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REVERSAL OF POLARISATION OF MICROWAVES FROM SUN-SPOTS

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ABSTRACT. The Zwaan-Kemble method for the calculation of the reflection coefficient of a barrier has been used to obtain the reflection coefficients of the ordinary and the extra-ordinary waves for a parabolic ion barrier. These results have been applied to explain the observed reversal of polarisation of microwaves escaping from sunspots.

I INTRODUCTION

Different workers at different stations have observed that microwaves received from sun-spots usually consist of a mixture of right-handed and left-handed circularly polarised components. Another characteristic feature observed is that when the sun-spot appears at an edge of the visible solar disc, one of the components is much stronger than the other, that is, the polarisation is almost purely right handed (or left-handed), but as the spot moves near the centre of the solar disc both the components become equally strong and as it disappears at the other edge of the disc the polarisation again becomes roughly pure but of the opposite kind, that is, left-handed (or right-handed). In a paper on the conditions of escape of micro-waves from sun-spots, Saha, Banerjea and Guha (1947) have theoretically shown that the polarisation of the escaping wave should be circular and that the magnetic field of the spot helps in the escape of one of the components. Ryle (1948) has also reached similar conclusions. But the characteristic feature mentioned above, namely, the remarkable change in the relative intensities of the two polarised components, has not been explained. In the present article we shall attempt to explain this feature by calculating the transmission co-efficients of a parabolic ion-barrier in a magnetic field. Since the details of the actually existing conditions in a sun-spot region are likely to be very complicated, the general conclusion reached in sec. VII has only qualitative significance; we have, therefore, based our treatment only on simple assumptions which nevertheless take account of the essential characteristics of the problem.

II WAVE EQUATIONS

For plane electromagnetic waves travelling in the direction of the concentration gradient of a non-homogeneous friction-free ionosphere, the following differential equations hold (Saha, Banerjea and Guha, 1947).

(A) Transverse case :

$$\frac{d^2E_x}{dz^2}+\frac{p^2}{c^2}(1-r)E_x=0 \quad \text{(ordinary wave)} \tag{2.1}$$

$$\frac{d^2E_y}{dz^2}+\frac{p^2}{c^2}\left(1-\frac{r}{1-\frac{\omega^2}{1-r}}\right)E_y=0 \quad \text{(extraordinary wave)} \tag{2.2}$$

(B). Longitudinal case : $\frac{d^2}{dZ^2}(E_x \pm iE_y)+\frac{p^2}{c^2}\left(1-\frac{r}{1\pm\omega}\right)(E_x \pm iE_y)=0$ ... (2.3)

where $(E_x, E_y)e^{ipt}$ = components of the electric vector, p being the pulsatance

z = distance measured along the direction of propagation

$$r=\frac{4\pi Ne^2}{mp^2}$$

N = ion density

e and m = charge and mass of the electron

$$\omega=\frac{p_h}{p}, \qquad p_h=\frac{eH}{mc}$$

H = strength of the external magnetic field.

III PARABOLIC LAYER

If the ion-barrier is parabolic with half width l, that is, if

$$N=N_m\left(1-\frac{z^2}{l^2}\right), \qquad \text{[see Fig.1.]}$$

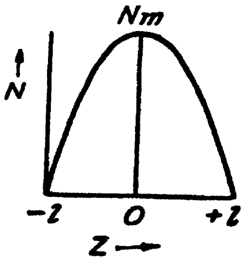

FIG. 1

then equations (2.1) and (2.2) respectively reduce to

$$\phi''+K^2(z^2-a^2)\phi=0 \tag{3.1}$$

and

$$\phi''+K^2\frac{(z^2-b^2)(z^2-c^2)}{Z^2-d^2}\phi=0 \quad \ldots \quad (3.2)^*$$

* This equation for the extra-ordinary wave was given to the author by Prof. M. N. Saha. It does not appear to have been discussed by any previous worker.

where $K^2=\frac{p_0^2}{c^2l^2}$, $a^2=\left(1-\frac{p^2}{p_0^2}\right)l^2$, $b^2=\left(1-\frac{p^2}{p_0^2}+\frac{pp_h}{p_0^2}\right)l^2$,

$$c^2=\left(1-\frac{p^2}{p_0^2}-\frac{pp_h}{p_0^2}\right)l^2, \qquad d^2=\left(1+\frac{p_h^2}{p^2}-\frac{p^2}{p_0^2}\right)l^2,$$

p_0 = critical pulsatance of the ordinary wave.

We shall now calculate the transmission coefficients of the barrier for waves satisfying equations (3.1) & (3.2) respectively. It will be atonce noticed that this problem is similar to the quantum-mechanical problem of calculating the transmission coefficient of a potential barrier for matter waves, which, as is well known, has been tackled by different workers in different ways, sometimes yielding different results for the same problem. We shall here adopt a method used by Kemble (1935) but with different approximating functions in place of the B.W.K. approximations used by him. This simplifies the calculations in our case, but to be more sure about the validity of our method, we shall also calculate the transmission coefficient of the ordinary wave in an alternative way. It is assumed that the width of the barrier is sufficiently large for the application of the following methods.

IV TRANSVERSE CASE
(Ordinary wave)

(A) *Kemble's method*:

Taking equation (3.1) we consider asymptotic representations of $\phi(z)$ of the form $z^\sigma e^{(Pz)}$ where σ is a constant and $P(z)$ a polynomial in z. These are found to be

$$\left.\begin{aligned} f_1(z) &= e^{\frac{1}{2}iKz^2}z^{-\frac{1}{2}-iKa/2} \\ f_2(z) &= e^{-\frac{1}{2}iKz^2}z^{-\frac{1}{2}+iKa^2/2} \end{aligned}\right\} \qquad \dots \quad (4.1)$$

and

The differential equations (in the normal form) satisfied by these functions are

$$\left.\begin{aligned} &f''_1+\left[K^2(z^2-a^2)-\frac{\sigma_1(\sigma_1-1)}{z^2}\right]f_1=0, \text{ where } \sigma_1=-\tfrac{1}{2}-\frac{iKa^2}{2} \\ \text{and } &f''_2+\left[K^2(z^2-a^2)-\frac{\sigma_2(\sigma_2-1)}{z^2}\right]f_2=0, \text{ where } \sigma_2=-\tfrac{1}{2}+\frac{iKa^2}{2} \end{aligned}\right\} \qquad \dots \quad (4.2)$$

Hence f_1 and f_2 will give good approximation for ϕ whenever z is large. We now propose to fit the linear combination $a_1(z)f_1(z)+a_2(z)f_2(z)$ to an exact solution $\phi(z)$ of the equation (3.1), so that we take

$$\left.\begin{aligned} a_1(z)f_1(z)+a_2(z)f_2(z) &= \phi(z) \\ a_1()zf_1'(z)+a_2(z)f_2'(z) &= \phi'(z) \end{aligned}\right\} \qquad \dots \quad (4.3)$$

Solving these equations for $a_1(z)$ and $a_2(z)$, we obtain

$$a_1(z) = \frac{1}{-2iK + \dfrac{iKa^2}{z}} (\phi f_2' - \phi' f_2)$$

$$\approx \frac{i}{2K}\left(\phi f_2' - \phi' f_2\right) \text{ for large } |z|$$

$$\frac{da_1}{dK} \approx \frac{i}{2K}\left(\phi f_2'' - \phi'' f_2\right)$$

$$= \frac{i}{2K}\,\frac{\sigma_2(\sigma_2 - 1)}{z^2}\,\phi f_2 \text{ from (3.1) and (4.2)}$$

$$= \frac{i\sigma_2(\sigma_2 - 1)}{2Kz^2}\left(\frac{a_1}{z} + \frac{e^{-iKz^2}}{z^{1-iKa^2}}\, a_2\right) \text{ from } \phi = a_1 f_1 + a_2 f_2 \text{ and equation (4.1)}$$

Hence
$$\left|\frac{da_1}{dz}\right| \leq \frac{A_1}{|z|^3}\left[E_1 |a_1| + |e^{-iKz^2}|\,|a_2|\right] \qquad \ldots \quad (4.4)$$

where A_1 and B_1 are positive constants of the problem.

Similarly
$$\left|\frac{da_2}{dz}\right| \leq \frac{A_2}{|z|^3}\left[B_2 |a_2| + |e^{iKz^2}|\,|a_1|\right] \qquad \ldots \quad (4.5)$$

We notice that on both sides of the the barrier $f_1(z)$ and $f_2(z)$ respectively represent waves entering and leaving the barrier. Now consider a path Γ in the lower half of the complex z-plane (Fig. 2), starting at $+l$, the right hand end of the barrier and terminating at $-l$ the left hand end, such that $|z|$ always remains large on Γ. Now, supposing that waves are incident on the left hand side of the barrier and transmited through the right hand side, we put $a_1 = o$ and $a_2 = 1$ at $z = +l$. From equation (4.4) it follows that

FIG. 2

throughout the first half of the path Γ (*i.e.* the portion in the 4th quadrant) a_1 remains practically constant (*i.e.* $a_1 = o$). Since $a_1 = o$ in this portion of the path it follows from equation (4.5) that a_2 also remains constant (*i.e.* $a_2 = 1$). These equations also show that a_2 remains constant throughout the rest of the path but a_1 may change. Thus, at $z = -l$ we have $a_1 = c$, say, and $a_2 = 1$. Hence we have established a connection formula $f_2 + cf_1 \longleftarrow f_2$

where
$$\left.\begin{aligned} f_2(+l) &\doteq e^{-\frac{1}{2}iKl^2}\, l^{-\frac{1}{2}+\frac{1}{2}iKa^2} \\ f_2(-l) &= i.e^{-\frac{1}{2}iKl^2}\, l^{-\frac{1}{2}+\frac{1}{2}iKa^2}\, e^{\frac{1}{2}\pi Ka^2} \\ f_1(-l) &= i.e^{\frac{1}{2}iKl^2}\, l^{-\frac{1}{2}-\frac{1}{2}iKa^2}\, e^{-\frac{1}{2}\pi Ka^2} \end{aligned}\right\} \qquad \ldots \quad (4.6)$$
and

Since damping has been neglected, we have

$$|c|^2 |f_1(-l)|^2 = |f_2(l)|^2 + |f_2(-l)|^2$$

$$\therefore \qquad |c|^2 = \frac{1 + e^{\pi K a^2}}{e^{-\pi K a^2}} \quad \text{using (4.6)}$$

If T_0 denotes the transmission coefficient, then

$$T_0^2 = \left| \frac{f_2(+l)}{c f_1(-l)} \right|^2$$

$$= \frac{1}{1 + e^{\pi K a^2}} = \frac{1}{1 + e^{\pi K l^2 (1 - p^2/p_0^2)}} \qquad \ldots \quad (4.7)$$

(B) *Alternative method* :

We look for a solution $\phi(z)$ of the differential equation (3.1) behaving asymptotically as follows :

$\phi(z) \sim \psi_3(z)$ in the neighbourhood of $z = +l$

and $\phi(z) \sim \psi_1(z) + \psi_2(z)$,, ,, ,, $z = -l$,

where ψ_2 and ψ_3 both represent waves leaving the barrier, and ψ_1 represents waves entering the berrier, so that

$\psi_1(z)$ represents the incident weve

$\psi_2(z)$,, ,, reflected wave

and $\psi_3(z)$,, ,, transmitted wave

The reflection coefficient R_0 will therefore be given by $\left| \frac{\psi_2(z)}{\psi_1(z)} \right|$

Change the independent variable in equation (3.1) from z to ξ where $\xi = \sqrt{2K} e^{i\pi/4} z$ and put $n = \frac{iKa^2}{2} - \frac{1}{2}$. We get

$$\frac{d^2\phi}{d\xi^2} + \left(n + \frac{1}{2} - \frac{\xi^2}{4} \right) \phi = 0 \qquad \ldots \quad (4.8)$$

Moreover, $\arg \xi = \frac{\pi}{4}$ for z real and positive

and $= -\frac{3\pi}{4}$ for z real and negative.

The differential equation (4.8) is known as Weber's equation. From the properties of its solution $D_n(\xi)$ discussed in Whittaker and Watson's book on Modern analysis (pp. 347-349) we get

for $\arg \xi = \frac{\pi}{4}$, $D_n(\xi) \approx e^{-\xi^2/4} \xi^n$ (transmitted wave) and for $arg\,\xi = -\frac{3\pi}{4}$, $D_n(\xi) \approx e^{-\xi^2/4}\,\xi^n$ (reflected wave)

$$+\frac{\sqrt{2\pi}}{\Gamma(-n)}e^{-n\pi i}e^{\xi^2/4}\,\xi^{-n-1} \text{ (incident wave)}$$

Hence, from the remarks made above, it follows that

$$R_0 = \left| \frac{e^{-\xi^2/4}\,\xi^n}{\frac{\sqrt{2\pi}}{\Gamma(-n)}e^{-n\pi i}\,e^{\xi^2/4}\,\xi^{-n-1}} \right|$$

$$= \frac{1}{\sqrt{2\pi}}\left|\Gamma(-n)e^{-\xi^2/2}\,\xi^{2n+1}\,e^{n\pi i}\right|$$

$$= \frac{e^{\pi Ka^2/4}}{\sqrt{2\pi}}\left|\Gamma\left(-\frac{iKa^2}{2}+\frac{1}{2}\right)\right|$$

$$= \frac{e^{\pi Ka^2/4}}{\sqrt{2\cosh \pi Ka^2/2}}$$

$$\therefore \qquad T_0^2 = 1 - R_0^2$$

$$= \frac{1}{1+e^{\pi Ka^2}}$$

$$= \frac{1}{1+e^{\pi Kl^2(1-p^2/p_0^2)}}$$

which is the same as equation (4.7).

V. TRANSVERSE CASE
(Extra-ordinary wave)

Taking equation (3.2) and adopting method (A) of sec. IV, we find that in this case the asymptotic representations of $\phi(z)$ are

$$f_1(z) = e^{\frac{iKz^2}{2}}\,z^{-\frac{1}{2}-iK(b^2+c^2-d^2)}$$

and

$$f_2(z) = e^{-\frac{iKz^2}{2}}\,z^{-\frac{1}{2}+iK(b^2+c^2-d^2)}$$

In this case also it can be easily shown that the coefficient function occurring in equation (3.2) differs from the coefficient functions occurring in the differential equations (in the normal form) satisfied by $f_1(z)$ and $f_2(z)$ by terms of the order of $1/z^2$. Hence, the method applied in (A) of sec IV., can be adopted here without any alteration. Thus, if T_e denotes the transmission coefficient in this case, then

$$T_e^2 = \frac{1}{1+e^{\pi K(b^2+c^2-d^2)}}$$

$$T_e^2 = \frac{1}{1+e^{\pi K\left(1-\frac{p^2}{p_0^2}-\frac{p_h^2}{p_0^2}\right)}} \qquad \dots (5.1)$$

$$\simeq 1$$

for large p_h, that is for large magnetic field.

Had we assumed that the magnetic field also varies parabolically, so that $H = H_m\left(1-\frac{z^2}{l'^2}\right)$, say, then in place of equation (3.2) we would have obtained

$$\phi'' + K^2 \frac{(z^2-a'^2)(z^2-b'^2)}{(z^2-c'^2)(z^2-d'^2)}\phi = 0 \qquad \dots (5.2)$$

This gives $T_e^2 = 1.$

Thus when the magnetic field is large and constant, or when it varies as above, the barrier is almost transparent to the extra-ordinary wave.

VI. LONGITUDINAL CASE

If we consider a parabolic ion-barrier $N = N_m\left(1-\frac{z^2}{l^2}\right)$ and a large constant magnetic field so that, $\omega_c \gg 1$, then equation (2.3) gives two circularly polarised waves of opposite senses determined by equations of the type

$$\phi_1'' + K^2(z^2-a^2)\phi_1 = 0 \qquad \dots (6.1)$$

and

$$\phi''_2 - K^2(z^2-a^2)\phi_2 = 0 \qquad \dots (6.2)$$

Since the disturbance satisfying equation (6.2) has no wave character for $|z|>a$, it can not leak through the barrier as a wave. Hence, only the component satisfying equation (6.1) leaks through the barrier with a certain transmission coefficient.

But if we assumed a parabolic variation for the magnetic field as well, so that $H = H_m\left(1-\frac{z^2}{l'^2}\right)$, then we would have obtained in place of the equations (6.1) and (6.2) the equations

$$\phi_1'' + K^2 \frac{(z^2-a^2)}{(z^2-b^2)}\phi_1 = 0 \qquad \dots (6.3)$$

and

$$\phi_2'' + K^2 \frac{(z^2-a'^2)}{(z^2-b'^2)}\phi_2 = 0 \qquad \dots (6.4)$$

The equations (6.3) and (6.4) are easily seen, by the methods already discussed, to give values of the transmission coefficients

as
and

$$\left.\begin{array}{l} T_1^2 = 1 \\ T_2^2 = 1 \end{array}\right\} \qquad \dots (6.5)$$

The assumed variation of the magnetic field thus makes the barrier equally transparent to both the circularly polarised components.

VII. MICROWAVES FROM SUNSPOTS

Now consider a sunspots S moving along the central equator from one edge of the visible solar disc to the other edge. Consider the plane containing spot S the direction SE towards the earth and the normal SN to the sun's surface at S (Fig.3), and suppose that projected on this planes the

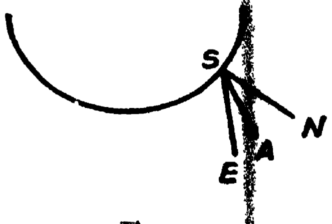

FIG. 3

positive direction of the magnetic lines of force make on the average a mean angle $\alpha(\angle NSA)$ with the positive direction of SN. Then it follows that the angle θ between the direction of propagation of the microwaves towards the earth and the positive direction of the magnetic lines of force varies in magnitude from $\left|\frac{\pi}{2}-\alpha\right|$ to $\left| \quad +\alpha\right.$. The cases $\theta=0, <\frac{\pi}{2}, =\frac{\pi}{2}, >\frac{\pi}{2}, =\pi$ correspond respectively to south pole, southern hemisphere, equator, northern hemisphere, north pole (in the language of propagation in the earth's ionosphere). Hence, the propagation of the microwaves through the ionosphere of the spot, as the latter moves from one edge of the solar disc to the other edge, has one-to-one correspondence with propagation of radio waves through the earth's ionosphere when the source on the earth traverses an angle π along a meridian from a point P to a point Q, say.

From Secs. IV and V it follows that when this propagation becomes tranverse, $T_e^2 \simeq 1$ and is much greater than T_o^2, so that one of the polarised components is much stronger than the other, where as when the propagation becomes longitudinal, we have $T_1^2=T_2^2=1$ from equation (6.5), so that the two polarised components are equally strong. It is natural to expect that similar results will hold for the quasi-transverse and the quasi-longitudinal cases. This gives the reason for the change in the relative intensities of the two polarised components mentioned in sec. I. The reversal of the senses of polarisation of the two components obviously depends upon the angle α, because the senses of polarisation of the two components are opposite in the two hemispheres.

For example, in the case of a uni-polar spot we may take $\alpha=0$ or π, so that the path on the meridian from P to Q lies on the same hemisphere and there should therefore be no reversal of the sense of polarisation. On the other hand if the spot is a member of a bi-polar spot group, then α should be different from 0 and π, so that P and Q lie in different hemispheres. Hence, there will be a reversal of the senses of polarisation, the actual duration through which the stronger component remains left-handed or right-handed depending upon the actual value of α.

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