RADIATION RESISTANCE OF SKEW-WIRE RADIO FREQUENCY TRANSMISSION LINES

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ABSTRACT. An equation has been developed for calculating the radiation resistance of skew-wire high frequency transmission lines, by the method of 'induced e.m.f.' It has been shown that the general formula derived can be applied for any orientation of the transmission lines with respect to each other. In view of the practical layout design of transmission lines, six special cases of orientation of the lines have been considered, for which the expressions for radiation resistance have been deduced mathematically and verified by independent process. Finally, for practical verification of the expressions obtained, the variation of radiation resistance of skew-wire transmission lines without taper when the angle of elevation of one of the wires is increased, has been experimentally determined.

INTRODUCTION

The study of the radiation resistance of transmission lines has recently gained considerable importance due to their wide applications at very high frequencies. Radiation resistance of parallel and coplanar tapered-wire in high frequency transmission lines and the effects of bends and curves of various types on the same have been studied by one of us (Banerjee, 1935; Banerjee and Singh, 1936) previously. It may be mentioned however, that the transmission lines instead of always being coplanar may happen to be skew owing to the unavoidable layout of the transmission lines. In the present communication we have developed a general formula for the calculation of radiation resistance for half wave length long skew-wire transmission lines, with any kind of orientation of the two wires with respect to each other. For the practical use of the problems six special cases have been considered for different orientations of the lines for which the expressions for radiation resistance have been deduced from the general equation and verified independently by the method of 'induced e.m.f.'

THEORETICAL CONSIDERATIONS

Let BP and CR in Fig. 1 represent two skew-wires half wavelength long. The total radiation resistance R_T of the two wires of transmission lines when the currents in them are in antiphase is given by

$$R_T = 2(R_{11} - R_{12}) \tag{1}$$

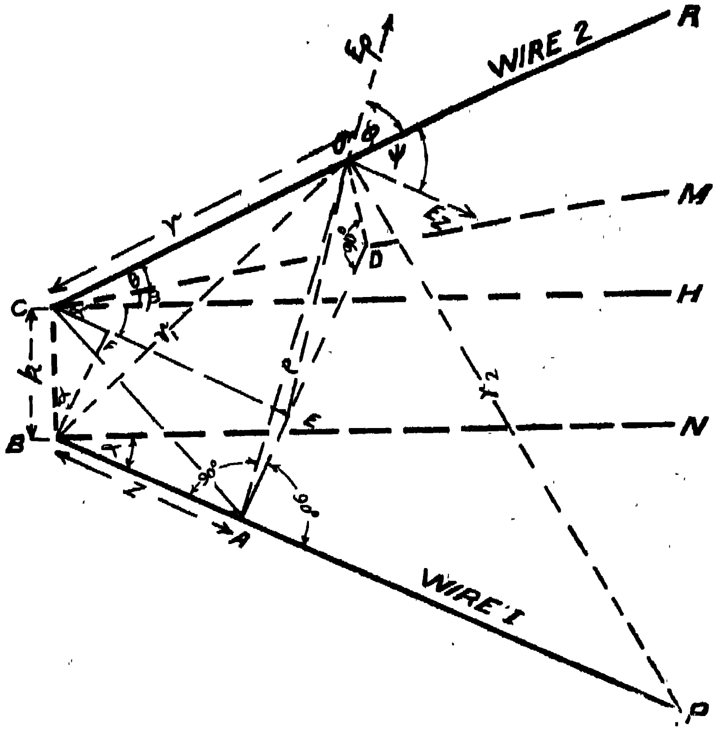

FIG. 1

where R_{11} is the radiation resistance of one of the wires and R_{12} is the radiation resistance of one wire due to the other. R_{11} can be calculated after Pistolkors (1929) and Banerjee (1935). R_{12} can be calculated from the relation,

$$R_{12} = -\int^{l} \frac{E_r}{I} \sin (mr)dr \qquad (2)$$

where E_r is the value of electric field at any point O along the wire 2 due to the current in the wire 1 (Fig. 1). For the calculation of E_r we have to find out the value of electric intensity E_z parallel to the wire 1, and E_ρ perpendicular to this direction as shown in Fig. 1. We will then have,

$$E_r = E_z \cos \psi + E_\rho \cos \phi \qquad \ldots (3)$$

where ψ is the angle between E_z and the wire 2, and ϕ is the angle between E_ρ and the same wire. By evaluating E_z and E_ρ after Carter (1932) it may be shown that

$$R_{12} = 30\int_0^{l} \frac{\sin mr_2}{r_2} \cos \psi \sin (mr)dr + 30\int_0^{l} \frac{\sin mr_1}{r_1} \cos \psi . \sin (mr)dr$$

$$-30\int \frac{\sin mr_2}{r_2} \cdot \frac{Z-l}{\rho} \cdot \cos \phi \sin (mr)dr$$

$$-30\int \frac{\sin mr_1}{r_1} \cdot \frac{Z}{\rho} \cos \phi \sin (mr)dr. \qquad \ldots (4)$$

where $m = 2\pi/\lambda$, $l =$ length of the wires, Z and ρ are the coordinates of the point O with reference to wire 1, and r_1 and r_2 are the distances of the point O from the input and output ends respectively of wire 1. Thus, knowing the values of Z, ρ, r_1, r_2, ϕ and ψ from the orientation of the lines, the radiation resistance, can be calculated with the help of equation (4). For representing the orientation of the lines for the general case of skew-wires a reference plane is taken which is defined as the plane containing one wire and the line joining the input ends of the wires. The angle which the other wire makes with the reference plane is the angle of elevation. The reference plane mentioned above is the plane of the paper in Fig. 1. Wire 1 lies on this plane and wire 2 makes an angle of elevation θ with this plane. CM represents the projection of wire 2 on the reference plane and therefore angle MCR = angle θ. The tapering angles for these wires may be represented by the angles which these wires, or their projections on the reference plane, make with the lines drawn on the reference plane perpendicular to the line joining the input ends of the transmission lines. In Fig. 1, angle PBN and angle MCH, represented by α and β respectively, are the tapering angles of wires 1 and 2.

Now, for evaluating Z, ρ, r_1, r_2, ϕ and ψ take any point O on the wire 2, at a distance r from the input end of this wire. From O draw a line OA perpendicular to wire 1. AD is the projection of the line OA on the reference plane. Draw CE perpendicular to AD and BF perpendicular to CE. It will be seen from the diagram that $\angle CBF = \angle ECH = \angle\alpha$.

Now $\qquad Z = AB = CE - CF = CD\cos(\alpha+\beta) - k\sin\alpha,$

and, as $\qquad CD = r\cos\theta,\ Z = r\cos\theta\cos(\alpha+\beta) - k\sin\alpha \qquad \ldots (5)$

$$\rho = OA = \sqrt{AD^2 + OD^2} = \sqrt{(AE+ED)^2 + OD^2}$$

As $\qquad AE = BF = k\cos\alpha$, and $ED = CD\sin(\alpha+\beta) = r\cos\theta\sin(\alpha+\beta)$

we get
$$\rho = \sqrt{\{r\cos\theta\sin(\alpha+\beta) + k\cos\alpha\}^2 + r^2\sin^2\theta} \qquad \ldots (6)$$

$$r_1 = OB = \sqrt{\rho^2 + Z^2} \qquad \ldots (7)$$

$$r_2 = OP = \sqrt{\rho^2 + (l-Z)^2} \qquad \ldots (8)$$

For calculating the angles ϕ and ψ, join AC and OE. In the ΔACO the angle $AOC = \phi$ and

$$\cos\phi \quad \frac{r^2+\rho^2-AC^2}{2r\rho},$$

Now $\qquad AC^2 = CB^2 + BA^2 - 2CB.BA\cos(CBA) = k^2 + Z^2 + 2kZ\sin\alpha$

$$\phi = \cos^{-} \frac{r^2+\rho^2-k^2-Z^2-2kZ\sin\alpha}{2r\rho} \qquad \ldots (9)$$

For calculating ψ triangle CEO is considered in which angle $OCE = \psi$

$$\therefore \quad \cos\psi = \frac{OC^2 + CE^2 - OE^2}{2.OC.CE} = \frac{r^2 + r^2\cos^2\theta\cos^2(\alpha+\beta) - OD^2 - DE^2}{2r^2\cos\theta\cos(\alpha+\beta)}$$

$$= \frac{r^2 + r^2\cos^2\theta\cos^2(\alpha+\beta) - r^2\sin^2\theta - r^2\cos^2\theta\sin^2(\alpha+\beta)}{2r^2\cos\theta\cos(\alpha+\beta)}$$

$$= \cos(\alpha+\beta)\cos\theta$$

$$\therefore \quad \psi = \cos^{-1}[\cos(\alpha+\beta)\cos\theta] \qquad \ldots \quad (10)$$

The six special cases mentioned previously have been considered below for calculation of radiation resistance. It may be mentioned, however, that as r_1 and r_2 are functions of ρ and Z, we will consider the expressions for Z, ρ, ϕ and ψ only in the following cases.

Case 1. Skew wires with single taper, ($\alpha = 0$).

Substituting $\alpha = 0$ in equations (5), (6), (9) and (10) we get

$$Z = r\cos\theta\cos\beta$$

$$\rho = \sqrt{[r\cos\theta\cos\beta + k]^2 + r^2\sin^2\theta}$$

$$\phi = \cos^{-1}\left[\frac{r^2 + \rho^2 - k^2 - Z^2}{2r\rho}\right]$$

$$\psi = \cos^{-1}[\cos\beta\cos\theta]$$

Case 2. Skew-wires without taper.

For this case, $\alpha = \beta = 0$, which gives from the same equations as mentioned in case 1,

$$Z = r\cos\theta$$

$$\rho = \sqrt{k^2 + r^2\sin^2\theta}$$

$$\phi = \cos^{-1}\left[\frac{r\sin^2\theta}{\rho}\right], \quad \text{and } \psi = \theta$$

Case 3. Coplanar tapered wire.

For coplanar wires $\theta = 0$ and substituting this in equations (5), (6), (9) and (10) we get,

$$Z = r\cos(\alpha+\beta) - k\sin\alpha$$

$$\rho = k\cos\alpha + r\sin(\alpha+\beta)$$

$$\phi = [\pi/2 - (\alpha+\beta)] \text{ and } \psi = (\alpha+\beta)$$

Case 4. Coplanar wires with equal taper.

In this case $\theta = 0$ and $\alpha = \beta$ and we have

$$Z = r \cos 2\alpha - k \sin \alpha$$

$$\rho = r \sin 2\alpha + k \cos \alpha$$

$$\phi = (\pi/2 - 2\alpha) \text{ and } \psi = 2\alpha$$

Case 5. Coplanar wires with single taper ($\theta = \alpha = 0$).

This will give,

$$Z = r \cos \beta, \ \rho = k + r \sin \beta$$

$$\phi = \pi/2 - \beta \text{ and } \psi = \beta$$

Case 6. Parallel wires.

This is obtained by making $\theta = \alpha = \beta = 0$ which leads to the usual expressions given as,

$$Z = r, \ \rho = k, \ \phi = \pi/2 \text{ and } \psi = 0$$

values of Z, ρ, ϕ and ψ obtained in the above equations have been verified by evaluating them from their actual orientations, the diagramatic representations of which have not been shown separately.

EXPERIMENTAL

The radiation resistance of skew wire transmission lines with any orientation can be determined experimentally by the method adopted by Banerjee (1935) and subsequently by Banerjee and Singh (1936) for measuring the radiation resistance of parallel and coplanar tapered transmission lines. In the present communication, however, the variation of radiation resistance of untapered skew-wires as described in case 2, in the previous section, has been experimentally determined, when the angle of elevation of one of the wires is increased.

In the case of skew-wires the mean radiation resistance per unit length is obtained from the relation

$$R = 2\alpha\sqrt{L/C},$$

where α is the attenuation constant, and, L and C are the mean values of inductance and capacity per unit length of the wires. The mean value of $\sqrt{L/C}$ can be written as,

$$\frac{120}{\sqrt{2l^2(1-\cos\theta)+k^2}-k}\int_k^{\sqrt{2l^2(1-\cos\theta)+k^2}} \log_e \frac{2x}{d}\,dx$$

where d is the diameter of the wire and x is the separation between the wires, denoted by the distance between any two points on the two transmission lines which are equidistant from their respective input ends. After integrating the above expression, we get,

$$\sqrt{L/C} = \frac{120}{\sqrt{2l^2(1-\cos\theta)+k^2}-k}\left[\sqrt{2l^2(1-\cos\theta)+k^2}\left(\log_e \frac{2\sqrt{2l^2(1-\cos\theta)+k^2}}{d}\right)\right.$$
$$\left. - \sqrt{2l^2(1-\cos\theta)+k^2} - k\log_e \frac{2k}{d} + k\right]$$

The attenuation oonstant α is determined experimentally as stated above, by drawing resonance curves obtained by noting the deflections in a sensitive galvanometer at the input end of the transmission lines as a short circuiting bridge is moved along the lines. It should be mentioned, however, that the short circuiting metal bridge is moved along the length of the wires in such a way that the points of contact of the bridge with the wires may be equidistant from their respective input ends. Thus finding out α and the mean value of $\sqrt{L/C}$, radiation resistance is obtained for various angles of elevation of one of the wires.

TABLE I

Wavelength—254.0 cm.

Distance between input ends—5.0 cm.

Angle of elevation	Total radiation resistance in ohms (observed)	Total radiation resistance in ohms (calculated)
0°	5.96	5.92
2°	10.49	10.19
4°	14.11	14.01
6°	15.39	15.34

Table I shows the radiation resistance of untapered skew-wire transmission lines half wave-length long when angle of elevation, θ, of one of the lines is increased. Column 2 contains the experimentally observed values of radiation resistance and column 3 gives the calculated values for comparison. It may be mentioned, however, that as the experimentally observed values of radiation resistance include the ohmic resistance of the lines, the values shown in column 2 contain the sum of radiation resistance calculated from equation (4) and the high frequency ohmic resistance of the lines. Fig. 2 shows the variation of radiation resistance of untapered skew-wire transmission lines, as the angle of elevation of one of the lines is increased. The dotted line shows the calculated variation of radiation resistance and the continuous one represents the experimental variation.

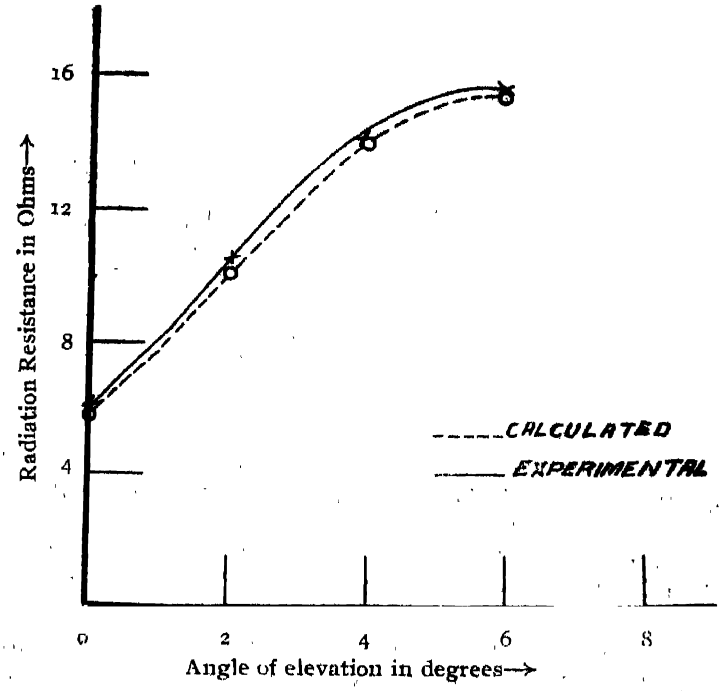

FIG. 2

SUMMARY AND CONCLUSION

A general formula for calculating radiation resistance of skew-wire radio-frequency transmission lines has been obtained by the method of 'induced e.m.f.' It has been shown that with the help of this formula radiation resistance can be calculated for any orientation of the transmission lines with respect to each other, and equation for six special cases of orientation of the lines have been derived from the general formula and verified independently. For the sake of practical interest, the variation of radiation resistance in one of the above cases, namely, for skew-wire transmission lines without taper, when the angle of elevation of one of the wires is increased, has been verified experimentally.

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