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Research Article

Robust Adaptive Neural Backstepping Control for a Class of Nonlinear Systems with Dynamic Uncertainties

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This paper is concerned with adaptive neural control of nonlinear strict-feedback systems with nonlinear uncertainties, unmodeled dynamics, and dynamic disturbances. To overcome the difficulty from the unmodeled dynamics, a dynamic signal is introduced. Radical basis function (RBF) neural networks are employed to model the packaged unknown nonlinearities, and then an adaptive neural control approach is developed by using backstepping technique. The proposed controller guarantees semiglobal boundedness of all the signals in the closed-loop systems. A simulation example is given to show the effectiveness of the presented control scheme.

1. Introduction

In the past decades, much attention has been paid on the control design of complex nonlinear systems [1–11]. Many remarkable control approaches in this area have been developed, including adaptive backstepping technique [1–3], fault tolerant control [12–17], and and fuzzy control [18–29]. In particular, adaptive backstepping approach has played an important role in the control of strict-feedback nonlinear systems. Generally, adaptive backstepping provides a systematic control approach to solve the tracking or regulation control problems of uncertain nonlinear systems, in which the classic adaptive control is applied to deal with the unknown parameter and backstepping technique is used to construct controller. The main feature of adaptive backstepping control is that it can handle the control problems of nonlinear systems without the requirement of matching condition. Adaptive backstepping technique was provided in [1] to obtain global stability and asymptotic tracking performance for parametric strict-feedback systems with overparameterization, and the overparameterization was overcome by applying the tuning functions in [2]. Then, a backstepping-based design was extensively utilized to control different types of nonlinear systems [30–35]. All the above control methods, however, assume that the nonlinear functions of the control systems are either known or bounded by known functions multiplying uncertain parameters. This restriction makes the aforementioned methods inapplicable to the control of the systems with unknown continuous nonlinear functions.

On the other hand, approximation-based adaptive neural (or fuzzy) backstepping control has received increasing attention in recent years. In general, approximation-based adaptive backstepping technique is an effective control approach for handling the control problem of highly uncertain complex nonlinear strict-feedback systems, in which neural networks or fuzzy systems are utilized to model the unknown nonlinear functions. So far, there exist some elegant results; see, for example, [36–54] and the references therein. By applying adaptive neural control together with backstepping, in [36–43], many control approaches are developed for single-input and single-output (SISO) nonlinear systems or multi-input and multioutput (MIMO) nonlinear systems. Alternatively, several fuzzy adaptive control strategies [19, 44–55] were developed to deal with the control problem of uncertain nonlinear systems with strict-feedback form. However, the above adaptive neural or fuzzy backstepping control approaches required the controlled strict-feedback nonlinear systems to be free of the unmodeled dynamics and dynamic disturbances. As stated in [56, 57], the unmodeled dynamics and dynamic disturbances often appear in practical systems [58, 59] due to the measurement noise, modeling errors, external disturbances, modeling simplifications, or changes with time variations, and they are also the resources of the instability of the considered systems. Therefore, some researchers have concentrated on the problem of control design for nonlinear systems with unmodeled dynamics and dynamic disturbances. In [56, 57], the problem of adaptive backstepping control was investigated for a class of nonlinear systems with dynamics uncertainties, in which the nonlinear functions were assumed to be linear combinations of the known functions with unknown parameters. Furthermore, by using the approximation properties of fuzzy logic systems, Tong et al. [58, 59] developed several fuzzy adaptive control approaches for nonlinear systems in strict-feedback form, where the number of adaptation laws depends on the number of fuzzy base functions. The more fuzzy rules are applied to improve approximation accuracy, the more adaptive parameters will be needed, and, in this way, the online learning time may be very large.

Inspired by previous works, this paper focuses on the problem of adaptive neural control for nonlinear strict-feedback systems with unmodeled dynamics and dynamic disturbances. During the controller design, a dynamic signal is introduced to handle the unmodeled dynamics and RBF neural networks are used to approximate the unknown nonlinearities, and then an adaptive neural control scheme is systematically derived via backstepping. The proposed controller guarantees that all the signals in the closed-loop systems are semiglobally uniformly ultimately bounded (SGUUB) in the sense of mean square. Compared with the control approaches [58, 59], the main contributions of this paper are summarized as follows: (1) the strict limitation to the dynamic disturbances is relaxed, which can refer to Remark 3; (2) by estimating the norm of the weight vector of neural networks basis functions, the number of adaptive parameters is not more than the order of the considered nonlinear system. As a result, the burdensome computation is significantly alleviated, which makes our control design more suitable for the practical applications.

The remainder of the paper is organized as follows. Section 2 begins with the problem formulation and some preliminaries. A backstepping-based adaptive control scheme is design in Section 3. In Section 4, a numerical example is given. Finally, the conclusion of this paper is shown in Section 5.

2. Problem Formulation and Preliminaries

In this paper, we consider a class of nonlinear strict-feedback systems described by

$$\begin{aligned}\dot{z} &= q(z,x),\\ \dot{x}_i &= g_i(\bar{x}_i)x_{i+1} + f_i(\bar{x}_i) + \Delta_i(x,z,t), \quad i = 1,\dots,n-1,\\ \dot{x}_n &= g_n(\bar{x}_n)u + f_n(\bar{x}_n) + \Delta_n(x,z,t),\end{aligned} \tag{1}$$

where $x = [x_1, x_2, \dots, x_n]^T \in R^n$, $u \in R$, and $y \in R$ are the system state, control input, and system output, respectively, $\bar{x}_i = [x_1, x_2, \dots, x_i]^T \in R^i$, $f_i(\cdot)$, and $g_i(\cdot)$ are unknown smooth functions, and Δ_i $(i = 1, 2, \dots, n)$ are nonlinear dynamic disturbances. The z-dynamics in (1) denotes the unmodeled dynamics.

Remark 1. It is worth noting that many practical systems such as the electromechanical system [59] transformable into (1) have been investigated extensively during the last decades from both theoretical and practical viewpoints; see, for example, [56–59].

In order to facilitate the control design later, the below assumptions are imposed on the system (1).

Assumption 2. For the dynamic disturbances Δ_i $(i = 1, 2, \dots, n)$ in (1), there exist unknown nonnegative smooth functions $\phi_{i1}(\cdot)$ and $\phi_{i2}(\cdot)$, such that

$$\left|\Delta_i(x,z,t)\right| \le \phi_{i1}\left(\left|(x_1,\dots,x_i)\right|\right) + \phi_{i2}(|z|). \tag{2}$$

Remark 3. This assumption is similar to the one in [58, 59] in which $\phi_{i1}(\cdot)$ and $\phi_{i2}(\cdot)$ are known. Assumption 2, however, does not require them to be known. Therefore, Assumption 2 relaxes the restriction in the existing results.

Assumption 4 (see [59]). The unmodeled dynamics in (1) is exponentially input-to-state practically stable (exp-ISpS); that is, for the system $\dot{z} = q(z,x)$, there exists an exp-ISpS Lyapunov function $V(z)$ such that

$$\begin{gathered}\alpha_1(|z|) \le V(z) \le \alpha_2(|z|),\\ \frac{\partial V(z)}{\partial z}q(z,x) \le -c_0V(z) + \mu(|x|) + d_0,\end{gathered} \tag{3}$$

where α_1, α_2, and μ are of class K_∞-functions and c_0 and d_0 are known positive constants.

Assumption 5 (see [50]). For $1 \le i \le n$, the signs of $g_i(\bar{x}_i)$ are known, and there exists unknown positive constant b such that

$$0 < b \le \left|g_i(\bar{x}_i)\right| < \infty, \quad \forall \bar{x}_i \in R^i. \tag{4}$$

Remark 6. Equation (4) implies that $g_i(\bar{x}_i)$ are either strictly positive or negative. Without loss of generality, it is supposed that $0 < b \le g_i(\bar{x}_i)$. In addition, since b is not required in the designed controller, its true value is not required to be known.

Lemma 7 (see [59]). *If V is an exp-ISpS Lyapunov function for a control system, that is,* (3) *hold, then, for any constants $\bar{c}$ in $(0, c_0)$, any initial condition $x_0 = x_0(0)$, and any function $\bar{\mu}(x_1) \ge \mu(|x_1|)$, there exists a finite time $T_0 = T_0(\bar{c}, r_0, z_0)$, a nonnegative function $D(t)$ defined for all $t \ge 0$, and a signal described by*

$$\dot{r} = -\bar{c}r + \bar{\mu}(x_1(t)) + d_0, \quad r(0) = r_0, \tag{5}$$

such that $D(t) = 0$ for all $t \ge T_0$,

$$V(z(t)) \le r(t) + D(t). \tag{6}$$

For all $t \geq 0$, the solutions are defined. Without loss of generality, this paper takes $\overline{\mu}(\cdot)$ as $\overline{\mu}(s) = s^2\mu_0(s^2)$, where $\overline{\mu}(\cdot)$ is a nonnegative smooth function. Therefore, the dynamical r defined by (5) *becomes*

$$\dot{r} = -\bar{c}r + x_1^2\mu_0\left(\left|x_1^2\right|\right) + d_0, \quad r(0) = r_0, \tag{7}$$

where μ_0 is a nonnegative smooth function.

Throughout this paper, RBF neural networks are applied to model the unknown continuous nonlinear functions. In [60], it has been indicated that, with enough node number l, the RBF neural networks $\phi^{*T}\xi(X)$ can model the continuous function $f(X)$ within a compact set $\Omega_X \subset R^q$ to arbitrary accuracy $\varepsilon > 0$ as

$$f(X) = \phi^{*T}\xi(X) + \delta(X), \quad \forall X \in \Omega_X \in R^q, \tag{8}$$

in which ϕ^* denotes the ideal weight vector and is specified as

$$\phi^* := \arg\min_{\phi \in \overline{R}^N}\left\{\sup_{X \in \Omega_X}\left|f(X) - \phi^T\xi(X)\right|\right\} \tag{9}$$

$\delta(X)$ depicts the approximation error satisfying $|\delta(X)| \leq \varepsilon$, $\phi^* = [\phi_1, \phi_2, \ldots, \phi_l]^T \in R^l$ is the weight vector, and $\xi(X) = [\xi_1(X), \xi_2(X), \ldots, \xi_l(X)]^T$ is the basis function vector with l being the number of the neural networks nodes and $l > 1$. The basis function $\xi_i(X)$ is taken as the Gaussian function in the below form:

$$\xi_i(X) = \exp\left[-\frac{(X-\mu_i)^T(X-\mu_i)}{\eta_i^2}\right], \quad (i = 1, 2, \ldots, l), \tag{10}$$

where $\mu_i = [\mu_{i1}, \mu_{i2}, \ldots, \mu_{iq}]^T$ and η_i are the center of the receptive field and the width of the Gaussian function, respectively.

3. Adaptive Neural Control Design

In this section, the adaptive backstepping control design for system (1) is proposed. As usual, in the backstepping approach, the following coordinate transformation is made:

$$z_i = x_i - \alpha_{i-1}, \quad i = 1, 2, \ldots, n, \tag{11}$$

where $\alpha_0 = 0$, α_i is the virtual control signal and will be constructed at Step i, and the actual controller u will be designed at Step n. Now, we begin the controller design procedure.

Step 1. Consider the following subsystem:

$$\begin{aligned} \dot{z} &= q(z, x), \\ \dot{x}_1 &= g_1(x_1)x_2 + f_1(x_1) + \Delta_1(x, z, t). \end{aligned} \tag{12}$$

Based on $z_1 = x_1$, then choose Lyapunov functions as

$$V_1 = \frac{1}{2}z_1^2 + \frac{1}{\lambda_0}r + \frac{b}{2\gamma_1}\widetilde{\theta}_1^2, \tag{13}$$

where $\overline{\mu}(x_1) = x_1^2\mu_0(x_1^2)$, λ_0 and γ_1 are positive design parameters, and $\widetilde{\theta}_1 = \theta_1 - \widehat{\theta}_1$ is the parameter error with $\widehat{\theta}_1$ being the estimation of θ_1 which is defined later.

By taking (12) into account, we have

$$\begin{aligned} \dot{V}_1 &= z_1\left(g_1(x_1)x_2 + f_1(x_1) + \Delta_1\right) + \frac{1}{\lambda_0}\dot{r} - \frac{b}{\gamma_1}\widetilde{\theta}_1\dot{\widehat{\theta}}_1 \\ &\leq z_1\left(g_1(x_1)x_2 + f_1(x_1)\right) + |z_1||\Delta_1| + \frac{1}{\lambda_0}\dot{r} - \frac{b}{\gamma_1}\widetilde{\theta}_1\dot{\widehat{\theta}}_1. \end{aligned} \tag{14}$$

By Assumption 2, it follows that

$$\begin{aligned} \dot{V}_1 &\leq z_1\left(g_1(x_1)x_2 + f_1(x_1)\right) + |z_1|\phi_{11}(|x_1|) \\ &\quad + |z_1|\phi_{12}(|z|) + \frac{1}{\lambda_0}\left(x_1^2\mu_0\left(x_1^2\right) + d_0\right) - \frac{\bar{c}}{\lambda_0}r - \frac{b}{\gamma_1}\widetilde{\theta}_1\dot{\widehat{\theta}}_1. \end{aligned} \tag{15}$$

Then, we will deal with the third and fourth terms in (15), respectively. By using $0 \leq |x| - x\tanh(x/\epsilon) \leq 0.2785\epsilon = \epsilon'$, for $\forall\epsilon > 0$, we have

$$\begin{aligned} &|z_1|\phi_{11}(|x_1|) \\ &\quad = |z_1\phi_{11}(|x_1|)| - z_1\phi_{11}(|x_1|)\tanh\left(\frac{z_1\phi_{11}(|x_1|)}{\epsilon_{11}}\right) \\ &\quad\quad + z_1\phi_{11}(|x_1|)\tanh\left(\frac{z_1\phi_{11}(|x_1|)}{\epsilon_{11}}\right) \\ &\quad \leq \epsilon_{11}' + z_1\phi_{11}(|x_1|)\tanh\left(\frac{z_1\phi_{11}(|x_1|)}{\epsilon_{11}}\right) \\ &\quad \leq z_1\widehat{\phi}_{11}(x_1) + \epsilon_{11}', \end{aligned} \tag{16}$$

where $\epsilon_{11}' = 0.2785\epsilon_{11}$ and $\widehat{\phi}_{11}(x_1) = \phi_{11}(|x_1|)\tanh(z_1\phi_{11}(|x_1|)/\epsilon_{11})$ is a smooth function.

By using the same derivations as [58], one has

$$\begin{aligned} |z_1|\phi_{12}(|z|) &\leq |z_1|\overline{\phi}_{12}(r) + \frac{1}{4}z_1^2 + d_1(t) \\ &\leq z_1\overline{\phi}_{12}(r)\tanh\left(\frac{z_1\overline{\phi}_{12}(r)}{\epsilon_{12}}\right) + \epsilon_{12}' \\ &\quad + \frac{1}{4}z_1^2 + d_1(t) \\ &\leq z_1\widehat{\phi}_{12}(x_1, r) + \epsilon_{12}' + \frac{1}{4}z_1^2 + d_1(t), \end{aligned} \tag{17}$$

where $\epsilon_{12}' = 0.2785\epsilon_{12}, \overline{\phi}_{12}(r) = \phi_{12} \circ \alpha_1^{-1}(2r)$, $d_1(t) = (\phi_{12} \circ \alpha_1^{-1}(2D(t)))^2$, and $\widehat{\phi}_{12}(x_1, r) = \overline{\phi}_{12}(r)\tanh(z_1\overline{\phi}_{12}(r)/\epsilon_{12})$.

Subsequently, substituting (16) and (17) into (15) gives

$$\begin{aligned} \dot{V}_1 &\leq z_1\left(g_1(x_1)z_2 + g_1(x_1)\alpha_1 + \widehat{f}_1(Z_1)\right) - \frac{z_1^2}{2} \\ &\quad + \frac{d_0}{\lambda_0} - \frac{\bar{c}}{\lambda_0}r - \frac{b}{\gamma_1}\widetilde{\theta}_1\dot{\widehat{\theta}}_1 + \epsilon_{11}' + \epsilon_{12}' + d_1(t), \end{aligned} \tag{18}$$

where the function $\hat{f}_1(Z_1)$ is defined as

$$\hat{f}_1(Z_1) = f_1(x_1) + \hat{\phi}_{11}(x_1) + \hat{\phi}_{12}(r) + \frac{3}{4}z_1 + \frac{1}{\lambda_0}\left(x_1\mu_0\left(x_1^2\right)\right), \tag{19}$$

where $Z_1 = [x_1, r]^T \in \Omega_{Z_1} \subset R^2$.

Since the smooth function $\hat{f}_1(Z_1)$ is unknown, it cannot be implemented in practice. By employing RBF neural network in $\phi_1^T\xi(Z_1)$ to approximate $\hat{f}_1(Z_1)$, we have

$$\hat{f}_1(Z_1) = \phi_1^T\xi_1(Z_1) + \delta_1(Z_1), \quad |\delta_1(Z_1)| \le \epsilon_{13}, \tag{20}$$

where $\delta_1(Z_1)$ denotes an approximation error and $\epsilon_{13} > 0$ is a given positive constant.

Next, the following result can be obtained by substituting (20) into (18):

$$\dot{V}_1 \le g_1(x_1)z_1z_2 + g_1(x_1)z_1\alpha_1 + z_1\phi_1^T\xi_1(Z_1) + z_1\delta_1(Z_1) - \frac{z_1^2}{2} + \frac{d_0}{\lambda_0} - \frac{\bar{c}}{\lambda_0}r - \frac{b}{\gamma_1}\tilde{\theta}_1\dot{\hat{\theta}}_1 + \epsilon'_{11} + \epsilon'_{12} + d_1(t). \tag{21}$$

By using

$$z_1\phi_1^T\xi_1(Z_1) \le \frac{b\theta_1}{2\eta_1^2}\xi_1^T(Z_1)\xi_1(Z_1)z_1^2 + \frac{\eta_1^2}{2},$$
$$z_1\delta_1(Z_1) \le \frac{z_1^2}{2} + \frac{\epsilon_{13}^2}{2}, \tag{22}$$

one has

$$\dot{V}_1 \le g_1(x_1)z_1z_2 + g_1(x_1)z_1\alpha_1 + \frac{b\theta_1}{2\eta_1^2}\xi_1^T(Z_1)\xi_1(Z_1)z_1^2 + \frac{\eta_1^2}{2} + \frac{\epsilon_{13}^2}{2} + \frac{d_0}{\lambda_0} - \frac{\bar{c}}{\lambda_0}r - \frac{b}{\gamma_1}\tilde{\theta}_1\dot{\hat{\theta}}_1 + \epsilon'_{11} + \epsilon'_{12} + d_1(t) \tag{23}$$

with $\theta_1 = b^{-1}\|\phi_1\|^2$ being an unknown parameter.

Construct the virtual control signal α_1 as

$$\alpha_1 = -k_1z_1 - \frac{\hat{\theta}_1}{2\eta_1^2}z_1\xi_1^T(Z_1)\xi_1(Z_1), \tag{24}$$

where k_1 and η_1 are positive design constants.

By taking Assumption 5 into account, one has

$$g_1(x_1)z_1\alpha_1 \le -k_1bz_1^2 - \frac{\hat{\theta}_1}{2\eta_1^2}bz_1^2\xi_1^T(Z_1)\xi_1(Z_1). \tag{25}$$

Further, by substituting (25) into (23), we obtain

$$\dot{V}_1 \le -bk_1z_1^2 - \frac{\bar{c}}{\lambda_0}r + \frac{b\tilde{\theta}_1}{\gamma_1}\left(\frac{\gamma_1}{2\eta_1^2}\xi_1^T(Z_1)\xi_1(Z_1)z_1^2 - \dot{\hat{\theta}}_1\right) + g_1(x_1)z_1z_2 + \frac{\eta_1^2}{2} + \frac{\epsilon_{13}^2}{2} + \frac{d_0}{\lambda_0} + \epsilon'_{11} + \epsilon'_{12} + d_1(t). \tag{26}$$

Next, we choose the adaptive law in the following form:

$$\dot{\hat{\theta}}_1 = \frac{\gamma_1}{2\eta_1^2}\xi_1^T(Z_1)\xi_1(Z_1)z_1^2 - \sigma_1\hat{\theta}_1, \tag{27}$$

where η_1 and σ_1 are design parameters.

By using (27), we can rewrite (26) as

$$\dot{V}_1 \le -bk_1z_1^2 - \frac{\bar{c}}{\lambda_0}r + g_1(x_1)z_1z_2 + \frac{b\sigma_1\tilde{\theta}_1\hat{\theta}_1}{\gamma_1} + \frac{\eta_1^2}{2} + \frac{\epsilon_{13}^2}{2} + \frac{d_0}{\lambda_0} + \epsilon'_{11} + \epsilon'_{12} + d_1(t). \tag{28}$$

Noting

$$\frac{b\sigma_1\tilde{\theta}_1\hat{\theta}_1}{\gamma_1} = -\frac{b\sigma_1\tilde{\theta}_1^2}{\gamma_1} + \frac{b\sigma_1\tilde{\theta}_1\theta_1}{\gamma_1} \le -\frac{b\sigma_1\tilde{\theta}_1^2}{2\gamma_1} + \frac{b\sigma_1\theta_1^2}{2\gamma_1}, \tag{29}$$

then the following inequality holds:

$$\dot{V}_1 \le -bk_1z_1^2 - \frac{\bar{c}}{\lambda_0}r - \frac{b\sigma_1\tilde{\theta}_1^2}{2\gamma_1} + \frac{d_0}{\lambda_0} + D_1 + g_1(x_1)z_1z_2, \tag{30}$$

where $D_1 = \eta_1^2/2 + b\sigma_1\theta_1^2/2\gamma_1 + \epsilon_{13}^2/2 + \epsilon'_{11} + \epsilon'_{12} + d_1(t)$.

Step 2. Based on $z_2 = x_2 - \alpha_1$, then the time derivative of z_2 is given by

$$\begin{aligned}\dot{z}_2 &= \dot{x}_2 - \dot{\alpha}_1 \\ &= g_2(\bar{x}_2)x_3 + f_2(\bar{x}_2) + \Delta_2 - \frac{\partial\alpha_1}{\partial x_1}\left(g_1(x_1)x_2 + f_1(x_1) + \Delta_1\right) - \frac{\partial\alpha_1}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1 - \frac{\partial\alpha_1}{\partial r}\dot{r} \\ &= g_2(\bar{x}_2)x_3 + \left(f_2(\bar{x}_2) - \frac{\partial\alpha_1}{\partial x_1}f_1(x_1)\right) + \bar{\Delta}_2 - \frac{\partial\alpha_1}{\partial x_1}g_1(x_1)x_2 - \frac{\partial\alpha_1}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1 - \frac{\partial\alpha_1}{\partial r}\dot{r},\end{aligned} \tag{31}$$

where $\bar{\Delta}_2 = \Delta_2 - (\partial\alpha_1/\partial x_1)\Delta_1$.

Construct the Lyapunov function

$$V_2 = V_1 + \frac{1}{2}z_2^2 + \frac{b}{2\gamma_2}\tilde{\theta}_2^2. \tag{32}$$

The derivative of V_2 is

$$\begin{aligned}\dot{V}_2 \le& -bk_1 z_1^2 - \frac{b\sigma_1\tilde{\theta}_1^2}{2\gamma_1} - \frac{\bar{c}}{\lambda_0}r \\ &+ z_2\left[g_2(\bar{x}_2)x_3 + g_1(x_1)z_1 + f_2(\bar{x}_2) - \frac{\partial\alpha_1}{\partial x_1}f_1(x_1)\right. \\ &\left.-\frac{\partial\alpha_1}{\partial x_1}g_1(x_1)x_2 - \frac{\partial\alpha_1}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1 - \frac{\partial\alpha_1}{\partial r}\dot{r}\right] \\ &+ \left|z_2\overline{\Delta}_2\right| + D_1 + \frac{d_0}{\lambda_0} - \frac{b}{\gamma_2}\tilde{\theta}_2\dot{\hat{\theta}}_2.\end{aligned} \tag{33}$$

By Assumption 2, we have

$$\begin{aligned}\left|z_2\overline{\Delta}_2\right| &= |z_2|\left|\Delta_2 - \frac{\partial\alpha_1}{\partial x_1}\Delta_1\right| \\ &\le |z_2|\left(|\Delta_2| + \left|\frac{\partial\alpha_1}{\partial x_1}\right||\Delta_1|\right) \\ &\le |z_2|\left(\phi_{21} + \left|\frac{\partial\alpha_1}{\partial x_1}\right|\phi_{11}\right) \\ &\quad + |z_2|\left(\phi_{22}(|z|) + \left|\frac{\partial\alpha_1}{\partial x_1}\right|\phi_{12}(|z|)\right).\end{aligned} \tag{34}$$

Similar to the estimation methods in (35), the following results can be obtained:

$$\begin{aligned}&|z_2|\left(\phi_{21} + \left|\frac{\partial\alpha_1}{\partial x_1}\right|\phi_{11}\right) \le z_2\hat{\phi}_{21}\left(\bar{x}_2,\hat{\theta}_1,r\right) + \epsilon'_{21}, \\ &|z_2|\left(\phi_{22}(|z|) + \left|\frac{\partial\alpha_1}{\partial x_1}\right|\phi_{12}(|z|)\right) \\ &\quad\le |z_2|\left(\phi_{22}\circ\alpha_1^{-1}(2r) + \left|\frac{\partial\alpha_1}{\partial x_1}\right|\phi_{12}\circ\alpha_1^{-1}(2r)\right) \\ &\qquad + |z_2|\,\phi_{22}\circ\alpha_1^{-1}(2D(t)) \\ &\qquad + |z_2|\left|\frac{\partial\alpha_1}{\partial x_1}\right|\phi_{12}\circ\alpha_1^{-1}(2D(t)) \\ &\quad\le |z_2|\,\overline{\phi}_{22}\left(x_1,\hat{\theta}_1,r\right) + \frac{z_2^2}{4}\left(1 + \left(\frac{\partial\alpha_1}{\partial x_1}\right)^2\right) + d_2(t) \\ &\quad\le z_2\hat{\phi}_{22}\left(x_1,\hat{\theta}_1,r\right) + \epsilon'_{22} + \frac{z_2^2}{4}\left(1 + \left(\frac{\partial\alpha_1}{\partial x_1}\right)^2\right) + d_2(t),\end{aligned} \tag{35}$$

where $\hat{\phi}_{21}(\bar{x}_2,\hat{\theta}_1,r) = (\phi_{21} + |\partial\alpha_1/\partial x_1|\phi_{11})\tanh(z_2(\phi_{21} + |\partial\alpha_1/\partial x_1|\phi_{11})/\epsilon_{21})$, $\epsilon'_{21} = 0.2785\epsilon_{21}$, $\overline{\phi}_{22}(x_1,\hat{\theta}_1,r) = \phi_{22}\circ\alpha_1^{-1}(2r) + |\partial\alpha_1/\partial x_1|\phi_{12}\circ\alpha_1^{-1}(2r)$, $\hat{\phi}_{22}(x_1,\hat{\theta}_1,r) = \overline{\phi}_{22}(x_1,\hat{\theta}_1,r)\tanh(z_2\overline{\phi}_{22}(x_1,\hat{\theta}_1,r)/\epsilon_{22})$, and $d_2(t) = \sum_{j=1}^{2}(\phi_{j2}\circ\alpha_1^{-1}(2D(t)))^2$.

Substituting (35) into (33) gives

$$\begin{aligned}\dot{V}_2 \le& -bk_1 z_1^2 - \frac{b\sigma_1\tilde{\theta}_1^2}{2\gamma_1} - \frac{\bar{c}}{\lambda_0}r \\ &+ z_2\left[g_2(\bar{x}_2)x_3 + g_1(x_1)z_1 + f_2(\bar{x}_2)\right. \\ &- \frac{\partial\alpha_1}{\partial x_1}f_1(x_1) - \frac{\partial\alpha_1}{\partial x_1}g_1(x_1)x_2 - \frac{\partial\alpha_1}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1 \\ &- \frac{\partial\alpha_1}{\partial r}\dot{r} + \hat{\phi}_{21}\left(\bar{x}_2,\hat{\theta}_1,r\right) + \hat{\phi}_{22}\left(x_1,\hat{\theta}_1,r\right) \\ &\left.+\frac{z_2}{4}\left(1 + \left(\frac{\partial\alpha_1}{\partial x_1}\right)^2\right)\right] + \epsilon'_{21} + \epsilon'_{22} + d_2(t) \\ &+ D_1 + \frac{d_0}{\lambda_0} - \frac{b}{\gamma_2}\tilde{\theta}_2\dot{\hat{\theta}}_2 \\ \le& -bk_1 z_1^2 - \frac{b\sigma_1\tilde{\theta}_1^2}{2\gamma_1} - \frac{\bar{c}}{\lambda_0}r \\ &+ z_2\left[g_2(\bar{x}_2)z_3 + g_2(\bar{x}_2)\alpha_2 + \hat{f}_2(Z_2)\right] - \frac{z_2^2}{2} + \epsilon'_{21} \\ &+ \epsilon'_{22} + d_2(t) + D_1 + \frac{d_0}{\lambda_0} - \frac{b}{\gamma_2}\tilde{\theta}_2\dot{\hat{\theta}}_2,\end{aligned} \tag{36}$$

where $z_3 = x_3 - \alpha_2$ and the function $\hat{f}_2(Z_2)$ is specified as

$$\begin{aligned}\hat{f}_2(Z_2) =& g_1(x_1)z_1 + f_2(\bar{x}_2) - \frac{\partial\alpha_1}{\partial x_1}f_1(x_1) + \hat{\phi}_{21}\left(x_1,\hat{\theta}_1,r\right) \\ &+ \hat{\phi}_{22}\left(\bar{x}_2,\hat{\theta}_1,r\right) + \frac{z_2}{4}\left[1 + \left(\frac{\partial\alpha_1}{\partial x_1}\right)^2\right] + \frac{z_2}{2} \\ &- \frac{\partial\alpha_1}{\partial x_1}g_1(x_1)x_2 - \frac{\partial\alpha_1}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1 - \frac{\partial\alpha_1}{\partial r}\dot{r}\end{aligned} \tag{37}$$

with $Z_2 = [\bar{x}_2,\hat{\theta}_1,r]^T \in \Omega_{Z_2} \in R^4$ and Ω_{Z_2} being some known compact set in R^4.

To compensate for the unknown nonlinear function $\hat{f}_2(Z_2)$, a neural network $\phi_2^T\xi_2(Z_2)$ is utilized to model $\hat{f}_2(Z_2)$ such that, for any given positive constant ϵ_{23},

$$\hat{f}_2(Z_2) = \phi_2^T\xi_2(Z_2) + \delta_2(Z_2), \quad \left|\delta_2(Z_2)\right| \le \epsilon_{23}, \tag{38}$$

where ϵ_{23} denotes approximation error.

Then, substituting (38) into (36), one has

$$\dot{V}_2 \le -bk_1 z_1^2 - \frac{b\sigma_1\tilde{\theta}_1^2}{2\gamma_1} - \frac{\bar{c}}{\lambda_0}r + g_2(\bar{x}_2)z_2z_3 + g_2(\bar{x}_2)z_2\alpha_2$$

$$+ z_2\phi_2^T\xi_2(Z_2) + z_2\delta_2(Z_2) - \frac{z_2^2}{2} + \epsilon'_{21} + \epsilon'_{22} + d_2(t) + D_1 + \frac{d_0}{\lambda_0} - \frac{b}{\gamma_2}\widetilde{\theta}_2\dot{\widehat{\theta}}_2. \tag{39}$$

By using

$$z_2\phi_2^T\xi_2(Z_2) \le \frac{b\theta_2}{2\eta_2^2}\xi_2^T(Z_2)\xi_2(Z_2)z_2^2 + \frac{\eta_2^2}{2}, \qquad z_2\delta_2(Z_2) \le \frac{z_2^2}{2} + \frac{\epsilon_{23}^2}{2}, \tag{40}$$

it can be easily verified that

$$\begin{aligned}\dot{V}_2 \le{}& -bk_1z_1^2 - \frac{b\sigma_1\widetilde{\theta}_1^2}{2\gamma_1} - \frac{\bar{c}}{\lambda_0}r + g_2(\bar{x}_2)z_2\alpha_2 \\ &+ \frac{b\theta_2}{2\eta_2^2}\xi_2^T(Z_2)\xi_2(Z_2)z_2^2 + g_2(\bar{x}_2)z_2z_3 \\ &+ \frac{\eta_2^2}{2} + \epsilon'_{21} + \epsilon'_{22} + \frac{\epsilon_{23}^2}{2} + d_2(t) + D_1 + \frac{d_0}{\lambda_0} - \frac{b}{\gamma_2}\widetilde{\theta}_2\dot{\widehat{\theta}}_2\end{aligned} \tag{41}$$

with $\theta_2 = b^{-1}\|\phi_2\|^2$ being an unknown constant.

Furthermore, the virtual control α_2 is constructed as

$$\alpha_2 = -k_2z_2 - \frac{\widehat{\theta}_2}{2\eta_2^2}z_2\xi_2^T(Z_2)\xi_2(Z_2), \tag{42}$$

where $k_2 > 0$ and η_2 are the design constants.

Then, the following result can be easily obtained:

$$g_2(\bar{x}_2)z_2\alpha_2 \le -k_2bz_2^2 - \frac{\widehat{\theta}_2}{2\eta_2^2}bz_2^2\xi_2^T(Z_2)\xi_2(Z_2). \tag{43}$$

By applying (43), (41) can be rewritten as

$$\begin{aligned}\dot{V}_2 \le{}& -\sum_{i=1}^{2}k_ibz_i^2 - \frac{b\sigma_1\widetilde{\theta}_1^2}{2\gamma_1} - \frac{\bar{c}}{\lambda_0}r \\ &+ \frac{b\widetilde{\theta}_2}{\gamma_2}\left(\frac{\gamma_2}{2\eta_2^2}\xi_2^T(Z_2)\xi_2(Z_2)z_2^2 - \dot{\widehat{\theta}}_2\right) + g_2(\bar{x}_2)z_2z_3 \\ &+ \frac{\eta_2^2}{2} + \epsilon'_{21} + \epsilon'_{22} + \frac{\epsilon_{23}^2}{2} + d_2(t) + \frac{d_0}{\lambda_0} + D_1.\end{aligned} \tag{44}$$

Define the adaptive law as

$$\dot{\widehat{\theta}}_2 = \frac{\gamma_2}{2\eta_2^2}\xi_2^T(Z_2)\xi_2(Z_2)z_2^2 - \sigma_2\widehat{\theta}_2, \tag{45}$$

where η_2, γ_2, and σ_2 are design parameters.

Combining (44) with (45) produces

$$\begin{aligned}\dot{V}_2 \le{}& -\sum_{i=1}^{2}k_ibz_i^2 - \frac{b\sigma_1\widetilde{\theta}_1^2}{2\gamma_1} - \frac{\bar{c}}{\lambda_0}r + \frac{b\sigma_2\widetilde{\theta}_2\widehat{\theta}_2}{\gamma_2} + g_2(\bar{x}_2)z_2z_3 \\ &+ \frac{\eta_2^2}{2} + \epsilon'_{21} + \epsilon'_{22} + \frac{\epsilon_{23}^2}{2} + d_2(t) + D_1 + \frac{d_0}{\lambda_0} \\ \le{}& -\sum_{i=1}^{2}\left(k_ibz_i^2 + \frac{b\sigma_i\widetilde{\theta}_i^2}{2\gamma_i}\right) - \frac{\bar{c}}{\lambda_0}r + \frac{d_0}{\lambda_0} \\ &+ \sum_{i=1}^{2}D_i + g_2(\bar{x}_2)z_2z_3,\end{aligned} \tag{46}$$

where $D_i = \eta_i^2/2 + b\sigma_i\theta_i^2/2\gamma_i + \epsilon_{i3}^2/2 + \epsilon'_{i1} + \epsilon'_{i2} + d_i(t)$, $i = 1, 2$, and the result $b\sigma_2\widetilde{\theta}_2\widehat{\theta}_2/\gamma_2 \le -b\sigma_2\widetilde{\theta}_2^2/2\gamma_2 + b\sigma_2\theta_2^2/2\gamma_2$ has been used in the above equation.

Step i $(3 \le i \le n-1)$. According to $z_i = x_i - \alpha_{i-1}$, the dynamics of z_i is

$$\begin{aligned}\dot{z}_i ={}& g_i(\bar{x}_i)x_{i+1} + f_i(\bar{x}_i) + \Delta_i \\ &- \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\left(g_j(\bar{x}_j)x_{j+1} + f_j(\bar{x}_j) + \Delta_j\right) \\ &- \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\widehat{\theta}_j}\dot{\widehat{\theta}}_j - \frac{\partial\alpha_{i-1}}{\partial r}\dot{r} \\ ={}& g_i(\bar{x}_i)x_{i+1} + f_i(\bar{x}_i) - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\left(g_j(\bar{x}_j)x_{j+1} + f_j(\bar{x}_j)\right) \\ &+ \overline{\Delta}_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\widehat{\theta}_j}\dot{\widehat{\theta}}_j - \frac{\partial\alpha_{i-1}}{\partial r}\dot{r},\end{aligned} \tag{47}$$

where $\overline{\Delta}_i = \Delta_i - \sum_{j=1}^{i-1}(\partial\alpha_{i-1}/\partial x_j)\Delta_j$.

Consider the Lyapunov function V_i as

$$V_i = V_{i-1} + \frac{1}{2}z_i^2 + \frac{b}{2\gamma_i}\widetilde{\theta}_i^2. \tag{48}$$

By using the derivations similar to those used in the former steps, we can obtain

$$\begin{aligned}\dot{V}_i \le{}& -\sum_{j=1}^{i-1}\left(k_jbz_j^2 + \frac{b\sigma_j\widetilde{\theta}_j^2}{2\gamma_j}\right) - \frac{\bar{c}}{\lambda_0}r \\ &+ z_i\left(g_i(\bar{x}_i)x_{i+1} + f_i(\bar{x}_i) - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}f_j(\bar{x}_j)\right. \\ &\left. - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}g_j(\bar{x}_j)x_{j+1} - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\widehat{\theta}_j}\dot{\widehat{\theta}}_j\right.\end{aligned}$$

$$-\frac{\partial \alpha_{i-1}}{\partial r}\dot{r} + g_{i-1}\left(\overline{x}_{i-1}\right) z_{i-1}\Bigg)$$

$$+\left|z_i\overline{\Delta}_i\right| + \frac{d_0}{\lambda_0} + \sum_{j=1}^{i-1} D_j - \frac{b}{\gamma_i}\widetilde{\theta}_i\dot{\widehat{\theta}}_i. \tag{49}$$

Similar to (34), we have

$$\begin{aligned}
\left|z_i\overline{\Delta}_i\right| &\leq \left|z_i\right|\left(\left|\Delta_i\right| + \sum_{j=1}^{i-1}\left|\frac{\partial \alpha_{i-1}}{\partial x_j}\right|\left|\Delta_j\right|\right) \\
&\leq \left|z_i\right|\left(\phi_{i1} + \sum_{j=1}^{i-1}\left|\frac{\partial \alpha_{i-1}}{\partial x_j}\right|\phi_{j1}\right) \\
&\quad + \left|z_i\right|\left(\phi_{i2}\left(|z|\right) + \sum_{j=1}^{i-1}\left|\frac{\partial \alpha_{i-1}}{\partial x_j}\right|\phi_{j2}\left(|z|\right)\right).
\end{aligned} \tag{50}$$

Furthermore, the following inequalities can be easily verified by repeating the same arguments as (35):

$$\left|z_i\right|\left(\phi_{i1} + \sum_{j=1}^{i-1}\left|\frac{\partial \alpha_{i-1}}{\partial x_j}\right|\phi_{j1}\right) \leq z_i\widehat{\phi}_{i1}\left(\overline{x}_i, \overline{\widehat{\theta}}_{i-1}, r\right) + \epsilon_{i1}', \tag{51}$$

$$\begin{aligned}
&\left|z_i\right|\left(\phi_{i2}\left(|z|\right) + \sum_{j=1}^{i-1}\left|\frac{\partial \alpha_{i-1}}{\partial x_j}\right|\phi_{j2}\left(|z|\right)\right) \\
&\quad\leq z_i\widehat{\phi}_{i2}\left(\overline{x}_i, \overline{\widehat{\theta}}_{i-1}, r\right) + \frac{z_i^2}{4}\left[1 + \sum_{j=1}^{i-1}\left(\frac{\partial \alpha_{i-1}}{\partial x_j}\right)^2\right] \\
&\qquad + \epsilon_{i2}' + d_i\left(t\right),
\end{aligned} \tag{52}$$

where $\widehat{\phi}_{i1}(\overline{x}_i, \overline{\widehat{\theta}}_{i-1}, r) = (\phi_{i1} + \sum_{j=1}^{i-1} |\partial\alpha_{i-1}/\partial x_j|\phi_{j1}) \tanh(z_i(\phi_{i1} + \sum_{j=1}^{i-1} |\partial\alpha_{i-1}/\partial x_j|\phi_{j1})/\epsilon_{i1})$, $\epsilon_{i1}' = 0.2785\epsilon_{i1}$, $\widehat{\phi}_{i2}(\overline{x}_i, \overline{\widehat{\theta}}_{i-1}, r) = \overline{\phi}_{i2}(\overline{x}_i, \overline{\widehat{\theta}}_{i-1}, r) \tanh(z_i\overline{\phi}_{i2}(\overline{x}_i, \overline{\widehat{\theta}}_{i-1}, r)/\epsilon_{i2})$, $\overline{\phi}_{i2}(\overline{x}_i, \overline{\widehat{\theta}}_{i-1}, r) = \phi_{i2} \circ \alpha_1^{-1}(2r) + \sum_{j=1}^{i-1} |\partial_{i-1}/\partial x_j|\phi_{j2} \circ \alpha_1^{-1}(2r)$, $\epsilon_{i2}' = 0.2785\epsilon_{i2}$, and $d_i(t) = \sum_{j=1}^{i} (\phi_{j2} \circ \alpha_1^{-1}(2D(t)))^2$, noting that $d_i(t) \geq 0$ for all $t \geq 0$.

Substituting (51) and (52) into (49) results in

$$\begin{aligned}
\dot{V}_i &\leq -\sum_{j=1}^{i-1}\left(k_j b z_j^2 + \frac{b\sigma_j\widetilde{\theta}_j^2}{2\gamma_j}\right) - \frac{\overline{c}}{\lambda_0}r \\
&\quad + z_i\left(g_i\left(\overline{x}_i\right) z_{i+1} + g_i\left(\overline{x}_i\right)\alpha_i + \widehat{f}_i\left(Z_i\right)\right) \\
&\quad - \frac{z_i^2}{2} + \epsilon_{i1}' + \epsilon_{i2}' + d_i\left(t\right) + \frac{d_0}{\lambda_0} + \sum_{j=1}^{i-1} D_j - \frac{b}{\gamma_i}\widetilde{\theta}_i\dot{\widehat{\theta}}_i,
\end{aligned} \tag{53}$$

where the function $\widehat{f}_i(Z_i)$ is defined by

$$\begin{aligned}
\widehat{f}_i\left(Z_i\right) &= f_i\left(\overline{x}_i\right) - \sum_{j=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial x_j} f_j\left(\overline{x}_j\right) - \sum_{j=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial x_j} g_j\left(\overline{x}_j\right) x_{j+1} \\
&\quad - \sum_{j=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial \widehat{\theta}_j}\dot{\widehat{\theta}}_j - \frac{\partial \alpha_{i-1}}{\partial r}\dot{r} + g_{i-1}\left(\overline{x}_{i-1}\right) z_{i-1} \\
&\quad + \widehat{\phi}_{i1}\left(\overline{x}_i, \overline{\widehat{\theta}}_{i-1}, r\right) + \widehat{\phi}_{i2}\left(\overline{x}_i, \overline{\widehat{\theta}}_{i-1}, r\right) \\
&\quad + \frac{z_i}{4}\left[1 + \sum_{j=1}^{i-1}\left(\frac{\partial \alpha_{i-1}}{\partial x_j}\right)^2\right] + \frac{z_i}{2},
\end{aligned} \tag{54}$$

where $Z_i = [\overline{x}_i, \overline{\widehat{\theta}}_{i-1}, r]^T \in \Omega_{Z_i} \in R^{2i}$, $\overline{x}_i = [x_1, \ldots, x_i]^T$, and Ω_{Z_i} is some known compact set in R^{2i}.

Currently, a neural network $\phi_i^T\xi_i(Z_i)$ is utilized to model $\widehat{f}_i(Z_i)$ such that, for a given $\epsilon_{i3} > 0$, $\widehat{f}_i(Z_i)$ can be expressed as

$$\widehat{f}_i\left(Z_i\right) = \phi_i^T\xi_i\left(Z_i\right) + \delta_i\left(Z_i\right), \quad \left|\delta_i\left(Z_i\right)\right| \leq \epsilon_{i3}. \tag{55}$$

Further, similar to (40), we can obtain

$$\begin{aligned}
z_i\phi_i^T\xi_i\left(Z_i\right) &\leq \frac{b\theta_i}{2\eta_i^2}\xi_i^T\left(Z_i\right)\xi_i\left(Z_i\right) z_i^2 + \frac{\eta_i^2}{2}, \\
z_i\delta_i\left(Z_i\right) &\leq \frac{z_i^2}{2} + \frac{\epsilon_{i3}^2}{2},
\end{aligned} \tag{56}$$

where $\theta_i = b^{-1}\|\phi_i\|^2$ is an unknown constant.

Now, construct the virtual control signal α_i as

$$\alpha_i = -k_i z_i - \frac{\widehat{\theta}_i}{2\eta_i^2} z_i\xi_i^T\left(Z_i\right)\xi_i\left(Z_i\right), \tag{57}$$

with $k_i > 0$ and η_i being design constants.

Then, by substituting (55)–(57) into (53), choosing the adaptive law

$$\dot{\widehat{\theta}}_i = \frac{\gamma_i}{2\eta_i^2}\xi_i^T\left(Z_i\right)\xi_i\left(Z_i\right) z_i^2 - \sigma_i\widehat{\theta}_i \tag{58}$$

with η_i, γ_i, and σ_i being the design parameters, and then following the same line as the procedures from (43) to (46), we have

$$\begin{aligned}
\dot{V}_i &\leq -\sum_{j=1}^{i}\left(k_j b z_j^2 + \frac{b\sigma_j\widetilde{\theta}_j^2}{2\gamma_j}\right) - \frac{\overline{c}}{\lambda_0}r + \frac{d_0}{\lambda_0} + \sum_{j=1}^{i} D_j \\
&\quad + g_i\left(\overline{x}_i\right) z_i z_{i+1},
\end{aligned} \tag{59}$$

where $D_j = \eta_j^2/2 + b\sigma_j\theta_j^2/2\gamma_j + \epsilon_{j3}^2/2 + \epsilon_{j1}' + \epsilon_{j2}' + d_j(t)$, $j = 1, 2, \ldots, i$.

Step n. In this step, the actual controller u is designed. According to $z_n = x_n - \alpha_{n-1}$, then we have

$$\dot{z}_n = g_n(\bar{x}_n)u + f_n(\bar{x}_n) - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}f_j(\bar{x}_j) - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}g_j(\bar{x}_j)x_{j+1} + \bar{\Delta}_n - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}_j}\dot{\hat{\theta}}_j - \frac{\partial\alpha_{n-1}}{\partial r}\dot{r}, \tag{60}$$

where $\bar{\Delta}_n = \Delta_n - \sum_{i=1}^{n-1}(\partial\alpha_{n-1}/\partial x_i)\Delta_i$. Similarly, choose the following Lyapunov function as

$$V_n = V_{n-1} + \frac{1}{2}z_n^2 + \frac{b}{2\gamma_n}\tilde{\theta}_n^2. \tag{61}$$

From (53) and (54), we have

$$\begin{aligned}\dot{V}_n \le & -\sum_{j=1}^{n-1}\left(k_j b z_j^2 + \frac{b\sigma_j\tilde{\theta}_j^2}{2\gamma_j}\right) - \frac{\bar{c}}{\lambda_0}r + \frac{d_0}{\lambda_0} + \sum_{j=1}^{n-1}D_j + g_{n-1}(\bar{x}_{n-1})z_{n-1}z_n \\ & + z_n\left(g_n(\bar{x}_n)u + f_n(\bar{x}_n) - \sum_{i=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_i}f_i(\bar{x}_i) - \sum_{i=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_i}g_i(\bar{x}_i)x_{i+1} - \sum_{i=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}_i}\dot{\hat{\theta}}_i - \frac{\partial\alpha_{n-1}}{\partial r}\dot{r}\right) - \frac{b}{\gamma_n}\tilde{\theta}_n\dot{\hat{\theta}}_n + \left|z_n\bar{\Delta}_n\right|.\end{aligned} \tag{62}$$

Using the same estimation methods as (42)–(44), we have

$$\left|z_n\bar{\Delta}_n\right| \le z_n\hat{\phi}_{n1}\left(\bar{x}_n,\bar{\hat{\theta}}_{n-1},r\right) + \epsilon_{n1}' + z_n\hat{\phi}_{n2}\left(\bar{x}_n,\bar{\hat{\theta}}_{n-1},r\right) + \frac{z_n^2}{4}\left[1 + \sum_{j=1}^{n-1}\left(\frac{\partial\alpha_{n-1}}{\partial x_j}\right)^2\right] + \epsilon_{n2}' + d_n(t), \tag{63}$$

where $\hat{\phi}_{n1}(\bar{x}_n,\bar{\hat{\theta}}_{n-1},r)$, ϵ_{n1}', $\hat{\phi}_{n2}(\bar{x}_n,\bar{\hat{\theta}}_{n-1},r)$, ϵ_{n2}', and $d_n(t)$ are defined in (51) or (52) with $i = n$. By substituting (63) into (62), one has

$$\begin{aligned}\dot{V}_n \le & -\sum_{j=1}^{n-1}\left(k_j b z_j^2 + \frac{b\sigma_j\tilde{\theta}_j^2}{2\gamma_j}\right) - \frac{\bar{c}}{\lambda_0}r + \frac{d_0}{\lambda_0} + \sum_{j=1}^{n-1}D_j + \epsilon_{n1}' \\ & + \epsilon_{n2}' + d_n(t) + z_n\left(g_n(\bar{x}_n)u + \hat{f}_n(Z_n)\right) - \frac{z_n^2}{2} - \frac{b}{\gamma_n}\tilde{\theta}_n\dot{\hat{\theta}}_n,\end{aligned} \tag{64}$$

where the function $\hat{f}_n(Z_n)$ is defined by

$$\begin{aligned}\hat{f}_n(Z_n) = & \, g_{n-1}(\bar{x}_{n-1})z_{n-1} + f_n(\bar{x}_n) - \sum_{i=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_i}f_i(\bar{x}_i) - \sum_{i=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_i}g_i(\bar{x}_i)x_{i+1} - \sum_{i=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}_i}\dot{\hat{\theta}}_i - \frac{\partial\alpha_{n-1}}{\partial r}\dot{r} \\ & + \hat{\phi}_{n1}\left(\bar{x}_n,\bar{\hat{\theta}}_{n-1},r\right) + \hat{\phi}_{n2}\left(\bar{x}_n,\bar{\hat{\theta}}_{n-1},r\right) + \frac{z_n}{4}\left[1 + \sum_{j=1}^{n-1}\left(\frac{\partial\alpha_{n-1}}{\partial x_j}\right)^2\right] + \frac{z_n}{2}\end{aligned} \tag{65}$$

with $Z_n = [\bar{x}_n,\bar{\hat{\theta}}_{n-1},r]^T \in \Omega_{Z_n} \in R^{2n}$ and Ω_{Z_n} being some known compact set in R^{2n}. Similarly, neural network $\phi_n^T\xi_n(Z_n)$ is employed to model $\hat{f}_n(Z_n)$ such that $\hat{f}_n(Z_n) = \phi_n^T\xi_n(Z_n) + \delta_n(Z_n)$, $|\delta_n(Z_n)| \le \epsilon_{n3}$. Then, following the same line as used in (40), we have

$$\begin{aligned} z_n\phi_n^T\xi_n(Z_n) &\le \frac{b\theta_n}{2\eta_n^2}\xi_n^T(Z_n)\xi_n(Z_n)z_n^2 + \frac{\eta_n^2}{2}, \\ z_n\delta_n(Z_n) &\le \frac{z_n^2}{2} + \frac{\epsilon_{n3}^2}{2},\end{aligned} \tag{66}$$

where $\theta_n = b^{-1}\|\phi_n\|^2$ denotes an unknown constant and η_n is a design constant.

Subsequently, by combining (64) together with (66), the inequality below holds:

$$\begin{aligned}\dot{V}_n \le & -\sum_{j=1}^{n-1}\left(k_j b z_j^2 + \frac{b\sigma_j\tilde{\theta}_j^2}{2\gamma_j}\right) - \frac{\bar{c}}{\lambda_0}r + \frac{d_0}{\lambda_0} + \sum_{j=1}^{n-1}D_j + \epsilon_{n1}' + \epsilon_{n2}' + d_n(t) + z_n g_n(\bar{x}_n)u \\ & + \frac{b\theta_n}{2\eta_n^2}\xi_n^T(Z_n)\xi_n(Z_n)z_n^2 + \frac{\eta_n^2}{2} + \frac{\epsilon_{n3}^2}{2} - \frac{b}{\gamma_n}\tilde{\theta}_n\dot{\hat{\theta}}_n.\end{aligned} \tag{67}$$

At the present stage, construct the real controller u and adaptive law $\dot{\hat{\theta}}_n$ in the following forms:

$$u = -k_n z_n - \frac{\hat{\theta}_n}{2\eta_n^2}z_n\xi_n^T(Z_n)\xi_n(Z_n), \tag{68}$$

$$\dot{\hat{\theta}}_n = \frac{\gamma_n}{2\eta_n^2}\xi_n^T(Z_n)\xi_i(Z_n)z_n^2 - \sigma_n\hat{\theta}_n, \tag{69}$$

where k_n, η_n, γ_n, and σ_n are design constants.

Then, repeating the similar procedures as (43)–(46), we can obtain

$$\dot{V}_n \le -\sum_{j=1}^{n}\left(k_j b z_j^2 + \frac{b\sigma_j\tilde{\theta}_j^2}{2\gamma_j}\right) - \frac{\bar{c}}{\lambda_0}r + \frac{d_0}{\lambda_0} + \sum_{j=1}^{n}D_j, \tag{70}$$

where $D_j = \eta_j^2/2 + b\sigma_j\theta_j^2/2\gamma_j + \epsilon_{j3}^2/2 + \epsilon_{j1}' + \epsilon_{j2}' + d_j(t)$, $j = 1, 2, \ldots, n$.

Now, the main result of this research is summarized as follows.

Theorem 8. *Consider the system* (1) *consisting of Assumptions 2–5, the control input* (68)*, and the adaptive laws* (58) *and* (69)*. Assume that the packaged unknown functions $\hat{f}_i(Z_i)$ ($i = 1, 2, \ldots, n$) could be modeled by neural networks $\phi_i^T \xi_i(Z_i)$ with the bounded approximation errors. Then, for bounded initial values with $\hat{\theta}_i(0) \geq 0$, all the signals in the closed-loop system are semiglobally boundedness in mean square.*

Proof. To give the stability analysis for the closed-loop system, consider the Lyapunov function in the form $V = V_n$, and define

$$a_0 = \min\{2k_i b, \bar{c}, \sigma_i, i = 1, 2, \ldots, n\}, \qquad b_0 = \frac{d_0}{\lambda_0} + \sum_{j=1}^{n} D_j. \tag{71}$$

Furthermore, we can rewrite (70) as

$$\dot{V} \leq -a_0 V + b_0. \tag{72}$$

Next, from (72), the following inequality can be easily verified:

$$V(t) \leq \left(V(0) - \frac{b_0}{a_0}\right) e^{-a_0 t} + \frac{b_0}{a_0}, \quad \forall t > 0, \tag{73}$$

which means that

$$V(t) \leq V(0) + \frac{b_0}{a_0}, \quad \forall t > 0. \tag{74}$$

Therefore, based on the definition of V in (61), z_j, $\tilde{\theta}_j$ ($j = 1, 2, \ldots, n$), and r are bounded. Because the signal r is bounded, the trajectory $z(t)$ is bounded. Since θ_i are constants, $\hat{\theta}_i$ are bounded. Consequently, α_i are also bounded because z_i and $\hat{\theta}_i$ are bounded variables. Hence, we conclude that the signals x_i are bounded. □

4. Simulation Example

A simulation example is presented to show the effectiveness of the proposed control scheme. Consider the second-order nonlinear system as

$$\begin{aligned}
\dot{z} &= -z + x_1^2 + 0.5, \\
\dot{x}_1 &= x_2 + x_1^2 e^{-0.5x_1} + z x_1 \sin(x_1), \\
\dot{x}_2 &= u + x_1 x_2^2 + x_1 x_2 z,
\end{aligned} \tag{75}$$

where $f_1(x_1) = x_1^2 e^{-0.5x_1}$, $f_2(x_1, x_2) = x_1 x_2^2$, $\Delta_1 = z x_1 \sin(x_1)$, and $\Delta_2 = x_1 x_2 z$. It can be easily verified that Assumption 2 is satisfied. In order to check Assumption 4 holds for z-subsystem in (75), consider $V_z(z) = z^2$, and then

$$\begin{aligned}
\dot{V}_z(z) &= 2z\left(-z + x_1^2 + 0.5\right) \\
&\leq -2z^2 + \frac{1}{4\varepsilon}(2z)^2 + \varepsilon x_1^4 + \frac{\varepsilon}{4} + \frac{z^2}{\varepsilon}.
\end{aligned} \tag{76}$$

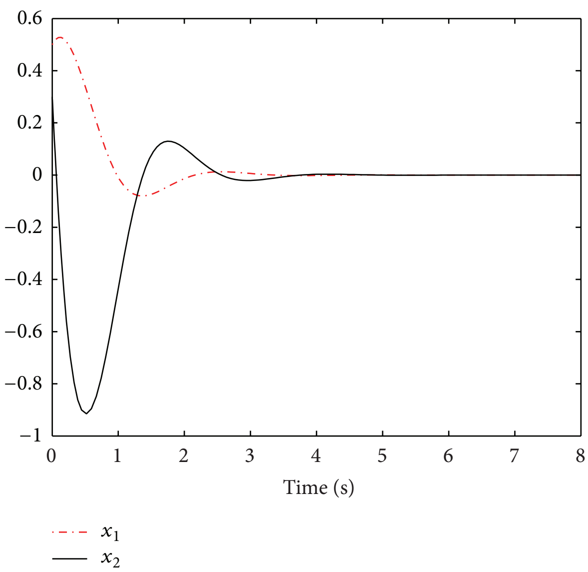

Figure 1: State variables x_1 and x_2.

By choosing $\varepsilon = 2.5$, we have

$$\dot{V}_z(z) \leq -1.2z^2 + 2.5x_1^4 + 0.625. \tag{77}$$

By Defining $\alpha_1(|z|) = 0.5z^2$, $\alpha_2(|z|) = 2z^2$, $c_0 = 1.2$, $d_0 = 0.625$, and $\mu(|x_1|) = 2.5x_1^4$, Assumption 4 is satisfied. Take $\bar{c} = 1 \in (0, c_0)$ and define the dynamic signal as follows:

$$\dot{r} = -r + 2.5x_1^4 + 0.625. \tag{78}$$

By using Theorem 8, the virtual control signal, the real control input, and the adaptive laws are constructed as follows:

$$\begin{aligned}
\alpha_1 &= -k_1 z_1 - \frac{\hat{\theta}_1}{2\eta_1^2} \xi_1^T(Z_1) \xi_2(Z_1) z_1, \\
u &= -k_2 z_2 - \frac{\hat{\theta}_2}{2\eta_2^2} \xi_2^T(Z_2) \xi_2(Z_2) z_2, \\
\dot{\hat{\theta}}_i &= \frac{\gamma_i}{2\eta_i^2} \xi_i^T(Z_i) \xi_i(Z_i) z_i^2 - \sigma_i \hat{\theta}_i, \quad i = 1, 2.
\end{aligned} \tag{79}$$

In the simulation, the design constants are chosen as $k_1 = k_2 = 3$, $\eta_1 = \eta_2 = 1$, $\gamma_1 = \gamma_2 = 2$, and $\sigma_1 = \sigma_2 = 1$. The simulation is carried out with the initial conditions $[x_1(0), x_2(0), \hat{\theta}_1(0), \hat{\theta}_2(0)]^T = [0.5, 0.3, 0, 0]$.

The simulation results are shown in Figures 1–3. Figure 1 shows the trajectories of states x_1 and x_2. Figure 2 displays the trajectory of control input u. Figure 3 shows the trajectories of adaptive parameters $\hat{\theta}_1$ and $\hat{\theta}_2$. From Figures 1–3, we can see that the proposed control approach can guarantee the boundedness of the variables x_1, x_2, u, $\hat{\theta}_1$, and $\hat{\theta}_2$.

5. Conclusion

In this research, a backstepping-based adaptive neural control scheme has been developed for strict-feedback nonlinear systems with unmodeled dynamics and dynamic disturbances.

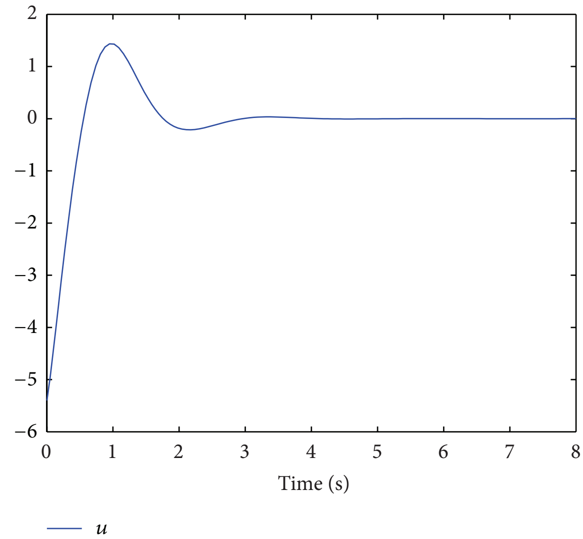

Figure 2: The actual control input u.

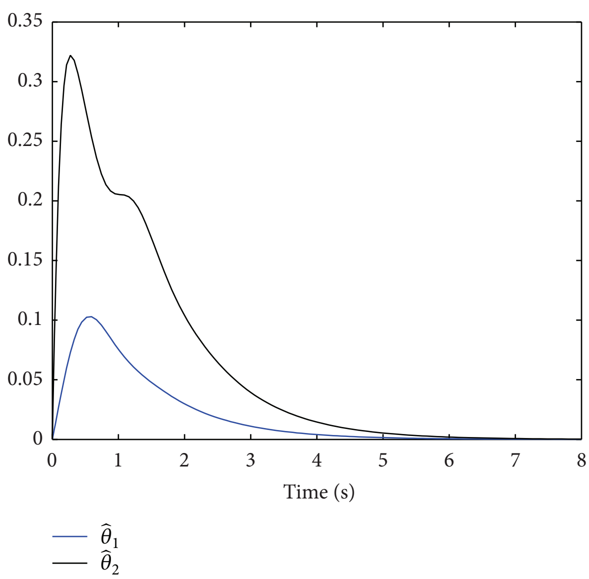

Figure 3: The adaptive parameters $\hat{\theta}_1$ and $\hat{\theta}_2$.

The proposed adaptive neural controller guarantees that all the signals of the resulting closed-loop system remain semi-globally uniformly ultimately bounded in the sense of mean square. Simulation results have been provided to illustrate the effectiveness of the proposed control scheme. It should be pointed out that the work in this paper does not consider the problem of input nonlinearity and time-delay. Then, they may occur in practical engineering. So, how to control a nonlinear system with input nonlinearity and time-delay is our future research direction.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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