EFFECT OF BACKGROUND INTENSITY ON RESOLUTION

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ABSTRACT The paper discusses the effect of background intensity on the resolving power Tables, illustrated by graphs, have been given for the variation of resolving power with background intensity in case of Fabry-Perot etalon, prism, grating and reflecting echelon, and when the instrumental width is negligible

INTRODUCTION

This paper discusses the effect of background intensity on the resolving power of spectroscopic instruments, on the Rayleigh's criterion of resolution of spectral lines. The two cases, viz., when the instrumental width is negligible as compared to the Doppler line-width and vice versa have been distinguished.

RESOLVING POWER WHEN INSTRUMENTAL WIDTH IS NEGLIGIBLE

The intensity distribution of a spectral line of wave number ν_0 due to Doppler effect is given by

$$I' = I_0 e^{-\beta(\nu-\nu_0)^2}$$

where $\beta = \dfrac{\mu c^2}{2RT\nu_0^2}$, μ being the mass of radiant atoms.

The intensity distribution of another spectral line of wave number $\nu_0 + \Delta\nu$ and same intensity is

$$I'' = I_0 e^{-\beta(\nu-\nu_0-\Delta\nu)^2}$$

if $\Delta\nu$ is small. (β same for both lines).

Putting $\sqrt{\beta}\,(\nu-\nu_0)=x$ and $\sqrt{\beta}\,\Delta\nu=a$, the resultant intensity pattern, in the presence of a background intensity kI_0 is given by

$$I = kI_0 + I_0 e^{-x^2} + I_0 e^{-(x-a)^2} \qquad \dots \quad (1)$$

Oldenburg (1922) gives the position of maximum as

$$x_{max} = \frac{a}{e^{a^2} + 1 - 2a^2}$$

In this section we will discuss cases with $a^2 > 3.524$ and we may assume, as a good approximation, $x_{max} \approx 0$.

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Hence the intensity maxima and minimum ($x=a/2$) are given by

$$I_{max}/I_0=1+k+e^{-a^2} \quad \cdots \quad (2)$$

and

$$I_{min}/I_0=k+2e^{-a^2/4} \quad \cdots \quad (3)$$

Rayleigh's criterion for resolution states

$$I_{min}/I_{max}=0.8$$

Hence we get

$$k=4(1+e^{-a^2})-10e^{-a^2/4} \quad \cdots \quad (4)$$

We find that $k=0$ when $a^2=3.524$. If the resolving power be denoted by R and its value, when $k=0$ by R_0 we have

$$R/R_0=\sqrt{(3.524/a^2)} \quad \cdots \quad (5)$$

Table I gives R/R_0 for a few values of k.

TABLE I

Variation of R/R_0 with k when instrumental width is negligible.

a^2	3.524	4.000	4.400	4.800	5.600	6.400	7.600	9.200	12.000	∞
R/R_0	1.00	0.939	0.895	0.857	0.793	0.742	0.671	0.619	0.532	0.00
k	0.00	0.39	0.72	1.02	1.55	1.99	2.50	3.00	3.50	4.00

RESOLVING POWER OF GRATING, REFLECTING ECHELON AND PRISM

The intensity of a spectral line diffracted by a grating or reflecting echelon is given by

$$I'=B\frac{\sin^2 N\beta}{\sin^2 \beta}$$

where N is the number of lines of the grating or the number of steps in the reflecting echelon and 2β the phase difference between two adjacent beams. If the intensity maximum be denoted by I_0 we have

$$I'/I_0=\frac{B}{BN^2}\frac{\sin^2 N\beta}{\sin^2 \beta}=\frac{\sin^2 x}{x^2} \quad \cdots \quad (6)$$

where $x=N\beta$, and β is small.

The above expression also represents the intensity distribution in a prism if $x=\pi l \sin \theta/\lambda$.

The intensity distribution of another line of the same intensity and an angular separation corresponding to $\Delta x=a$ is represented by

$$I''/I_0=\frac{\sin^2 (x-a)}{(x-a)^2} \quad \cdots \quad (7)$$

The resultant intensity distribution of the two lines when the background intensity is kI_0 is given by

$$I/I_0 = k + \frac{\sin^2 x}{x^2} + \frac{\sin^2 (x-a)}{(x-a)^2} \qquad \ldots \quad (8)$$

The intensity maxima ($x \approx 0$ or a) and central minimum ($x = a/2$) are given by

$$I_{max} = 1 + k + \frac{\sin^2 a}{a^2} \qquad \ldots \quad (9)$$

and

$$I_{min} = k + 2\frac{\sin^2 a/2}{(a/2)^2} \qquad \ldots \quad (10)$$

Applying Rayleigh's criterion for resolution we get

$$k = 4\left(1 + \frac{\sin^2 a}{a^2}\right) - 10\frac{\sin^2 (a/2)}{(a/2)^2} \qquad \ldots \quad (11)$$

when $a = 1.006\pi$, $k = 0$. Hence if R_0 denotes the resolving power for $k = 0$ we have

$$R/R_0 = 1.006\pi/a \qquad \ldots \quad (12)$$

Table II gives R/R_0 for various values of k.

TABLE II

Variation of R/R_0 with k for grating, reflecting echelon and prism.

a	1.006 π	1.10 π	1.20 π	1.30 π	1.40 π	1.50 π	1.60 π	2.00 π
k	0.00	0.77	1.55	2.25	2.83	3.28	3.60	4.00
R/R_0	1.00	0.91	0.84	0.77	0.72	0.66	0.63	0.50

For $k > 4$ we get imaginary values for a showing thereby that no resolution is possible. Physically it means that even single lines are not visible because $I_{min}/I_{max} > 0.8$.

RESOLVING POWER OF FABRY-PEROT ETALON

The intensity of a spectral line in the order $n_0 + n$, where n is small and n_0 an integer, is given in the case of Fabry-Perot etalon by

$$I' = I_0/\{1 + F \sin^2 \pi(n_0 + n)\} = I_0/(1 + x^2)$$

where $x = \pi n F^{1/2}$, F being the coefficient of fineness.

The intensity of another spectral line of equal intensity separated by an order Δn is given by

$$I'' = I_0/\{1 + F \sin^2 \pi(n_0 + n - \Delta n)\} = I_0/\{1 + (x-a)^2\}$$

where $a = \pi F^{1/2} \Delta n$.

The resultant intensity pattern is

$$I/I_0 = k + [1/(1+x^2)] + [1/\{1+(x-a)^2\}] \quad \dots (13)$$

The maximum ($x \approx o$ or a) and minimum ($x = a/2$) of the resultant intensity pattern, when the background intensity is kI_0, are given by

$$I_{min}/I_0 = k + 2/\{1 + a^2/4\} \quad \dots (14)$$

and

$$I_{max}/I_0 = k + 1 + 1/(1 + a^2) \quad \dots (15)$$

Rayleigh's criterion for resolution requires

$$I_{min} = 0.8 I_{max}$$

or

$$(4-k)a^4 - (5k+16)a^2 - (4k+8) = 0$$

or

$$a^2 = \frac{(5k+16) + \sqrt{(5k+16)^2 + 16(4-k)(k+2)}}{2(4-k)}$$

since a is real.

The resolving power of the etalon is given by

$$R = \lambda/d\lambda = n_0/\Delta n = \frac{\pi n_0 F^{1/2}\{2(4-k)\}^{1/2}}{\left[(5k+16) + \sqrt{(5k+16)^2 + 16(4-k)(k+2)}\right]^{\frac{1}{2}}} \quad \dots (16)$$

The resolving power with $k=0$ (Meissner, 1941) is

$$R_0 = 1.49 n_0 F^{1/2}$$

Hence $R/R_0 = 2.109\left[2(4-k)/\{(5k+16) + \sqrt{(5k+16)^2 + 16(4-k)(k+2)}\}\right]^{\frac{1}{2}}$ ... (17)

Table III gives R/R_0 for some values of k.

TABLE III

Variation of R/R_0 with k for Fabry-Perot etalon.

k	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5
R/R_0	1.00	0.88	0.77	0.67	0.57	0.47	0.38	0.26

For higher values of k which give high values for a, eqn. (13) does not hold because sin a differs appreciably from a for large values of a.

DISCUSSION

The variation of R/R_0 with k, in the three cases discussed earlier is represented graphically in figure 1. It can be seen that the background intensity has an important bearing on the choice of an instrument for a particular investigation. For example, a reflecting echelon, having half the resolving power as a Febry Perot etalon for $k=0$ is superior to it for $k=3.5$.

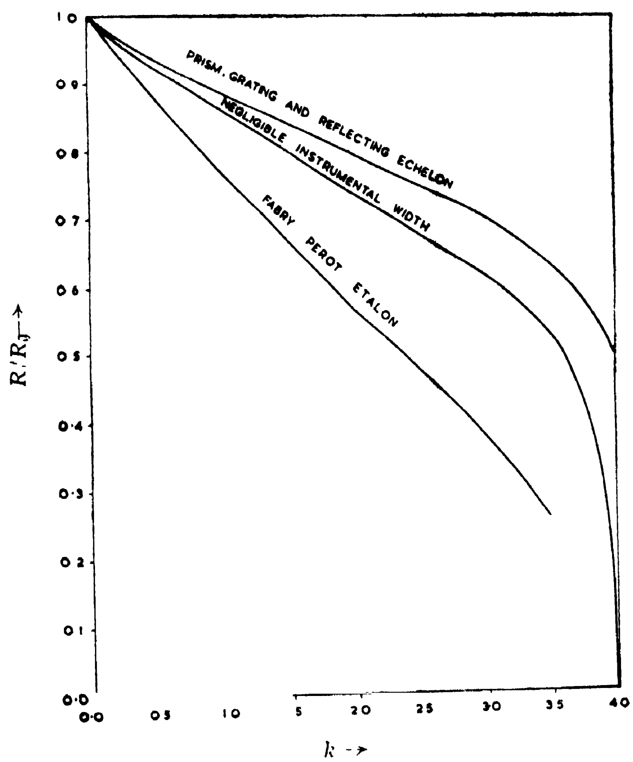

FIG. 1

Variation of R'/R_I with k

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