_3 = C_6 = C_{10} = 5000pf$	$= \frac{1}{2}6SL7$
$= R_{25} = 50K$	$C_7 = 500 \rightarrow 1000pf$, variable	$T-8 = \frac{1}{2}6H6$
$R_7 = R_{21} = 100K$, variable	$C_{12} = 2000pf$	$T-11 = 6H6$
$R_3 = 0.33K$	$C_{13} = 1000pf$	$T-12 = 6AC7$
$R_6 = R_{10} = R_{20}$, $R_{23} = 5K$		
$R_{12} = 10K$		
$R_8 = R_{24} = 5K$		
$R_{15} = R_{19} = 47K$		

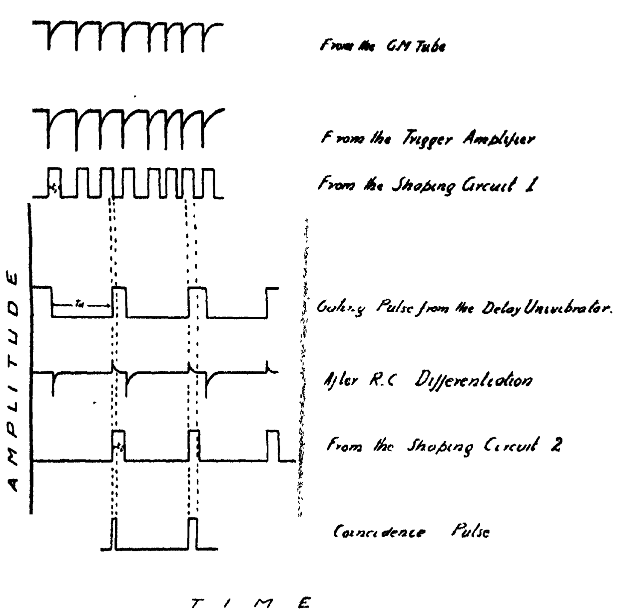

FIG. 3. Wave-shapes at different stages (not to scale)

3. THEORY OF THE METHOD OF RETARDED COINCIDENCES

Let n_r be the real number of particles entering the counter per sec,

n_1', the average number of particles recorded per second by the counter,

n_1, the average number of particles at the output of the amplifier per sec,

τ, the dead time of the counter,

τ', the recovery time of the conventional amplifier. It is the minimum time interval that must elapse so that the amplifier amplifies the next pulse after the discharge of the G-M counter.

Then the loss of counting rate, incurred by the counter due to its finite dead time is $n_1' n_r \tau$.

Therefore $$n_r - n_1' = n_1' n_r \tau$$

so that $$n_r = n_1'(1 + n_r \tau)$$

Therefore the efficiency of the counter

$$\frac{n_1'}{n_r} = \frac{1}{1 + n_r \tau} \tag{1}$$

Now consider an interval of time τ' following an amplifier pulse at time zero. There can be no additional pulse from the counter within its dead time τ. The probability that none occurs in the interval τ to τ' can be computed as follows:

Let the probability that no pulse occurs in time $\tau'-\tau=\tau''$ be $P(\tau'')$. The probability that the amplifier will receive a pulse in additional time $\partial\tau''$ is $n_r\partial\tau''$; therefore the probability that no pulse is received in time $\partial\tau''$ will be $(1-n_r\partial\tau'')$. The probability of not receiving a pulse in $\tau''+\partial\tau''$ is $P(\tau''+\partial\tau'')$.

$$P(\tau''+\partial\tau'')=P(\tau'')\{1-n_r\partial\tau''\}$$

Applying Taylor's theorem $\quad P'(\tau'')=-P(\tau'')n_r$

therefore $\quad P(\tau'')=Ce^{-n_r\tau''}.$

But when $\quad \tau''=0,\ P(\tau'')=1$

Therefore $\quad C=1$

Hence $\quad P(\tau'')=e^{-n_r(\tau'-\tau)}$

This then is the probability that the amplifier receives a next pulse after τ'. Hence the efficiency of the amplifier is

$$\frac{n_1}{n'}=e^{-n_r(\tau'-\tau)} \qquad \ldots \quad (2)$$

Therefore
$$n_1=\frac{n_r}{1+n_r}e^{-n_r(\tau'-\tau)} \qquad \ldots \quad (2a)$$

For small counting rates, $n_r(\tau'-\tau)\ll 1$, therefore, expanding $e^{-n_r(\tau'-\tau)}$ in power series and neglecting the higher terms

$$n_1=\frac{n_r}{1+n_r\tau}\{1+n_r\tau-n_r\tau'\}$$

$$n_r\left\{1-\frac{n_r\tau}{1+n_r\tau}\right\} \qquad \ldots \quad (3)$$

But from (2a)

$$\frac{n_r}{1+n_r\tau}=n_1e^{n_r(\tau'-\tau)}$$

Substituting it in eq. 3, we obtain

$$n_1=n_r\{1-\tau'n_1e^{n_r(\tau'-\tau)}\}$$

and since $\quad n_r(\tau'-\tau)\ll 1$

$$n_1=n_r\left\{1-\tau'n_1\left[1+n_r(\tau'-\tau)+\frac{n_r^2(\tau'-\tau)^2}{2!}+\ldots\right]\right\}$$

$$=n_r\{1-n_1\tau'-n_rn_1\tau'(\tau'-\tau)\ldots\}$$

$$=n_r\{1-n_1\tau'\} \qquad \ldots \quad (4)$$

Furthermore, these n_1 pulses are fed to the delay univibrator whose recovery time is T_d, which is the duration of the square wave. Strictly speaking, the recovery time of the delay circuit is slightly longer than T_d since C_7 is to get recharged through R_{15} with the consequence that the trailing edge of the pulse is not very sharp (figure 2).

If n_2 impulses per sec are given as output by the univibrator, then the efficiency

$$\frac{n_2}{n_1} = e^{-n_1(T_d - \tau')}$$

Therefore
$$n_2 = n_r(1 - n_1\tau')e^{-n_1(T_d - \tau')}$$
$$= n_r(1 - n_1\tau')\left\{1 - n_1(T_d - \tau') + \frac{n_1^2(T_d - \tau')^2}{2!} - \right\}$$

Substituting from eq. (4) we get

$$n_2 = n_1\{1 - n_1(T_d - \tau')\} \qquad \dots \quad (5)$$

If t_1 is the duration of the direct pulse after it has been put into form by the shaping circuit I and t_2 the duration of the retarded pulse from shaping circuit II, then the coincidence rate n_c is given by

$$n_c = n_1 n_2 (t_1 + t_2)$$
$$= n_1^2\{1 - n_1(T_d - \tau')\} \qquad \dots \quad (6)$$

The two series of impulses, direct and retarded, are fed into the coincidence circuit, whose resolving time is $(t_1 + t_2)$. Since no pulse can occur during the dead time of the counter, no coincidence will be observed for time T_d less than τ'. Such a circuit is ideal for the rapid determination of τ'.

If any parasitic effect does not disturb the quasistatic distribution of the two series of pulses, the average frequency of coincidence n_c reduces to

$$n_c = n_1^2(t_1 + t_2)$$

which is the frequency of the coincidences for $T_d > \tau'$.

From the above equation n_c should have a constant value if there were no spurious effects present. The circuit detects the presence of the spurious pulses, if there is an excess of the intervals of short duration of the order of 10^{-4} sec. The circuit can be used for :

(*i*) measuring the dead-time and recovery time of the counter.

(*ii*) analysing its variation with overvoltage.

(*iii*) detecting spurious pulses if present, and indicating their origin, and

(*iv*) detecting and measuring the half life of a radio element of large disintegration constant, half life 50×10^{-6} sec. to 10^{-1} sec.

EVALUATION OF K, THE COEFFICIENT OF SECONDARY EMISSION

The after-discharges are systematically related in time to the impulses which generate them. The interval at which spuriousness occurs is roughly equal to the recovery time T of the counter in case of secondary emission caused by the impact of positive ions on the cathode, and is equal to $2T$ in case of negative ion formation at the cathode as in case of carbon dioxide-filled counters.

Let r be the rate of genuine discharges, n_1, the observed rate and K_1 the probablity that any discharge creates an after-discharge.

Then

$$n_1 = r + Kr + K^2r + K^3r + \dots$$

$$\frac{r}{1-K} \qquad (7)$$

If n_p is the frequency of parasitic impulses [Curran and Rae (1947); Picard and Rogoziuski (1953)],

$$n_p = rK + rK^2 + rK^3 + - \qquad \dots \quad (8)$$

$$= \frac{rK}{1-K}$$

Let us proceed to calculate the probability of the formation of the first peak, supposing there is a sequence of several parasitic impulses separated from each other by an interval of time T.

Actually the recovery time of the delay univibrator is more than T_d the imposed delay, a direct pulse giving a coincidence with a delayed pulse cannot itself initiate a delayed pulse. Consequently the coincidence will be between the alternate parasitic impulses.

For $T_d = T$, the probability of the formation of the first peak

$$K_1 = K + K^3 + K^5 + \dots$$

$$= \frac{K}{1-K^2} \qquad \dots \quad (9)$$

But, of the n_1 counts r are random, thus the frequency of real coincidences

$$r_c = r^2(t_1 + t_2) \qquad \dots \quad (10)$$

The area A_1 under the first peak represents the contribution of the parasitic impulses whose probability of formation is K_1. Normalizing this peak

$$A_1 = rK_1(t_1 + t_2) \qquad \dots \quad (11)$$

Thus $$\frac{A_1}{r_c} = \frac{K}{r(1-K^2)}$$

But $$r = n_1(1-K) \text{ vide eq. (7)}$$

Therefore

$$\frac{A_1}{r_c} = \frac{K}{n_1(1-K^2-K+K^3)}$$

which reduces to $$A_1 = r_c\frac{K}{n_1} \quad (\text{where } k \ll 4)$$

Thus $$K = \frac{A_1}{r_c} n_1 \qquad \dots \quad (12)$$

Putting the values of n_1, A_1 and r_c, which are read out from the experimental curve, the coefficient of secondary emission and of any other mechanism which gives spurious count at $T_d = T$ can be evaluated.

OBSERVATIONS

1. *Determination of Dead and Recovery Times.* Figure 4 represents the plateau characteristic of the counter having a diameter of 1.5 cm and 13 cm long. The central wire had .003 inch diameter. The copper cathode was oxidized. It was filled with spectroscopically pure argon to a pressure of 9 cm of mercury, mixed with 1.1 cm of petroleum ether vapour (B. P. 40–60°C). It had already furnished 5×10^5 counts in its previous run and showed a slope of 7.04% per 100 volts.

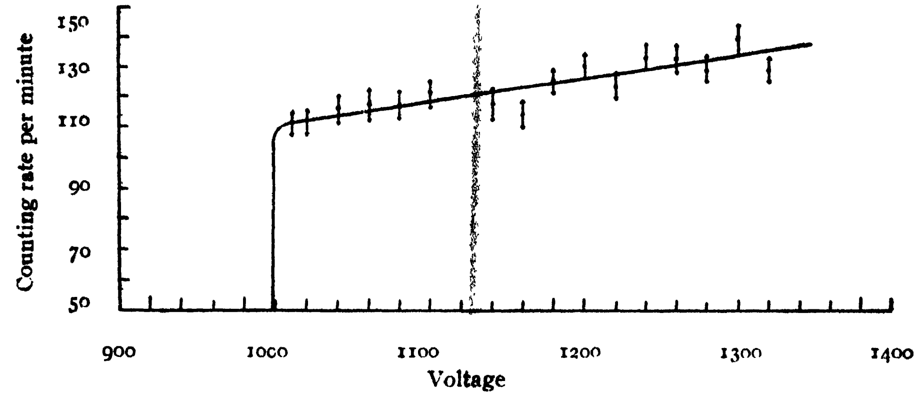

FIG. 4. Plateau characteristic of the counter investigated

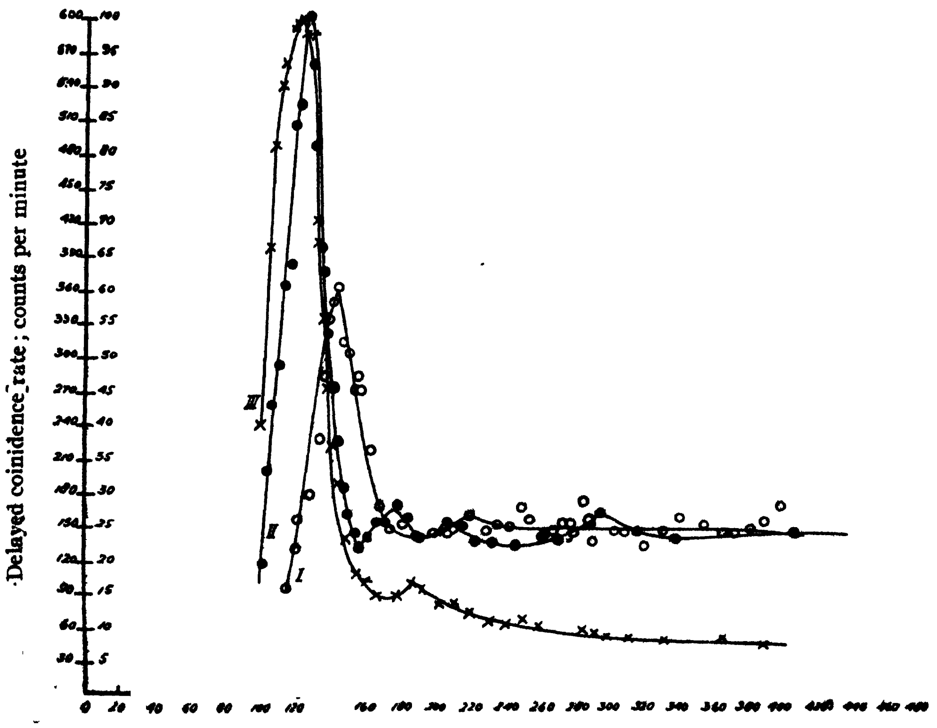

FIG. 5a Curves I, II and III represent the coincidence *vs* delay time for over-voltages of 30, 150 and 230 volts respectively. In each case the resolving time is $t_1+t_2=40.50$ microseconds. The counting rates used for curves I, II and III were 9478, 9940 and 13, 120 per minute respectively

In figure 5 curves (I), (II), (III) represent the delayed coincidence rate (counts per minute) *vs* delay time for three different overvoltages of 30, 150, and 230 volts respectively. Counting rates of the order of 10,000 counts per minute were employed so that the spurious effects, if any, may become pronounced and thus afford a greater accuracy of measurement. The coincidence rate was recorded with a statistical accuracy of 4%.

In a counter with effective dead-time τ', no interval less than τ' is found and there will not be any coincidence. The dead time τ' can be easily measured. The observed values of dead and recovery times are of the order of previously reported values, (Korff, 1946) for counters of these dimensions. The recovery time, that is, the time required for the transit of the positive ions from the central wire to the cathode can be read out under the peak of the curve, since it is at this value of $T_d = T$, that the maximum effects of the secondary emission from the cathode are expected.

Curves of figure 6 give plots of the variation of dead and recovery times with overvoltage. An exponential decrease with overvoltage is indicated.

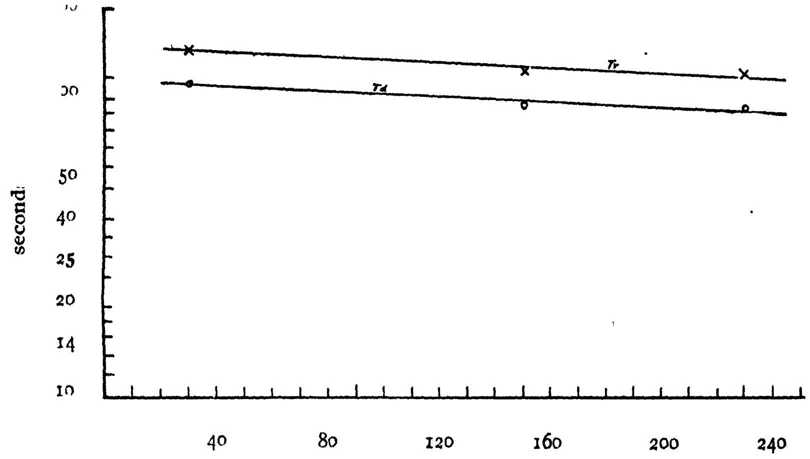

FIG. 6. Variation of dead and recovery times with over voltage. The curves may be represented by $T_d = 117\, e^{-.00107(v-v_s)}$ and $T_r = 152 e^{-.00107(v-v_s)}$

2. *Investigation of Spuriousness.* Curves (I), (II), (III) of figure 5 represent coincidence rate *vs* delay time. All the three curves show a pronounced maximum which occurs at the time of transit of positive ions. The peaks show a shift towards the origin at higher overvoltages. This shift is due to the increased mobility of the positive ions with the increase of overvoltages and consequently less time is required to reach the cathode.

The values of K, the coefficient of secondary emission, as calculated by employing equation (12), give pretty high values for it. This would mean that G-M tube will give more and more spurious counts with higher and higher overvoltages. As a result a correction must be made to the counting rate while carrying out precision measurements.

The values of K seem to increase exponentially with overvoltage (see figure 7). Curves (I) and (II) show that the spurious counts occur at the time of recovery whereas, curve (III) shows that some of them also occur at a later time, $T_d = 192$ microseconds. There is, however, an indication of second maxima in curve (II) as well. The counter under investigation was nearing its end, since it had furnished about 4×10^7 counts by the end of the present investigation. At the end, it was found to have the following characteristics :

(i) that threshold had gone up by 180 volts.

(ii) the plateau length diminished to about 40 volts only.

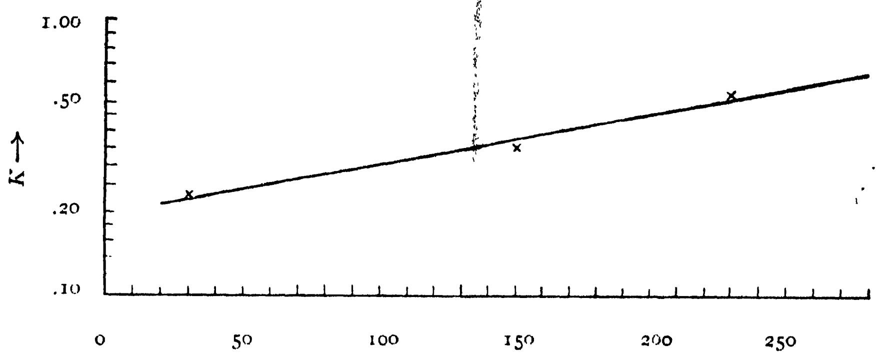

FIG. 7 Variation of coefficient of secondary emission with overvoltage. The curve may be represented by $K = 198\, e^{.00428(v - v_t)}$

DISCUSSION

The exponential decrease of dead and recovery times with overvoltage fits in with "Stevers" analysis of the motion of the positive ion sheath. As the overtension increases, the drift velocity of the sheath must increase and consequently the times required to travel to the critical radius and the cylinder must decrease. It is exponential, because the distributed capacity of the counter is to get recharged through the leakage resistance to the threshold from the starting voltage. If the final voltage to which it is to get charged is higher, as is the case at higher overvoltage, the time to reach the threshold falls exponentially.

Since the average life of a petroleum ether G-M tube is between 10^7 and 10^8 counts, it is concluded that the deterioration is due to the combination of two processes :

(i) Alteration in the optimum gas composition resulting from decomposition of petroleum ether into hydrogen, which is non-self-quenching and saturated and unsaturated hydrocarbons [Farmer and Brown (1948) ; Yaddanapalli (1942)].

(ii) The deposition on the cathode and anode of dielectric polymers formed from unsaturated hydrocarbons.

It is also evident from figure 5 that the spurious counts occur at the recovery time of the counter (Montgomery and Montgomery, 1940).

The high value of K may be arising from a number of causes. The most important cause seems to be that the insufficient amount of the self-quenching constituent is not acting as a complete trap for the vehicular gas positive ions. Only very few charge transfer collisions occur, with the consequence that some vehicular gas ions reach the cathode. Since the quenching resistance employed is only 1 meg, the secondary electrons resulting from the impact of positive ions on the cathode will re-ignite discharges and thus cause spurious counts.

In the case figure 5 curve (III), the presence of a second peak of spurious counts at $T_a = 192$ microseconds which is almost twice the recovery time is evident. The mechanism for the production of this peak may be :

(i) the cathodic emission and

(ii) the formation of negative ions as dissociation fragments at the wall or in the gas discharge and if formed will reach the vicinity of the wire after time $2T$.

The petroleum ether had not been subjected to its ionization and dissociation study by electron impacts, but in the case of methane, which is the parent hydrocarbon of petroleum ether, the negative ions had been detected (Smith, 1937).

The presence of spurious discharges cannot be explained by assuming the survival by a few metastable ions, of the dead-time; and ionization by positive ions, since both these hypotheses are unlikely and improbable for the observed effect.

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