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A numerical investigation on the use of the virtual element method for topology optimization on polygonal meshes

Aim

Moving along the lines of the pioneering works [1,2] explore the potentiality of polygonal meshes and the virtual element method to solve topology optimization problems governed by: a) elasticity equation - b) Stokes equation

VEM discretization

$\mathcal{T}_h$: decomposition into polygonal elements E

$\mathbb{Q}_{ad} = \{\rho_h \in \mathcal{Q}_{ad} : \ \rho_{h|E} \in \mathbb{P}_0(E) \ \ \forall E \in \mathcal{T}_h\} \rightarrow$ discrete controls

$\mathbf{V}_h \subset \mathcal{V} \rightarrow$ low-order VEM space of discrete displacements [3]

- a) minimum compliance problem:

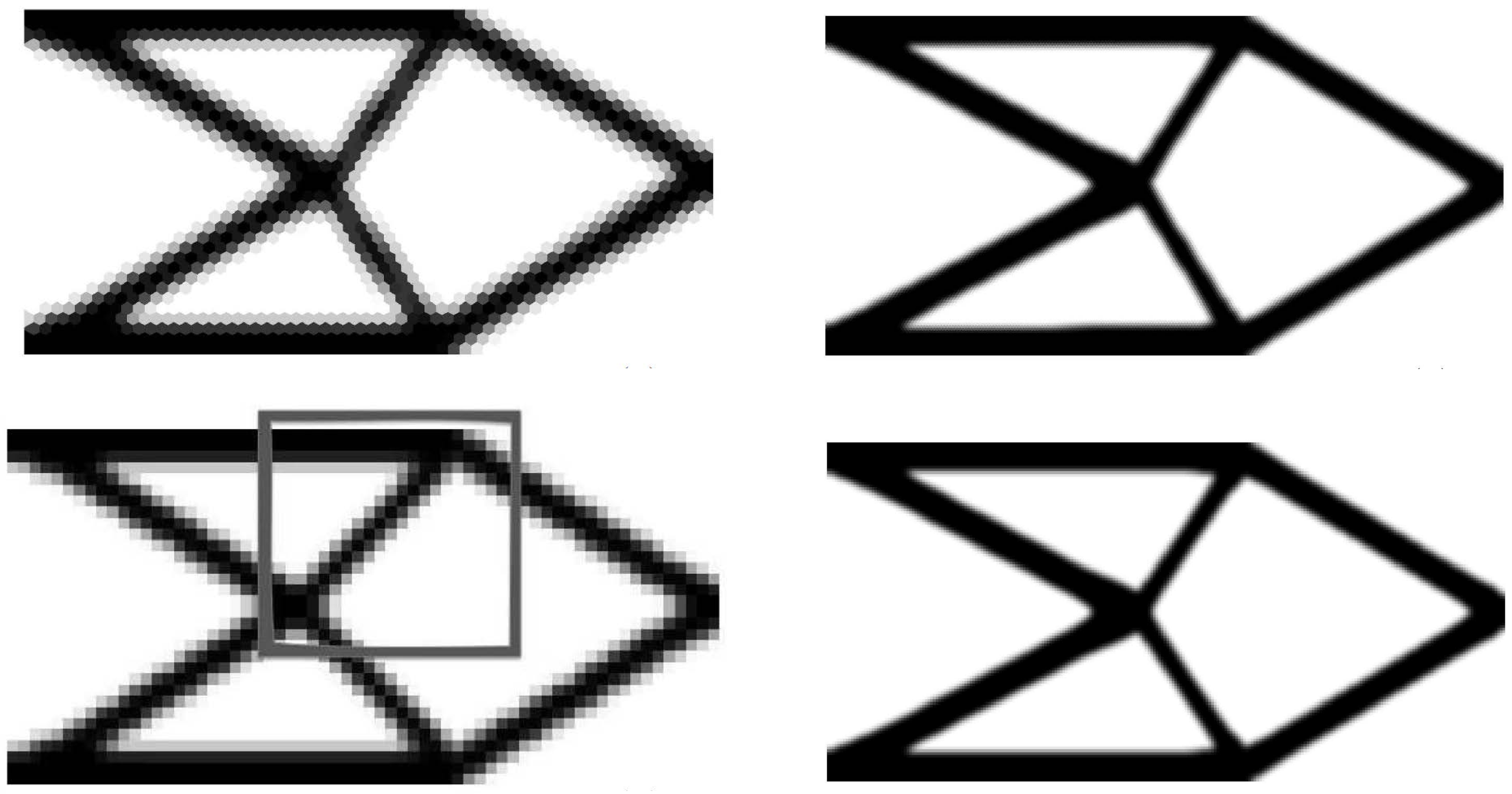
$$\begin{cases} \min\limits_{\rho_h \in \mathbb{Q}_{ad}} & \mathcal{C}(\rho_h, \mathbf{u}_h) \ = \mathcal{F}_h(\mathbf{u}_h) \\ \text{s.t.} & a_h(\rho_h; \mathbf{u}_h, \mathbf{v}_h) = \mathcal{F}_h(\mathbf{v_h}) \quad \forall \mathbf{v_h} \in \mathcal{V}_{\mathbf{0,h}} \\ & \dfrac{1}{V} \displaystyle\int_\Omega \rho_h dx \ \leq V_f \end{cases}$$

- $a_h(\rho_h; \mathbf{u}_h, \mathbf{v}_h) = \sum_{E \in \mathcal{T}_h} \rho_E^p a_h^E(\mathbf{u}_h, \mathbf{v}_h) \rightarrow$ discrete form
- $a_h^E(\mathbf{u}_h, \mathbf{v}_h) \simeq 2\mu_0 \int_E \epsilon(\mathbf{u}) : \epsilon(\mathbf{v}) \ \mathbf{dx} + \lambda_0 \int_\mathbf{E} \mathrm{div}\mathbf{u} \ \mathrm{div}\mathbf{v} \ \mathbf{dx}$
- $\mathcal{F}_h(\mathbf{v_h}) \rightarrow$ discrete load

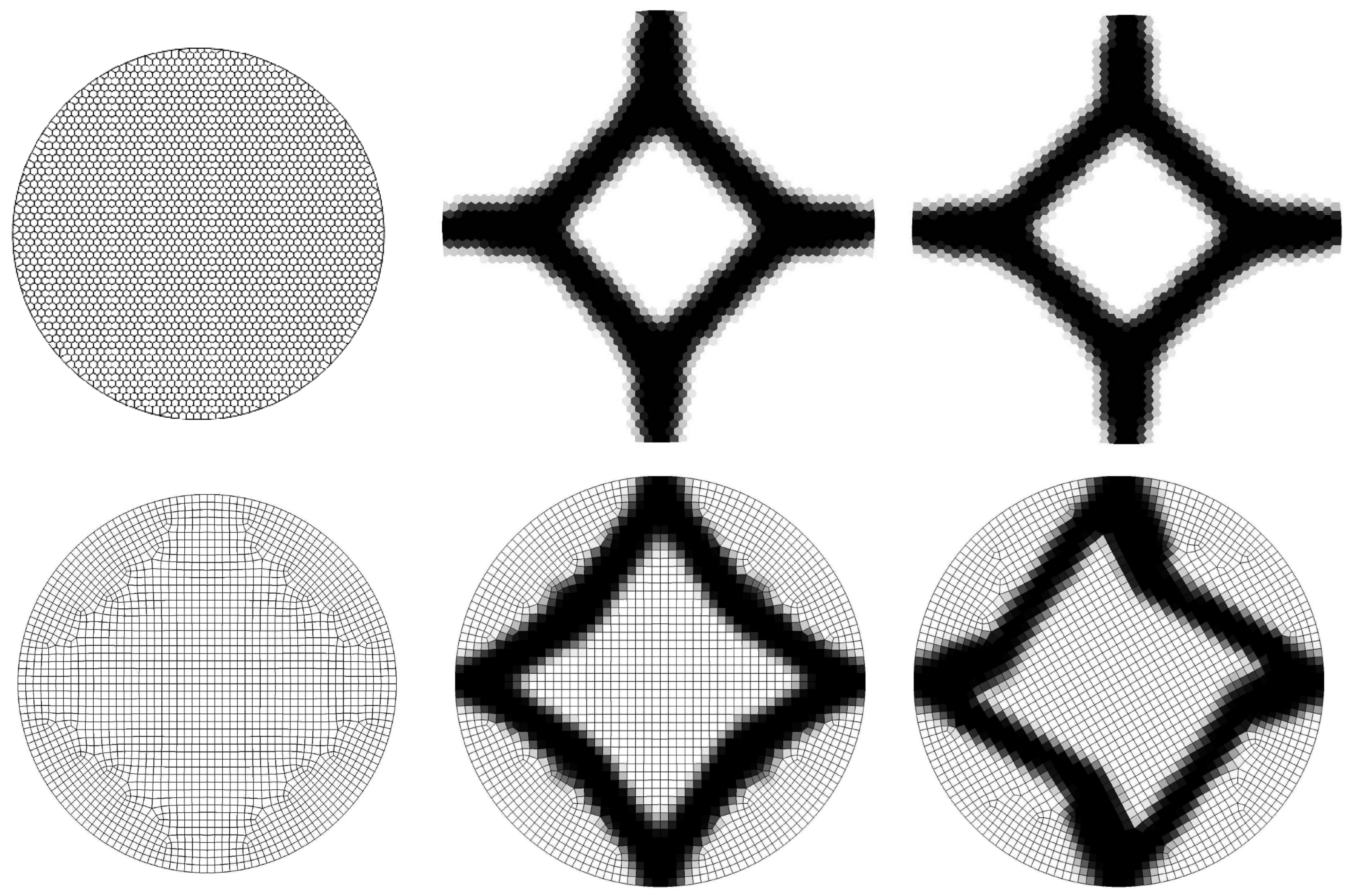
- b) minimum dissipated energy problem:

$$\begin{cases} \min\limits_{\rho_h \in \mathbb{Q}_{ad}} & a_h(\rho_h; \mathbf{u}_h, \mathbf{u}_h) \\ \text{s.t.} & a_h(\rho_h; \mathbf{u}_h, \mathbf{v}_h) = 0 \quad \forall \mathbf{v_h} \in \mathcal{V}_{\mathbf{0,h}} \\ & \dfrac{1}{V} \displaystyle\int_\Omega \rho_h dx \ \leq V_f \end{cases}$$

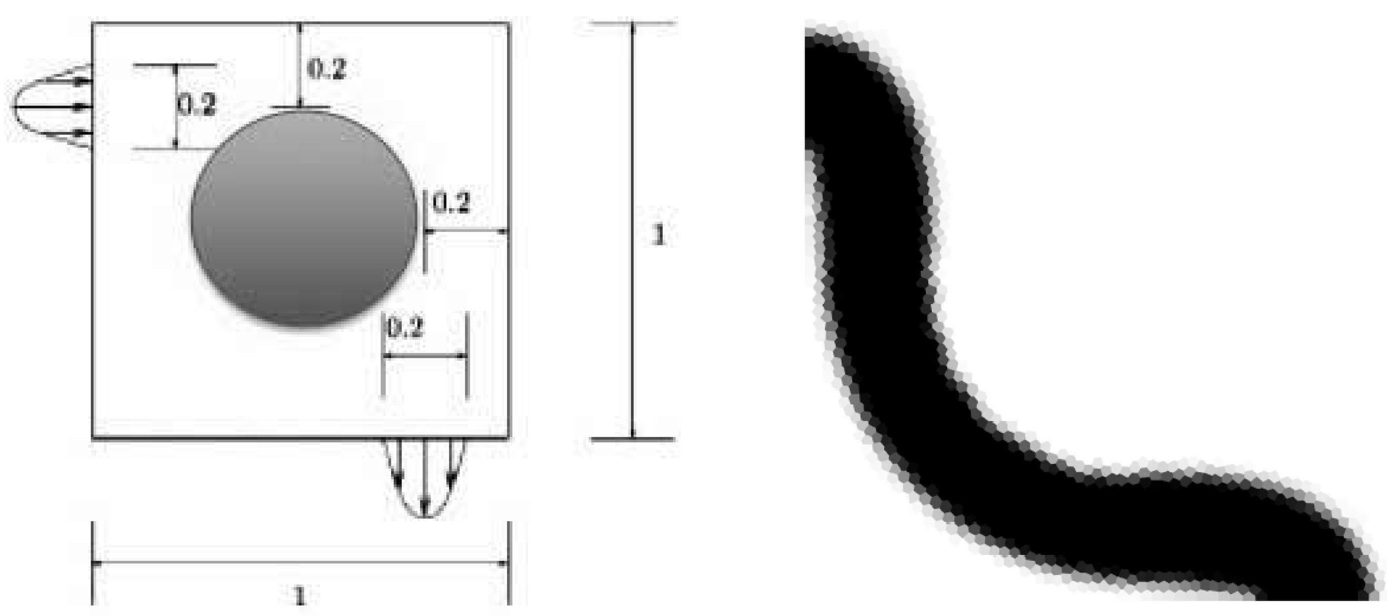
- $a_h(\rho_h; \mathbf{u}_h, \mathbf{v}_h) = \sum_{E \in \mathcal{T}_h} a_h^E(\rho_h; \mathbf{u}_h, \mathbf{v}_h) \rightarrow$ discrete form
- $a_h^E(\rho_h; \mathbf{u}_h, \mathbf{v}_h) \simeq 2\mu_0 \int_E \epsilon(\mathbf{u}) : \epsilon(\mathbf{v}) \ \mathbf{dx} + \lambda_0 \int_\mathbf{E} \mathrm{div}\mathbf{u} \ \mathrm{div}\mathbf{v} \ \mathbf{dx} + \int_\Omega \frac{5\mu}{\rho^2} \mathbf{uv}$



Clamped cantilever. Optimal layouts achieved using structured VEM grids (top - 2006 and 7990 polygonal elements) vs. FEM grids (bottom - 2048 and 8192 square elements)
→ Coarse quadrilateral grids can lead to sub-optimal solutions



Four-point load specimen on unrotated / 30°-rotated meshes. Optimal layouts achieved on a VEM grid (top - 2168 elements) vs. a FEM grid (bottom - 2068 four-node elements)
→ Rotated quadrilateral grids can lead to unphysical optima



Pipe with obstacle. Optimal layout on an unstructured grid with 4096 polygons → Accurate geometrical description

References

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[3] L. Beirão da Veiga, F. Brezzi, and L. D. Marini. Virtual elements for linear elasticity problems. SIAM J. Numer. Anal., 51(2):794-812, 2013

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