ANALYSIS OF THE RELAXATION PERIOD OF A MULTIVIBRATOR

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ABSTRACT. An improved mathematical analysis of the relaxation periods of a free running symmetrical multivibrator has been made showing expressions for different electrode potentials and time periods. The theoretical values obtained have been compared with experimental data, part of which is also presented here.

INTRODUCTION

As a result of the many common as well as specialized applications of the multivibrator circuits and their derivatives, a large number of papers have appeared on the subject in recent years. The analyses made generally pertain either to the relaxation periods (e.g. Kiebert and Inglish, 1945[1] or to the switching periods between two states of passive relaxation (e.g. Williams et ai, 1950; Rais Ahmed, 1950). However, it is noticed that although the voltages obtained in the relaxation period are used as boundary conditions in the switching period, it is not always possible to carry over the assumptions implicit in one analysis into the other. For example, with reference to the waveforms shown in figure 2, the maximum negative grid voltage appearing at the grid has to be computed without ignoring the positive drive of the other grid, and the voltage division between the shunting and coupling capacitances. A common relaxation analysis assuming no positive grid swing and negligible shunting capacitance will lead not only to erroneous values of the relaxation period but also to unreliable boundary conditions for the switching period, especially for high frequency multivibrators.

In the present analysis both these shortcomings of the older discussions have been removed so that the results can be safely applied to the more minute observations of the switching period.

THE ANALYSIS

e_0 = positive swing of the grid voltage,

E_{co} = static cut-off grid voltage for supply potential E_b,

E_b = the plate supply potential.

E_0 = voltage drop across the load resistance R_L when the grid voltage of the same tube is zero with coupling condenser removed,

E_1 = voltage drop across R_L when the grid. voltage of the same tube is e_c,

C_1 and C_2 are shunting capacitances from plate to cathode and grid to cathode respectively of each tube,

e_{gm} = maximum negative grid voltage,

r_p = plate resistance of the tube, and

k = non-linearity factor of the tube characteristic.

With the notations indicated in figures 1 and 2, E_o and the current through the load resistance R_L, namely i_{bo} will be :

$$i_{bo}=\frac{E_b-K}{r_p+R_L};\quad E_o=\left(\frac{E_b-K}{r_p+R_L}\right)R_L \qquad (1)$$

The voltage E_1 is greater than E_0 because of the fact that a greater current i_{b1} flows through R_L when the grid of the tube is positive. The current in the tube is then

$$i_{b_1}=\frac{\mu e_c+E_b-K}{r_p+R_L}$$

and

$$E_1=\left(\frac{\mu e_c+K_b-K}{r_p+R_L}\right)R_L \qquad (2)$$

where μ stands for $g_m r_p$, both the latter factors having a value different from the normal value of the tube transconductance and plate resistance when taken in the region of positive grid drive. Since the difference in the value of μ is usually not more than 15 % of the normal mean value, this difference may be ignored (when e_c is large μ decreases sharply). The grid voltage e_c can be found from the equivalent circuit of figure 3 which represents the conditions just at the instant when the grid voltage jumps from $-E_{co}$ to e_c, the positive value.

At the instant τ_1 (figure 2), the tube V2 begins to conduct and V1 is suddenly cut off. Just before τ_1, V1 was conducting and its plate voltage was E_b-E_o, so that the condenser was charged to E_b-E_o at the plate side of the tube V1, and to $-E_{co}$ at the grid side of V2, because V2 just reaches cut-off at that instant, so that the condenser was charged to $(E_b-E_o+E_{co})$ volts.

From a consideration of the total driving voltage and the potential drop in the parallel combination of r_g and R_g of figure 3, we get

$$e_c=\frac{[E_b-(E_b-E_o+E_{co})]r'_g}{R_L+r'_g}$$

where r_g = grid cathode resistance of V_2 when it is conducting

and

$$=\frac{r_g R_g}{r_g+R_g} \quad \text{or} \quad e_c=\frac{(E_o-E_{co})r'_g}{R_L+r'_g} \qquad \ldots (3)$$

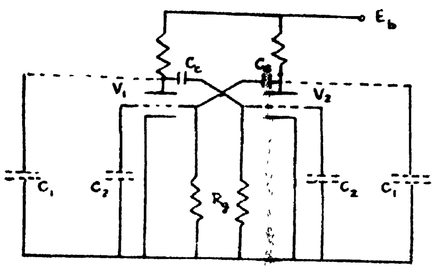

FIG. 1. A symmetrical multivibrator

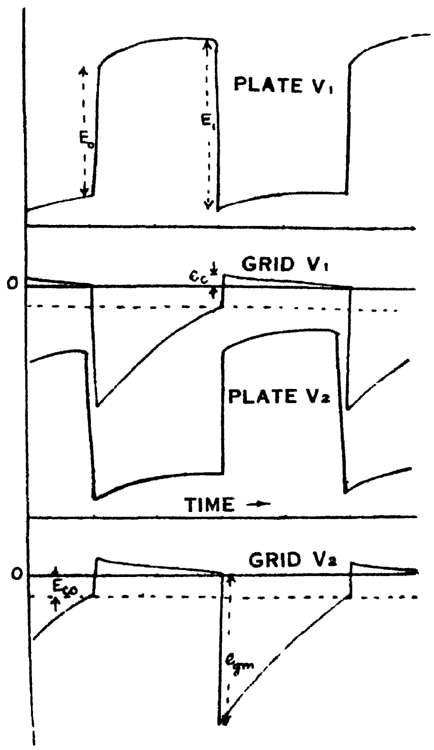

FIG. 2. The voltage waveform of a symmetrical multivibrator

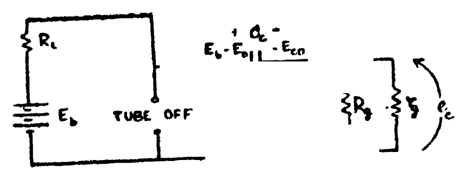

FIG. 3. Equivalent circuit for the calculation of the positive grid drive

If E_o is substituted in (3) from (1), we obtain

$$e_c = \frac{\left[\dfrac{R_L(E_b - K)}{r_p + R_L} - E_{co}\right] r'_g}{R_L + r'_g}$$

or

$$e_c = \frac{[R_L(E_b - K) - (R_L + r_p)E_{co}]r'_g}{(R_L + r'_g)(R_L + r_p)} \quad \ldots \quad (4)$$

On substitution of (4) in (2), we find

$$E_1 = \frac{R_L\left[\mu\left\{\dfrac{R_L(E_b - K) - (R_L + r_p)E_{co}}{(R_L + r'_g)(R_L + r_p)}\right\} r'_g + E_b - K\right]}{R_L + r_p}$$

or

$$E_1 = \frac{R_L[\mu r'_g\{(E_b - K)R_L - (R_L + r_p)E_{co}\} + (E_b - K)(r_p + R_L)(R_L + r'_g)]}{(R_L + r_p)^2(R_L + r'_g)} \quad \ldots \quad (5)$$

It is obvious that when the shunting capacitance C_2 is small compared to the coupling capacitance C_c, the voltage E_1 is also equal to e_{gm}. At higher frequencies, where C_2 is not negligible, the maximum negative grid voltage is given by

$$e_{gm} = \frac{C_c}{C_c + C_2} E_1 \quad \ldots \quad (6)$$

The above equation will give an idea of the maximum possible frequency attainable by the multivibrator. So long as $e_{gm} = \dfrac{C_c}{C_c + C_2} E_1 > E_{co}$ the multivibrator will oscillate. When C_2 is so large compared with C_c that e_{gm} is less than E_{co}, the multivibrator will stop oscillating.

Once the maximum negative grid voltage is known accurately, the period of relaxation can be calculated from the equivalent circuit of figure 4.

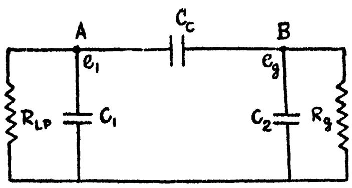

FIG. 4. Equivalent circuit for the calculation of relaxation time.

The equations for the circuit are :

$$R_{Lp}=\frac{r_p R_L}{r_p+R_L}=\frac{1}{G_1}\;;\quad R_g=\frac{1}{G_2}$$

at the junction A,

$$C_c\frac{d(e_1-e_g)}{dt}=-C_1\frac{de_1}{dt}-\frac{e_1}{R_{Lp}}=-\;C_1\frac{de_1}{dt}+\frac{e_1}{R_{Lp}}$$

or $$C_c\frac{de_1}{dt}+C_1\frac{de_1}{dt}+\frac{e_1}{R_{Lp}}=C_c\frac{de_g}{dt} \qquad \dots (7)$$

and at the junction B,

$$C_c\frac{d(e_1-e_g)}{dt}=C_2\frac{de_g}{dt}+\frac{e_g}{R_g}$$

or $$C_c\frac{de_1}{dt}=C_c\frac{de_g}{dt}+C_2\frac{de_g}{dt}+\frac{e_g}{R_g} \qquad \dots (8)$$

Denoting d/dt by p, (7) and (8) become

$$C_cpe_1+C_1pe_1+e_1G_1=C_cpe_g$$

$$C_cpe_1=C_cpe_g+C_2pe_g+e_gG_2$$

or $$(pC_c+pC_1+G_1)e_1-pC_ce_g=0 \qquad \dots (9)$$

$$C_cpe_1-(pC_c+pC_2+G_2)e_g=0 \qquad \dots (10)$$

From (9) and (10) we get $(pC_{e1}+G_1)(pC_{e2}+G_2)-p^2C_c^2=0$

where $\quad C_{e1}=C_c+C_1$ and $C_{e2}=C_c+C_2$

or $$p^2(C_{e1}C_{e2}-C_c^2)+p(C_{e1}G_2+C_{e2}G_1)+G_1G_2=0 \qquad \dots (11)$$

so $$p=\frac{-(C_{e1}G_2+C_{e2}G_1)\pm\left[(C_{e1}G_2+C_{e2}G_1)^2-4(C_{e1}C_{e2}-C_c^2)G_1G_2\right]^{1/2}}{2(C_{e1}C_{e2}-C_c^2)}$$

The above two values of p may be denoted by $-\alpha$ and $-\beta$. The grid voltage during the relaxation period may now be written as:

$$e_g=Ae^{-\alpha t}+Be^{-\beta t} \qquad \dots (12)$$

where A and B are the constants which are to be determined by the boundary conditions that:

(*i*) $e_g=e_{gm}$ at $t=0$,

and (*ii*) $\frac{de_g}{dt}=0$ at $t=0$.

Thus at $t=0$

$$e_{gm}=A+B$$

$$\frac{de_g}{dt}=0=A\alpha+B\beta$$

The above equations give $A=\frac{-\beta e_{gm}}{\alpha-\beta}$, $\quad B=\frac{\alpha e_{gm}}{\alpha-\beta}$.

After substituting the values A and B in (12), we obtain

$$e_g = \frac{e_{gm}}{\alpha-\beta}\left[\alpha e^{-\beta t} - \beta e^{-\alpha t}\right] \quad \ldots \quad (13)$$

By putting $e_g = E_{co}$ in this expression, the time required by the grid voltage to go from its most negative value to the tube current cut-off voltage E_{co} can be found. In this way the relaxation time is found with positive grid drive as well as shunting capacitances taken into account.

There are two special cases of interest.

Case I. When $C_1 = C_2 = 0$ and $C_{c1} = C_c = C_{c2}$, Eqn. (11) becomes

$$pC_c(G_1 + G_2) + G_1G_2 = 0$$

or
$$p = \frac{-G_1G_2}{(G_1+G_2)C_c} = \frac{-1}{(R_{Lp}+R_g)C_c}$$

or
$$e_g = Ae^{-t/(R_{Lp}+R_g)C_c}$$

In this case from (6), $e_{gm} = E_o$; and at $t = 0$, $e_g = e_{gm} = E_o$,

so that
$$e_g = E_oe^{-t/(R_{Lp}+R_g)C_c}.$$

and
$$\text{relaxation time} = \left(\frac{R_Lr_p}{R_L+r_p} + R_g\right)C_c \ln \frac{E_o}{E_{co}}$$

This is the conventional formula.

Case II. When $C_1 = 0$, $R_{Lp} = 0$; Eqn. (11) becomes

$$p(C_c+C_2)R_g + 1 = 0, \quad \text{or} \quad p = \frac{-1}{(C_c+C_2)R_g}$$

$$e_g = Ae^{-t/(C_c+C_2)R_g}$$

when
$$t = 0,\ e_g = {}_ge_m = A = \frac{C_c}{C_c+C_2}E_1$$

or
$$e_g = \frac{C_c}{C_c+C_2}E_1e^{-t/(C_c+C_2)R_g}$$

In this case the relaxation time $= (C_c+C_2)R_g \ln \frac{C_c}{C_c+C_2}\cdot\frac{E_1}{E_{co}}$

This is the formula given by Puckles (1951).

The practical method of calculating the relaxation time is given below.

Suppose in a particular case $C_c = 200$ mmf, $C_1 = 50$ mmf. $C_2 = 50$ mmf. ; $C_{c1} = C_{c2} = 250$ mmf. ; $R_L = 100\text{K}\Omega$; $R_g = 1\ \text{M}\Omega$; $r_p = 8\text{K}\Omega$; $R_{LP} = 7 \times 10^{3}$ $= 1/G_1$; $e_{gm} = 96v$. (measured) ; $E_{co} = 9.5v$.

$$C_{c1}C_{c2} - C_c^2 = 22.5 \times 10^{-21} \ ; \ C_{c1}G_2 + C_{c2}G_1 = 35.96 \times 10^{-15} \ ;$$
$$\text{and } G_1G_2 = 1.43 \times 10^{-10}$$

Equation (11) becomes $22.5 \times 10^{-21}p^2 + 35.96 \times 10^{-15}p + 1.43 \times 10^{-10} = 0$

or
$$p = \frac{-35.96 \pm 35.78}{45 \times 10^{-6}}$$

which gives $\alpha = -4 \times 10^3$ and $\beta = -1.594 \times 10^6$

Since $\quad |\alpha| \leqslant |\beta|; \; \frac{\beta}{\alpha - \beta} e^{-\alpha t} \geqslant \frac{\alpha}{\alpha - \beta} e^{-\beta t}$ and $\frac{-\beta}{\alpha - \beta} = 1$

so $\quad = e_{gm}{}^{-\alpha t} = 96 e^{-4 \times 10^3 t}$;

Relaxation time $= \frac{1}{4 \times 10^3} \ln \frac{96}{9.5} = 575$ microseconds

The time period will be twice the relaxation time and so is 1150 microseconds. An experimental value in this case was found to be 1020 microseconds.

EXPERIMENTAL RESULTS

A double beam Cossor oscillograph was used to measure the different electrode potentials and time periods. The measurement of time was checked by means of a standard oscillator.

TABLE I

$E_b = 110$v, $R_L = 100$KΩ; $R_g = 1$ M Ω; tube 6C5; $C_1 = 0$; $C_2 = 46$ mmf.

C_c in mmf	E_0 in volts	E_1 in volts	e_{gm} in volts	e_{gm} in volts (cal)
1000	84	94	90	90.0
500	86	94	87	86.1
250	85	94	80	79.3
100	84	94	68	64.3
50	86	94	44	48.9
25	86	94	30	33.0

By applying formula (6) in the first observation the value of C_2 was found to be 46 mmf. This value of C_2 was used in the calculation of e_{gm} for different values of the coupling condenser.

Equation (6) was further verified by taking a fixed value of C_c and observing the variation of e_{gm} with C_2.

TABLE II

C_2 in mmf.	E_1 in volts	E_0 in volts	e_{gm} in volts	e_{gm} calc
550	120	111	33	32.0
350	120	111	45	43.6
250	120	111	57	53.4
150	120	111	70	68.5
50	120	111	96	96.0

The constans for this set of observations were $E_b=150$ v., tube 6C5 ; $R_g=1$ megohm., $R_L=100$ KΩ and $C_o=200$ mmf. The value of C_2 inherent in the circuit was estimated from the last reading when no external shunting capacitance was used, and e_{gm} was calculated on the basis of equation (6).

TABLE III

$E_b=150$v., tube 6C5 ; $R_L=100$KΩ ; $R_g=1$ megohm., $r_p=8$KΩ ; $C_o=200$ mmf. $C_1=50$ mmf. (assumed) ; τ stands for time period in microseconds.

C_2 in mmf.	e_{gm} in volts	E_{co} in volts	τ obs.	τ cal
550	33	9 5	1750	1880
350	45	9 5	1500	1620
250	57	9 5	1450	1508
150	70	9.5	1200	1280
50	96	9.5	1020	1150

TABLE IV

$E_b=130$ v. ; tube 6SN7 ; $R_L=20$KΩ ; $R_g=1$ megohm ; $r_p=8$KΩ ; $C_2=80$ mmf ; $C_1=50$ mmf (assumed); $\tau/2$ measured in microseconds =relaxation period.

C_c in mmf.	E_1 in volts	e_{gm} in volts	E_{co} in volts	$\tau/2$ obs.	$\tau/2$ cal.
1000	92	85	8	2500	2510
500	92	80	8	1350	1298
250	92	68	8	550	611
100	92	56	8	340	328
50	90	37	8	130	137
25	90	21	8	70	86

TABLE V

$E_b=125$ v. ; tube 6C5 ; $R_g=1$ megohm ; $r'_g=1000$ ohm ; $r_p=10$ KΩ ; $E_{oo}=6$ v. ; $k=15$ v.; $C_o=500$ mmf.; μ nearly 10.

R_L in KΩ	E_0 obs in volts	E_0 cal. in volts	E_1 obs, in volts	E_1 cal in volts	e_c obs.	e_c cal.
100	83	100	94	109	1 5	1.5
30	70	82	92	100	3 0	2 4
10	50	55	78	77	5 0	4.5
5	30	36	58	54	5.0	5.1
2	18	18	38	28	3.0	4 0

CONCLUSION

It can be concluded from these tables that the set of equations derived here gives a reliable manner of calculating the relaxation period and the various electrode voltages of a multivibrator running at fairly high frequencies. These equations, therefore, can be safely taken to yield the boundary conditions for an accurate determination of the switching periods in a multivibrator.

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REFERENCES

Ahmed, Rais 1950, *Ind. Jour. Phys.*, **24**, 281.
Kiebert, M. V., and Inglis A. F, 1945, *Proc I R E*, **33**, 534.
M I. T. Staff 1946, Principles of Radar, McGraw Hill
Puckle, O. S., 1951, Time Bases, Chapman and Hall
Williams, F. M, Aldrich, D. F., and Woodford, J B., 1950, *Proc. I. & E.*. **38**, 65.