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PULSE WIDTH MEASUREMENTS IN RADAR

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Plate II

ABSTRACT. A very simple method of measuring short duration radar pulses, positive or negative has been developed. The theory of calculation of pulse duration after taking a photograph of the pulse superposed on an elliptical time base on an oscilloscope screen is completely given. The method is not only very accurate but also less costly than the existing elaborate method. Range marker units in radar equipments can also be calibrated easily using this method.

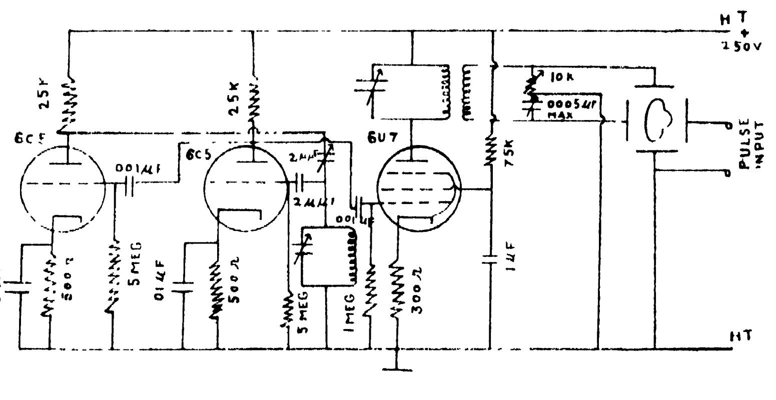
There are at present only a few accurate methods of measuring short duration pulses. The duration of the pulse used in radar transmission must be determined accurately so that not only the range of a target but also the peak power and the minimum detectable distance of the radar unit can be determined accurately.

Ludman (1945) uses a high speed linear time-base of duration 3 or 4 microseconds. The pulse, whose width is to be determined, is applied to the vertical deflection plates of a cathode-ray tube. Accurate measurements of pulse width can be made with this method provided the time rate of change per unit of deflection is known. To determine this, a damped sine-wave calibrating signal from a shock excited oscillator syncronised with the time-base is fed to the vertical deflection plates. The time interval between successive peaks depends upon the resonant frequency. The constanty of the frequency of the resonant circuit under continuous wave condition as well as under shock excitation is doubtful. Further, the time interval between any two successive cycles for damped waves is not the same. To overcome some of these disadvantages Ludman later on used a 5 megacycle crystal oscillator for calibration. Even then there are some disadvantages in this method. The sweep position on the oscilloscope screen should be kept the same for all measurements. Measurements as well as calibration are to be done near the centre of the screen in the same portion of the sweep.

Neglecting the above method of measuring pulse width using a linear saw-tooth sweep, Allan Easton (1946) uses a circular sweep with the pulse superposed on it. The frequency of the oscillator producing the sweep is continuously varied until a stationary pattern results on the screen of the oscilloscope. He also measured the pulse width by superposing on a circular sweep, the differentiated pulses obtained as a result of passing the pulse, whose width is to be measured, through a short CR circuit. The spacing between the differentiated pulses on the circular sweep gives an idea of the duration of the pulse. An elliptical time-base has also been used with considerable accuracy. But the method has got several inherent disadvantages. The resolution of the coincidence sets the maximum accuracy of measurements. In this method it is absolutely essential to have the pulse on the maximum velocity portion of the sweep and it is assumed that two-thirds of the horizontal deflection on the elliptical sweep corresponds to the maximum velocity portion.

The following method eliminates almost all the above restrictions. The calculations are simple and easy. The resolution error can be reduced to zero by taking the corresponding points at the feet of each pulse and thereby the pulse width can be calculated twice for the same pulse. The pulse may be displayed on any part of the elliptical sweep and yet its duration can be evaluated easily. The equipment is simple and easily operated.

The whole equipment consists of a Franklin oscillator whose frequency can be varied. The output of this is amplified by means of a tuned R.F. amplifier having a tuned circuit on the plate of the valve. The use of the amplifier suppresses any harmonic present. The amplified output is phase-shifted through 90° by means of a suitable condenser, and a potentiometer and the R.f. across the condenser and the potentiometer are applied to one horizontal and one vertical deflection plate, the junction of the condenser and the potentiometer which is earthed, is connected to another vertical deflection plate. The pulse output from the range calibrator unit TS-102/AP obtained from Army disposals is fed between the other horizontal deflection plate and earth. By varying the frequency of the oscillator a steady picture is obtained. To make the frequency of the oscillator and the R.F. amplifier steady without any variation due to voltage fluctuations of the high tension, a specially constructed voltage-stabilised power pack, giving less than 1 % variation in the output voltage even if there is a change of 20 volts on either side



FIG

of the 110 volts input to the primary of the power transformer is used. The circuit diagram with the values of components is given in figure 1.

THEORY OF CALCULATIONS

Let us consider an ellipse (figure 2) of major axis $2a$, minor axis $2b$ and having a pulse superposed on it. The ellipse is the resultant of two simple harmonic motions at right angles of same frequency but of different amplitudes. The co-ordinates of any point on the ellipse with respect to the centre of the ellipse as the origin can be represented as

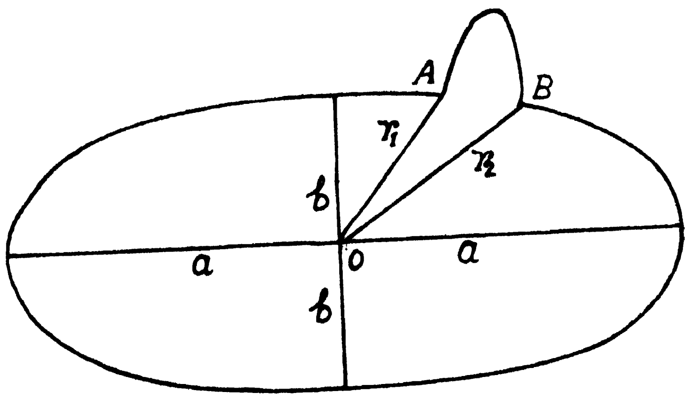


FIG 2

$$X = a \cos \omega t$$

and $$Y = b \sin \omega t$$

Let x_1 and y_1 be the co-ordinates of A, the foot of the leading edge of the pulse on the ellipse.

Then $$x_1 = a \cos \omega t_1 \qquad \dots (1)$$

and $$y_1 = b \sin \omega t_1 \qquad \dots (2)$$

But the polar co-ordinates of the same point can be represented by $r_1 \cos \theta_1$ and $r_1 \sin \theta_1$ where r_1 is the radius vector at A and θ the angle which it makes with the X-axis.

Then $$r_1 \cos \theta_1 = a \cos \omega t_1 \qquad (3)$$

$$r_1 \sin \theta_1 = b \sin \omega t_1 \qquad (4)$$

Similarly for the point B, the foot of the trailing edge of the pulse, the co-ordinates are

$$r_2 \cos \theta_2 = a \cos \omega t_2 \qquad (5)$$

$$r_2 \sin \theta_2 = b \sin \omega t_2 \qquad (6)$$

Taking equation (3), the time t_1 taken to describe that arc of the ellipse between the point A and the perihelion is given by

$$t_1 = 1/\omega \left[\cos^{-1} \frac{r_1}{a} \cos \theta_1 \right] \qquad \dots (7)$$

Taking the corresponding equation (5) for B

we have $$t_2 = 1/\omega \left[\cos^{-1} \frac{r_2}{a} \cos \theta_2 \right] \qquad \dots (8)$$

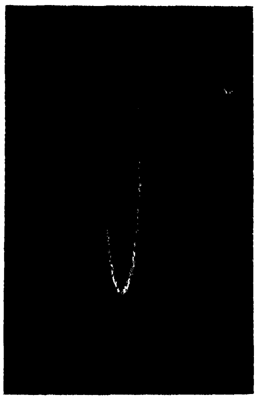


Fig. *a*.

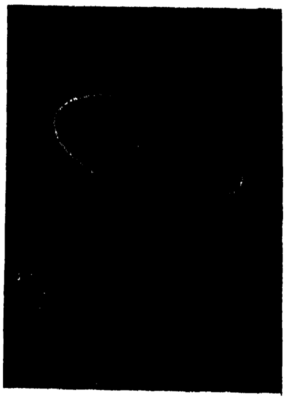


Fig. *b*.

Oscillograms of pulses superposed on eliptical time base.

The actual pulse duration, which is equal to $t_2 - t_1$, is given as

$$t_2 - t_1 = \frac{1}{\omega}\left[\cos^{-1}\frac{r_2}{a}\cos\theta_2 - \cos^{-1}\frac{r_1}{a}\cos\theta_1\right]$$

$$= \frac{1}{2\pi f}\left[\cos^{-1}\frac{r_2}{a}\cos\theta_2 - \cos^{-1}\frac{r_1}{a}\cos\theta_1\right] \quad \ldots (9)$$

where f is the frequency of the elliptical sweep.

Similarly the pulse duration can be determined taking into consideration the Y co-ordinates of the points A and B on the ellipse.

$$\therefore\ t_2 - t_1 = \frac{1}{2\pi f}\left[\sin^{-1}\frac{r_2}{b}\sin\theta_2 - \sin^{-1}\frac{r_1}{b}\sin\theta_1\right] \quad \ldots (10)$$

These formulae can further be simplified by substituting the values of $\cos\theta$ and $\sin\theta$ in terms of a, b and r.

The polar equation to the ellipse is

$$\frac{r^2\cos^2\theta}{a^2} + \frac{r^2\sin^2\theta}{b^2} = 1 \quad \ldots (11)$$

$$\frac{r^2}{a^2}\cos^2\theta + \frac{r^2}{b^2}(1 - \cos^2\theta) = 1$$

$$r^2\frac{(b^2 - a^2)}{a^2b^2}\cos^2\theta = \frac{b^2 - r^2}{b^2}$$

$$\cos\theta = \frac{a}{r}\sqrt{\frac{b^2 - r^2}{b^2 - a^2}} \quad \ldots (12)$$

similarly

$$\sin\theta = \frac{b}{r}\sqrt{\frac{r^2 - a^2}{b^2 - a^2}} \quad \ldots (13)$$

Equation (9) becomes

$$t_2 - t_1 = \frac{1}{2\pi f}\left[\cos^{-1}\sqrt{\frac{b^2 - r^2_2}{b^2 - a^2}} - \cos^{-1}\sqrt{\frac{b^2 - r^2_1}{b^2 - a^2}}\right] \quad \ldots (14)$$

Equation (10) becomes

$$t_2 - t_1 = \frac{1}{2\pi f}\left[\sin^{-1}\sqrt{\frac{r^2_2 - a^2}{b^2 - a^2}} - \sin^{-1}\sqrt{\frac{r^2_1 - a^2}{b^2 - a^2}}\right] (15)$$

Thus for the same pulse the duration can be caculated twice. One important precaution to be observed here is that when A and B are on either side of the centre of the ellipse, the supplement of the angle which the first factor gives or the complement of the angle given by the second factor in the above equations will have to be considered for proper signs for the displacements.

Measurements were taken on the enlargement of the photograph of the trace shown in Plate II for the two resolved pulses.

TABLE I

Frequency 329.4 kilocycles per second

Pulse	$2a$	$2b$	r_1	r_2	$\cos^{-1}\sqrt{\frac{b^2-r^2}{b^2-a^2}}$	$\sin^{-1}\sqrt{\frac{r^2-a^2}{b^2-a^2}}$	t_2-t_1 in μ second
I	29.2 cm	5.0 cm	10.9 cm	2.6 cm	42°29′ 87°9′	—	.38
I	29.2 „	5.0 „	10.9 „	2.6 „	—	42°28′ 87°12′	.38
II	29.2 „	5.0 „	9.5 „	2.8 „	50°25′ 95°2′	—	.38
II	29.2 „	5.0 „	9.5 „	2.8 „	—	50°25′ 95°5′	.38
Fig *b*, Plate II	28.7 „	10.0 „	8.6 „	5.9 „	58 9′ 103°28′	—	.38
	28.7 „	10.0 „	8.6 „	5.9 „	—	58°39′ 103°28′	.38

This method can also be used for calibrating range marker units employed in the radar equipments. When a nearly steady pattern of the ellipse having the pulse superposed on it is obtained by controlling the frequency of the oscillator, the duration of the sweep, which is equal to the pulse spacing, is obtained by finding the reciprocal of the frequency. This gives the total time taken by the electro-magnatic waves to travel from the radar unit to the target and from the target to the unit. Knowing the velocity of the electro-magnatic waves to be 984 ft. per microsecond, the range between two successive pulses can be calculated. In the present experiment the frequency of the sweep as measured by a precision GE wavemeter is obtained as 329.4 Kc/s. Therefore the duration is 3.042 micro-seconds. The distance travelled by electro-magnatic waves through half this time is equal to 1496 ft. But the actual spacing between the consecutive range marker pulses for the unit TS—102/AP is equal to 1500 ft. The error of calibration is less than .3% which is very small. The method therefore offers solution not only for measuring small duration pulses but also for calibrating range marker units.

ACKNOWLEDGMENT

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