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ON THE ORIGIN OF THE THIRD IONOSPHERIC ECHO*

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ABSTRACT. There is still some difference of opinion regarding the rav, ordinary or extraordinary, to which the so-called third or Z-component of the ionospheric echoes correspond. It is pointed out in the paper that there need be no controversy on the subject because, as first clearly shown by Bhar, the branch of the dispersion curve which passes through the "point" of reflection of the Z-component is really an ordinary branch. Calculations are further carried out to determine the retardation that the extraordinary ray would suffer, if it were able to partially penetrate its first point of reflection, in its passage into regions of higher electronic densities. It is found that the retardation on reaching the asymptote of the dispersion curve would become infinite, so that, the extraordinary ray, under no circumstances, is able to penetrate the barrier. One is thus forced to the alternative that it is the ordinary ray which provides the Z-component in the ionospheric echo pattern.

INTRODUCTION

In the usual C. R. tube displays of ionospheric echoes, split echoes are of common occurence. Generally, two split echoes are observed due to the breaking up of the incident ray into two component rays—ordinary and extraordinary—by the anisotropy introduced by the earth's magnetic field. According to the ray theory (based on the Appleton-Hartree magneto-ionic theory) these echoes are due to reflections of the extraordinary ray at the level where the electron concentration is that represented at δE_1 (figure 1) and of the ordinary ray the same as at O. The conditions of reflection at these points are $\dfrac{Ne^2}{\pi m} = f^2 - ff_H$ and $\dfrac{Ne^2}{\pi m} = f^2$ respectively, where N is the number density of the electrons, f the sounding frequency and e and m have their usual significance. Occasionally, however, three split echoes are observed. The third, the so-called Z component, is assumed to be due to the reflection of a ray which has partially penetrated its usual reflection level (at E_1 or O) and has suffered reflection from the level of electron concentration as at the point E_2. The reflection condition at this point is given by $\dfrac{Ne^2}{\pi m} = f^2 + ff_H$.

On rare occasions echoes are obtained which cannot be identified with any of the three echoes described above. This echo is sometimes described us an extraordinary echo due to a fourth reflection condition (Pant and Bajpai, 1936-7) corresponding to the level where the electron concentration

is   as   represented   at D.   The   reflection   condition   is   given   by

$$\frac{Ne^2}{\pi m} = f^2 \, \frac{f^2 - f_H{}^2}{f^2 - f_L{}^2} \, .$$

Now, regarding the Z-component there is some controversy as to which of the two rays—ordinary or extraordinary—this component corresponds. For example, while according to Seaton (1948), the Z-component corresponds to the extraordinary ray, according to Scott (1950) and others, it belongs to the ordinary ray. But there need be no confusion if it be remembered that the so-called third point of reflection (E_2) really belongs to the ordinary ray. Considering figure 1 it will be noticed that the portion FE_2 of the
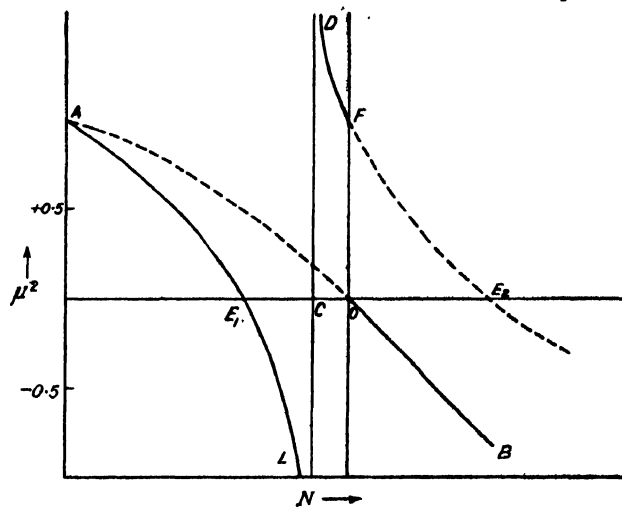

FIG. 1

dispersion curve which meets the N-axis at the point E_2 is continuous with the portion DF. It has, however, been pointed out by Bhar (1936-7) that while the portion DF belongs to the extraordinary branch, the portion FE_2 really belongs to the ordinary branch. In fact, it has been shown that the ordinary branch AO gives a discontinuous jump at O and proceeds along FE_2 instead of along OB while the latter is really a portion of the extraordinary branch.

It is the purpose of the present work to provide further evidence in corroboration of the above. By actually calculating the retardation suffered by the extraordinary ray before it reaches the third point of reflection, it is found that the ray suffers infinite retardation at the asymptote long before it can reach the point E_2.

Incidentally, the fact that the extraordinary ray suffers infinite retardation in reaching the assymptote shows that the fourth reflection

condition $\dfrac{Ne^2}{\pi m} = f^2 . \dfrac{f^2 - f_H{}^2}{s^2 - f_L{}^2}$ is inoperative.   It is, however, occasionally reported that evidence of a "fourth" echo has been obtained (Martyn and Munroe, 1938 ; Pant and Bajpai, 1936-7). This can only be explained as due to some ionospheric conditions in which, on account of increased

collision frequency, the infinities in dispersion curve are substantially reduced.

The calculations of retardation that are to follow are all based on the ray theory. This is not strictly justified, because, according to the ray theory, there is no room for partial reflection and penetration. However, it is found that the results of retardation according to the ray theory differ only by a few per cent from those calculated from the wave theory (Poeverlein, 1951). It is to be noted that a parabolic gradient of ionisation has been assumed. But results obtained in respect of infinite retardation do not depend to any marked extent on the shape of the rising ionisation gradient.

GROUP RETARDATION IN A PARABOLIC LAYER

In carrying out the calculation for retardation for any ray beyond the point of reflection, one has to use the portion of the dispersion curve below the N axis. The expression for μ for this portion of the curve, however, is purely imaginary and as such has no physical significance. It is also simply understood from the ray theory that for such cases the ray cannot penetrate beyond the reflection point and the portion of the curve below the N axis loses its meaning. However, if effect of collision is taken into account, the refractive index becomes, complex. The real part of this complex expression is everywhere positive and the numerical value of μ, as obtained from this positive part, is not greatly different from the modulus of μ in the collision-free case, Hence, the branch E_1 D of the dispersion curve (figure 2) may be considered as the mirror image of the part below the N axis in figure 1. The correctness of this qualitative approach is corroborated by Booker (1935) and also by the similarity with the dispersion curves (Ghosh, 1938 ; Taylor, 1933) calculated by taking collision into account. The analysis that is to follow is based on the first order correctness of figure 2. In all other respects the propagation properties are assumed to be the same as that given by or derived from the Appleton-Hartree formula for the collision-free case.
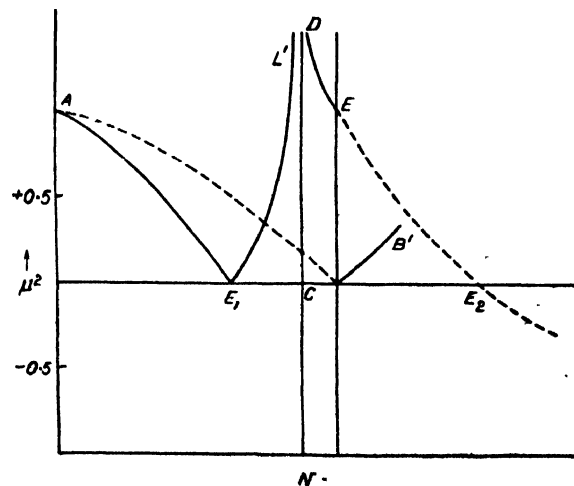

FIG. 2

For the group velocity U, we use the expression deduced by Rai (1937) :

$$\frac{c}{U} = \left\{ \left(\frac{f^2}{f_0^2} - 1 - \frac{f_T^2}{2f_0^2} \pm \sqrt{\left(\frac{f}{f_0}-\frac{f_0}{f}\right)^2 \frac{f_L^2}{f_0^2} + \frac{f_T^4}{4f_0^4}}\right) \left(\left(\frac{f}{f_0}-\frac{f_0}{f}\right)^2 - \frac{f_T^2}{2f_0^2} \pm \sqrt{\left(\frac{f}{f_0}-\frac{f_0}{f}\right)^2 \frac{f_L^2}{f_0^2} + \frac{f_T^4}{4f_0^4}}\right) \right\}^{1/2}$$

$$\left\{\left(\frac{f^2}{f_0^2} - 1 - \frac{f_T^2}{2f_0^2} \pm \sqrt{\left(\frac{f}{f_0}-\frac{f_0}{f}\right)^2 \frac{f_L^2}{f_0^2} + \frac{f_T^4}{4f_0^4}}\right)\right\}^{9/2}$$

$$+ \left[\frac{f^2}{f_0^2} - 1 - \frac{f_T^2}{2f_0^2} \pm \sqrt{\left(\frac{f}{f_0}-\frac{f_0}{f}\right)^2 \frac{f_L^2}{f_0^2} + \frac{f_T^4}{4f_0^4}}\right]^{9/2} \left(\frac{f}{f_0}-\frac{f_0}{f}\right)^2 + \frac{f_T^2}{2f_0^2} \mp \frac{f_0^2}{8f\delta^2}\sqrt{\left(\frac{f}{f_0}-\frac{f_0}{f}\right)^2 \frac{f_L^2}{f_0^2} + \frac{f_T^4}{4f_0^4}}$$

$$\left(\frac{f}{f_0}-\frac{f_0}{f}\right)^2 \left(1 + \frac{f_0^2}{f^2}\right) \frac{f_L^2}{f_0^2}$$

$$\pm 2\sqrt{\left(\frac{f}{f_0}-\frac{f_0}{f}\right)^2 \frac{f_L^2}{f_0^2} + \frac{f_T^4}{4f_0^4}} \left(\frac{f^2}{f_0^2} - 1 - \frac{f_T^2}{4f_0^2}\right)^{3/2} \left\{\left(\frac{f}{f_0}-\frac{f_0}{f}\right)^2 - \frac{f_T^2}{2f_0^2} \pm \sqrt{\left(\frac{f}{f_0}-\frac{f_0}{f}\right)^2 \frac{f_L^2}{f_0^2} + \frac{f_T^4}{4f_0^4}}\right\}^{3/2}$$

in which

$f \equiv$ the sounding frequency

$f_0 \equiv$ the penetration frequency of the ordinary ray

$f_T = f_H \sin \theta$

$f_L = f_H \cos \theta$

where f_H is the gyro-frequency, and θ is the angle of inclination of the direction of propagation to the magnetic field. For convenience we can re-write the above as

$$\frac{c}{U} = \left(1 - \frac{f_0^2}{f^2} - \frac{f_T^2}{2f^2} \pm \sqrt{\left(1-\frac{f_0^2}{f^2}\right)^2 \frac{f_L^2}{f^2} + \frac{f_T^4}{4f^4}}\right)^{3/2} \left\{\left(1-\frac{f_0^2}{f^2}\right)\left(1 - \frac{f_0^2}{f^2} - \frac{f_T^2}{2f^2} \pm \sqrt{\left(1-\frac{f_0^2}{f^2}\right)^2 \frac{f_L^2}{f^2} + \frac{f_T^4}{4f^4}}\right)\right.$$

$$\left\{\left(1-\frac{f_0^2}{f^2}\right) - \frac{f_T^2}{2f^2} \pm \sqrt{\left(1-\frac{f_0^2}{f^2}\right)^2 \frac{f_L^2}{f^2} + \frac{f_T^4}{4f^4}}\right\}^{1/2}$$

$$+\frac{f_0^2}{f^2}\cdot\frac{\left(1-\frac{f_0^2}{f^2}\right)+\frac{1}{2}\cdot\frac{f_T^2}{f^2}\mp\sqrt{\left(1-\frac{f_0^2}{f^2}\right)^2\frac{f_L^2}{f^2}+\frac{f_T^4}{4f^4}}}{\left(1-\frac{f_0^2}{f^2}-\frac{f_T^2}{2f^2}\pm\sqrt{\left(1-\frac{f_0^2}{f^2}\right)^2\frac{f_L^2}{f^2}+\frac{f_T^4}{4f^4}}\right)^{3/2}\left\{\left(1-\frac{f_0^2}{f^2}\right)^2\frac{f_L^2}{f^2}-\frac{f_T^2}{2f^2}\pm\sqrt{\left(1-\frac{f_0^2}{f^2}\right)^2\frac{f_L^2}{f^2}+\frac{f_T^4}{4f^2}}\right\}^{1/2}}$$

$$\pm\frac{f_0^2}{f^2}\cdot\frac{\sqrt{\left(1-\frac{f_0^2}{f^2}\right)^2\frac{f_L^2}{f^2}+\frac{f_T^4}{4f^4}}\left(1-\frac{f_0^2}{f^2}-\frac{f_T^2}{2f^2}\pm\sqrt{\left(1-\frac{f_0^2}{f^2}\right)^2\frac{f_L^2}{f^2}+\frac{f_T^4}{4f^4}}\right)^{3/2}}{2\sqrt{\left(1-\frac{f_0^2}{f^2}\right)^2\frac{f_L^2}{f^2}-\frac{f_T^2}{2f^2}\pm\sqrt{\left(1-\frac{f_0^2}{f^2}\right)^2\frac{f_L^2}{f^2}+\frac{f_T^4}{4f^4}}}\left(1-\frac{f_0^2}{f^2}\right)^2\left(1+\frac{f_0^2}{f^2}\right)}$$

Now in a parabolic layer of semi-thickness y_m, the ionic density can be represented by

$$f_0^2=f_c^2\left(2\frac{y}{y_m}-\frac{y^2}{y_m^2}\right)$$

or, putting

$$\xi=1-\frac{y}{y_m}$$

$$f_0^2=f_c^2(1-\xi^2).$$

so that, representing c/U, the retardation per unit path as a function of y or, of ξ we have, putting

$$\frac{f_c^2}{f^2}=\theta_c;\ \frac{f_T^2}{f^2}=\theta_T;\ \frac{f_L^2}{f^2}=\theta_L$$

$$\frac{c}{U}=\frac{\left\{1-\theta_c(1-\xi^2)-\frac{\theta_T}{2}\pm\sqrt{\{1-\theta_c(1-\xi^2)\}^2\theta_L+\frac{\theta_T^2}{4}}\right\}^{1/2}}{\left\{1-\theta_c(1-\xi^2)-\frac{\theta_T}{2}\pm\sqrt{\{1-\theta_c(1-\xi^2)\}^2\theta_L+\frac{\theta_T^2}{4}}\right\}^{3/2}\left[\{1-\theta_c(1-\xi^2)\}^2-\frac{\theta_T}{2}\pm\sqrt{\{1-\theta_c(1-\xi^2)\}^2\theta_L+\frac{\theta_T^2}{4}}\right]^{1/2}}$$

$$+ \frac{\theta_c(1-\xi^2)\{1-\theta_c(1-\xi^2)\}^2 + \theta_c(1-\xi^2)^2(1-\xi^2)\frac{\theta_T}{2} \mp \theta_c{}^2(1-\xi^2)^2\sqrt{\{1-\theta_c(1-\xi^2)\}^2\theta_L + \frac{\theta_T{}^2}{4}}}{\{1-\theta_c(1-\xi^2)\} - \frac{\theta_T}{2} \pm \sqrt{\{1-\theta_c(1-\xi^2)\}^2\theta_L + \frac{\theta_T{}^2}{4}}} \left[\{1-\theta_c(1-\xi^2)\}^2 - \frac{\theta_T}{2} \pm \sqrt{\{1-\theta_c(1-\xi^2)\}^2\theta_L + \frac{\theta_T{}^2}{4}} \right]^{1/2}$$

$$\pm \frac{\theta_c(1-\xi^2)\{1-\theta_c(1-\xi^2)\}^2\{1+\theta_c(1-\xi^2)\}\theta_L}{2\left\{1-\theta_c(1-\xi^2) - \frac{\theta_T}{2} \pm \sqrt{\{1-\theta_c(1-\xi^2)\}^2\theta_L + \frac{\theta_T{}^2}{4}}\right\}^{3/2}} \left[\{1-\theta_c(1-\xi^2)\}^2 - \frac{\theta_T}{2} \pm \sqrt{\{1-\theta_c(1-\xi^2)\}^2\theta_L + \frac{\theta_T{}^2}{4}} \right]^{1/2}$$

If we now put $1-\theta_c(1-\xi^2)=\zeta$, the above equation reduces to

$$\frac{c}{U} = \frac{\left[\zeta - \frac{\theta_T}{2} \pm \sqrt{\zeta^2\theta_L + \frac{\theta_T{}^2}{4}}\right]\left[\zeta^2 - \frac{\theta_T}{2} \pm \sqrt{\zeta^2\theta_L + \frac{\theta_T{}^2}{4}}\right]^{1/2} + \zeta(1-\zeta)^2 + (1-\zeta)\frac{\theta_T}{2} \mp (1-\zeta)^2\sqrt{\zeta^2\theta_L + \frac{\theta_T{}^2}{4}}}{\left[\xi - \frac{\theta_T}{2} \pm \sqrt{\zeta^2\theta_L + \frac{\theta_T{}^2}{4}}\right]\left[\zeta^2 - \frac{\theta_T}{2} \pm \sqrt{\zeta^2\theta_L + \frac{\theta_T{}^2}{4}}\right]^{3/2}}$$

$$\pm \frac{\theta_L(1-\zeta)(2-\zeta)\zeta^2}{2\sqrt{\zeta^2\theta_L + \frac{\theta_T{}^2}{4}}\left[\zeta - \frac{\theta_T}{2} \pm \sqrt{\zeta^2\theta_L + \frac{\theta_T{}^2}{4}}\right]^{3/2}\left[\zeta^2 - \frac{\theta_T}{2} \pm \sqrt{\zeta^2\theta_L + \frac{\theta_T{}^2}{4}}\right]^{1/2}}$$

Reflection occurs from points where the group-velocity vanishes, i.e. retardation becomes infinite. The above expression becomes infinite for the extraordinary wave when (with the lower signs) :

(a)
$$\zeta - \frac{\theta_T}{2} - \sqrt{\zeta^2 \theta_L + \frac{\theta_T{}^2}{4}} = 0$$

or
$$\zeta = \frac{\theta_T}{1 - \theta_L} \quad i.e. f_v{}^2 = f^2 \frac{f^2 - f_{II}{}^2}{f^2 - f_L{}^2}$$

(b)
$$\zeta^2 - \frac{\theta_T}{2} - \sqrt{\zeta^2 \theta_L + \frac{\theta_T{}^2}{4}} = 0$$

or
$$\zeta = \pm \sqrt{\theta_{II}} \quad \text{where} \quad \theta_{II} = \theta_T + \theta_L$$

i.e.   (bi) $f_v{}^2 = f^2 - ff_{II}$ and   (bii) $f_v{}^2 = f^2 + ff_{II}$

We have to integrate the above expression over y from $y = 0$ to values of ζ given by the above expressions.

Now, from our previous work (Banerji, 1951)

$$f^2 + ff_{II} > f^2 \frac{f^2 - f_{II}{}^2}{f^2 - f_L{}^2} > f^2 - ff_{II}$$

When $y = 0$, $\zeta = 1$ giving us the starting point of integration. We have to carry out the integration from $\zeta = 1$ to $\zeta = \sqrt{\theta_{II}}$ for first reflection and from $\zeta = 1$ to $\zeta = -\sqrt{\theta_{II}}$ for second reflection. We will first concentrate upon the first part.

Since $dy = -y_m d\xi$

and $d\zeta = -2\theta_c \xi d\xi$

i.e.
$$d\xi = -\frac{d\zeta}{2\theta_c \sqrt{1 - \frac{1 - \zeta}{\theta_c}}} = -\frac{d\zeta}{2\theta_c{}^{1/2} \sqrt{\theta_c - 1 + \zeta}}$$

we have

$$\int_0^{y} \frac{c}{U} dy = -\frac{y_m}{2\theta_c{}^{/2}} \int_1^{\zeta} \frac{\left[\zeta^2 - \frac{\theta_T}{2} - \sqrt{\zeta^2 \theta_L + \frac{\theta_T{}^2}{4}}\right]^{\frac{1}{2}}}{\left[\zeta - \frac{\theta_T}{2} - \sqrt{\zeta^2 \theta_L + \frac{\theta_T{}^2}{4}}\right]^{\frac{1}{2}}} \frac{d\zeta}{\sqrt{\zeta - (1 - \theta_L)}}$$

$$-\frac{y_m}{2\theta_c{}^{1/2}} \int_1^{\zeta} \frac{(1 - \zeta)\zeta^2 + (1 - \zeta_c)\frac{\theta_T}{2} + (1 - \zeta)^2 \sqrt{\zeta^2 \theta_L + \frac{\theta_T{}^2}{4}}}{\left[\zeta - \frac{\theta_T}{2} - \sqrt{\zeta^2 \theta_L + \frac{\theta_T{}^2}{4}}\right]^{3/2} \left[\zeta^2 - \frac{\theta_T}{2} - \sqrt{\zeta^2 \theta_L + \frac{\theta_T}{4}}\right]^{\frac{1}{2}}}$$

$$\times \frac{d\zeta}{\sqrt{\zeta - (1 - \theta_c)}}$$

$$+ \frac{y_m}{4\theta_c^{1/2}} \int_1^\zeta \frac{\theta_L(1-\zeta)(2-\zeta)\zeta^2}{\left[\zeta - \frac{\theta_T}{2} - \sqrt{\zeta^2\theta_L + \frac{\theta_T^2}{4}}\right]^{3/2}\left[\zeta - \frac{\theta_T}{2} - \sqrt{\zeta^2\theta_L + \frac{\theta_L^2}{4}}\right]^{\frac{1}{2}}\sqrt{\zeta^2\theta_L + \frac{\theta_T^2}{4}}}$$

$$\times \frac{d\zeta}{\sqrt{\zeta - (1-\theta_c)}}$$

For the first reflection we integrate from 1 to $\sqrt{\theta_H}$. In this range, the only disontinuity occurs when $\zeta = \sqrt{\theta_H}$. But, at this point the integral has the same order of infinity as $\frac{1}{\sqrt{\zeta^2 - \theta_H}}$ and hence the integial converges and the retardation is finite. This, however, does not hold if $\sqrt{\theta_H} = 1 - \theta_c$, when the integral, due to two vanishing terms of order in the $\frac{1}{2}$ denominator, diverges. This is clearly in accordance with the shape of the $P' - f$ curve observed.

Integrating for the second reflction we note that the part of the expression under the radical changes sign beyond the point $\zeta = \sqrt{\theta_H}$ and this continues till $\zeta = -\frac{\theta_T}{1-\theta_L}$ when the expression having exponent $3/2$ changes sign again, the condition continuing till $\zeta = -\sqrt{\theta_H}$. The remarks made below are with this reservation that these changes of sign are to be avoided by considering the mirror images of the parts below the N axis as mentioned above.

In this range of integration it is to be noted that a discontinuity occurs at the point $\zeta = -\frac{\theta_T}{1-\theta_L}$ in addition to that at the end point $\zeta = -\sqrt{\theta_H}$. At the former discontitinuity the expression is of same order of infinity as $1/\left(\zeta - \frac{\theta_T}{1-\theta_L}\right)^{\frac{3}{2}}$, and hence diverges, giving infinite retardation. This happens at all frequencies, i.e. irrespective of the value of θ_c. The $P' - f$ curve, however, indicates that the Z-component is not retarded any further than the other two traces, retardation setting in only near the penetration frequency, i.e. when $1 - \theta_c = -\sqrt{\theta_H}$ as above. This absence of retardation definitely indicates that the mode of propagation of the Z-component is not extraordinary.

From the above it seems reasonable to conclude that, as far as the ray theory holds valid, the ordinary, rather than the extraordinary ray would give rise to the Z-component.

CONCLUDING REMARKS

The above analysis shows that the possibility of the extraordinary ray being reflected from the third level (point E_2, figure 1) after partial reflection

at the first reflection level (point E_1, figure 1) is completely ruled out. The only other possibility, therefore, is that the ordinary ray is responsible for the production of the Z-component. This latter possibility is corroborated by the recent polarisation measurements (Hogarth, 1951) which shew that the sense of rotation of the Z-component is the same as that of the ordinary ray. Regarding the manner in which the Z-splitting occurs, Eckersely (1950) suggests that at the stage of transition from one mode of propagation to the other there is a "coupling" between the ordinary and the extraordinary rays near the point of reflection of the ordinary ray. A part of the energy of the ordinary ray thereby follows the path of the extraordinary ray and goes up to the third point of reflection. Scott (1950), however, favours the view that the Z-component is due to rays which are inclined slightly to the vertical and which, thus, following a longitudinal path, penetrate into the region of third reflection and are scattered back obliquely. This hypothesis presupposes the presence of scattering irregularities in the ionosphere. Rivault's (1950) observations corroborate this supposition. According to him the incidence of the Z-component generally coincides with the presence of a large amount of scattering in the ionosphere.

Both the above points of view consider the Z-component possible only at high latitudes. The observation of triple splitting at Allahabad (Toshni-wall, 1950) and at Calcutta (Banerji, 1951, appears to be in direct contradiction to this. However, it is to be pointed out that, if the angle of scattering is large, then the explanation, as suggested by Scott, may be operative. It may be recalled in this connccion that scattcrting being of frequent occurence n the tropical latitudes, the occasional incidence of large angles is understood.

It is, however, more natural to consider the Z-component as reflection of the ordinary ray, which has partially penetrated the point O from E_2 (figure 1). Because, as mentioned in the introduction, the portion of the curve FE_2 is really a portion of the ordinary branch of the dispersion curve, as has been pointed out by Bhar (1936-7). It may be pointed out that Poeverlein (1949) has recently mentioned the possibility of the ordinary ray going over to the branch EF_2 at the point O though only for a certain oblique angle of incidence.

R E F E R E N C E S

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