Robust Motion Control of a Robot Manipulator via Integral Suboptimal Second Order Sliding Modes

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Abstract— **This paper deals with the formulation of an *Integral Suboptimal Second Order Sliding Mode* control algorithm oriented to solve motion control problems for robot manipulators, taking into account the presence of unavoidable modelling uncertainties and external disturbances affecting the systems. The proposed algorithm is designed so that the so-called *reaching phase*, normally present in the evolution of a system controlled via a Sliding Mode controller, is reduced to a minimum. Moreover, since the relative degree of the relevant system output is suitably augmented through the use of an integrator, the control action affecting the robotic system is continuous, with a significant benefit, in terms of chattering alleviation, for the overall controlled electromechanical system. The verification and validation of our proposal have been performed by simulating the motion control scheme relying on a model of the considered robot, i.e. a COMAU SMART3-S2 anthropomorphic industrial robot manipulator, identified on the basis of real data.**

I. Introduction

Sliding Mode (SM) control is a widely used control methodology which ensures good performance of the controlled system even in presence of a significant class of uncertainties [1], [2]. Yet, because of the discontinuous nature of the Sliding Mode control law, it can produce the so-called chattering effect [3], [4], [5], [6], [7], i.e. high frequency oscillations of the controlled variable, which can be disruptive for the controlled plant or significantly limit the life cycle of the actuators. This is the reason why the use of Sliding Mode control in robotics is quite limited. Spong and Hutchinson in [8, subsection 8.4.11] suggested, in order to control robotic systems, to implement a continuous approximation to the discontinuous control, which however could only guarantee the uniformly ultimately boundedness of the tracking error system. This, in practice, diminishes the efficacy of Sliding Mode control, since a pseudo-sliding mode is generated, rather than an ideal sliding mode, and the robustness features of the methodology are lost.

Nowadays, a well-established method to perform chattering alleviation is that consisting in confining the discontinuity to a derivative of the control variable, so that the control signal actually fed into the system is continuous. This approach, called Higher Order Sliding Mode (HOSM) control [9], [10], [11], [12], [13], [14], [15], after a transient phase, enforces a sliding mode, involving not only the sliding variable but also their time derivatives up to the order $r-1$ in case of the so-called r-sliding mode.

This is the final version of the accepted paper submitted for inclusion in the Proceedings of the 52nd Conference on Decision and Control, Florence, Italy, Dec., 2013.

Because of the continuous nature of the control action, the HOSM control approach is appropriate to be applied even to electromechanical or mechanical systems [16], [17], as testified by [18], [19], [20]. Yet, as highlighted in [11], some problems remain during the transient phase, i.e. the so-called *reaching phase*, since in that time interval, which proves to be of finite, but, in general, unpredictable length, the transient duration being affected by the uncertainty terms, the robustness properties of the control approach do not hold.

In this paper, inspired by [21], we propose a modification of the control algorithm considered in [18], which belongs to the class of the so-called Suboptimal Second Order Sliding Mode algorithms (see [9], [11]), which gives rise to a new version of the algorithm, herein named *Integral Suboptimal Second Order Sliding Mode* algorithm. The proposed algorithm maintains the good properties of the original Suboptimal Second Order Sliding Mode approach, in terms of chattering alleviation, but also assigns a transient dynamics to the controlled system, so that the reaching phase occurs with a prescribed transient time. This feature is highly beneficial in robotics since it limits the time periods during which the 2-sliding mode on the selected sliding manifold is not enforced.

The integral 2-sliding mode is kept on a suitably modified sliding manifold from the initial time instant (this time instant being the time instant when the adopted Levant's differentiator [22], [23], involved in the scheme, converges) and from that time instant the robustness of the controlled system is proved. The effectiveness of the proposed approach has been assessed in simulation, relying on a model identified on the basis of the data collected on a COMAU SMART3-S2 anthropomorphic industrial robot manipulator, and experimentally, using the actual robotic system which is present in our lab. Beacuse of space limitations, only simulation results are hereafter reported and discussed.

The present paper is organized as follows. In Section II, higher order sliding modes and integral higher order sliding modes are reviewed with reference to a SISO uncertain dynamical system. In Section III, the Integral High Order Sliding Mode control approach is extended to the Suboptimal control approach, and the new algorithm is presented. In Section IV, the kinematical and dynamical models of a three joints planar robot manipulator are introduced, and the proposed motion control scheme is described. The final part of the paper is devoted to present simulation results. Some conclusions (Section V) end the paper.

II. Some Preliminary Issues

Consider the SISO system given by

$$\begin{cases} \dot{x}_i(t) = x_{i+1}(t) \quad i = 1, \dots, n-1 \\ \dot{x}_n(t) = f(x(t)) + g(x(t))u(t) \\ y(t) = s(x(t)) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}$ is the control variable, $s : \mathbb{R}^n \to \mathbb{R}$ is a smooth output function, named *sliding variable* in the subsequent analysis. System (1) is an *uncertain system* since $f(\cdot)$ and $g(\cdot)$ are unknown smooth functions. The *relative degree* of the system, i.e. the minimum order r of the time derivative $s^{(r)}$ of the sliding variable in which the control u explicitly appears, is considered well defined, uniform and time invariant. In the following, the dependence of s on $x(t)$ and of all the variables on t is omitted in some cases, when it is obvious, for the sake of simplicity.

A. Higher Order Sliding Modes

The Higher Order Sliding Mode (HOSM) control problem is based on the definition of an *auxiliary system* associated with the original uncertain system. The auxiliary system is a perturbed chain of integrators built starting from the sliding variable and its time derivatives. Thus, the original control objective, attained in conventional sliding mode control by zeroing the sliding variable in finite time, is transformed into the aim of regulating the auxiliary system. This means, for any r-th order sliding mode control, to force the system state to reach in finite time and remain on the subspace named *r-sliding manifold* $s = \dot{s} = \cdots = s^{(r-1)} = 0$. The time derivative $s^{(r)}$ is the bounded function which, relying on (1), can be expressed as follows

$$s^{(r)}(x(t)) = F(x(t), u(t)) + g(x(t))u(t) \tag{2}$$

where $F(\cdot) = s^{(r)}|_{u=0}$ and $g(\cdot) = (\partial s^{(r)}/\partial u) \neq 0$ are unknown functions. More precisely, it is assumed that there exist positive constants $G_1,\ G_2,\ F$, such that

$$0 < G_1 \leq g(x(t)) \leq G_2 \tag{3}$$

$$|F(x(t), u(t))| \leq F \tag{4}$$

Note that instead of (3), one could analogously have the opposite inequality

$$-G_2 \leq g(x(t)) \leq -G_1 < 0 \tag{5}$$

i.e. it is required that $g(\cdot)$ has constant known sign. Since the information about the bounds of $F(\cdot)$ and $g(\cdot)$ are assumed to be available, the original dynamical system (1) implies the differential inclusion [24]

$$s^{(r)} \in [-F, F] + [G_1, G_2]\, u \tag{6}$$

The problem of making the r-sliding manifold associated with (6) finite-time attractive, generating a sliding mode of order r (r-sliding mode), can be solved by any r-sliding mode controller of the type

$$u(t) = U_{max} \Psi\left(s, \dot{s}, \dots, s^{(r-1)}\right) \tag{7}$$

see for instance [10], [11], [12], [13], [14], [15], where Ψ is a discontinuous function, and $U_{max} > 0$ is chosen so as to ensure the finite time convergence of the sliding variable to the equilibrium $s = 0$, which is one of the strong points of sliding mode control and is particularly useful in specific applications.

B. Second Order Sliding Mode Algorithms

Second Order Sliding Mode (SOSM) control is a particular case of HOSM control. Several algorithms, such as the Twisting, the Super-Twisting [25] and the Suboptimal algorithm [9], [11], have been proposed in the last years. In this paper, we refer to the Suboptimal approach, and we assume that the sliding variable is expressed as

$$s(x(t)) = x_n(t) + \sum_{i=1}^{n-1} m_i x_i(t) \tag{8}$$

where $m_i,\ i = 1, \cdots,\ n-1$ are real positive constants. According to this choice of the sliding variable the states tends to zero asymptotically.

To construct the auxiliary system, one has to consider the first and the second-time derivative of the sliding variable, i.e.

$$\dot{s}(x(t)) = f(x(t)) + g(x(t))u(t) + \sum_{i=1}^{n-1} m_i x_{i+1}(t) \tag{9}$$

$$\ddot{s}(x(t)) = \frac{d}{dt} f(x(t)) + \frac{d}{dt} g(x(t))u(t) + g(x(t))\dot{u}(t) + \\ + m_{n-1}\left[f(x(t)) + g(x(t))u(t)\right] + \sum_{i=1}^{n-2} m_i x_{i+2}(t) \tag{10}$$

By defining $\xi_1(t) = s(x(t))$ and $\xi_2(t) = \dot{s}(x(t))$, it yields

$$\begin{cases} \dot{\xi}_1(t) = \xi_2(t) \\ \dot{\xi}_2(t) = F(x(t), u(t)) + g(x(t))w(t) \end{cases} \tag{11}$$

where $\xi_2(t)$ is assumed to be unmeasurable, $F(\cdot)$ and $g(\cdot)$ have the bounds indicated in (3) and (4), $w(t)$ is the auxiliary control law which has to be designed so that $\xi_1(t)$ and $\xi_2(t)$ are steered to zero in a finite time in spite of the uncertainties. Note that the Suboptimal algorithm, reported in [9], requires that the control $w(t) = \dot{u}(t)$ is discontinuous. Yet, $w(t)$ only affects $\ddot{s}$, but not $\dot{s}$ so that the undesired high frequency oscillations, called chattering, are alleviated. Indeed, the control actually fed into the plant is continuous, which is highly appreciable in case of mechanical or electromechanical plants.

C. Integral Sliding Mode Algorithms

Recent research has been devoted to study Integral Sliding Mode (ISM) methods, which enables to generate an ideal sliding mode of the controlled system starting from the initial time instant t_0. A sliding mode is defined Integral Sliding Mode if the system, while sliding, is of the same order as the original system [26]. ISM requires to split the control variable into two parts

$$u(t) = u_0(t) + u_1(t) \tag{12}$$

where $u_0(t)$ is generated by any suitably designed high level controller, and $u_1(t)$ is a discontinuous control action designed to compensate the uncertainties affecting the system. A particular sliding manifold is defined, named *integral sliding manifold*, as

$$\Sigma(t) = s(x(t)) - \varphi(t) = 0 \tag{13}$$

where Σ is an auxiliary sliding variable, s can be chosen, for instance, as in (8), and the integral term φ is

$$\varphi(t) = s(x(t_0)) + \int_{t_0}^{t} \frac{\partial s}{\partial x}\dot{x}(\zeta)d\zeta \tag{14}$$

with the initial condition $\varphi(t_0) = s(x(t_0))$. By virtue of the choice of $\varphi(t)$ and $\varphi(t_0)$, it is apparent that the controlled system is in sliding mode on the manifold $\Sigma(t) = 0$ since the initial time instant.

D. Integral Higher Order Sliding Mode Algorithms

A novel trend of the research on sliding mode control has led to the formulation of a joint approach which can be named Integral Higher Order Sliding Mode (IHOSM) control. This approach consists in defining an auxiliary sliding variable as in (13), with the function $\varphi(t)$ suitably chosen to fulfill some restriction on the transient time during the reaching phase, so that, thanks to the existence of a sliding mode on the integral sliding manifold since the initial time instant, one has

$$s(t) = \varphi(t), \qquad \forall t,\ t_0 \le t \le t_f \tag{15}$$

where t_0 is the initial time instant, and t_f is the time to reach the condition $s = 0$. Moreover,

$$\varphi(t_0) = s(t_0), \ldots, \varphi^{(r-1)}(t_0) = s^{(r-1)}(t_0) \tag{16}$$

$$\varphi(t) \equiv 0, \quad \forall t \ge t_f \tag{17}$$

In analogy with (7), it yields that the sliding mode can be enforced by the following control law

$$u(t) = U_{max}\Psi\left(\Sigma, \dot{\Sigma}, \ldots, \Sigma^{(r-1)}\right) \tag{18}$$

with Ψ discontinuous function and $U_{max} > 0$ suitably chosen design parameter.

III. The New Proposal: Integral Suboptimal Second Order Sliding Mode

In this section, the IHOSM control methodology is coupled with the Suboptimal SOSM control approach, giving rise to a new algorithm, herein named *Integral Suboptimal Second Order Sliding Mode* (ISSOSM) algorithm. This new control approach maintains the good properties of the original Suboptimal algorithm in terms of capability of stabilizing in finite time a perturbed chain of integrators with bounded control, as well as in terms of chattering alleviation. Moreover, the reaching phase is reduced to a minimum, as will be clarified in a moment, by the introduction of a transient dynamics with a prescribed time.

The transient trajectory is realized as in (15)-(17). More specifically, an appropriate choice (see [21]) for the transient function $\varphi(t)$ is the following

$$\begin{cases} \varphi(t) = (t-t_f)^2(c_0 + c_1(t-t_0)), & \forall t,\ t_0 \le t \le t_f \\ \varphi(t) = 0, & \forall t > t_f \end{cases} \tag{19}$$

where $c_0,\ c_1$ are found from (16) as

$$c_0 = s(t_0)T^{-2} \tag{20}$$

$$c_1 = \dot{s}(t_0)T^{-2} + 2s(t_0)T^{-3} \tag{21}$$

while $T = t_f - t_0$ is the *prescribed time*. Note that the knowledge of $\dot{s}(t_0)$ is necessary to define the transient function, and, since $\dot{s}$ is unmeasurable, the well-known Levant's differentiator

$$\dot{z_0} = -\lambda_0|z_0 - s|^{1/2}\operatorname{sgn}(z_0 - s) + z_1 \tag{22}$$

$$\dot{z_1} = -\lambda_1 \operatorname{sgn}(z_0 - s) \tag{23}$$

where $z_0,\ z_1$ are the estimated values of $s,\ \dot{s}$, respectively, and $\lambda_0 = L^{1/2}$, $\lambda_1 = 1.1L$, $L \ge F + \sup|\ddot{s}|$, is a possible choice of the differentiator parameters [22], is used for an initialization time period ending in t_0, with $t_0 \ge t_d$, t_d being the differentiator convergence time or an upper bound of it.

With reference to the Suboptimal SOSM algorithm, described in the previous section, considering the sliding variable used to define the integral sliding manifold $\Sigma = s - \varphi$, and the auxiliary system (11) with $\xi_1 = \Sigma$ and $\xi_2 = \dot{\Sigma}$, the following control algorithm can be written.

ISSOSM Algorithm:

1) Set $\Sigma(t) = s(x(t)) - \varphi(t)$.
   Repeat for any $t > t_0$, the following steps.
2) Set $\alpha^* \in (0,1] \cap (0, 3G_1/G_2)$.
3) Set $\xi_{1_{max}} = \xi_1(t_0)$.
4) If $t_0 \le t \le t_f$, then set $\varphi(t) = (t-t_f)^2(c_0 + c_1(t-t_0))$, else set $\varphi = 0$.
5) If $\left[\xi_1(t) - \frac{1}{2}\xi_{max}\right][\xi_{max} - \xi_1(t)] > 0$, then set $\alpha = \alpha^*$, else set $\alpha = 1$.
6) If $\xi_1(t)$ is extremal, the set $\xi_{max} = \xi_1(t)$.
7) Apply the control law

$$w(t) = -\alpha U_{max}\operatorname{sgn}\left(\xi_1(t) - \tfrac{1}{2}\xi_{max}\right) \tag{24}$$

with

$$U_{max} > \max\left(\frac{F}{\alpha^* G_1}; \frac{4F}{3G_1 - \alpha^* G_2}\right) \tag{25}$$

A possibility to evaluate the extremal values ξ_{max} can be to use again a Levant's differentiator. In practice, this means that, even if the state $\dot{\Sigma}$ is not available for measurements, it can be estimated by the differentiator, the structure of which is reported in (22) and (23), and the extremal values of $\Sigma(t)$ can be stored at the time instant when $\dot{\Sigma}(t)$ changes its sign. In alternative, one can deduce $\dot{\Sigma}$ relying on the definition of Σ and on the estimate of $\dot{s}$ obtained through (22) and (23).

Remark 1: Note that, if $\dot{\Sigma}$ were measurable, i.e. if $\dot{s}$ were available, the controlled system would be in sliding mode independently of the choice of the initial time instant t_0. Since instead a Levant's differentiator is used to determine the unavailable quantities, in spite of the choice of the integral approach, it is necessary to provide sufficient time for the differentiator convergence. Since the differentiator proves to converge in a finite time t_d, then one can claim that, in our case, the sliding mode is enforced for any $t \geq t_0$, with $t_0 \geq t_d$.

According to [11], [21], [26], the finite time convergence of the sliding variable to the origin of the state plane and the asymptotically convergence of the states trajectories can be proved. By virtue of the fact that the sliding mode on the integral sliding mode manifold is enforced since the initial time instant, to be selected according to Remark 1, the reaching phase can be definitely reduced, which produces a clear benefit in terms of robustness of the controlled system.

IV. A Case Study

In this section, the results of the verification and validation of the proposed algorithm based on simulation are reported. Simulations have been run using a model of the actual robot identified on the basis of real data. In order to formulate the model of a n-joints rigid robot manipulator, kinematical and dynamical aspects have to be considered. During our tests, for the sake of simplicity, only vertical planar motions of the robot manipulator were enabled, by locking three of the six joints of the robot (see Fig. 1).

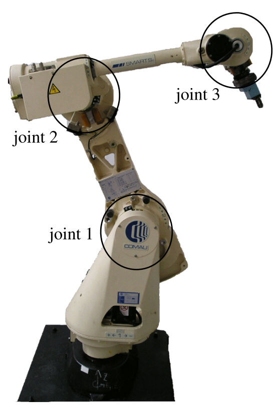

Fig. 1. The COMAU SMART3-S2 anthropomorphic industrial robot manipulator used for the tests with the joints numeration.

A. The Robot Model

Consider Fig. 2. Let $l_i,\ i = 1,2,3$ denote the length of the i-th link, q_1 the orientation of the first link with respect to y-axis clockwise positive, and let $q_j,\ j = 2,3$ denote the displacement of the j-th link with respect to the $(j-1)$-th one clockwise positive. Let $O - \{x,y,z\}$, denote the base-frame of the robotic manipulator, so that the center O is placed in the centre of the first joint of the robot. Let $O_e - \{n,s,a\}$ denote the end-effector frame of the robot manipulator, so that the center O_e is placed on its end-effector, and the axes $\{n,s,a\}$ are indicated in Fig. 2.

The direct kinematics of a 3-joints planar manipulator describes the relationship between the joint variables $q = [q_1\ q_2\ q_3]^T$ and the end-effector position and orientation $x = [p_x\ p_y\ \phi]^T$ in the planar workspace. With reference to Fig. 2, where the joint variables $q_i,\ i = 1,2,3$ are indicated, the direct kinematics equations in our case can be written as

$$\begin{cases} p_x = -l_1 \sin(q_1) - l_2 \sin(q_1 + q_2) - l_3 \sin(\phi) \\ p_y = l_1 \cos(q_1) + l_2 \cos(q_1 + q_2) + l_3 \cos(\phi) \\ \phi = q_1 + q_2 + q_3 \end{cases} \tag{26}$$

The dynamics of the robot can be described in the joint space, by using the Lagrangian approach, as

$$B(q)\ddot{q} + n(q,\dot{q}) = \tau \tag{27}$$

$$n(q,\dot{q}) = C(q,\dot{q})\dot{q} + F_v\dot{q} + F_s sgn(\dot{q}) + g(q) \tag{28}$$

where $B(q) \in \mathbb{R}^{3\times3}$ is the inertia matrix, $C(q,\dot{q}) \in \mathbb{R}^{3\times3}$ represents centripetal and Coriolis torques, $F_v \in \mathbb{R}^{3\times3}$ is the viscous friction matrix, $F_s \in \mathbb{R}^{3\times3}$ is the static friction matrix, $g(q) \in \mathbb{R}^3$ is the vector of gravitational torques and $\tau \in \mathbb{R}^3$ represents the motors torques. Note that the static friction is

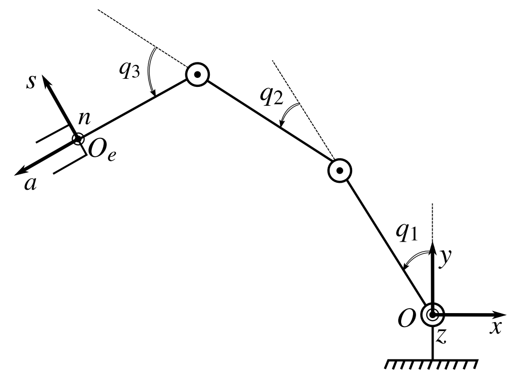

Fig. 2. A schematic planar view of the robot manipulator with the joint variables.

neglected during the design of the ISSOSM controller but it is present in the simulation model of the robot and, obviously, in the actual industrial robot.

B. The Motion Control Scheme

In Fig. 3 the proposed control scheme for the robot is illustrated. The feedback loop is designed for the position

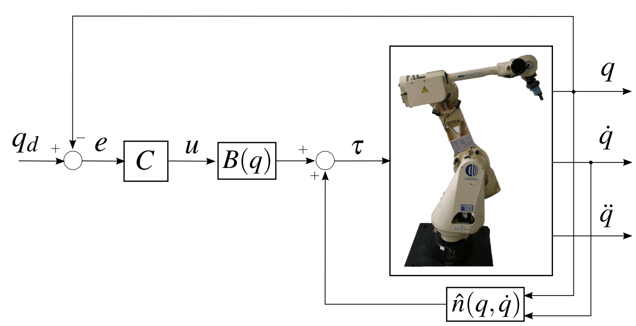

Fig. 3. The motion control scheme with the inverse dynamics-based feedback linearization applied to the robot system.

tracking control, the desired position being compared with the real position. Controller C computes the control variable $u \in \mathbb{R}^3$ starting from the position error $e \in \mathbb{R}^3$ with $e = q_d - q$.

To feedback linearize the nonlinear system (27), the classical inverse dynamics control approach [27] has been adopted. The inverse dynamics of a rigid robot manipulator can be written in the joint space as a non linear relationship between the plant inputs and the plant outputs, relying on (27) and (28), so that the control law results in being

$$\tau = B(q)u + \hat{n}(q,\dot{q}) \tag{29}$$

where u is an auxiliary control variable. Note that $B(q)$ and $\hat{n}$ need to be identified on the basis of experimental tests. In our work, we assume that the identified $B(q)$ coincides with the actual one (or it is a quite accurate replica), which, on the basis of our experience, is often true in practice, while $\hat{n}$ is an estimate of n, which does not necessarily coincide with n. In the following we make reference to the experimentally identified $B(q)$ and $\hat{n}$ in [28].

By applying the feedback linearization to system (27), (28), one obtains

$$\ddot{q} = u + B^{-1}(q)\tilde{n}(q,\dot{q}) = u - \eta(q,\dot{q}) \tag{30}$$

where $\eta(q,\dot{q})$ takes into account modelling uncertainties and dynamical effects, and

$$\tilde{n}(q,\dot{q}) = \hat{n}(q,\dot{q}) - n(q,\dot{q}) \tag{31}$$

Therefore, the whole system is reduced to the union of three decoupled uncertain smooth SISO systems.

Now we design the controller C relying on the previously described Integral Suboptimal Second Order Sliding Mode control approach. According to this latter, a sliding variable for each SISO system is selected as

$$s_i(t) = m_i e_i + \dot{e}_i, \quad i = 1,2,3 \tag{32}$$

where $m_i \in \mathbb{R}$ is scalar, and e_i is the position error of the i-th joint. The relative degree of each SISO system involving a single sliding variable s_i, considering s_i as the relevant system output, is 1 so that the control variable appears in the first time derivative of s_i as follows

$$\dot{s}_i(t) = m_i\dot{e}_i + \ddot{e}_i = m_i\dot{e}_i + \ddot{q}_{d_i} + \eta_i - u_i \tag{33}$$

According to the proposed ISSOSM Algorithm, the transient function φ_i is chosen as in (19), with (16), and (17), while the auxiliary sliding variable defining the integral sliding manifold for each joint is $\Sigma_i = s_i - \varphi_i$. The auxiliary system can be written as

$$\begin{cases} \dot{\xi}_{1_i}(t) = \xi_{2_i}(t) \\ \dot{\xi}_{2_i}(t) = m_i(\ddot{q}_{d_i}(t) + \eta_i(t) - u_i(t)) + \frac{d^3 q_{d_i}(t)}{dt^3} \\ \qquad + \dot{\eta}_i(t) - \dot{u}_i(t) - \ddot{\varphi}_i(t) \end{cases} \tag{34}$$

where $\xi_{1_i}(t) = \Sigma_i$ and $\xi_{2_i}(t) = \dot{\Sigma}_i$. Since the relative degree turns out to be raised, the auxiliary control law $\dot{u}_i$ is discontinuous according to the algorithm, while the effective control variable is

$$u_i(t) = \int_0^t \alpha_i U_{max_i} \operatorname{sgn}\left(\xi_{1_i}(\zeta) - \tfrac{1}{2}\xi_{max_i}\right) d\zeta \tag{35}$$

which is continuous, so that the chattering alleviation is attained.

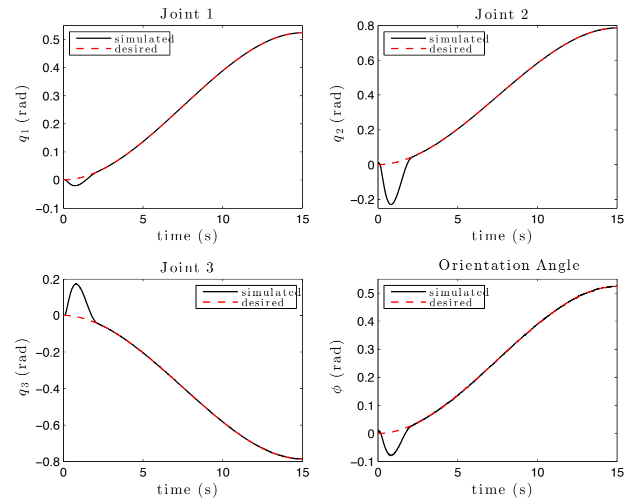

Fig. 4. Angular position of joints and end-effector orientation angle ϕ (simulation results). From the top on the left: the desired trajectory (dotted red line) and the real one (solid black line) for each joint.

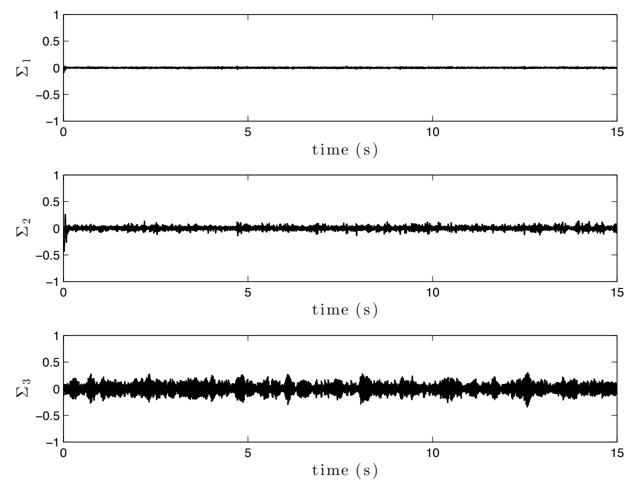

Fig. 5. From the top: the auxiliary sliding variable Σ_i for joint 1,2 and 3, respectively (simulation results).

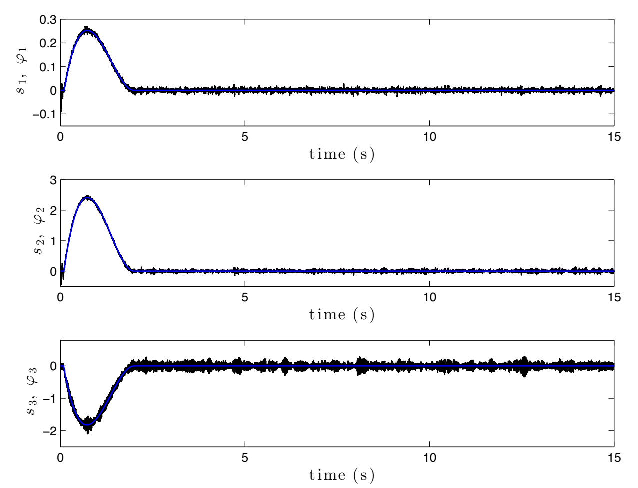

Fig. 6. From the top: the sliding variable s_i (solid black line) and the transient function φ_i (solid blue line) for joint 1,2 and 3, respectively (simulation results).

C. Simulation Results

In this subsection we assess the performance of the ISSOSM algorithm in simulation. The goal of the control system is to steer the joint angles from a given initial position $(q_{1_0},\ q_{2_0},\ q_{3_0}) = (0,\ 0,\ 0)$ to the final position $(q_{1_f},\ q_{2_f},\ q_{3_f}) = (\pi/6,\ \pi/4,\ -\pi/4)$, following the trajectory $q_i = a_3t^3 + a_2t^2 + a_1t + a_0$, where $a_0,\ a_1,\ a_2,\ a_3$ are coefficients depending on q_{i_0} and $q_{i_f},\ i = 1,2,3$. The parameters of the function φ_i are calculated at $t_0 = 0.1$ s, while the final time is $t_f = 2$ s. Moreover, to verify the robustness properties of the controller, random noise with uniform distribution $\eta = [\eta_1\ \eta_2\ \eta_3]^T$ with the following upper bounds

$$|\eta_1| \leq 20 \tag{36}$$

$$|\eta_2| \leq 80 \tag{37}$$

$$|\eta_3| \leq 100 \tag{38}$$

has been added to the angular accelerations of the joints of the simulated robot. Table I reports the control parameters,

TABLE I
CONTROL PARAMETERS (SIMULATION).

i	m_i	α_i^*	U_{max_i}
1	10	0.9	630
2	10	0.9	2130
3	10	0.9	10250

used for each joint. Fig. 4 shows the evolution of the joint variables and of the orientation angle of the end-effector $\phi = q_1 + q_2 + q_3$. Fig. 5 and 6 respectively show the corresponding auxiliary sliding variables Σ_i maintained to zero from the initial time instant, and the sliding variables s_i steered to zero. The root mean square error $e_{RMS} = 3.7996 \times 10^{-4}$ rad is obtained with the sampling time of $t_s = 0.0001$ s.

V. CONCLUSIONS

In the paper the good features of the Integral Sliding Mode control approach are extended to the so-called Suboptimal algorithm, also ensuring chattering alleviation and robustness with respect to matched uncertainties. A new version of the Suboptimal algorithm named Integral Suboptimal Second Order Sliding Mode control algorithm has been formulated. Some theoretical results have been first discussed: the finite time regulation of the auxiliary system state, the reduction of the reaching phase, as well as the robustness of the proposed approach guaranteed since the initial time instant to. Then, the proposed algorithm has been used to design a motion control scheme for robot manipulators. The scheme has been tested in simulation, relying on the data from a real industrial robot manipulator. The effectiveness of the proposed algorithm in terms of convergence and robustness is confirmed by the satisfactory simulation results.

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