A NOTE ON THE PROBLEM OF DISLOCATION IN A SEMI-INFINITE PLATE CONTAINING A CIRCULAR HOLE*

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ABSTRACT. A stress function giving the stress distribution due to dislocation in a semi-infinite plate containing an unstressed circular hole near the straight edge of the plate is obtained in bipolar co-ordinates after the work of Ghosh on the problems of dislocation in a circular plate containing an eccentric hole. The circumferential stresses over the hole boundary and the straight edge are calculated.

INTRODUCTION

Solutions to two problems of dislocation in a circular plate containing an eccentric circular hole are given by Ghosh (1926). The dislocations that he has considered are due to (1) a fissure of constant width joining the two boundaries and (2) a wedge shaped fissure with its wider end on the hole boundary. He has solved the problems in bipolar co-ordinates taking the apex of the wedge shaped fissure at the origin of the relevant Cartesian co-ordinates. It can be easily verified that his solutions hold good in cases of similar dislocations in a semi-infinite plate containing a circular hole near its straight edge, while the fissures are between the hole boundary and the straight edge or between the hole boundary and infinity, along the axis of symmetry of the hole. The apex of the wedge shaped fissure still remains at the origin of the Cartesian co-ordinates and the wider end of the wedge on the hole boundry. In the present paper a solution is given to the problem of dislocation due to a wedge shaped fissure in a semi-infinite plate containing a circular hole, where the apex of the wedge may be anywhere on the axis of symmetry. The solution is obtained by choosing a stress function equal to the sum of the stress functions required for the cases of a parallel fissure and a wedge shaped fissure with its apex at the origin.

THE SOLUTION

In the solution we shall use bipolar co-ordinates in the same notations as those used by Jeffery (1921), so that $\alpha=0$ represents the straight edge and $\alpha=\alpha_1$ represents the hole boundary. Let us choose a stress function

$$h\chi_0 = A\alpha \sinh \alpha + B\alpha \cosh \alpha \qquad \dots \quad (1)$$

which gives the following multiple valued terms in the displacements.

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$$\left.\begin{aligned} u &= \frac{\lambda+2\mu}{2\mu(\lambda+\mu)}(A\sinh\alpha + B\cosh\alpha)\frac{\beta\sin\beta}{\cosh\alpha-\cos\beta} \\ v &= \frac{\lambda+2\mu}{2\mu(\lambda+\mu)}\{A(1-\cosh\alpha\cos\beta) - B\sinh\alpha\cos\beta\}\frac{\beta}{\cosh\alpha-\cos\beta} \end{aligned}\right\} \quad \ldots \quad (2)$$

Then at the barrier $\beta = \pm\pi$, u is continuous and v has a discontinuity equal to

$$v_0 = \frac{\lambda+2\mu}{\mu(\lambda+\mu)}\left\{A + B\frac{\sinh\alpha}{\cosh\alpha+1}\right\}\pi$$

$$= C + Dy_0 \quad \ldots \quad (3)$$

where

$$\left.\begin{aligned} y_0 &= y\text{-ordinate at } \beta = \pm\pi \\ C &= \frac{\lambda+2\mu}{\mu(\lambda+\mu)}A\pi \\ D &= \frac{\lambda+2\mu}{\mu(\lambda+\mu)}B\pi \end{aligned}\right\} \quad \ldots \quad (4)$$

Therefore, th e chosen stress function will suit the state of dislocation due to a fissure bounded by the planes

$$x = \pm(C + Dy) \quad \ldots \quad (5)$$

By adjusting the values of C we shall be able to get point of intersection of these two planes at any desired point on the y-axis, and the value of D will determine the apex angle of the wedge. When $C=0$ we get a wedge shaped fissure with its apex at the origin and when $D=0$ we get a parallel fissure.

To obtain the complete solution to the problem we must add a stress function $h\chi_1$ to $h\chi_0$ such that $h\chi_1$ produces on stress over the straight boundary ($\alpha=0$) and at infinity ($\alpha=0$, $\beta\to 0$), where no stresses are produced by $h\chi_0$ either ; and the sum ($h\chi_0 + h\chi_1$) produces no stress over the boundary of the hole ($\alpha=\alpha_1$). It can be verified that all these requirements are satisfied by the stress function

$$h\chi_1 = B_0\alpha(\cosh\alpha - \cos\beta) + \{A_1(\cosh 2\alpha - 1) + C_1\sinh 2\alpha\}\cos\beta \quad \ldots \quad (6)$$

where

$$\left.\begin{aligned} B_0 &= 2C_1 \\ &= -\{A + B(\cosh\alpha_1\sinh\alpha_1 - \alpha_1)\operatorname{cosech}^2\alpha_1\}\coth\alpha_1 \\ A_1 &= \tfrac{1}{2}\{A + B(\cosh\alpha_1\sinh\alpha_1 - \alpha_1)\operatorname{cosech}^2\alpha_1) \end{aligned}\right] \quad \ldots \quad (7)$$

Calculating the stresses $\widehat{\beta\beta}$ over the boundaries from the complete stress function $(h\chi_0 + h\chi_1)$, we get over $\alpha = \alpha_1$

$$a\widehat{\beta\beta}_1 = 2(\cosh\alpha_1 - \cos\beta)\operatorname{cosech}^2\alpha_1 \qquad \ldots \quad (8)$$

$$\times\left[B(\alpha_1\cosh\alpha_1 - \sinh\alpha_1) - \{A\sinh^2\alpha_1 + B(\cosh\alpha_1\sinh\alpha_1 - \alpha_1)\}\cos\beta\right]$$

and over $\alpha = 0$

$$a\widehat{\beta\beta}_0 = 2(1 - \cos\beta)\left[A + \{A + B(\cosh\alpha_1\sinh\alpha_1 - \alpha_1)\operatorname{cosech}^2\alpha_1\}\cos\beta\right] \quad \ldots \quad (9)$$

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