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AN INSTRUMENT FOR DIRECT MEASUREMENTS OF CAPACITANCE AND POWER FACTOR *

By J. C. BHATTACHARYA

INSTITUTE OF RADIO PHYSICS AND ELECTRONICS, CALCUTTA UNIVERSITY

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ABSTRACT. The paper gives the design details of a new type of capacitance meter by which the capacitances and power factors of small condensers (maximum value 0.02 μF) can be directly measured on two calibrated meters, no adjustment or calculation being necessary. The design is based on the principle that when a small condenser (the test capacitor) is connected across the tuned circuit of an oscillator, both its frequency and magnification factor change. The change in frequency is indicated by a frequency-discriminator. This gives the value of the added capacitance. The change in magnification factor gives the power factor. By suitable discriminator and computor circuits, the meter readings are made proportional to capacitances and power factors.

1. INTRODUCTION

Various methods are employed for the determination of the capacitance and the power factor of a condenser. For the static condition the capacitance can be measured by charging the condenser to a given voltage and then discharging it through a leak resistance of high value. From the rate of fall of the voltage and the value of the leak resistance (R), the capacitance (C) can be calculated. The power factor (P) at any frequency ($\omega/2\pi$) is obtained from the value of $\frac{1}{\sqrt{1+\omega^2C^2R^2}}$. At audio frequencies the two quantities are usually determined by a bridge circuit. By inserting a parallel combination of a resistor and a capacitor in one of the arms of the bridge and adjusting it for balance, both the capacitance and the leak resistance can be calculated. In some types of impedance bridge, the resistance dial is calibrated in decrement $D\left(\text{which is } \frac{1}{\omega CR}\right)$ and, from the reading of this, the power factor is calculated. At radio frequencies, the capacitor value and the leak resistance (and hence the power factor) can also be measured by the so-called substitution method. In all these methods, however, several adjustments and/or computations are needed. And, when quick measurements, as in commercial tests are needed, these methods fail to satisfy the need. It is to be noted that the value of C can be measured directly by a frequency discriminator circuit. But any method by which P can be determined without making adjustments for balancing etc. is still

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unknown. In the present communication an instrument is described by which both capacitance and power factor of the test capacitor can be determined directly, no adjustment or calculation being necessary. By merely connecting the test capacitor across two terminals of the instrument provided for the purpose, the values of the capacitance and power factor are obtained from the readings of two meters mounted on the panel.

2. THE METHOD OF MEASUREMENT

Figure 1 shows schematically the method of working of the instrument for direct measurements of capacitance and power factor of a condenser.

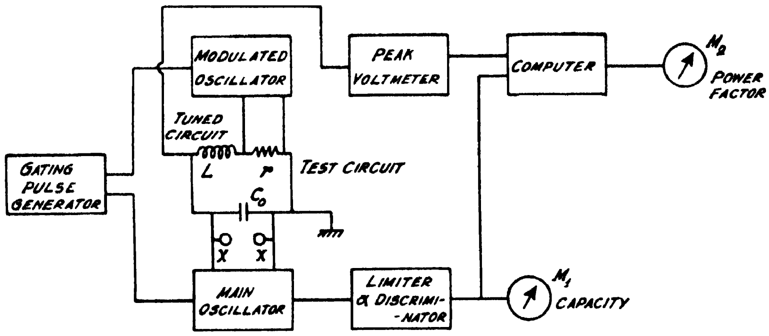

Fig. 1. Schematic diagram of the instrument which measures the value and power factor of capacitor automatically ; XX are the points where the test capacitor is connected. The value of the capacitor is indicated by meter M_1 and the power factor by meter M_2.

(i) Measurement of C.

The LC_0r oscillatory circuit shown is the frequency determining network. When a test capacitor C (small compared to C_0) is connected a cross the terminals (XX), the oscillation frequency changes by Δf, given by

$$\Delta f = \frac{f_0}{C_0} \cdot C \qquad (1)$$

where $f_0 = \frac{1}{2\pi\sqrt{LC_0}}$ is the original resonant frequency of the circuit. As $\frac{f_0}{2C_0}$ is a constant for the oscillator used, the test capacitor C is directly proportional to Δf. Δf is measured by means of a Foster-Seeley type discriminator as follows :

The centre frequency of the distriminator is set at f_0, so that when the capacitor is not connected the discriminator has zero output. This output is recorded in a meter directly calibrated to give the capacitance value.

(ii) Measurement of power factor.

The power factor of the test capacitor is obtained as follows: Let the magnification factor of the tuned circuit LC_0 be Q_0. When the test capacitor

C is connected across the terminals, let the new magnification factor of the circuit be Q. The power factor of the capacitor C is then given by

$$P = \frac{C_0}{C} \cdot \frac{Q_0 - Q}{Q_0 Q} \quad (2)$$

Here C_0 and Q_0 are constants. Hence,

$$P = K \cdot \frac{Q_0 - Q}{CQ} \quad (3)$$

where $K = \frac{C_0}{Q_0}$ is a known constant and C is also known from discriminator output as indicated above. To determine Q, an r.f. voltage (of varying frequency) is injected across the resistance r, the frequency sweeping over the entire range of possible resonant frequency of the circuit. When the frequency of the injected voltage coincides with the actual resonant frequency (with the test capacitor connected), the instantaneous voltage developed across the circuit is Q times the input voltage. The voltage which is proportional to Q is measured by means of a peak voltmeter and is fed into a subtractor circuit, the output of which gives a voltage proportional to $(Q_0 - Q)$. Since the discriminator output is proportional to C, one can obtain by means of a multiplier and divider, a voltage proportional to $Q_0 - Q/QC$ which, in its turn, is proportional to the required power factor. The computor output is calibrated to read the power factors directly.

Gating of the various stages is necessary, because, during measurements of Q, the oscillator has to be shut off, and during measurements of C, the peak voltmeter should be inactive. This is done by measuring these two quantities alternately at the rate of 50 c/s.

3. THE DESIGN OF THE INSTRUMENT

The oscillator circuit LC_0r is the test circuit of the instrument (figure 1). L is an air-core coil of a few turns of thick wire tuned to a frequency of about 1 Mc/s by means of a high grade mica condenser of value 2000 $\mu\mu$F. The diagram of the oscillator circuit is shown in figure 2. The small

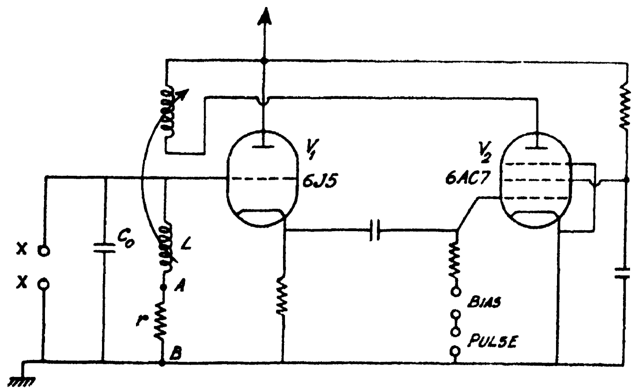

Fig. 2. The detail circuit diagram of the main oscillator.

resistance r, introduced in series with the coil is for injecting an r. f. voltage of constant amplitude in the circuit.

The oscillator assembly is slightly different from the conventional feed-back type. It consists of two valves. To the grid of the first valve is connected one end of the tuned circuit ; the other end is earthed. The first valve (V_1) acts as a cathode follower with resistive load. The output of this valve is amplified by the second valve (V_2) which has an inductive load. For oscillator, the final output is fed back in proper phase to the tuned circuit by mutual inductive coupling. If the amplifier tube is biased to cut-off, the oscillation stops without any fraction of the bias voltage appearing across the tuned circuit (Whitehead and Rueggaberg, 1949). The oscillator is switched on and off by applying suitable pulses to the second tube (V_2). The repetition frequency of the pulse is 50 c/s and the pulse width is approximately 6 milliseconds. These pulses or bias voltages do not appear in the tuned circuit, and this is necessary as pointed out later. Normally, the bias of V_2 is such that the system does not oscillate. When positive pulses of suitable amplitude are applied, oscillation occurs, the frequency being controlled by the tuned circuit.

The oscillator voltage is fed into a limiter followed by a Foster Seeley type discriminator, the circuit diagram of which is shown in figure 3. The

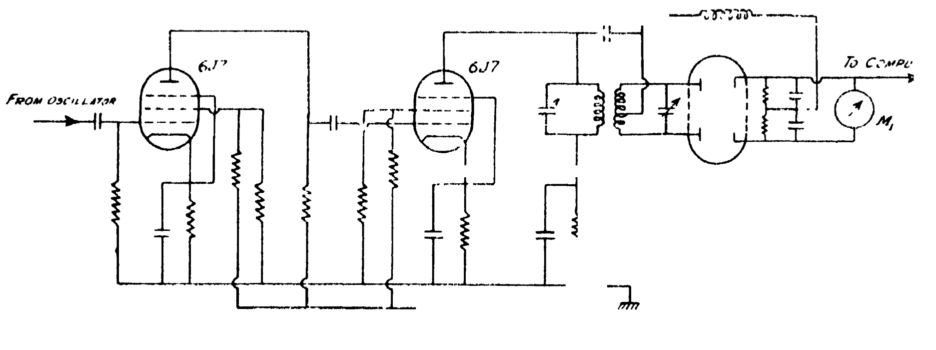

Fig 3. The circuit diagram of the limiter and discriminator

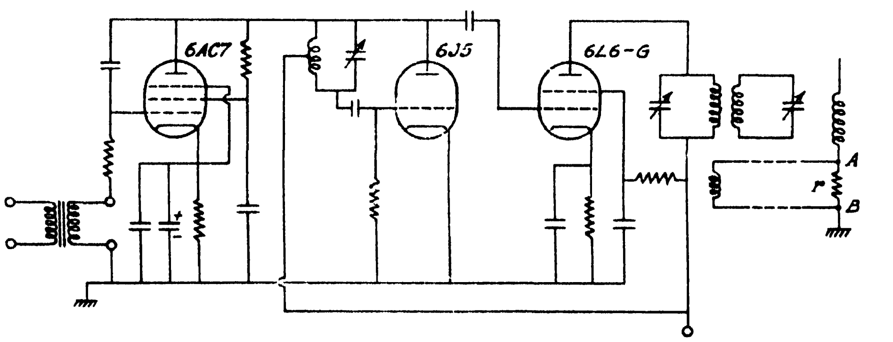

Fig. 4. The circuit diagram of the frequency-modulated oscillator and the amplifier for injecting constant voltage at variable frequency across the points AB (figure 2).

discriminator output is measured by a sensitive voltmeter which is calibrated directly in terms of capacity value. The output voltage is also fed into a computor system to be described later.

A separate auxiliary oscillator (figure 4) with a power amplifier is used to inject r. f. voltage of variable frequency across *r*, the points *AB* in figure 2. The oscillator is frequency-modulated by means of a reactance tube. The modulating signal to the reactance tube is obtained from the 50 c/s mains. The oscillator voltage is fed into a wide band power amplifier which is designed so as to give constant voltage across the load (*r*), over the entire frequency range at which the test circuit may be at resonance.

The power amplifier is gated by 50 c/s pulses with a duty cycle of 300 milliseconds. The gating pulses are obtained from the 50 c/s mains and the power amplifier is operated at the positive half cycles. Since the test circuit oscillator operates during the negative half cycles, the r. f. vlotage across *r* is injected only when the oscillator is not working. During this period the effect of V_1 (figure 2) on the tuned circuit is equivalent to that of a fixed small capacitance, since the cathode of V_1 is at a constant voltage (Jones and Ward, 1950).

The voltage across the tuning condenser appears twice in a full cycle of 50 c/s signal. During the negative half cycle the oscillator operates and r. f. voltage is developed across the tuned circuit. During the positive half cycle the frequency-modulated signal is applied across the small series resistance *r* in the tuned circuit, and when the frequency of injected voltage coincides with the resonant frequency of the tuned circuit, a magnified voltage (*Q* times the injected voltage) is produced across it. This voltage is measured by a vacuum tube peak-voltmeter. The voltmeter connections are so arranged that it is inoperative when the main oscillator (in the test circuit) operates and is operative only during the positive half cycle when the r. f. voltage is being injected across the resistance *r*. The circuit diagram of this arrangement is shown in figure 5.

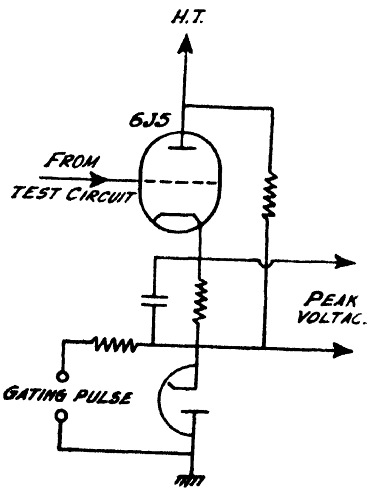

Fig. 5. The circuit diagram of the gated peak voltmeter which measures the voltage across the tuned circuit due to injection of voltage across *AB* (figure 2). The peak voltmeter is inoperative when the main oscillator is working.

The voltage at the cathode of the detector tube is maintained at such a value that during the positive half cycle, the tube is biased to exact cut-off. During this time any small voltage applied to the grid makes the tube conducting. During the negative cycle the tube is biased far beyond cut-off and the detector becomes inoperative. No output is, therefore, obtained from the detector when the main oscillator operates.

Since the voltage injected across the series resistance r is maintained constant (for all the frequencies), the peak voltmeter reading is a direct measure of the effective magnification factor of the circuit. When the test capacitor is included, the magnification factor of the circuit is reduced to a value, say Q, from the unloaded value Q_0. From the reduction of this magnification factor (Q_0-Q), the power factor of the test capacitor is calculated by means of the computor.

The Computor :

The purpose of the computor is to obtain from the two voltages, e. g. the output voltages of the peak voltmeter and of the discriminator, meter readings proportional to the power factor of the test capacitor. The schematic diagram of the computor circuit is given in figure 6. It consists of the multiplier, the subtractor and the divider.

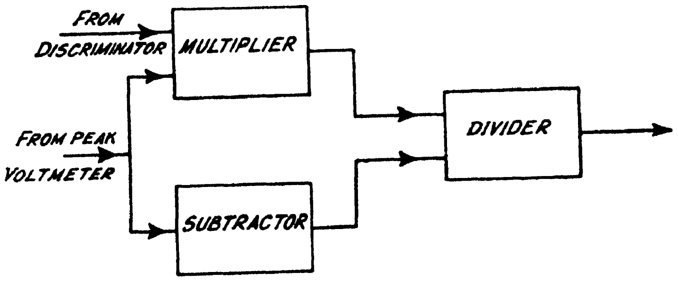

Fig. 6 Schematic diagram of the computor which produces a voltage proportional to the power factor of the capacitor.

(i) *The Multiplier*—It is known that when a multigrid tube is properly adjusted, its plate current varies linearly with the voltage injected in the control grid when the signal grid is kept at a constant voltage and vice versa. So, when both control grid and the signal grid voltages vary, the variation of plate current is proportional to the product of the two voltages injected in the two grids (Chance, et al).

This fact forms the basis of the design of the multiplier circuit. The plate current *versus* signal and control grid voltage characteristics were first studied with different voltages applied to the other electrodes of a pentagrid tube (1A7-GT) and the optimum condition for linear characteristics determined. It was found that with the given supply voltages to the plates and the electrodes G_2, G_3 and G_5, the plate current was proportional to the product of the voltages applied to the control (G_1) and signal (G_4) grids when the bias applied to these two grids were −14.0 and −4.5 volts respectively.

A resistance, small compared to the plate resistance of the tube, is included in the plate circuit. The output voltage appears across this resistance as shown in figure 7.

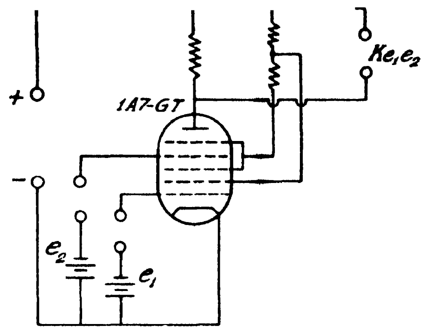

Fig. 7 Circuit diagram of the multiplier.

(ii) *The Subtractor.* For a triode with resistive load, operating in the linear portion of its grid voltage plate current characteristics, the drop in plate voltage for a voltage e applied to the grid is ke, where k is a constant depending on the plate current and the load value. Thus, if E_0 be the initial plate voltage, then the plate voltage when a signal e is applied to the grid is

$$E_0 - ke = k(e_0 - e)$$

where E_0 is equal to ke_0, e_0 being another constant.

In our case, the value of e_0 is made proportional to Q_0 and that of e to Q and thus the subtraction is effected (figure 8).

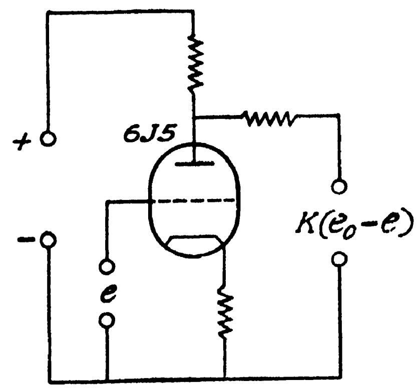

Fig. 8. Circuit diagram of the subtractor.

(iii) *The Divider.* Division is accomplished by first determining a quantity proportional to the reciprocal of the divisor and then multiplying it with the dividend. The principle of determining the inverse quantity of a given voltage is as follows :

If a voltage E is applied to the two resistances R_1 and R_2 in series, then the voltage e' across R_2 is given by

$$e' = E . \frac{R_2}{R_1 + R_2}$$

If $R_2 << R_1$, then,

$$e' \simeq E . \frac{R_2}{R_1}$$

Now, if R_1 be such that it can be varied linearly with any input voltage, *i.e.*, $R_1 = Ke_1$ then,

$$e' = E . \frac{R_2}{Ke_1} = K_1 . \frac{1}{e_1}$$

where

$$K_1 = \frac{ER_2}{K}$$

Now, ordinary triodes and pentodes do not show linear variation of plate resistance with grid voltage throughout their characteristics but do so only over very small ranges. This range, however, can be increased by using some sort of feed back. In our case, the pentode is used as the variable resistance whose screen voltage is controlled by another triode which in its turn is controlled by the voltage e_1.

Such a circuit used in conjunction with a multiplier circuit will give in its output a quantity proportional to the quotient of the two quantities. The circuit is shown schematically in figure 10. All interstage couplings used here are direct, and, therefore, the cathodes of succeeding tubes in the chain have to be maintained at higher and higher potentials.

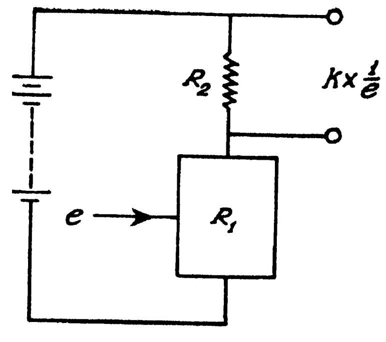

Fig. 9. Circuit for obtaining inverse voltage of a d.c. voltage

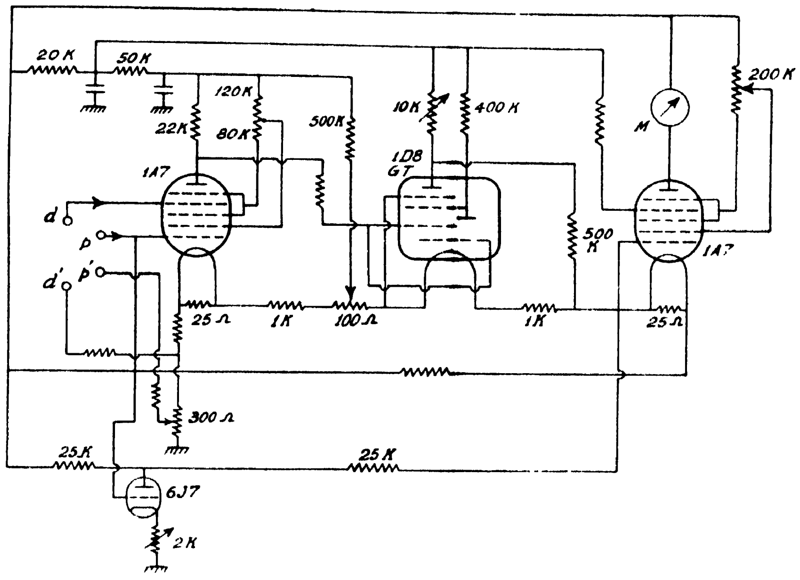

Fig. 10. Circuit diagram of the computor for calculating the power factor. It has two inputs. (1) The points *pp'* where the peak voltmeter output (figure 5) is fed and (2) the points *dd'* where the output of the discriminator is fed. The meter *M* reads the power factor

4. RESULTS OF MEASUREMENTS : CONCLUDING REMARKS

The capacitances and power factors of a number of condensers were measured by means of the instrument constructed and the values were also determined with the help of a Q-meter. The two sets of results are shown in Tables I and II for comparison.

TABLE I

Nominal value ±10%	Measured by Q-meter	Measured by the instrument (100μμF=0.95 volts)
10*pF* (or μμF)	9.5 pF	10.0 pF
10 ,,	10.5 ,,	10.5 ,,
30 ,,	31.0 ,,	30.5 ,,
30 ,,	30.0 ,,	30.0 ,,
52 ,,	54.0 ,,	54.0 ,,
52 ,,	55.0 ,,	55.0 ,,
82 ,,	82.0 ,,	82.0 ,,
82 ,,	82.5 ,,	82.5 ,,
130 ,,	127 ,,	126 ,,
130 ,,	128 ,,	127 ,,
170 ,,	170 ,,	167 ,,
170 ,,	168 ,,	165 ,,
220 ,,	220 ,,	215 ,,
220 ,,	222 ,,	218 ,,

TABLE II

Capacity (nominal value)	Calculated power factor	Measured power factor (.01 = 26μA)
80 pF	.0004	.0004
	.0011	.0011
	.0040	.0039
	.0073	.0073
	.0105	.0104
120 pF	.0013	.0013
	.0037	.0037
	.0065	.0064
	.0089	.0089

The apparatus is designed for measurements of condensers of small values (maximum 0.02 μ F) with power factors not exceeding 0.02. The accuracy of the measured values as obtained from direct readings of this instrument, is comparable to that obtained with any standard instrument for the purpose. It is possible to increase the ranges of both the values, but with some loss in accuracy. The instrument can also be modified to measure the inductance and Q values of coils at the working frequency of the instrument. With a little modification, measurement at other frequencies is also possible. Introduction of tuned circuits, tuned to different frequencies serves the purpose. By ganging them together, it is possible to switch over to other frequencies with a single control.

ACKNOWLEDGMENTS

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