Switched Sliding Mode Control Strategy for Networked Systems

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***Abstract*— In this paper, a networked switched control strategy based on Sliding Mode Control is presented. The idea pursued in this work is to reduce to a minimum the packet rate over the network, in order to limit the problems induced by the transmission of the state measurement between the sensor and the controller, while providing performance comparable with that of a non networked Sliding Mode Control scheme. The proposed scheme includes a model based controller which contains the nominal model of the plant, and relies on a suitably defined triggering condition. The latter considers the amplitude of a sliding variable determined relying on nominal model, and enables the actual state transmission only when the sliding variable is within a predefined boundary layer. When the plant state is not transmitted, the model state is used to determine the control action. In this way, it is possible to guarantee the same robustness with respect to matched uncertainties as in conventional sliding mode control schemes, as well as the exponential stability of the origin of the controlled system state space, even if the actual system state is not always used to close the feedback. Moreover, in steady-state, when the boundary layer is reached, in order to avoid a continuous transmission of the actual state measurement, a mechanism based on a moving average of the current sliding variable is adopted, which allows to suitably deactivate the state transmission even within the boundary layer, yet maintaining some robustness. Simulation results demonstrates the effectiveness of the proposed strategy.**

I. Introduction

Sliding Mode Control (SMC) is a widely appreciated strategy because of its capability of guaranteeing satisfactory performance of the controlled system under critical uncertainty conditions [1], [2], [3], [4]. For this reason, it can be regarded as a good candidate to be used in Networked Control Systems (NCSs), i.e., feedback systems including data networks. NCSs present several advantages compared with traditional configurations, such as reconfigurability, low installation costs, and the possibility to create a wide interconnected grid to transmit information [5]. However, the presence of the network in the control loop can cause the occurrence of packet loss, jitter, and delayed transmissions, which deteriorate the performance of control systems designed in the conventional way. As a consequence, suitable data communication protocols, new fault detection strategies, and control schemes designed explicitly taking into account the network presence have been proposed in the literature in recent years [5], [6], [7].

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The aim of this paper is the design of a networked control scheme, which is able to solve the trade-off between the stability of the controlled system and the bandwidth allocation over the network [8]. In order to obtain satisfactory performance, in this framework, the use of a robust control strategy is mandatory. In this paper, the SMC methodology is adopted to design the controller. This is of model based type [9], [10], so that, relying on a suitably defined event triggered strategy [11], [12], [13], [14], [15], [16], it is possible to switch between the use of the actual plant state and of the model state to close the feedback. More specifically, the model based controller contains three elements: the controller itself, the nominal model of the plant and a triggering condition. The triggering condition verifies if a sliding variable, determined relying on the nominal model state, belongs to a predefined boundary layer, and enables the actual state transmission only when this occurs. When the condition does not hold, then the model state is used to determine the control action.

As a result, the state of the nominal model is used during the reaching phase, i.e. the phase during which the sliding variable is steered to the boundary layer, while the actual plant state is used within the boundary layer. This could cause an intensification of state transmission in steady-state. To circumvent this drawback, and to keep the packet rate to a minimum, a mechanism based on a moving average of the current sliding variable is adopted. This mechanism enables to deactivate the state transmission even within the boundary layer. Indeed, if the moving average remains almost constant for a certain number of sampling time instants, then the model state is used again.

The theoretical assessment of the proposed scheme is provided in the paper, by addressing both robustness and stability issues. Assuming that the deactivation mechanism is switched off, our proposal proves to maintain the same robustness property with respect to matched uncertainties as conventional SMC. Moreover, the exponential stability of the origin of the controlled system state space can be proved, even if the actual system state is not always used to close the feedback, which implies a clear benefit in terms of bandwidth allocation. When the deactivation mechanism is on, then the robustness versus matched uncertainties is attenuated. Yet, it can be proved that the effect of such uncertainties is bounded.

Note that, in this scheme, a traditional sensor, with no particular computational capability, is required. Moreover, the network mathematical model is not considered in the design. Finally, for the sake of simplicity, the network presence is assumed only between the sensor and the model based controller, since, the controlled system being single-input, the control variable transmission is definitely less onerous than state transmission. Finally, it is worth noting that the nominal model of the plant receives the same control variable fed into the real plant and it is updated whenever a triggering event occurs.

The paper is organized as follows. Section II is devoted to the problem formulation. In Section III the overall control strategy is described, putting into evidence the event triggered and switched nature of the proposed networked control scheme. In Section IV the theoretical results are presented. Simulations on an inverted pendulum are reported in Section V to demonstrate the efficacy of the proposal, while some conclusions are gathered in Section VI.

II. Problem Formulation

Consider the following perturbed chain of integrators

$$\begin{cases} \dot{x}_i(t) = x_{i+1}(t) \qquad i = 1, \ldots, n-1 \\ \dot{x}_n(t) = f(x,t) + b(x,t)u(t) + h(x,t) \\ y(t) = \sigma(x(t)) \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}$ is the control variable, and $\sigma : \mathbb{R}^n \to \mathbb{R}$ is a smooth output function, called *sliding variable*. The functions $f(\cdot)$ and $b(\cdot)$ are known, while $h(\cdot)$ is the matched uncertainty affecting the system, such that

$$h(x,t) = b(x,t)u_m(t) \tag{2}$$

where it holds that

$$|h(x,t)| \le h_{max} \tag{3}$$

with h_{max} being a positive constant.

The *nominal model* of system (1), which is not affected by the uncertain terms by definition, is the following

$$\begin{cases} \dot{\hat{x}}_i(t) = \hat{x}_{i+1}(t) \qquad i = 1, \ldots, n-1 \\ \dot{\hat{x}}_n(t) = f(\hat{x},t) + b(\hat{x},t)u(t) \\ \hat{y}(t) = \hat{\sigma}(\hat{x}(t)) \end{cases} \tag{4}$$

where $\hat{x} \in \mathbb{R}^n$ is the state, u is the same control variable of the plant, while $\hat{\sigma} : \mathbb{R}^n \to \mathbb{R}$ is the sliding variable of the model.

Now, consider the control scheme illustrated in Fig. 1. The

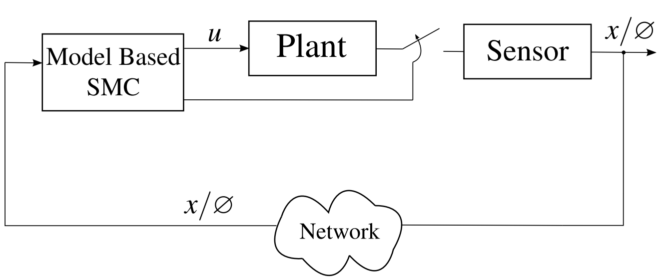

Fig. 1. The model based networked sliding mode control scheme.

Plant, which is represented by system (1), is connected to the sensor through a switch, controlled by the *model based Sliding Mode Control* (SMC) block. In particular, the latter generates the control law both for the plant and the model, on the basis of a suitably defined *triggering condition*

$$|\hat{\sigma}| \le \lambda_0 \tag{5}$$

where λ_0 is a positive constant. Moreover, we define *boundary layer* the following set

$$\mathcal{B}_{\lambda_0} \triangleq \{\hat{x}(t) : \ |\hat{\sigma}| \le \lambda_0\} \tag{6}$$

Note that, in this paper, the network is not described by a model and it is present only between the sensor and the model based controller. Moreover, in Fig. 1, the notation $x/\varnothing$ means that the actual state or no variable is received over the network, on the basis of condition (5).

Considering (1)-(5), and the control scheme in Fig. 1, the problem dealt with the proposal underlying this paper is that of steering the system state to the origin, while guaranteeing robustness properties in front of matched disturbances, and ensuring a significant reduction of the packet rate over the network with respect to a conventional (i.e., non networked) solution.

III. The Proposed Control Strategy

The key block of the proposed control scheme is the model based controller, depicted in Fig. 2. This block includes the sliding mode controller, the nominal model of the plant and the triggering condition. The proposed strategy consists of two operating modes.

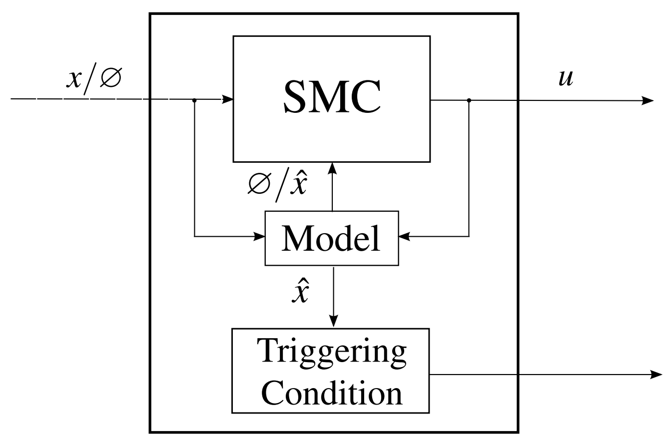

Fig. 2. A representation of the model based controller with SMC law.

Mode 1: The state of the model $\hat{x}$ is provided to the triggering condition block which computes the sliding variable as

$$\hat{\sigma}(t) = \hat{x}_n(t) + \sum_{i=1}^{n-1} m_i \hat{x}_i(t) \tag{7}$$

with m_i being positive constants. When condition (5) is violated, the state $\hat{x}$ is sent to the sliding mode controller and the following control law

$$u_{SMC}(t) = -U_{max}\,\mathrm{sgn}(\hat{\sigma}(t)) \tag{8}$$

is generated, with $U_{max} > 0$ being a suitably chosen control parameter in order to enforce a sliding mode on the selected sliding manifold $\hat{\sigma} = 0$. The control law (8) is fed both to the plant and to the nominal model.

Mode 2: Contrary to Mode 1, when condition (5) is verified, the switch between the plant and the sensor is closed and the measured state x is sent over the network. The nominal model is reinitialized with the plant and the following control variable

$$u_{SMC}(t) = -U_{max}\,\mathrm{sgn}(\sigma(t)) \tag{9}$$

where σ depends on the actual state and has the same form in (7), is fed both to the plant and to the nominal model. Note that, in Fig. 2 the notation $\varnothing/\hat{x}$ means that the state of the model is used only if the actual state has not been received through the network.

The use of the nominal model out of the boundary layer $\mathcal{B}_{\lambda_0}$ in (6), during the so-called reaching phase when also the plant is sensitive with respect to matched uncertainties, allows to force the sliding variable to the origin also in presence of unknown terms. In order to guarantee the robustness of the controlled system uniformly in time, the actual sliding variable is used in a neighborhood of the origin where the nominal model based control variable could not guarantee a complete disturbance rejection. Since the objective of this paper is also to reduce the packet rate over the network, an additional condition has been considered within the boundary layer $\mathcal{B}_{\lambda_0}$, that is, the state of the nominal model is used instead of the actual plant state also if

$$|\hat{\sigma}| \leq \lambda_0 \ \wedge \ |\sigma_{MA}| \leq \lambda_1 \tag{10}$$

where $0 < \lambda_1 < \lambda_0$, while σ_{MA} is the Moving Average (MA) of the current sliding variable, computed as

$$\sigma_{MA}(\tau_i) = \frac{1}{N}\sum_{k=1}^{N} \zeta(\tau_i - \tau_k) \tag{11}$$

$$\zeta = \begin{cases} \hat{\sigma} & \text{if } |\hat{\sigma}| \leq \lambda_0 \ \wedge \ |\sigma_{MA}(\tau_{i-1})| \leq \lambda_1 \\ \sigma & \text{if } |\hat{\sigma}| \leq \lambda_0 \ \wedge \ |\sigma_{MA}(\tau_{i-1})| > \lambda_1 \end{cases} \tag{12}$$

with τ_i being the current numerical integration step, and N being a suitably selected number of samples.

IV. Stability Analysis

With reference to the proposed strategy, the following results, here reported without proofs because of space limitation, can proved.

Theorem 1: Given system (1)-(4), the control laws (8) and (9) depending on the triggering condition (5), assume that the deactivation mechanism is off, then the state of the system is exponentially steered to zero in spite of matched uncertainty terms.

Theorem 2: Given system (1)-(4), the control laws (8) and (9) depending on the triggering condition (10), assume that the deactivation mechanism is on, then the effect of matched uncertainty terms $\bar{h} = \hat{\sigma} - \sigma$ in steady-state is bounded, i.e., $\lambda_1 < |\bar{h}(t)| \leq \lambda_0$, $\forall\, t \geq t_r$, t_r being the reaching time.

V. Illustrative Example

In this section an inverted pendulum is considered as an illustrative example. Consider Fig. 3, where x is the linear position of the cart, θ is the angular position of the pendulum with respect to y-axis clockwise positive, w is the control variable, while M =0.455 kg and m =0.21 kg denote the mass of the cart and the mass of the pendulum, respectively. Moreover, let l =0.305 m denote the distance

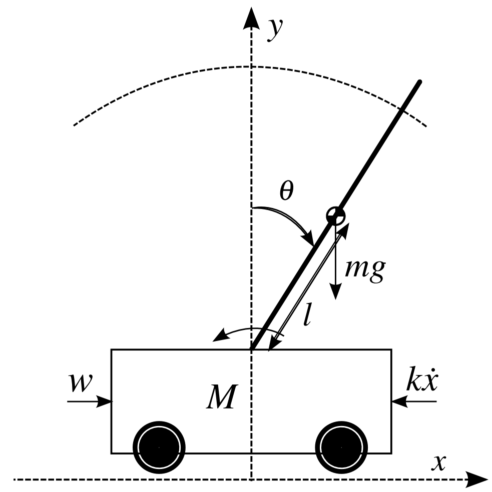

Fig. 3. A schematic view of the considered inverted pendulum.

from the pivot to the center of mass of the pendulum, let k be the friction coefficient of the cart, let g =9.81 m s^{-2} be the gravitational acceleration, and let J the moment of inertia of the pendulum with respect to its center of mass. Now, by posing $[x_1\ x_2\ x_3\ x_4]^T = [x\ \dot{x}\ \theta\ \dot{\theta}]^T$, the nonlinear coupled system is the following

$$\begin{cases} \dot{x}_1 = & x_2 \\ \dot{x}_2 = & \frac{\left(J+ml^2\right)\left(w+ml\sin x_3 \dot{x_3}^2 - k\dot{x_1}\right) - m^2l^2 g\sin x_3 \cos x_3}{\left(J+ml^2\right) - m^2l^2\cos^2 x_3} \\ \dot{x}_3 = & x_4 \\ \dot{x}_4 = & -\frac{ml\cos x_3}{J+ml^2}\,\frac{\left(J+ml^2\right)\left(w+ml\sin x_3 \dot{x_3}^2 - k\dot{x_1}\right) - m^2l^2 g\sin x_3\cos x_3}{\left(J+ml^2\right)(M+m) - m^2l^2\cos^2 x_3} + \\ & + \frac{mlg\sin x_3}{J+ml^2} \end{cases} \tag{13}$$

Note that, in the following, k and J are considered equal to zero, for the sake of simplicity.

In order to transform system (13) into an equivalent decoupled system, we consider the *Lie Derivatives* [17] to partially linearize the nonlinear system (more precisely, the input-output map is linearized, while the original state system is only partially linearized). Considering the following control law

$$w = (M + m\sin^2 x_3)u - (mlx^2 \sin x_3 - mg\sin x_3 \cos x_3) \tag{14}$$

where u is an auxiliary control variable, system (13) can be expressed as

$$\begin{cases} \dot{x}_1 = & x_2 \\ \dot{x}_2 = & u \\ \dot{x}_3 = & x_4 \\ \dot{x}_4 = & \frac{g\sin x_3 - u\cos x_3}{l} \end{cases} \tag{15}$$

Define an auxiliary output variable z_1, such that

$$z_1 = x_1 + l\ln\left(\frac{1+\sin x_3}{\cos x_3}\right) \tag{16}$$

while its time derivatives up to order 4 are the following

$$\begin{cases} \dot{z}_1 & = & x_2 + \frac{lx_4}{\cos x_3} = z_2 \\ \ddot{z}_2 & = & \tan x_3\left(g + \frac{lx_4{}^2}{\cos x_3}\right) = z_3 \\ z_3^{(3)} & = & \left(\frac{2}{\cos^3 x_3} - \frac{1}{\cos x_3}\right) lx_4{}^3 \\ & & + \left(\frac{3g}{\cos^2 x_3} - 2g\right) x_4 - 2x_4\tan x_3 u = z_4 \\ z_4^{(4)} & = & f(x) + b(x)u \end{cases} \tag{17}$$

Note that, $f(\cdot)$ and $b(\cdot)$ are known functions defined as follows

$$\begin{aligned} f(x) = \frac{\sec x_3}{l}\Big(& g^2\tan x_3 + 9glx_4{}^2\sec x_3\tan x_3 + \\ & + 5l^2x_4{}^2\sec x_3{}^2\tan x_3 + g(3lx_4{}^2 + \\ & + 2g\cos x_3)\sin x_3\tan x_3{}^2 + l^2x_4{}^2\tan x_3{}^3\Big) \end{aligned} \tag{18}$$

$$\begin{aligned} b(x) = -\frac{1}{l}(& \sec x_3(g + 3lx_4{}^2\sec x_3 \\ & - (3lx_4{}^2 + 2g\cos x_3)\sin x_3\tan x_3)) \end{aligned} \tag{19}$$

Finally, the corresponding nominal model is

$$\begin{cases} \dot{\hat{x}}_1 = & \hat{x}_2 \\ \dot{\hat{x}}_2 = & \frac{ml^2\left(w + ml\sin\hat{x}_3\dot{\hat{x}}_3{}^2\right) - m^2l^2g\sin\hat{x}_3\cos\hat{x}_3}{ml^2 - m^2l^2\cos^2\hat{x}_3} \\ \dot{\hat{x}}_3 = & \hat{x}_4 \\ \dot{\hat{x}}_4 = & -\frac{ml\cos\hat{x}_3}{ml^2}\frac{ml^2\left(w + ml\sin\hat{x}_3\dot{\hat{x}}_3{}^2\right) - m^2l^2g\sin\hat{x}_3\cos\hat{x}_3}{\left(ml^2\right)(M+m) - m^2l^2\cos^2\hat{x}_3} + \\ & + \frac{mlg\sin\hat{x}_3}{ml^2} \end{cases} \tag{20}$$

where $[\hat{x}_1\ \hat{x}_2\ \hat{x}_3\ \hat{x}_4]^T$ is the state of the nominal model. In order to verify the effectiveness of the proposed control strategy, a matched uncertainty term $h = bu_m$ has been injected to the feedback linearized model in (16) and (17), such that $|h| \leq 5.4\,\text{N}$ (see Fig. 4).

The sliding variable has been chosen equal to

$$\sigma = m_1z_1 + m_2z_2 + m_3z_3 + z_4 \tag{21}$$

where $m_1 = m_2 = 27$, $m_3 = 9$, while the control parameter has been selected equal to $U_{max} = 12$.

The standard Eulero solver with numerical integration step $\tau_i = 0.001\,\text{s}$ has been used, while the simulation time is $T_s = 30\,\text{s}$. The initial conditions are $x(0) = [0\ 0\ 0.52\ 0]$, while the width of the boundary layer $\mathcal{B}_{\lambda_0}$ in (6) and the threshold of the MA in (11) are $\lambda_0 = 0.45$ and $\lambda_1 = 0.085$, respectively.

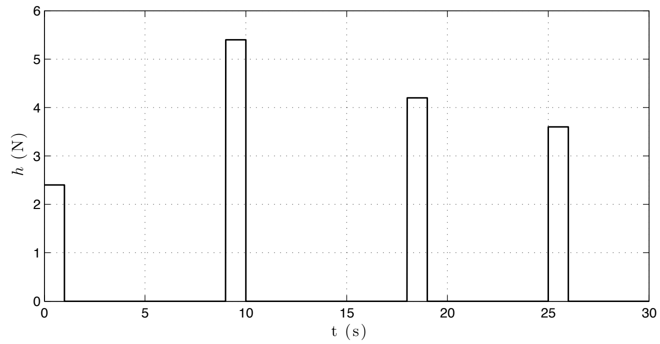

Fig. 4. The matched uncertainties affecting the system.

In order to evaluate the closed-loop performance, we have proposed the following indices:

$$x_{RMS} = \sqrt{\frac{\sum_{i=1}^{n_s}\sum_{j=1}^{n} x_{ji}^2}{n_s}}, \quad \sigma_{RMS} = \sqrt{\frac{\sum_{i=1}^{n_s}\sigma_i^2}{n_s}} \tag{22}$$
$$n_{up} = \frac{\sum_{i=1}^{n_s} f_{up}(\tau_i)}{n_s}$$

where x_{RMS} is the Root Mean Square (RMS) value of the state, σ_{RMS} is the RMS value of the sliding variable, and n_{up} is the number of updates of the plant state with respect to conventional SMC. Note that in the definitions above, we have denoted with n_s the total number of the integration steps of the simulation, with x_{ji} and σ_i the j-th component, respectively, of the state vector and of the sliding variable at the i-th integration step.

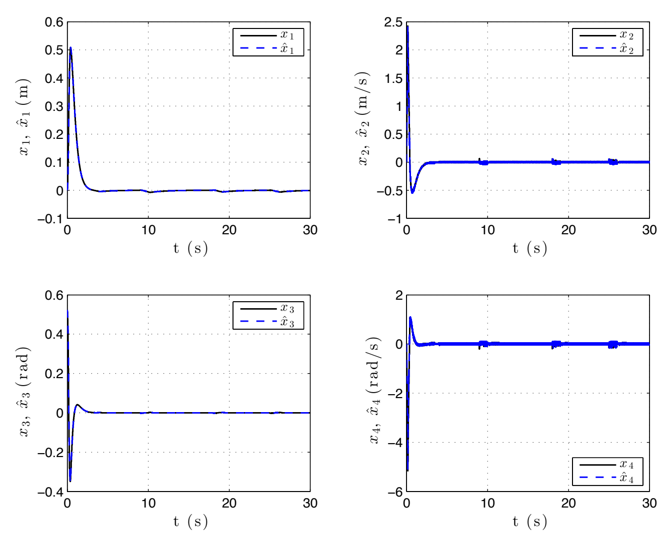

Fig. 5. Time evolution of the state variables of the plant (solid black line), and of the model (dashed blue line).

Fig. 5 shows the evolution of the state variables steered to the origin in spite of the matched disturbances injected

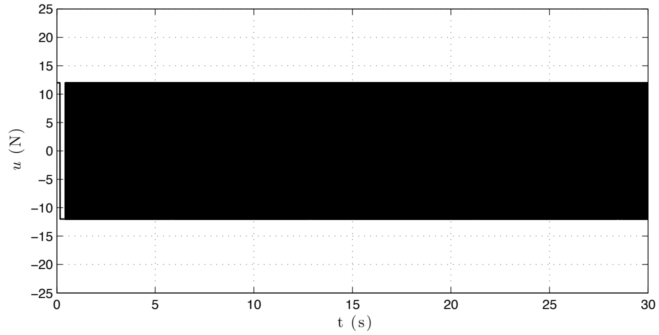

Fig. 6. Time evolution of the auxiliary control variable u of the partially feedback linearized system.

to the system. The corresponding control variable u of the feedback linearized system is illustrated in Fig. 6. Fig. 7

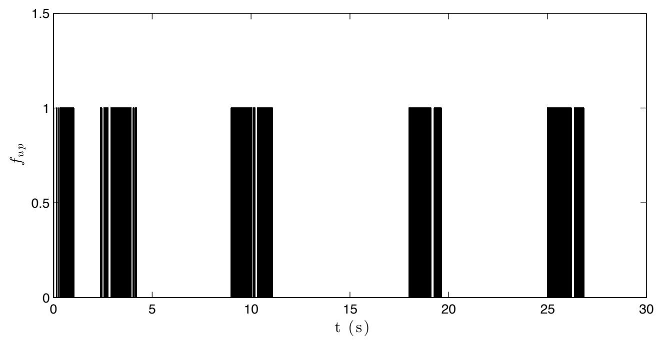

Fig. 7. The flag function of the number of updates when the actual state is sent over the network.

shows the flag function which is equal to 1 only when the plant state is transmitted over the network, while in Fig. 8 the sliding variables σ and $\hat{\sigma}$ are reported. Note that two different

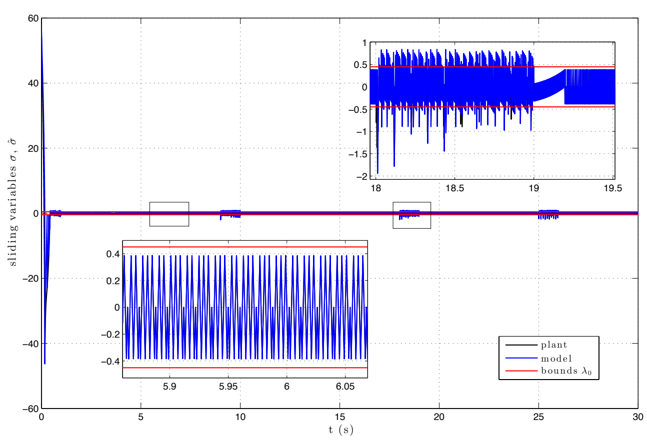

Fig. 8. Time evolution of the sliding variables of the plant (solid black line), and of the model (solid blue line) with bounds λ_0.

situations are highlighted: the case in which no uncertainties are present, and the case in which the external disturbances cause the violation of condition (5).

The RMS value of the state results in being equal to x_{RMS} =4.69×10^{-2}, while the RMS value of the sliding variable is σ_{RMS} =13.61×10^{-2}. Finally, Table I and Fig. 9 report the obtained value for the index n_{up} defined in (22), for different values of λ_1. Note that n_{up} provides a comparison in terms of bandwidth consumption with respect to a traditional SMC scheme in which the effective state is transmitted over the network at any sampling time instant, that is, in total n_s times.

TABLE I

NUMBER OF UPDATES (%).

λ_1	n_{up}
0.4	7.44
0.1	16.57
0.085	18.57
0.07	21.84
0.01	71.83

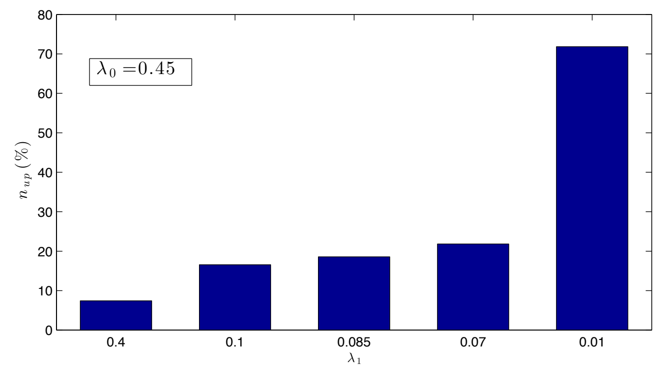

Fig. 9. Packet rate depending on the deactivation threshold.

Moreover, in order to investigate the effect of model mismatching in the unstable considered plant, and emulate a quite realistic set-up, unmatched uncertainty terms Δx_2, Δx_4 with $|\Delta x_2|, |\Delta x_4|$ <0.04 have been injected to the the linear velocity and to the angular velocity of the inverted pendulum, respectively (see Fig. 10). Fig. 11 shows the flag function

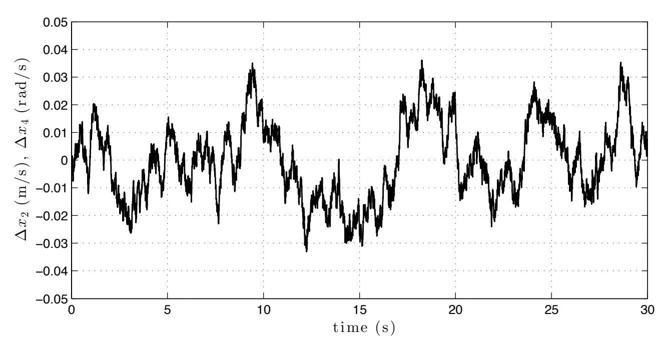

Fig. 10. The unmatched uncertainties affecting the system.

which is equal to 1 only when the plant state is transmitted through the network, while in Fig. 12 the sliding variables σ and $\hat{\sigma}$ are illustrated. The RMS value of the state also in

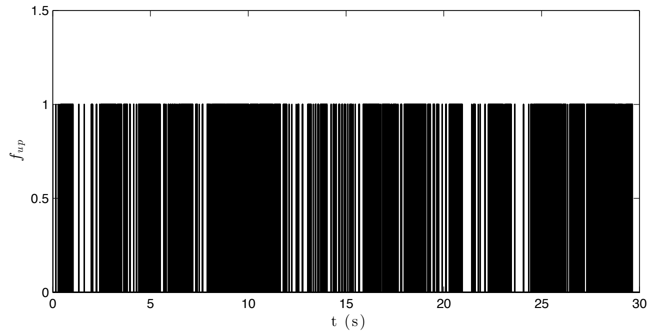

Fig. 11. The flag function of the number of updates when the actual state is sent over the network, in presence of modelling errors.

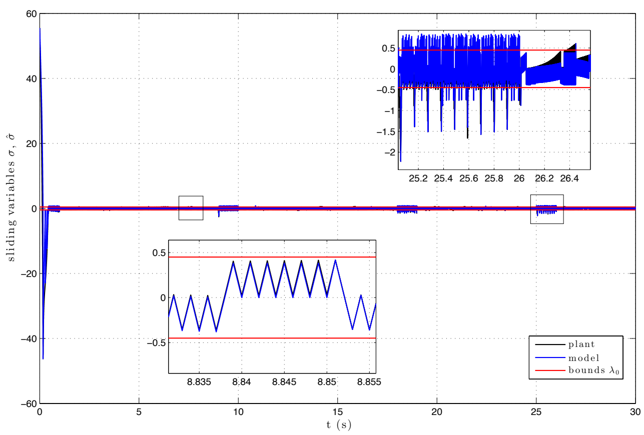

Fig. 12. Time evolution of the sliding variables of the plant (solid black line), and of the model (solid blue line) with bounds λ_0, in presence of modelling errors.

this case results in being equal to $x_{RMS} = 4.69 \times 10^{-2}$, while the RMS value of the sliding variable is $\sigma_{RMS} = 13.6 \times 10^{-2}$. The index n_{up} is greater than that obtained in the previous case due to the presence of some modelling errors and it is equal to 81%, causing, also in this case, a reduction of the packet rate over the network.

VI. Conclusions

In this paper, a networked control strategy based on sliding mode control is presented. The main objective is to reduce the number of transmissions of the actual plant state over the network, while guaranteeing performance analogous to the performance attainable in a non networked case. The proposed model based controller, equipped with a triggering condition, enables to handle the switch between the usage of the actual plant state and the nominal model state. A suitable mechanism to deactivate the plant state transmission in steady-state is also proposed. Robustness and stability results are formally addressed in the paper, and an illustrative example is reported to demonstrate the satisfactory performance of the proposal in simulation.

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