STUDIES ON THE SPORADIC E-LAYER*

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ABSTRACT. Attempt has been made to investigate the structure and properties of the sporadic E_s region of the ionosphere from a study of the fading of the echoes from this region. Statistical analysis shows that the echo consists of two components, one superposed on the other One of the components is due to random scattering and the other to a steady reflection. This shows that the E_s region consists of a regularly reflecting region and a region of ion-clouds A method for estimating the average number density of electrons in the clouds has been developed. It is found that the average number density is below that required for totally reflecting the exploring waves. Expressions for the variation of reflection coefficient with the variation of exploring frequency has been developed for a thin and for a semi-infinite layer (The electron distribution at the boundary is assumed to have a linear profile). The expression shows that it is possible from the observation of reflection-characteristics to distinguish between the two cases. Hence with the help of this expression one can investigate the structure of that part of the E_s region which gives rise to the steady echo.

1. INTRODUCTION

The irregularly occurring region of ionisation, known as the sporadic E-layer (E_s), presents many perplexing problems regarding its origin and structure. As is well-known, the E_s-echoes are characterised by the facts that they are obtained on frequencies much higher than the normal Elayer critical frequency (from about the same height as that of the normal Elayer) and that the 'reflections' are only partial. These phenomena are explained by Best, Farmer and Ratcliffe (1938) on the hypothesis that sporadic E region consists of ion-clouds and that the observed echoes are due to scattering, rather than reflection from these clouds. This view has been experimentally corroborated by Eckersely and Farmer (1945). Booker (1950) has recently given a theory of scattering from such clouds. (A similar theory applied to the case of propagation of microwaves through the troposphere shows that the observed variation of signal intensity with distance agrees with that calculated from the theory.) However, Appleton, Naismith and Ingram (1939) explain the E_s echoes as due to partial reflection from extremely thin layer of ionisation embedded within the Elayer. One of the ways of investigating the E_s region, *i.e.* determining how far it consists of irregularly distributed clouds and how far it has a thin-layer structure, is to make a statistical study of the fading of the E_s echoes. This is because, as pointed

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out by McNicol (1949), the probability distribution of the amplitude of the echoes, if they are only due to scattering, will be different from that if they have a steady component (due to partial reflection from a thin layer) superposed on them. Such studies have been made by the author and the results of the analysis show that even when there are echoes only from the sporadic E region, there is a strong component which can only be ascribed to regular reflection from a layer. A quasi-periodic variation of the ionospheric fading characteristics has also been noticed. This suggests the existence of a steady drift. Attempt has also been made, by extending Booker's formula, to determine the ionic density of the E_s clouds from the variation of scattered intensity with frequency. It is found that the observed ionic density of the clouds bear no relation to the normal E layer critical frequency.

2. EXPERIMENTAL SET-UP AND PROCEDURE

The apparatus used was the 500-watt ionospheric sounding equipment used for routine observations installed at the University College of Science, Calcutta. The intensity of the E_s echo was delineated on the C.R.O. screen and its amplitude was noted visually every five seconds to the nearest half-centimetre of length. This gave a measure of the fluctuation of the reflection coefficient on an arbitrary scale. Observations were taken continuously for five minutes (giving sixty values of the amplitude) on each of a number of sounding frequencies. Care was taken to keep the aerial current constant during the observations. As the radiation pattern of the inverted-L aerial which was used for transmission would not change much within the range of frequencies employed (0.5 to 1.0 Mc/sec.), the constancy of aerial current ensured an appreciably constant value of the radiated power. Observations extending over periods of 7 to 20 minutes were also taken to detect any variation of the scattering characteristics.

3. THE SCATTERED AND STEADY COMPONENTS

(*a*) *The probability distribution curves.*—If the returned echo consists of a steady component and a scattered component then the probability distribution of the amplitude of the echo follows the curve given by Rice (1945).

$$\psi(r) = \frac{2r}{R^2} I_0 \left(\frac{2rB}{R^2}\right) e^{-(B^2+r^2)/R^2} \quad \ldots \quad (1)$$

where I_0 is the Bessel function of zero order with imaginary argument, B is the amplitude of the steady component, and R the amplitude of the scattered component, *i.e.*, root mean square amplitude due to a large number of waves in random phase. McNicol has pointed out that if $B\sqrt{2}/R < 1$, the Rice distribution approximates to the Rayleigh distribution

$$\psi(r) = \frac{2r}{R^2} e^{-r^2/R^2} \quad \ldots \quad (2)$$

and, if $B\sqrt{2}/R > 3$, it approximates to the Gaussian distribution

$$\psi(r) = \frac{1}{\sqrt{\pi R^2}} e^{-(r-r_m)^2/R^2} \quad (3)$$

where

$$r_m^2 = B^2 + \frac{R^2}{2}. \quad \text{(see figure 1)}$$

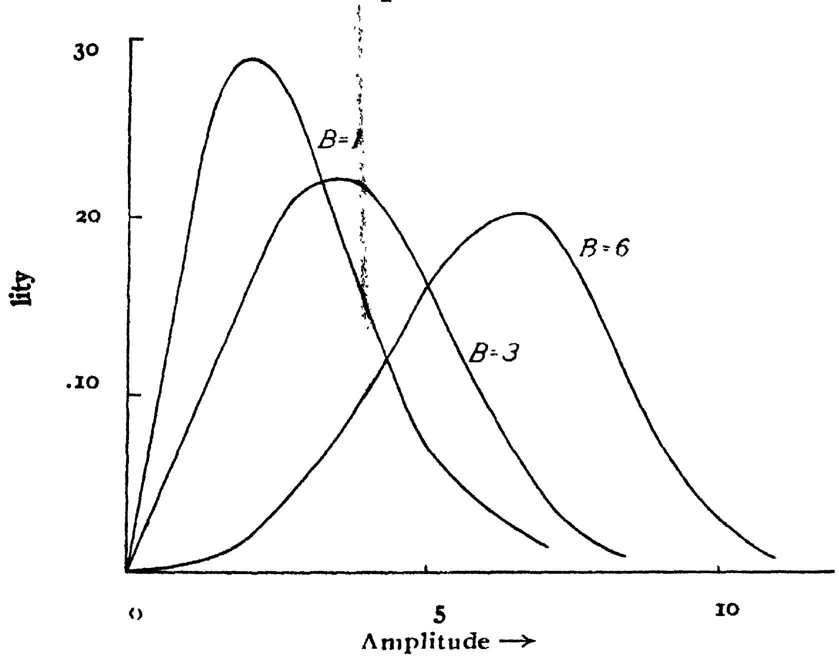

FIG. 1

Rice curves with $R=2\sqrt{2}$ and $B=1$, 3 and 6 as noted against each curve. Note how the form of the curve goes over to the Gaussian type from the Rayleigh type as the ratio $B\sqrt{2}/R$ increases from below 1 to above 3.

From a study of the closeness of fit of the observed distribution curves with the one or the other of the theoretical curves (1), (2) or (3), it is possible to estimate the respective proportions of R and B, *i.e.*, of the scattered and steady components in the received E_s echoes.

McNicol (1949) has analysed a large amount of fading data, both at high and medium frequencies at both vertical and oblique incidence. He found that echoes on high frequency and on medium frequencies at oblique incidence have a large steady reflection component.

(*b*) *Fitting of the curves.*—To draw the probability distribution curves from the fading data the amplitudes recorded were divided into groups differing from one another by 0.5 cm. The number of amplitudes falling into each group was plotted against the corresponding amplitude to give the frequency distribution or, when normalised, the probability distribution of the echo intensity. The form of the probability distribution gave an indication of the approximate values of the parameters of the distribution curve. It was found, however, that these values did not conform to either of the two limiting conditions (2) or (3), in which case the process of curve fitting could have been simplified by transformation to a case of straight line fitting. In this case intermediate values of $B\sqrt{2}/R$ were most frequent. Accurate determination of B and R was, therefore, very difficult and trial-and-error method was adopted. Values of the parameters differing by finite steps were taken and the theoretical curves corresponding to these values were fitted to the experimental distribution curves. A set of typical results is shown in Table I. Figure 2 shows one such curve.

TABLE I

A typical set of observations

Date: January 1, 1951; Time: 12.10 hrs. to 12.30 hrs.

$f^0E_s = 4.65$ Mc/sec; $f^0E = 3.60$ Mc/sec.

Frequency of observations	Number of data	Significance of fit	B	R	ν_c
4.55	61	0.60	2	$1\sqrt{2}$	4.15 Mc/sec.
4.45	61	0.85	5	$2.5\sqrt{2}$	4.33 Mc/sec.
4.30	56	0.70	2	$2\sqrt{2}$	3.50 Mc/sec.

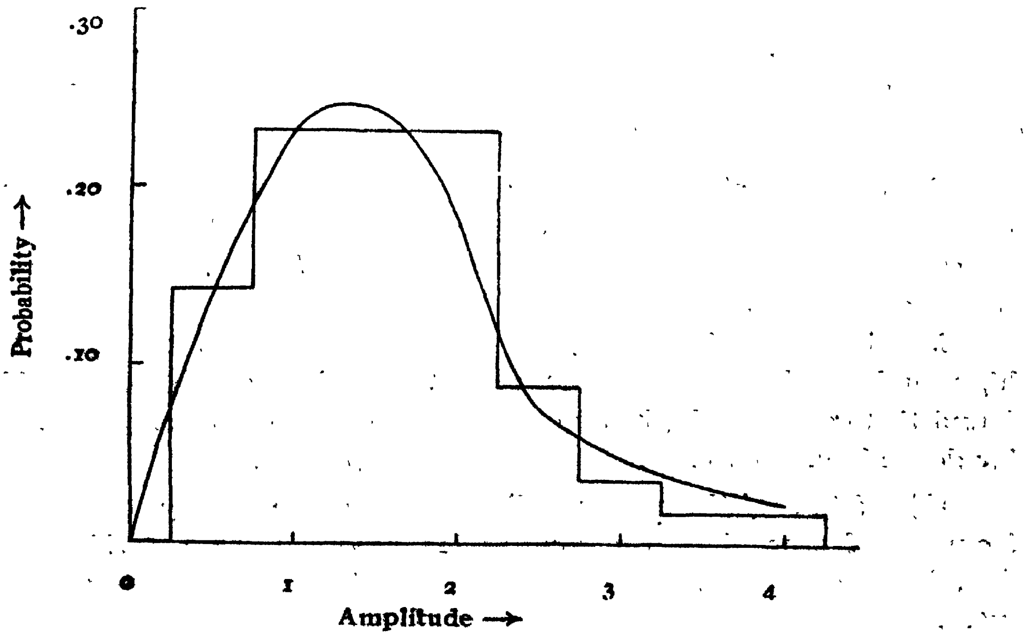

FIG. 2

Probability histogram of amplitude recorded on Jan. 1 at a frequency of 4.30 Mc/sec. The distribution curve with $B=2$ and $R=2\sqrt{2}$ (0.5 cm. being chosen as unit) is shown for comparison.

The significance of the closenesses of the fits could not, however, be well estimated by only a visual comparison of the theoretical and experimental histograms due to the statistical scatter of the points introduced by the smallness of the sample. Recourse was, therefore, taken to the well-known method of significance test due to Pearson. Out of 25 curves with short periods of observation (size of sample = 60) 22 were found to have probabilities of fit greater than .05. This is sufficient evidence to the effect that Rice distribution is being followed. Values of $B\sqrt{2}/R$ as found, varied generally from 1 to 3, showing that there was a distinct steady echo in addition to the scattered wavelets.

4. THE SCATTERED COMPONENT—CALCULATION OF ELECTRON DENSITY IN THE CLOUDS

Booker and Gordon (1950) have given a theory of scattering of microwaves in the troposphere which Booker later applied with appropriate modifications to the E_s layer. Their formula provides a means of estimating the mean electron density of the scattering clouds. It is shown below that the variation of the mean scattering coefficient (characterised by the parameter R of the probability distribution) with the frequency of the sounding wave can be used for this calculation.

According to Booker, the scattering coefficient (σ) of a bunch of clouds with a scale of fine structure l, mean dielectric constant ϵ and mean variation thereof $\Delta\epsilon$ is given by

$$= \frac{\left(\frac{\Delta\epsilon}{\epsilon}\right)^2 \left(\frac{2\pi l}{\lambda}\right)^3}{\lambda\left[1+\left(\frac{4\pi l}{\lambda}\right)^2\right]^2} \qquad (7)$$

for perpendicular incidence.

Replacing $4\pi l$ by λ_0, the penetration wavelength, we can write,

$$\sqrt{\sigma} = \frac{1}{\sqrt{8}} \cdot \frac{\frac{\Delta\epsilon}{\epsilon}\sqrt{\frac{\lambda_0^3}{\lambda^3}}}{\sqrt{\lambda}\left[1+\left(\frac{\lambda_0}{\lambda}\right)^2\right]}$$

Now, neglecting the magnetic field of the earth, the dielectric constant of an ionic cloud is given by

$$\epsilon = 1 - \frac{4\pi N e^2}{m p^2} \qquad \frac{\Delta\epsilon}{\epsilon} = \frac{-\frac{4\pi N e^2}{m p^2}}{1 - \frac{4\pi N e^2}{m p^2}} \cdot \frac{dN}{N}.$$

Putting $\frac{4\pi Ne^2}{m} = 4\pi^2\nu_c^2$ for convenience (ν_c having no significance except that this would be the penetration frequency of a thick layer having this ionic density)

$$\frac{\Delta\epsilon}{\epsilon} = \frac{-\frac{\nu_c^2}{\nu^2}}{1-\frac{\nu_c^2}{\nu^2}} \frac{dN}{N}.$$

Hence

$$\sqrt{\sigma} = \frac{1}{\sqrt{8}} \frac{\nu_c^2}{\sqrt{\nu_0^3}} \cdot \frac{1}{1+\left(\frac{\nu}{\nu_0}\right)^2} \cdot \frac{1}{1-\frac{\nu_c^2}{\nu^2}} \cdot \frac{dN}{N} \cdot \qquad \ldots (5)$$

If σ and σ be the scattering coefficients at two frequencies ν and ν (ν_c being the same under constant conditions) we have

$$\sqrt{\frac{\sigma}{\sigma'}} \cdot \frac{1+\left(\frac{\nu}{\nu_0}\right)^2}{1+\left(\frac{\nu'}{\nu_0}\right)^2} = \frac{1-\frac{\nu_c^2}{\nu'^2}}{1-\frac{\nu_c^2}{\nu^2}} \qquad \ldots (6)$$

Now $\sqrt{\sigma}$ is proportional to the amplitude of the echo, as depicted on the oscillograph screen. Thus by noting the mean amplitude of the echoes at two frequencies and also the penetration frequency, we can calculate the value of ν_c, which is a measure of the ionic density in the clouds.

In course of our observations we were able to measure ν_c on six days. For each of three of these days, only two values of σ could be obtained. The values of ν_c were found to be between 2 and 5 Mc/sec. For each of the three other days, three values of σ were obtained. These latter observations naturally allowed us to calculate three independent values of ν_c and hence afforded a means of checking the theory and observations. The general values of ν_c agreed with those given above and also among themselves within ± 0.5 Mc/sec. One such set of values is given in Table 1.

In addition to our own observations, we utilised some of the earlier data of Rawer (1949) on the variations of the reflection coefficient of the E_s layer with exploring frequency. Rawer noted the frequencies at which the ratios of the amplitudes of the E_s and simultaneous F-echoes were 100:1, 10:1, 1:1, 1:10 and 1:100. Some curves showing the hourly values of these frequencies were used to verify our analysis. In these calculations, the values of the frequencies for 1:100 were taken to be the penetration frequency. From the four other frequencies, six values of ν_c were deduced; these six agreeing among themselves in each case. The frequencies were of the order of 2—4 Mc/sec.

5. THE STEADY COMPONENT—REFLECTIONS FROM A THIN LAYER

The presence of a steady reflected component in the echo from the E_s region (Sec. 3) indicates the presence of a stratum of ionisation of horizontal extent much greater than the length of the exploring wave. Also, the phenomenon of partial reflection associated with E_s echoes indicates that the thickness of the stratum must be comparable to the wavelength. A part of the E_s region may thus be identified with a thin layer.

It is possible in principle, by measuring the variation of reflection coefficient with change of exploring frequency and comparing results with the deductions from the theoretical formulae for reflection coefficient under similar conditions [as given by Hartree (1929), Saha and Rai (1937) and Deb (1940)] to make an estimate of the thickness of the layer. Unfortunately, on account of experimental difficulties, reflection coefficients could not be obtained over a sufficiently large range of exploring frequencies. However, over our limited range of observations we obtained a result which indicates qualitative agreement with the variation of reflection coefficient with wave length as given by Hartree's formula. In two of our observations the parameter B of the distribution curve (which characterises the coefficient corresponding to the steady component) passed through a maximum with change of the exploring wave frequency having values higher than ν_c. According to Hartree's formula also the reflection coefficient should pass through a number of maxima and minima as the frequency of the exploring wave is changed in the range greater than the 'penetration' frequency (by this term we mean here a frequency that would be just reflected by a thick layer of same ionic density).

It may be mentioned in this connection that Best, Farmer and Ratcliffe (1938) suggest that the E_s reflections may be due to a sharp gradient of ionisation in the lower part of the E region. It is clear from the above that observations on the variation of the parameter B over a wide range of ν may yield sufficient data to descriminate between the two points of view (thin layer or sharp gradient).

An alternative and simpler method for determining the reflection coefficient of a sharp gradient or a thin layer, as developed by the author, is given below. The starting point of the analysis is the expression given by Rayleigh for reflection from a medium of varying refractive index. Rayleigh's expression for the reflection coefficient may be written as

$$\frac{B_1}{A_1} = -\int_0^{\mu_1} \frac{d\mu}{\mu} e^{-\frac{4\pi i}{\lambda}\int_0 \mu dZ}.$$

It is assumed that the reflection coefficient is small at all points.

Let the reflecting medium consist of electrons distributed in a thin layer of triangular profile having semi-thickness Z_1, and maximum ionic density N_1, (corresponding to the critical frequency ν_c of a thick layer). The number density of the electrons at any height is then given by

$$N = \frac{N_1}{Z_1} \qquad \text{for } 0 < Z < Z_1 \qquad \ldots (7)$$

and

$$N = \frac{N_1}{Z_1}\left(2 - \frac{Z}{Z_1}\right) \qquad \text{for } Z_1 < Z < 2Z_1 \qquad \ldots (8)$$

Then, since

$$\mu = \sqrt{1 - \frac{4\pi N e^2}{m p^2}},$$

we can split the integral into two parts and write,

$$\frac{B_1}{A_1} = \int_0^{Z_1} \frac{\frac{\nu_c^2}{\nu^2} \cdot \frac{1}{Z_1}}{4\left(1 - \frac{\nu_c^2}{\nu^2} \cdot \frac{Z}{Z_1}\right)} e^{-\frac{4\pi i}{\lambda} \int_0^Z \sqrt{1 - \frac{\nu_c^2}{\nu^2} \cdot \frac{Z}{Z_1}}\, dZ} dZ$$

$$- \int_{Z_1}^{2Z_1} \frac{\frac{\nu_c^2}{\nu^2} \cdot \frac{1}{Z_1}}{4\left\{1 - \frac{\nu_c^2}{\nu^2}\left(2 - \frac{Z}{Z_1}\right)\right\}} e^{-\frac{4\pi i}{\lambda} \int_0^Z \sqrt{1 - \frac{\nu_c^2}{\nu^2}\left(2 - \frac{Z}{Z_1}\right)}} dZ$$

In the second integral, the transformation $2 - \frac{Z}{Z_1} = \frac{x}{Z_1}$ yields, on making the running variable uniform,

$$\frac{B_1}{A_1} = \int_0^{Z_1} \frac{\frac{\nu_c^2}{\nu^2} \cdot \frac{1}{Z_1}}{4\left(1 - \frac{\nu_c^2}{\nu^2} \cdot \frac{Z}{Z_1}\right)} \left[e^{-\frac{4\pi i}{\lambda} \int_0^Z \sqrt{1 - \frac{\nu_c^2}{\nu^2} \cdot \frac{Z}{Z_1}}\, dZ} - e^{\frac{4\pi i}{\lambda} \int_{Z_1}^Z \sqrt{1 - \frac{\nu_c^2}{\nu^2} \cdot \frac{Z}{Z_1}}\, dZ} \right] dZ,$$

which yields

$$\frac{B_1}{A_1} = \frac{1}{4Z_1} \cdot \frac{\nu_c^2}{\nu^2} \left[e^{-\frac{8\pi i}{3\lambda} \cdot \frac{\nu^2}{\nu_c^2} Z_1} - e^{-\frac{8\pi i}{3\lambda} \cdot \frac{\nu^2}{\nu_c^2} Z_1 \left(1 - \frac{\nu_c^2}{\nu^2}\right)^{3/2}} \right]$$

$$\times \int_0^{Z_1} \frac{e^{-\frac{8\pi i}{3\lambda} \frac{\nu^2}{\nu_c^2} Z_1 \left(-1 \frac{\nu^2}{\nu_c^2} \cdot \frac{Z}{Z_1}\right)^{3/2}}}{\left(1 - \frac{\nu_c^2}{\nu^2} \cdot \frac{Z}{Z_1}\right)} dZ.$$

putting $\left(1 - \frac{\nu_c^2}{\nu^2} \cdot \frac{Z}{Z_1}\right)^{3/2} = \eta$, the above can be written as

$$\frac{B_1}{A_1} = -\frac{1}{6}\left[e^{-\frac{8\pi i}{3c} \cdot \frac{Z_1}{\nu_c^2} \nu^3} - e^{-\frac{8\pi i}{3c} \frac{Z_1}{\nu_c^2} \nu^3 \left(1 - \frac{\nu_c^2}{\nu^2}\right)^{3/2}} \right] \int_{\frac{8\pi}{3c} \frac{Z_1}{\nu_c^2} \nu^3 \left(1 - \frac{\nu_c^2}{\nu^2}\right)^{3/2}}^{1} \frac{e^{ix}}{x}\, dx \qquad \ldots (9)$$

This expression ceases to be valid when ν approaches ν_c *i.e.* near critical reflection, since the reflection here is very profuse and our initial assumption breaks down. Within the region of validity, the lower limit of integration is greater than 1. The modulus of the integral has no pronounced variation in this range (figure 3) and hence the maxima and minima of the reflection coefficient are determined by the first factor.

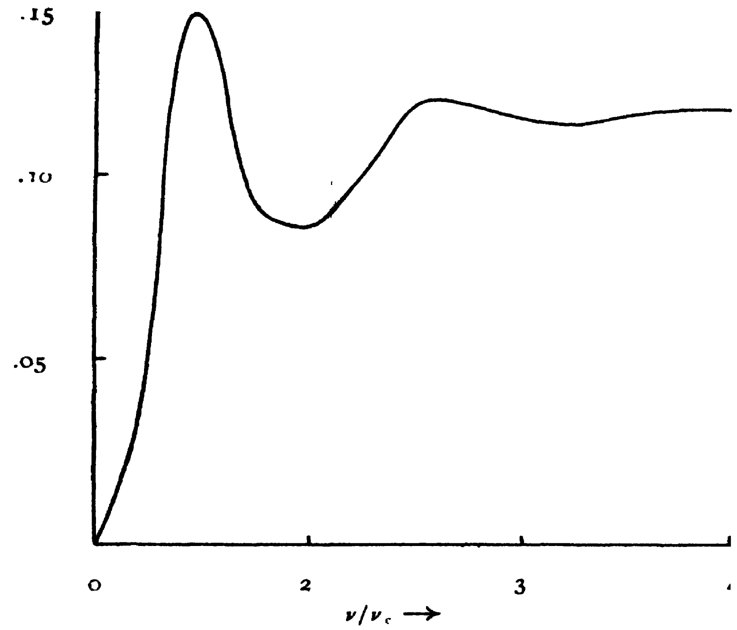

FIG. 3

Variations of the integral $R' = \frac{1}{8}\int_{\frac{8\pi}{3c}\frac{Z\nu}{\nu_c^2}\left(1-\frac{\nu_c^2}{\nu^2}\right)^{3/2}}^{\infty} \frac{e^{ix}}{x}\,dx$ with ν when $\frac{\nu_1}{\nu_c} = 2.5$

The first factor is maximum when the arguments of the two complex quantities differ by an odd multiple of π, *i.e.* when

$$\frac{8\pi}{3c}\cdot\frac{Z_1}{\nu_c^2}\,\nu^3 = (2n+1)\pi + \frac{8\pi}{3c}\cdot\frac{Z_1}{\nu_c^2}\,\nu^3\left(1-\frac{\nu_c^2}{\nu^2}\right)^{3/2},$$

$$i.\,e.\left(1-\frac{\nu_c^2}{\nu^2}\right)^{3/2} = 1 - \frac{(2n+1)3c}{8}\,\frac{\nu_c^2}{Z_1\nu^3}$$

Or, putting $c/Z_1 = \nu_1$,

$$\left(1-\frac{\nu_c^2}{\nu^2}\right)^{3/2} = 1 - \frac{3(2n+1)}{8}\,\frac{\nu_c^2\nu_1}{\nu^3} \qquad \ldots \quad (10)$$

Graphical solution of this equation shows that the function B_1/A_1 will go through a series of maxima and minima. The spacings between the maxima and the frequency for the first maximum are dependent on the ratio ν_1/ν_c, *i. e.* is a function of the thickness and density of the layer. However, the frequency separation of the maxima are of the same order as ν_c, (see figure 4).

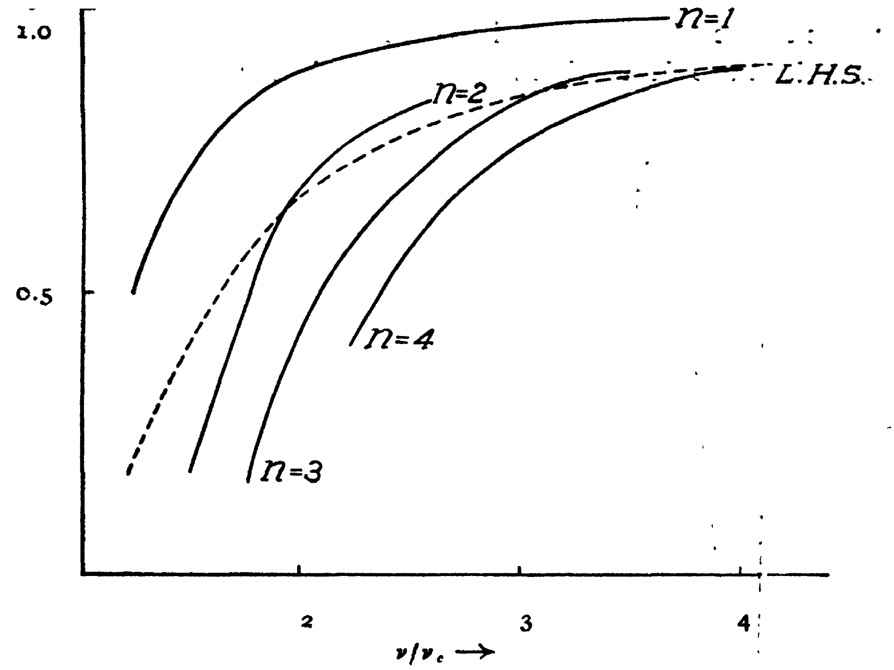

FIG. 4

Graphical solution of Eq. (10). The dotted lines represent the L. H. S. and the bold lines the R. H. S. for the various values of n noted against each. The intersections give values of ν/ν_c for maxima. $\nu_1/\nu_c = 2.5$

If instead of a layer of finite thickness we have a semi-infinite layer of steep boundary (the electron distribution at the boundary being given by eqn. 7, then the condition for maximum reflection coefficient is given by

$$\frac{8\pi}{3c}\,\frac{Z_1}{\nu_c{}^2}\,\nu^3 = (2n+1)\pi;$$

i.e., $$\nu^3 = \underline{3(2n+1)}\ \nu\ \nu^2 \qquad (11)$$

It is to be noted that the spacings between the cubes of the maxima are constant. Thus a measurement of the spacings between the maxima when the exploring wave frequency is varied can provide a test for the nature of the layer, whether it is a thin layer or has a thickness large compared to the wavelength having a steep lower boundary.

6. A NOTE ON THE OBSERVED VARIATION OF FIT

During our observations it was noticed that the fits for smaller samples (50 to 60 data) were more significant than those for larger samples (100 to

200 data). It was suspected that the misfit was being caused by a change in the ionospheric conditions (as characterised by the distribution parameters B and R) during the period of observation, which was perforce rather long (15 to 20 minutes) for the larger samples. Such changes would cause a variation of the closeness of the fit as the time of observation is gradually lengthened. It is evident that a functional relationship exists between the variation of fit and the variation of the distribution parameters.

To study this effect a number of long duration observations (ranging from 8 to 20 minutes) were taken. Instead of calculating the actual value of the Pearson's χ^2 function, the closeness of fit was calculated by the quantity χ^2/N, N being the total number of observations. This quantity is given by the equation

$$\frac{\chi^2}{N} = \sum_i \frac{(\psi_i - p_i)^2}{\psi_i}$$

where ψ_i is the theoretical probability of occurence of the i th amplitude range and p_i the experimentally observed probability. The analysis was carried out as follows :

The total number of data was first fitted to the nearest Rice curve. The set was then considered up to the first 40 observations and the value of χ^2/N calculated with the Rice curve deduced before. The number of data was then progressively increased and χ^2/N recalculated till the end of the set.

The investigation of the functional relationship between variations of χ^2/N and the characteristics is complicated by the finite size of each sample. As a first approximation we will assume that the whole deviation of the fit is due to the variation of the characteristics or, in effect, that the samples behave as infinitely large ones. A general analysis of the variation of Rayleigh fits is given below. The parallel analysis for Rice curves is made difficult by the comparatively complicated Bessel function with imaginary arguments.

The Rayleigh distribution

$$\psi_0 = \frac{2r}{R^2} e^{-\frac{r^2}{R^2}} dr,$$

can be written as

$$\psi_0 = -\frac{d}{dr}\left[e^{-\frac{r^2}{R^2}}\right] dr.$$

If $1/R^2$ be a function of time, given generally by

$$\frac{1}{R^2} = \frac{1}{R_0^2} + \phi(t)$$