Nº 30

AD-MINISTER

UNIVERSIDAD EAFIT · MEDELLÍN - COLOMBIA · JANUARY-JUNE 2017 · ISSN 1692-0279 · E-ISSN: 2256-4322

ZAHRA
ZARE

JEL: C29, I19

DOI: 10.17230/ad-minister.30.12
www.eafit.edu.co/ad-minister

UNIVERSIDAD EAFIT®

AD-minister Nº. 30 january-june 2017 pp. 237 - 265 · ISSN 1692-0279 · eISSN 2256-4322

SERVICE PERFORMANCE IN PUBLIC HEALTHCARE SYSTEM: DATA ENVELOPMENT ANALYSIS

LA PRESTACIÓN DEL SERVICIO EN EL SISTEMA PÚBLICO DE SALUD: ANÁLISIS ENVOLVENTE DE DATOS

ZAHRA ZARE[1]

JEL: C29, I19

RECEIVED: 5-09-2016

MODIFIED: 14-11-2016

ACCEPTED: 26-11-2016

DOI: 10.17230/ad-minister.30.12

www.eafit.edu.co/ad-minister

ABSTRACT

As well as companies that compete in market, in public health system organizations compete to satisfy customer's need, and therefore identifying those needs and delivering the value is critical in success. Return to Scale and Damage to Scale are measures. In this study data envelopment analysis is developed to measure the Return to Scale and Damage to Scale in public health organization. Two new assumptions for production possibility set are proposed as Weak Natural Disposability and weak managerial disposability. Then three types of models including efficiency evaluation, Return to Scale determination, and Damage to Scale determination are proposed based on radial and non-radial models. A case study is handled using real data of 33 hospitals in Tehran. Each hospital is assumed as a decision-making unit with 4 inputs, 2 desirable outputs, and 2 undesirable outputs. The proposed approaches are straightforward and applicable for real world problems.

KEYWORDS

Scale Efficiency; Return to Scale; Damage to Scale; Healthcare performance; Hospital Performance.

RESUMEN

Al igual que las empresas que compiten en el mercado, las organizaciones del sistema público de salud compiten para satisfacer las necesidades de los clientes y, por lo tanto, es fundamental identificar dichas necesidades y entregar valor para alcanzar el éxito. Los Rendimiento de Escala y Daños de Escala se utilizan como medidas. En este estudio, el análisis envolvente de datos se desarrolla para medir los Rendimientos y Daños de Escala en una organización pública de salud. Se proponen dos nuevos supuestos para la posibilidad de producción: Baja disponibilidad natural y baja disponibilidad de gestión. Seguidamente, se proponen tres tipos de modelos basados en modelos radiales y no radiales que incluyen la evaluación de la eficiencia, la determinación de los rendimientos de escala y la determinación de los daños de escala. Se maneja un estudio de caso que utiliza datos reales de 33 hospitales de Teherán, Irán. Cada hospital se asume como una unidad de toma de decisión de cuatro insumos (*inputs*), dos productos (*outputs*) deseables y dos productos (*outputs*) indeseables. Los enfoques propuestos son sencillos y aplicables a los problemas del mundo real.

PALABRAS CLAVE

Eficiencia de escala; Rendimientos de escala; daños de escala; desempeño del Sistema de salud.

INTRODUCTION

Data envelopment analysis (DEA), as a linear programming technique, serves several benefits in comparison with the other techniques for performance

1 MA and BA in Industrial Engineering at the Faculty of Industrial Engineering, Islamic Azad University (South Tehran Branch). Her main areas of interest are: Deterministic Models and Optimization, Operation Research, Health systems, Economic and Financial Models. Institutional Email Address: st_z_zare@azad.ac.ir

measurement. DEA requires no preference articulation on priority of inputs/outputs, and can estimate the production function as a non-parametric approach. DEA determines the efficient DMUs among others, and suggests the projection of efficiency for the inefficient DMUs. Moreover, Return to Scale (RTS) issues can be determined using an especial variants of DEA models. There are several successful applications of DEA models in the healthcare systems (Chowdhury and Zelenyuk, 2015). Thanassoulisa, Portelab and Graveneyc (2016) identify the usefulness of the length of stay of each episode and explores the differences between hospitals and between the care teams within the hospitals.

Thus, in this study DEA models are developed in order to measure the efficiency scores, RTS and Damage to Scale (DTS) issues of Iranian public health organizations in Tehran. The main objectives of this research are:

1. The development of DEA models to measure the efficiency scores of public health organization with multiple inputs/outputs.
2. The development of mathematical models in order to measure the return/DTSs in public health organizations.
3. And finally, the application of the proposed models in a real case study.

The structure of this paper is as follows. In Section 2, literature of past works is reviewed. In Section 3 the proposed models are developed. In Section 3, new models for assessing Tehran's health systems are presented, decision-making departments, factors would be defined. the solution algorithm that is applied for finding the application of scale in radial and non-radial position is presented. The justification of the application of such models will be discussed in Section 3. In Section 4, the case study and data analysis are presented. The results and findings are presented in Section 5. On the other hand, in Section 5, the Weak Natural Disposability (WND) and weak managerial disposability (WMD) models' results are presented in both radial and non-radial status. The models are used for the evaluation of public health organization of Medicinal Universities using desirable and undesirable outputs. And also, the economics of scale of public health organization has been defined using radial, non-radial models and the obtained results were compared using the collected data. Finally, the paper will be summarized in Section 6 with concluding remarks and future research directions.

LITERATURE OF PAST WORKS (APPLICATION OF DEA MODELS IN HEALTHCARE)

Although hospital efficiency analysis has attracted a large number of studies (e.g., see Goldstein et. al., 2002; Hollingsworth, 2003; O'Neill et al., 2008; Garcia-Lacalle and Martin, 2010; Rosko and Mutter, 2011; Mitropoulosa et al., 2015 and Thanassoulisa et al., 2016), there are less research whose focal point is on analyzing the determinants of hospital efficiency (e.g., Grosskopf et al., 2004; Lee et al., 2008; Blank and Valdmanis, 2010; Tsekouras et al., 2010; Cristian and Fannin, 2013; Ding, 2014). DEA is a method which evaluates service providers and defines a rate as the ratio of perceived performance to its closed potential. Or it shows the amount of the

AD-minister Nº. 30 january-june 2017 pp. 237 - 265 · ISSN 1692-0279 · eISSN 2256-4322

desirable efficiency of resources. In a juxtapose with the above results, Gok and Sezen (2013) showed that efficiency of small hospitals is relatively more, and satisfaction is higher compared to medium and large hospitals.

The theoretical improvement of the DEA approach commences with the influencing study of Charnes, Cooper and Rhodes (1979) towards the efficiency evaluation of DMUs. Mentioned in the first application of DEA in the health care sector was the works by Nunamaker (1983). Since then, DEA was vastly used for technical efficiency of public health organization in US and other parts of the world. Sherman (1984) was the first scholar who applied DEA for technical efficiency evaluation of public health organization in the US. Sherman (1984) analyzed factors like budget, day beds, number of full-time physicians, +65 years old patients, -65 years old patients, instructed nurses and interns as inputs and outputs. Butler and Li (2005) evaluated the benefits of RTS in DEA for rural public health organization of Michigan State. They used the BCC model and considered all the costs except salaries, the number of hospital beds, number of services and total number of employees as inputs, and the number of days of care for a patient, number of surgery operations, number of emergency rooms and number of hospitalized patients as outputs. Al-Shammari (1999) investigated the efficiency of 15 public health organization. He considered the number of beds' active days, number of physicians, and number of personnel as input and number of hospitalized days, number of emergency surgeries, and number of general surgeries as output.

Tsai and Mar (2002) surveyed the scale efficiency in five departments of public health organizations in Britain using the variable RTS assumption that considered sum of operational costs, number of hospitalized, and emergency patients as main criteria Nayar and Ozcan (2008) investigated efficiency of Virginia public health organization using DEA. The results demonstrated that in most cases, working on quality resulted in an increase in hospital costs and hence efficiency was decreased. Joses et al., (2008) measured the efficiency of 54 public health organization in Kenya - although they found that there was a big gap between scientific and real-life evaluation of health care departments in Africa's central Sahara. The obtained results showed that %26 of public health organization were inefficient. The projection of inefficient DMUs towards efficient frontier was calculated.

Kawaguchi, Tone and Tsutsui (2014) applied DEA for evaluating the efficiency of governmental public health organization. They applied both static and dynamic DEA models and categorized public health organization into two namely: managerial and operational departments. The obtained results demonstrated that there was a slight difference between static and dynamic models.

FORMULATION OF PROPOSED MODEL

There are several approaches and perspective when measuring efficiency scores using DEA models. In this section, some commonly used approaches are briefly presented.

WEAK AND STRONG DISPOSABILITY

Assumeas $Y \in R^m$ input vector, $Y \in R^s$ as desirable output vector, and $Y \in R^h$ as undesirable output vector. Therefore, the possibility production set (PPS) in a weak consumption environment would be defined as (1).

$$P^W = \{ (G,B): G \leq \sum_{j=1}^{n} \lambda_j G_j, B = \sum_{j=1}^{n} \lambda_j B_j, X \geq \sum_{j=1}^{n} \lambda_j X_j, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, (j = 1 \dots n\} \quad (1)$$

Where, DMU_j is assessed under weak disposability approach if it seeks to increase desirable outputs and decrease inputs while the undesirable outputs are assumed to be constant (Sueyoshi, and Goto, 2011a; Sueyoshi and Goto, 2012a).

In a strong disposability approach, DMU_j attempts to increase both desirable and undesirable outputs while the inputs are assumed to be decreased (Sueyoshi, and Goto, 2011b; Sueyoshi and Goto, 2012b). In this way, the PPS is defined as (2).

$$P^S = \left\{(G,B): G \leq \sum_{j=1}^{n} \lambda_j G_j, B \leq \sum_{j=1}^{n} \lambda_j B_j, X \geq \sum_{j=1}^{n} \lambda_j X_j, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, (j = 1 \dots n\}\right) \quad (2)$$

NATURAL AND MANAGERIAL DISPOSABILITY

The natural and managerial consumption approach was presented first in a study by Sueyoshi and Goto (2012c) and later in many other researches (Cooper, Seiford and Zhu, 2011; Sueyoshi and Goto,2012d; Sueyoshi and Goto,2012e; Sueyoshi and Goto,2012f). Under such conditions, the influence of the input vector on the increase and decrease of inputs is discussed. In the natural disposability approach, with an increase in input vectors, we attempt to decrease undesirable output vectors and increase desirable vectors. In this way, the PPS is defined as (3).

$$P^N = \{ (G,B): G \leq \sum_{j=1}^{n} \lambda_j G_j, B \geq \sum_{j=1}^{n} \lambda_j B_j, X \geq \sum_{j=1}^{n} \lambda_j X_j, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, (j = 1, \dots n)\} \quad (3)$$

AD-minister Nº. 30 january-june 2017 pp. 237 - 265 · ISSN 1692-0279 · eISSN 2256-4322

But in some cases, a DMU tries to develop available resources assuming an increase in inputs leads to an increase in desirable outputs and a decrease in undesirable outputs. In this way, the preliminary hypotheses of classic CCR and BCC models are not supported. In this way, the PPS would be shown as (4).

$$P^{M} = \{ (G,B): G \leq \sum_{j=1}^{n} \lambda_j G_j, B \geq \sum_{j=1}^{n} \lambda_j B_j, X \leq \sum_{j=1}^{n} \lambda_j X_j, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, (j = 1, \dots n)\} \quad (4)$$

NEW PROPOSED PPSS: NATURAL/ MANAGERIAL CONSUMPTION CONSIDERING WEAK DISPOSABILITY

In this study, based on the requirement of case study, a new condition for PPS is proposed in which a combination of natural and managerial approaches under weak disposability condition is used. Assume that DMU is not responsible for any changes made in undesirable outputs. Moreover, DMU tries to increase desirable outputs through increasing inputs, therefore the two types of PPS would be shown as follows.

- A PPS under natural consumption approach considering weak disposability can be shown as (5).

$$P^{WN} = \{ (G,B): G \leq \sum_{j=1}^{n} \lambda_j G_j, B = \sum_{j=1}^{n} \lambda_j B_j, X \geq \sum_{j=1}^{n} \lambda_j X_j, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, j = 1 \dots n \quad (5)$$

And a PPS under managerial consumption approach considering weak disposability can be shown as (6).

$$P^{WM} = \{ (G,B): G \leq \sum_{j=1}^{n} \lambda_j G_j, B = \sum_{j=1}^{n} \lambda_j B_j, X \leq \sum_{j=1}^{n} \lambda_j X_j, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, j = 1 \dots n \quad (6)$$

Thus, in this study, two new types of PPS are used to develop the radial and non-radial DEA models. The developed models under such circumstances are used to determine the return and DTSs at some public health organization in Tehran.

FORMULATION

In this subsection, the proposed models of this study, under several assumptions, are developed. All models are proposed in both radial and non-radial situations. In radial models, inputs and outputs are changing simultaneously but in non-radial models, one of the inputs or outputs would change per request and the changes are separate. In order to making a better sense for readers, Models are proposed in two main classes known as the WND models, and weak managerial disposability models. In each classes several models are discussed. On the other hand, in each classes radial and non-radial models are proposed, and for each case a model is proposed to calculate for efficiency evaluation, and a procedure is proposed to determine the scale efficiency and RTS situation.

Table 1 presents indices, parameters, decision variables, and sets which are used in this section.

Table1. Notations used in the proposed models.

Sets	
m	Number of inputs
n	Number of DMUs
s	Number of good outputs
h	Number of bad outputs
Indices	
j	Index of DMUs, j=1‹2‹…‹n
k	DMU under assessment
i	Index of Inputs i=1‹2‹…‹m
r	Index of good outputs, r=1‹2‹…‹s
f	Index of bad outputs, , f=1‹2‹…‹h
Parameters	
x_{ij}	i-th Input of DMU_j
G_{rj}	r-th good output of DMU_j
B_{fj}	f-th bad output of DMU_j
R_i^x	Range of input i
R_r^g	Rang of good output r
R_f^b	Rang of bad output f
	Decision Variables
u_r	Decision variable of good output r (multiplier form)
w_f	Decision variable of bad output f (multiplier form)
v_i	Decision variable of input i (multiplier form)
d_i^x	Surplus variable for input i
d_r^g	Slack variable for good output r

AD-minister Nº. 30 january-june 2017 pp. 237 - 265 · ISSN 1692-0279 · eISSN 2256-4322

Table1. Notations used in the proposed models. Continued

d_f^b	Slack variable for bad output *f*
λ	Decision Variable for envelopment form
θ	Efficiency score
SE	Scale Efficiency
SD	Scale of Damage
μ	RTS value under WND for Radial model
π	RTS value under WND for non-radial model
η	DTS value under WMD for radial model
δ	Index of DTS under WMD for non-radial model
ξ	Output oriented projection towards efficient frontier (multiplier form)
ε	A very small positive number between 0.0001 and 0.0000001

WND MODLES

Radial Model for Efficiency Evaluation under WND

In this scenario as shown in (5), efficiency would be investigated using a decrease in input level and an increase in desirable output level and keeping undesirable outputs fixed. To calculate Unified Efficiency (UE) under WND approach, the Model (7)-(14) is proposed.

$$Max\ UE = \xi + \varepsilon\left(\sum_{i=1}^{m} R_i^x\, d_i^x + \sum_{r=1}^{s} R_r^g\, d_r^g\right) \tag{7}$$

$s.t.$

$$\sum_{j=1}^{n} X_{ij}\lambda_j + d_i^x = X_{ik} \tag{8}$$

$$\sum_{j=1}^{n} G_{rj}\lambda_j - d_r^g = (1+\xi) * G_{rk} \tag{9}$$

$$\sum_{j=1}^{n} B_{fj}\lambda_j = B_{fk} \tag{10}$$

$$\sum_{j=1}^{n} \lambda_j = 1, \quad \lambda_j \geq 0 \quad (j = 1, \dots, n) \tag{11}$$

Unified Efficiency (θ*) under WND model would also be calculated as (15) using slack variables.

$$\theta^* = 1 - (\xi + \varepsilon(\sum_{i=1}^{m} R_i^x\, d_i^{x^*} + \sum_{r=1}^{s} R_r^g\, d_r^{g^*}) \tag{12}$$

The dual of WNDR[2] Model (7)-(14) can be written as multiplier form Model (16)-(22).

$$Min \sum_{i=1}^{m} v_i X_{ik} - \sum_{r=1}^{s} u_r G_{rk} + \sum_{f=1}^{h} w_f B_{fk} + \sigma \tag{13}$$

$s.t.$

$$\sum_{i=1}^{m} v_i X_{ij} - \sum_{r=1}^{s} u_r G_{rj} + \sum_{r=1}^{h} w_f B_{fj} + \sigma \geq 0 \tag{14}$$

$$\sum_{r=1}^{s} u_r G_{rk} = 1 \tag{15}$$

$$v_i \geq \varepsilon R_i^x (i = 1, \dots, m), \tag{16}$$

$$w_f > \varepsilon R_f^b \ (f = 1, \dots, h) \tag{17}$$

$$u_r \geq \varepsilon R_r^g (r = 1, \dots, s) \tag{18}$$

$$\sigma Free\ in\ Sign \tag{19}$$

It is notable that the optimum value of Model (7)-(14) is equal to optimum values of Models (16)-(22) due to the duality theorem in linear programming.

NON-RADIAL MODEL FOR EFFICIENCY EVALUATION UNDER WND

To calculate Unified Efficiency (UE) under the WNDNR[3] approach, Models (23)-(31) are proposed.

$$Max\ UE = \sum_{i=1}^{m} R_i^x d_i^x + \sum_{r=1}^{s} R_r^g d_r^g \tag{20}$$

$s.t.$

$$\sum_{j=1}^{n} X_{ij} \lambda_j + d_i^x = x_{ik} \tag{21}$$

$$\sum_{j=1}^{n} G_{rj} \lambda_j - d_r^g = G_{rk} \tag{22}$$

$$\sum_{j=1}^{n} B_{fj} \lambda_j = B_{fk} \tag{23}$$

$$\sum_{j=1}^{n} \lambda_j = 1, \tag{24}$$

Again, the unified Efficiency (θ^*) under WNDNR model can be calculated same as the WND Model.

2. WND Radial
3. WND Non-Radial (WNDNR)

AD-minister Nº. 30 january-june 2017 pp. 237 - 265 · ISSN 1692-0279 · eISSN 2256-4322

The duality of the WNDNR Model (23)-(31) can be written as multiplier form Model (33)-(38).

$$Min\ \sum_{i=1}^{m} v_i X_{ik} - \sum_{r=1}^{s} u_r G_{rk} + \sum_{f=1}^{h} w_f B_{fk} + \sigma \tag{25}$$

$s.t.$

$$\sum_{i=1}^{m} v_i X_{ij} - \sum_{r=1}^{s} u_r G_{rj} + \sum_{r=1}^{h} w_f B_{fj} + \sigma \geq 0 \tag{26}$$

$$v_i \geq \varepsilon R_i^x (i = 1, \dots, m) \tag{27}$$

$$u_r \geq \varepsilon R_r^g (r = 1, \dots, s) \tag{28}$$

$$w_f \geq \varepsilon R_f^b\ (f = 1, \dots, h) \tag{29}$$

$$\sigma Free\ in\ Sign \tag{30}$$

It is notable that due to the duality theorem in linear programming, the optimum objective values of both Models (23)-(31) and (33)-(38) are equal.

The range adjusted measure (RAM) of efficiency model which was first proposed by Cooper, Park and Pastor (2000), is used here in order to adjust the range of inputs and outputs.

SCALE EFFICIENCY AND RTS CALCULATIONS UNDER WND

Scale Efficiency (SE) score in both radial and non-radial states are the same, but the RTS sign would be calculated separately based on the following algorithm.

Now, in order to define economic scale of inefficient units, first an image of inefficiency was extracted and the rate of inefficiency obtained using (43).

$$SE = \frac{\sum_{i=1}^{m} v_i^* (x_{ik} - d_i^{x^*})}{\sum_{r=1}^{s} u_r^* (G_{rk} + d_r^{G^*})} = \frac{1}{1 + (\ \sigma^* / \sum_{i=1}^{m} v_i^* (x_{ik} - d_i^{x^*})} \tag{30}$$

In order to calculate RTS for radial models, first the two maximization and minimization models (44)-(59) should be solved.

$$Min\ (Max)\ \mu = \sigma \tag{31}$$

$s.t.$

$$\xi + \varepsilon\left(\sum_{i=1}^{m} R_i^x\, d_i^x + \sum_{r=1}^{s} R_r^g\, d_r^g\right) = \sum_{i=1}^{m} v_i X_{ik} - \sum_{r=1}^{s} u_r G_{rk} + \sum_{r=1}^{h} w_f B_{fk} + \sigma \quad (33)$$

$$\sum_{j=1}^{n} X_{ij}\lambda_j + d_i^x = x_{ik} \quad (34)$$

$$\sum_{j=1}^{n} G_{rj}\lambda_j - d_r^g = (1+\xi)G_{ik} \quad (35)$$

$$\sum_{j=1}^{n} B_{fj}\lambda_j = B_{ik} \quad (36)$$

$$\sum_{i=1}^{m} v_i X_{ij} - \sum_{r=1}^{s} u_r G_{rj} + \sum_{r=1}^{h} w_f B_{fj} + \sigma \geq 0 \quad (37)$$

$$\sum_{r=1}^{s} u_r G_{rj} = 1 \quad (38)$$

$$\sum_{j=1}^{n} \lambda_j = 1 \quad (39)$$

$$\lambda_j \geq 0, (j = 1, \dots, n) \quad (40)$$

$$v_i \geq \varepsilon R_i^x, (i = 1, \dots, m) \quad (41)$$

$$u_r \geq \varepsilon R_r^g, (r = 1, \dots, s) \quad (42)$$

$$w_f \geq \varepsilon R_f^b, (f = 1, \dots, h) \quad (43)$$

Using the calculated upper and lower limit of μ^* per each DMU, the RTS of DMUk is determined as follows: if $0 > \overline{\mu^*} \geq \underline{\mu^*}$ there is increasing return to scale, if $\overline{\mu^*} > 0 \geq \underline{\mu^*}$ there is constant return to scale, and if $\overline{\mu^*} \geq \underline{\mu^*} > 0$ there is decreasing return to scale.

In order to calculate RTS for non-radial models, first the two maximization and minimization models (60)-(75) should be solved.

Min (Max) $\pi=\sigma$

s.t.

$$\sum_{i=1}^{m} R_i^x\, d_i^x + \sum_{r=1}^{s} R_r^g\, d_r^g = \sum_{i=1}^{m} v_i X_{ik} - \sum_{r=1}^{s} u_r G_{rk} + \sum_{r=1}^{h} w_f B_{fk} + \sigma \quad (45)$$

$$\sum_{j=1}^{n} X_{ij}\lambda_j + d_i^x = x_{ik} \quad (46)$$

$$\sum_{j=1}^{n} G_{rj}\lambda_j - d_r^g = G_{rk} \quad (47)$$

$$\sum_{j=1}^{n} B_{fj}\lambda_j = B_{fk} \quad (48)$$

AD-minister Nº. 30 january-june 2017 pp. 237 - 265 · ISSN 1692-0279 · eISSN 2256-4322

$$\sum_{i=1}^{m} v_i X_{ij} - \sum_{r=1}^{s} u_r G_{rj} + \sum_{r=1}^{h} w_f B_{fj} + \sigma \geq 0 \quad (49)$$

$$\sum_{r=1}^{s} u_r G_{rj} = 1 \quad (50)$$

$$\sum_{j=1}^{n} \lambda_j = 1 \quad (51)$$

$$\lambda_j \geq 0, (j = 1, \dots, n) \quad (52)$$

$$v_i \geq \varepsilon R_i^x, (i = 1, \dots, m) \quad (53)$$

$$u_r \geq \varepsilon R_r^g, (r = 1, \dots, s) \quad (54)$$

$$w_f \geq \varepsilon R_f^b, (f = 1, \dots, h) \quad (55)$$

Using the calculated upper and lower limit of π^* per each DMU, the RTS of DMUk is determined as follows: if $0 > \overline{\pi^*} \geq \underline{\pi^*}$ there is increasing return to scale, if $\overline{\pi} > 0 \geq \underline{\pi^*}$ there is constant return to scale, and if $\overline{\pi^*} \geq \underline{\pi^*} > 0$ there is decreasing return to scale.

WMD MODELS

Radial Model under WMD

To calculate Unified Efficiency (UE) under WMD using radial approach, Models (76)-(84) is proposed.

$$Max\ UE = \xi + \varepsilon \left(\sum_{i=1}^{m} R_i^x d_i^x + \sum_{r=1}^{s} R_r^g d_r^g \right) \quad (56)$$

$s.t.$

$$\sum_{j=1}^{n} X_{ij} \lambda_j + d_i^x = X_{ik} \quad (57)$$

$$\sum_{j=1}^{n} G_{rj} \lambda_j - d_r^g = (1 + \xi) * G_{rk} \quad (58)$$

$$\sum_{j=1}^{n} B_{fj} \lambda_j = B_{fk} \quad (59)$$

$$\sum_{i=1}^{n} \lambda_j = 1 \quad (60)$$

Then, Unified Efficiency (θ^*) under WMDR model would be calculated again same as WND Model

The dual of WMDR Model (76)-(84) can be written as multiplier form Models (86)-(93).

$$Min \sum_{i=1}^{m} v_i X_{ik} - \sum_{r=1}^{s} u_r G_{rk} + \sum_{r=1}^{h} w_f B_{fk} + \sigma \quad (61)$$

$$s.t. \quad (62)$$

$$\sum_{i=1}^{m} v_i X_{ij} - \sum_{r=1}^{s} u_r G_{rj} + \sum_{r=1}^{h} w_f B_{fj} + \sigma \geq 0 \quad (63)$$

$$\sum_{r=1}^{s} u_r G_{rk} = 1 \quad (64)$$

$$v_i \geq \varepsilon R_i^x (i = 1, \dots, m), \quad (65)$$

$$u_r \geq \varepsilon R_r^g (r = 1, \dots, s) \quad (66)$$

$$w_f \geq \varepsilon R_f^b, (f = 1, \dots, h) \quad (67)$$

$$\sigma \text{ free in sign} \quad (68)$$

It is notable that due to duality theorem in linear programming the optimum objective values of both Models (76)-(84) and (86)-(93) are equal.

Non-radial Model under WMD

To calculate Unified Efficiency (UE) under Weak Managerial Disposability considering a non-radial situation, Models (94)-(101)are proposed.

$$Max\ UE = \sum_{i=1}^{m} R_i^x d_i^x + \sum_{r=1}^{s} R_r^g d_r^g \quad (69)$$

$$s.t.$$

$$\sum_{j=1}^{n} X_{ij} \lambda_j + d_i^x = x_{ik} \quad (70)$$

$$\sum_{j=1}^{n} G_{rj} \lambda_j - d_r^g = G_{rk} \quad (71)$$

$$\sum_{j=1}^{n} B_{fj} \lambda_j = B_{fk} \quad (72)$$

$$\sum_{j=1}^{n} \lambda_j = 1 \quad (73)$$

The dual of Model (94)-(101) can be written as Models (103)-(108).

$$Min \sum_{i=1}^{m} v_i X_{ik} - \sum_{r=1}^{s} u_r G_{rk} + \sum_{r=1}^{h} w_f B_{fk} + \sigma \quad (74)$$

$$s.t.$$

AD-minister Nº. 30 january-june 2017 pp. 237 - 265 · ISSN 1692-0279 · eISSN 2256-4322

$$\sum_{i=1}^{m} v_i X_{ij} - \sum_{r=1}^{s} u_r G_{rj} + \sum_{r=1}^{h} w_f B_{fj} + \sigma \geq 0 \quad (75)$$

$$v_i \geq \varepsilon R_i^x (i = 1, \ldots, m) \quad (76)$$

$$u_r \geq \varepsilon R_r^g (r = 1, \ldots, s) \quad (77)$$

$$w_f \geq \varepsilon R_f^b (f = 1, \ldots, h) \quad (78)$$

$$\sigma \; free \; in \; sign \quad (79)$$

It is clear that due to the duality theorem in linear programming the optimum objective values of both Models (94)-(101) and (103)-(108) are equal.

SCALE EFFICIENCY AND RTS CALCULATIONS UNDER WEAK MANAGERIAL DISPOSABILITY

The Scale Efficiency (SE) score in both radial and non-radial states are the same, but RTS sign would be calculated separately based on the following algorithm. If DMUk is efficient under the WMD assumption, the efficiency score would be calculated using (42). Now, in order to define economic scale of inefficient units, first an image of inefficiency was extracted and the rate of inefficiency obtained using (43). In order to calculate RTS for radial models, first the two maximization and minimization models (109)-(124) should be solved.

$$Min \; (Max) \; \mu = \sigma \quad (80)$$

$$s.t.$$

$$\xi + \varepsilon \left(\sum_{i=1}^{m} R_i^x d_i^x + \sum_{r=1}^{s} R_r^g d_r^g \right) = \sum_{i=1}^{m} v_i X_{ik} - \sum_{r=1}^{s} u_r G_{rk} + \sum_{r=1}^{h} w_f B_{fk} + \sigma \quad (81)$$

$$\sum_{j=1}^{n} X_{ij} \lambda_j - d_i^x = x_{ik} \quad (82)$$

$$\sum_{j=1}^{n} G_{rj} \lambda_j - d_r^g = (1 + \xi) G_{ik} \quad (83)$$

$$\sum_{j=1}^{n} B_{fj} \lambda_j = B_{ik} \quad (84)$$

$$\sum_{i=1}^{m} v_i X_{ij} - \sum_{r=1}^{s} u_r G_{rj} + \sum_{r=1}^{h} w_f B_{fj} + \sigma \geq 0 \quad (85)$$

$$\sum_{r=1}^{s} u_r G_{rj} = 1 \quad (86)$$

$$\sum_{j=1}^{n} \lambda_j = 1 \quad (87)$$

$$\lambda_j \geq 0, (j = 1, \ldots, n) \quad (88)$$

$$v_i \geq \varepsilon R_i^x, (i = 1, \ldots, m) \quad (89)$$

$u_r \geq \varepsilon R_r^g, (r = 1, \dots, s)$ (90)

$w_f \geq \varepsilon R_f^b, (f = 1, \dots, h)$ (91)

Similar to the natural disposability conditions, using the calculated upper and lower limit of μ^* per each DMU, the RTS of DMUk is determined as follows: if $0 > \overline{\mu^*} \geq \underline{\mu^*}$ there is increasing return to scale, if $\overline{\mu^*} > 0 \geq \underline{\mu^*}$ there is constant return to scale, and if $\overline{\mu^*} \geq \underline{\mu^*} > 0$ there is decreasing return to scale.

In order to calculate RTS for non-radial models, first the two maximization and minimization models (125)-(140) should be solved.

$Min\ (Max)\ \pi = \sigma$ (92)

$s.t.$

$\sum_{i=1}^{m} R_i^x d_i^x + \sum_{r=1}^{s} R_r^g d_r^g = \sum_{i=1}^{m} v_i X_{ik} - \sum_{r=1}^{s} u_r G_{rk} + \sum_{r=1}^{h} w_f B_{fk} + \sigma$ (93)

$\sum_{j=1}^{n} X_{ij}\lambda_j - d_i^x = x_{ik}$ (94)

$\sum_{j=1}^{n} G_{rj}\lambda_j - d_r^g = G_{rk}$ (95)

$\sum_{j=1}^{n} B_{fj}\lambda_j = B_{fk}$ (96)

$\sum_{i=1}^{m} v_i X_{ij} - \sum_{r=1}^{s} u_r G_{rj} + \sum_{r=1}^{h} w_f B_{fj} + \sigma \geq 0$ (97)

$\sum_{r=1}^{s} u_r G_{rj} = 1$ (98)

$\sum_{j=1}^{n} \lambda_j = 1$ (99)

$\lambda_j \geq 0, (j = 1, \dots, n)$ (100)

$v_i \geq \varepsilon R_i^x, (i = 1, \dots, m)$ (101)

$u_r \geq \varepsilon R_r^g, (r = 1, \dots, s)$ (102)

$w_f \geq \varepsilon R_f^b, (f = 1, \dots, h)$ (103)

$\sigma\ free\ in\ sign$ (104)

$\xi \geq 0$ (105)

$d_f^b \geq 0\ , (f = 1, \dots, h)$ (106)

$d_i^x \geq 0\ (i = 1, \dots, m)\ , d_r^g \geq 0\ (r = 1, \dots, s)$ (107)

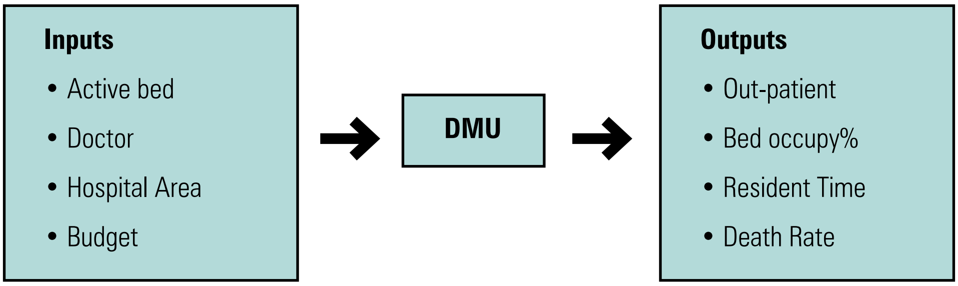
AD-minister Nº. 30 january-june 2017 pp. 237 - 265 · ISSN 1692-0279 · eISSN 2256-4322

Again, using the calculated upper and lower limit of π^* per each DMU, the RTS of DMUk is determined as follows: if $0 > \overline{\pi^*} \geq \underline{\pi^*}$ there is increasing return to scale, if $0 > \overline{\pi^*} \geq \underline{\pi^*}$ there is constant return to scale, and if $\overline{\pi^*} \geq \underline{\pi^*} > 0$ there is decreasing return to scale.

CASE STUDY AND ANALYSIS OF THE RESULTS

The proposed models are applied in a real case study involving 33 public health organization in Tehran, Iran. A hospital is assumed as a DMU which uses inputs in order to produce outputs. Both WND and WMD situations are assumed for DMUs and all models are run and the results are discussed in this section. On the other hand, the efficiency score, scale efficiency, and RTS are calculated for both WND and WMD situations, using both radial and non-radial models. The schematic vies of a DMU is shown in Figure 1.

Figure 1. Schematic View of DMU (Hospital).



It is notable that due to anonymity of results of this research the name of public health organization are not reported.

RESULTS OF WND MODELS

In this part, the results of WND models in radial and non-radial cases are presented. Table 2 presents the result of Radial WND models in radial case. The results include efficiency scores, RTS of each DMU, slack variables of each input, and slack variable of each output.

Table 2. Results of Radial Models considering WND.

DMU	Efficiency	RTS	Slack of Inputs				Slack of Outputs			
			Doctor	Active Bed	Hospital Area	Budget	Resident Time	Death Rate	Bed occupy	Out-Patient
DMU1	1.00	CRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU2	0.72	CRS	1.94	71.30	7.28	0.15	0.98	-12.57	0.00	0.00
DMU3	0.83	CRS	8.99	2.54	0.00	0.17	1.51	-2.81	0.00	0.00
DMU4	1.00	CRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU5	1.00	CRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU6	0.65	DRS	14.74	75.28	0.00	0.25	0.87	6.38	0.00	0.00
DMU7	1.00	CRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU8	0.89	CRS	0.00	237.51	0.00	0.00	-0.17	-6.54	0.00	0.00
DMU9	0.97	DRS	5.38	289.65	0.00	0.00	1.05	3.73	0.00	0.00
DMU10	1.00	CRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU11	0.86	CRS	0.00	75.90	0.00	0.00	0.83	-11.11	0.00	0.00
DMU12	1.00	CRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU13	0.69	CRS	0.00	0.00	6.84	0.41	0.36	-8.20	0.00	0.00
DMU14	1.00	CRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU15	1.00	CRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU16	1.00	CRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU17	1.00	CRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU18	0.56	CRS	2.28	0.00	10.23	1.23	0.62	8.76	0.00	0.00
DMU19	0.68	CRS	0.00	32.51	0.00	0.00	-1.38	7.21	0.00	0.00
DMU20	1.00	CRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU21	0.88	CRS	5.09	0.00	17.45	0.00	-0.07	-5.64	0.01	0.00
DMU22	1.00	CRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU23	0.95	CRS	0.00	0.00	0.00	1.48	-0.40	-4.72	0.00	0.00
DMU24	1.00	CRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU25	1.00	CRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU26	1.00	CRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU27	1.00	CRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU28	0.87	CRS	13.02	225.08	0.00	0.00	-0.84	-18.12	0.00	0.00
DMU29	1.00	DRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU30	1.00	CRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU31	0.92	CRS	0.00	21.85	0.00	0.00	0.10	7.70	0.00	0.63
DMU32	1.00	CRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU33	0.86	DRS	0.00	132.46	0.00	3.45	2.03	-13.80	0.00	0.00

AD-minister Nº. 30 january-june 2017 pp. 237 - 265 · ISSN 1692-0279 · eISSN 2256-4322

Based on Table 2, 19 public health organization are efficient, and 14 public health organization are inefficient. Seventeen out of 19 efficient public health organization have constant RTS while 2 of efficient public health organization have decreasing RTS. The slack variables for all 19 efficient public health organization are equal to zero, and this means that these public health organization are strong efficient. It is notable that slack variables can help an inefficient hospital to find its projection towards an efficient frontier. On the other hand, the slack variables represent extra input used, and shortage of production of outputs. For instance, consider DMU_2 which is inefficient among all 33 DMUs. The efficiency score of DMU_2 is equal to 0.72. DMU_2 can improve its efficiency score using projection. If DMU2 reduces 1.94 of its doctors, 71.30 of its active beds, 7.28 of its hospital area, and 0.15 of its budget the inputs will be set. Moreover, DMU2 should improve the resident time 0.98, the death rate 12.57 in order to set its outputs. Under such changes DMU2 will be an efficient DMU among the other DMUs. The same analysis can be done based on Table 2 for all other inefficient DMUs.

In a similar way, Table 3 presents the result of WND models in non-radial case. Again, the results include efficiency scores, RTS of each DMU, slack variables of each input, and slack variable of each output.

Table 3. Results of Non-Radial Models considering WND.

DMU	Efficiency	RTS	Slack of Inputs				Slack of Outputs			
			Doctor	Active Bed	Hospital Area	Budget	Resident Time	Death Rate	Bed occupy	Out-Patient
DMU1	1.00	DRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU2	0.83	DRS	12.44	16.78	3111.11	1.67	0.00	0.00	0.13	1.02
DMU3	0.88	CRS	1.00	0.00	0.00	2.37	1.54	0.00	0.13	0.55
DMU4	1.00	CRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU5	1.00	CRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU6	0.71	IRS	14.54	82.23	0.00	4.38	0.43	12.85	0.18	1.24
DMU7	1.00	IRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU8	1.00	IRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU9	0.84	IRS	5.00	360.71	16942.86	0.43	0.26	5.86	0.00	0.02
DMU10	1.00	CRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU11	1.00	CRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU12	1.00	CRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 3. Results of Non-Radial Models considering WND. Continued

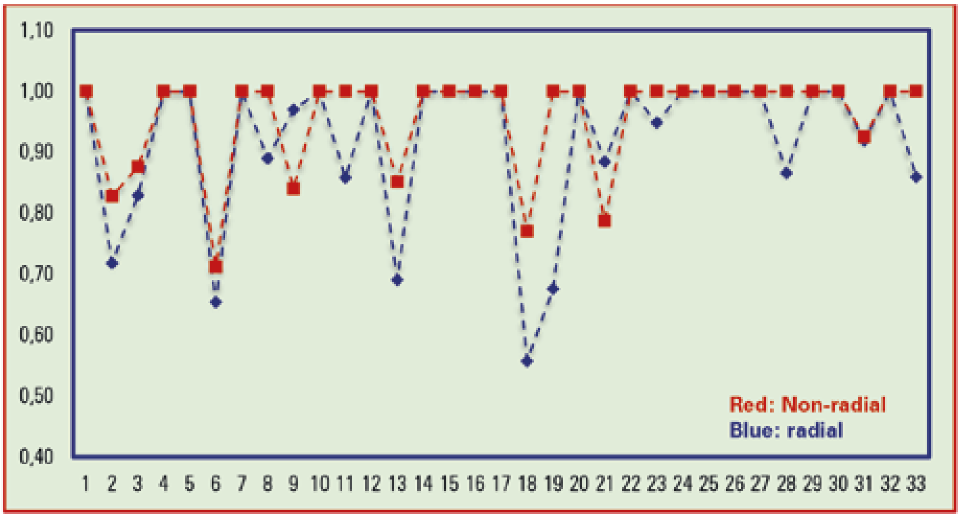
DMU	Efficiency	RTS	Slack of Inputs				Slack of Outputs			
			Doctor	Active Bed	Hospital Area	Budget	Resident Time	Death Rate	Bed occupy	Out-Patient
DMU14	1.00	CRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU15	1.00	DRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU16	1.00	CRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU17	1.00	CRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU18	0.77	DRS	19.19	19.10	2280.95	2.24	0.00	9.57	0.20	0.70
DMU19	1.00	IRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU20	1.00	CRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU21	0.79	DRS	13.59	48.75	22466.05	2.15	0.29	0.00	0.19	0.00
DMU22	1.00	CRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU23	1.00	CRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU24	1.00	CRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU25	1.00	DRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU26	1.00	CRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU27	1.00	CRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU28	1.00	CRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU29	1.00	DRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU30	1.00	CRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU31	0.93	DRS	0.00	36.13	9100.00	0.44	0.18	8.56	0.02	1.27
DMU32	1.00	DRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DMU33	1.00	CRS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

AD-minister Nº. 30 january-june 2017 pp. 237 - 265 · ISSN 1692-0279 · eISSN 2256-4322

Similar analysis can be made on contents of Table 3. Twenty five public health organization are efficient, and 8 public health organization are inefficient. Eighteen out of 25 efficient public health organization have constant RTS, 2 of them have increasing RTS and 5 of efficient public health organization have decreasing RTS. Again the slack variables of all 25 efficient public health organization are equal to zero, and this means that public health organization are strong efficient. Slack variables can help an inefficient hospital to find its projection towards efficient frontier.

As it is clear from the contents of Table 2 and Table 3, the number of efficient DMUs in non-radial models are more than radial models. In order to make a better sense of the results under WND conditions, the efficiency scores of all DMUs for both radial and non-radial models are plotted in Figure 2.

Figure 2. WND Case: Comparison of Efficiency Scores of Radial and Non-Radial Models.



It could be concluded that radial models measure efficiency in a more precise way than non-radial models and the discrimination power of DEA approaches is decreased when using non-radial models as DMU can change its inputs and outputs independently, so it has more opportunity to close towards efficient frontier.

Results of WMD Models

In this part, the results of WMD models in radial and non-radial cases are presented. Table 4 presents the result of Radial WMD models in radial case. The results include efficiency scores, DTS of each DMU, slack variables of each input, and slack variable of each output.

Table 4. Results of Radial Models considering WMD.

DMU	Efficiency	DTS	Slack of Inputs				Slack of Outputs			
			Doctor	Active Bed	Hospital Area	Budget	Resident Time	Death Rate	Bed occupy	Out-Patient
DMU1	0.92	CDS	17.48	67.678	2400.499	0.12	-1.7	1.5199	0	0
DMU2	0.81	IDS	0	30.457	0	0	0.842	-6.025	0	0.38
DMU3	0.88	IDS	0	25.946	8731.289	0	0.359	-1.173	0	0
DMU4	0.98	CDS	0	9	18200	0	-2.1	1	0	0
DMU5	0.87	IDS	10	264	7400	1	-1.4	20	0	0.59
DMU6	1.00	IDS	0	0	0	0	0	0	0	0
DMU7	0.91	IDS	11.32	37.965	12631.64	1.51	-1.5	-2.035	0	0
DMU8	0.76	CDS	12.09	0	3279.67	1.04	-0.8	5.9637	0	0
DMU9	1.00	IDS	0	0	0	0	0	0	0	0
DMU10	1.00	CDS	0	0	0	0	0	0	0	0
DMU11	0.62	IDS	10.6	0	1815.189	2.51	-0	-7.31	0	0
DMU12	1.00	CDS	0	0	0	0	0	0	0	0
DMU13	0.69	IDS	0	81.158	0	0	1.08	-5.215	0	0.49
DMU14	1.00	CDS	0	0	0	0	0	0	0	0
DMU15	1.00	IDS	0	0	0	0	0	0	0	0
DMU16	0.87	IDS	0.092	0	3516.389	3.98	0.424	-12.07	0	0
DMU17	0.89	IDS	5.334	12.303	18761.61	2.65	-1.8	8.3969	0	0
DMU18	0.58	IDS	0	58.555	0	0	-0.3	11.538	0	1.34
DMU19	0.52	CDS	5.314	0	13831.42	0.64	-1.5	5.6106	0	0
DMU20	1.00	CDS	0	0	0	0	0	0	0	0
DMU21	1.00	IDS	0	0	0	0	0	0	0	0
DMU22	0.96	CDS	0.333	191.33	16333.33	0	1.8	2	0	1.14
DMU23	0.84	IDS	4.396	61.888	4334.419	0	-0.9	1.0615	0	0
DMU24	1.00	IDS	0	0	0	0	0	0	0	0
DMU25	1.00	IDS	0	224	10000	0	0.2	10	0	0.86
DMU26	0.97	IDS	32	57	5000	0	0.2	0	0.02	0
DMU27	1.00	IDS	0	0	0	0	0	0	0	0
DMU28	1.00	CDS	0	0	0	0	0	0	0	0
DMU29	1.00	IDS	0	0	0	0	0	0	0	0
DMU30	0.85	IDS	23.05	0	4400.292	4.09	-1.1	2.5414	0	0
DMU31	0.81	IDS	2.935	204.64	0	0.99	-0.3	21.707	0	0.26
DMU32	1.00	IDS	0	0	0	0	0	0	0	0
DMU33	1.00	IDS	0	0	0	0	0	0	0	0

AD-minister Nº. 30 january-june 2017 pp. 237 - 265 · ISSN 1692-0279 · eISSN 2256-4322

Based on Table 4, 14 public health organization were efficient and 19 public health organization were inefficient. Among 14 efficient public health organization, 5 public health organization had constant DTS and 9 public health organization had increasing DTS. The slack variables for all 14 efficient public health organization are equal to zero, and this means these public health organization are strong efficient.

In a similar way, Table 5 presents the result of Radial WMD models in non-radial case. Again, the results include efficiency scores, DTS of each DMU, slack variables of each input, and slack variable of each output.

Table 5. Results of Non-Radial Models considering WMD.

			Slack of Inputs				Slack of Outputs			
DMU	Efficiency	DTS	Doctor	Active Bed	Hospital Area	Budget	Resident Time	Death Rate	Bed occupy	Out-Patient
DMU1	0.82	IDS	8.3	156.8	16694.2	0.5	-1.7	8.4	0.2	0.0
DMU2	0.85	IDS	0.0	194.0	10000.0	1.0	0.0	5.0	0.0	1.7
DMU3	0.85	IDS	4.0	263.0	22000.0	0.0	0.4	18.0	0.0	0.7
DMU4	0.97	IDS	0.0	9.0	18200.0	0.0	-2.1	1.0	0.0	0.2
DMU5	0.68	IDS	25.0	409.0	12400.0	0.0	-1.7	6.0	0.0	3.1
DMU6	1.00	IDS	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
DMU7	0.81	IDS	11.8	173.8	21270.1	1.4	-1.1	8.1	0.1	0.0
DMU8	0.76	IDS	24.0	42.0	12000.0	2.0	-1.4	13.0	0.0	2.4
DMU9	1.00	IDS	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
DMU10	1.00	IDS	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
DMU11	0.69	IDS	26.0	176.0	9000.0	2.0	0.1	6.0	0.1	2.1
DMU12	1.00	CDS	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
DMU13	0.78	IDS	13.0	183.9	0.0	0.3	0.2	0.6	0.1	3.0
DMU14	1.00	IDS	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
DMU15	1.00	IDS	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
DMU16	0.79	IDS	9.4	319.8	13600.0	2.2	0.4	-3.0	0.0	0.1
DMU17	0.80	IDS	10.9	188.6	25513.8	2.2	-1.2	21.4	0.1	0.0
DMU18	0.74	CDS	3.0	280.0	9500.0	1.0	-1.1	27.0	0.1	3.3
DMU19	0.72	CDS	20.0	235.0	22000.0	0.0	-1.3	21.0	0.1	2.4

Table 5. Results of Non-Radial Models considering WMD. Continued

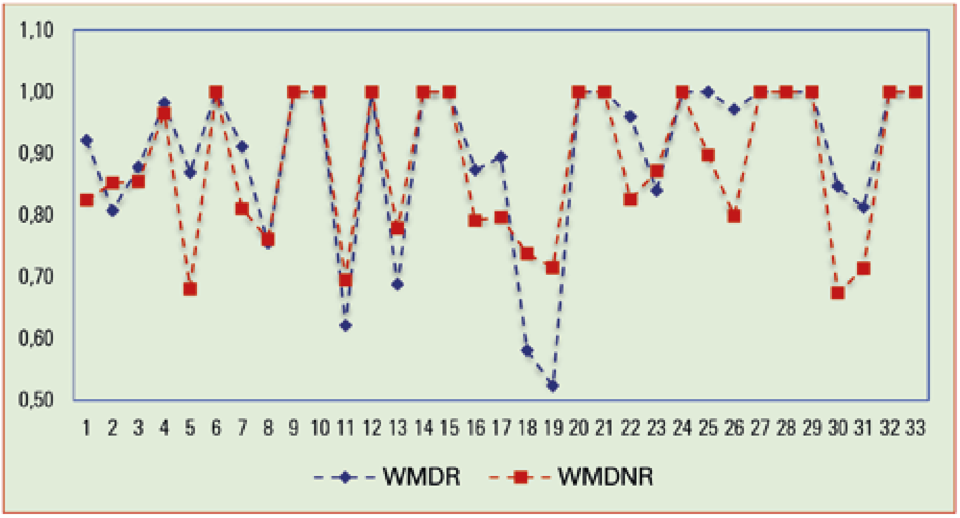
DMU	Efficiency	DTS	Slack of Inputs				Slack of Outputs			
			Doctor	Active Bed	Hospital Area	Budget	Resident Time	Death Rate	Bed occupy	Out-Patient
DMU21	1.00	IDS	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
DMU22	0.83	IDS	4.1	200.0	12700.0	0.8	2.2	-3.3	0.0	2.3
DMU23	0.87	IDS	4.9	0.0	12655.5	0.0	-2.2	-6.7	0.1	2.2
DMU24	1.00	IDS	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
DMU25	0.90	IDS	0.0	224.0	10000.0	0.0	0.2	10.0	0.0	0.9
DMU26	0.80	IDS	13.9	40.6	20341.8	0.8	-0.3	-0.8	0.3	0.0
DMU27	1.00	IDS	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
DMU28	1.00	IDS	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
DMU29	1.00	IDS	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
DMU30	0.67	IDS	24.0	205.0	16300.0	4.0	-0.6	18.0	0.1	0.1
DMU31	0.71	IDS	18.4	311.8	3600.0	1.2	-0.6	14.0	0.0	3.1
DMU32	1.00	IDS	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
DMU33	1.00	IDS	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Similar analysis can be made on contents of Table 5. Fourteen public health organization are efficient, and 19 public health organization are inefficient. One out of 14 efficient public health organization have constant DTS, and 13 of efficient public health organization have increasing DTS. Again the slack variables of all 19 efficient public health organization are equal to zero, and this means that public health organization are strong efficient. Slack variables can help an inefficient hospital to find its projection towards efficient frontier.

As it is clear from contents of Table 4 and Table 5, the number of efficient DMUs in non-radial and radial models are equal, although the average efficiency scores in non-radial model is quite higher than the average efficiency scores in radial model. In order to make a better sense of the results under WMD conditions, the efficiency scores of all DMUs for both radial and non-radial models are plotted in Figure 3.

AD-minister Nº. 30 january-june 2017 pp. 237 - 265 · ISSN 1692-0279 · eISSN 2256-4322

Figure 3. WMD Case: Comparison of Efficiency Scores of Radial and Non-Radial Models.



Discussion and Further investigation

Table 6 presents the summary results of WMD and WND cases for both radial and non-radial models.

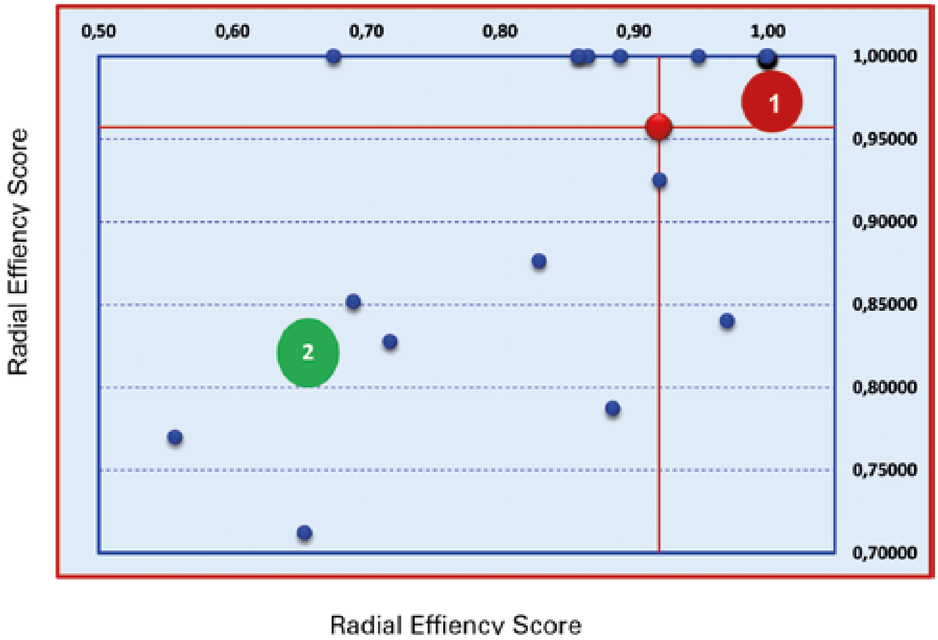
Table 6. Summary Results of WMD and WND cases for radial and non-radial models.

Radial/ Non-radial	Non-Radial models				Radial models			
PPS Assumption	WMD		WMD		WND		WND	
No. of Efficient DMUs	14		14		25		19	
No. of Increasing RTS	30	13	10	9	5	2	0	0
No. of Constant RTS	3	1	23	5	19	18	28	17
No. of Decreasing RTS	0	0	0	0	9	5	5	2

In order to determine the position of a DMU, the following analysis is conducted. In WND case, first the average efficiency score for each radial and non-radial models have been calculated. Then, a 2-dimentional plot, as in Figure 4, is prepared. In Figure 4, the efficiency score of a DMU under radial and non-radial model in presence of WND situation has been plotted.

In WND case, first the average efficiency score for each radial and non-radial models have been calculated. Then, a 2-dimentional plot, as in Figure 4, is prepared. In Figure 4, the efficiency score of a DMU under radial and non-radial model in presence of WND situation has been plotted.

Figure 4. Radial VS Non-Radial Efficiency Scores WND case.



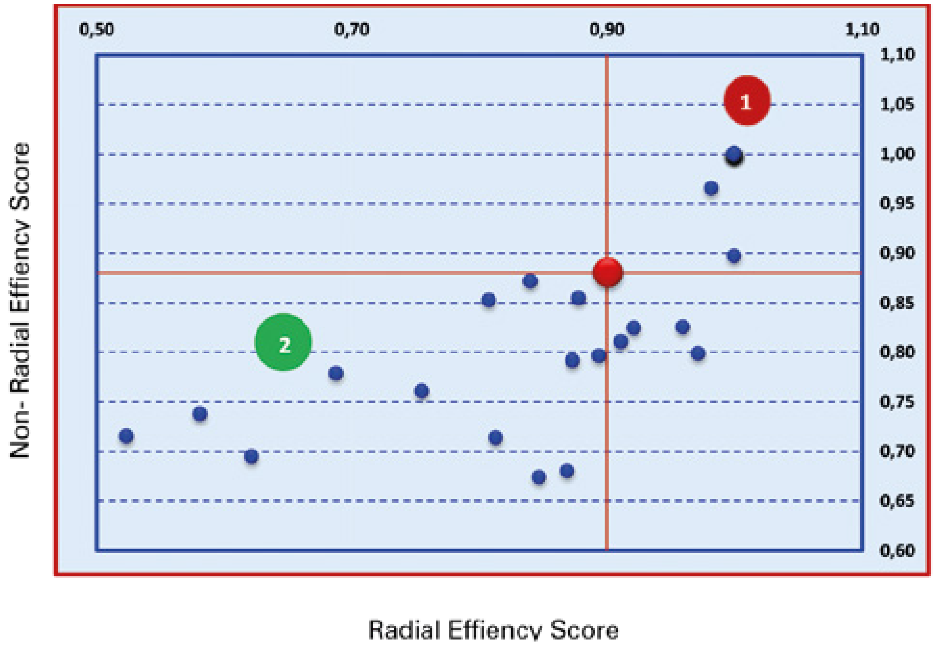
The red horizontal and vertical lines in figure 4, shows the average efficiency score values for both radial and non-radial models, respectively. The Figure 4 is divided into 4 regions. The DMUs in region 1, are the DMUs which their efficiency scores are higher than average in both radial and non-radial models under WND case. These DMUs are the best public health organization and the efficiency is stable in them. The DMUs in region 2, are the DMUs where their efficiency scores are lower than average in both radial and non-radial models under WND case. These DMUs are the worst public health organizations and the efficiency is very low in these organizations. This can give public health organization' managers proper insights for improvement.

The red point shows the average efficiency scores. The red horizontal and vertical lines in figure 4, shows the average efficiency score values for both radial and non-radial models, respectively. The Figure 4 is divided into 4 regions. The DMUs in region 1, are the DMUs which their efficiency scores are higher than average in both radial and non-radial models under WND case. These DMUs are the best public health organization and the efficiency is stable in them. The DMUs in region 2, are the DMUs which their efficiency scores are lower than average in both radial and non-radial models under WND case. These DMUs are the worst public health organization and the efficiency is very low in them. This can give public health organization' managers proper insights for improvement.

The analysis is conducted for WMD case. In Figure 5, the efficiency score of a DMU under radial and non-radial model in presence of WMD situation has been plotted.

AD-minister Nº. 30 january-june 2017 pp. 237 - 265 · ISSN 1692-0279 · eISSN 2256-4322

Figure 5. Radial VS Non-Radial Efficiency Scores WMD case.



Based on the information in Figure 4 and Figure 5, it is clear that the average efficiency score in both WND and WMD cases, when the non-radial mode is used to measure the efficiency, is low. Moreover, the efficiency scores of most of public health organization are less than the red horizontal line.

The situation is opposite when the radial model is used to measure the efficiency. Only few DMUs are settled down at the right side of vertical line of Figure 4 and Figure 5. This means that the average efficiency score in both WND and WMD cases, when the radial mode is used to measure the efficiency, is high.

The average efficiency scores for the WND situation are equal to 0.91 and 0.95 for radial and non-radial models, respectively. The average efficiency scores for WMD situation are equal to 0.90 and 0.88 for radial and non-radial models, respectively. Totally, it can be concluded from Figure 4, and Figure 5 that the average efficiency score is lower for WMD in comparison with WND situation

It is notable that the two PPS defined in this research (i.e., weak natural and weak managerial) have two completely different viewpoints towards efficiency. It means the natural viewpoint seeks to decrease inputs, to fix bad outputs, and to increase good inputs, while the managerial viewpoints tries to use more inputs, to produce more good outputs in a constant rate of bad outputs. So, as mentioned, in Table 2 and Table 3 the term RTS was used while in Table 4 and Table 5 the term DTS was used. It is clear that the strategies of these managers are different and opposite. Table 7 shows the associated strategy related to DTS and RTS.

Table 7. Associated Strategies related to DTS and RTS.

WND		WMD	
Associated Strategy	RTS	Associated Strategy	DTS
Increase in size is preferred	Increasing	Decrease in size is preferred	Increasing
No change in size is preferred	Constant	No change in size is preferred	Constant
Decrease in size is preferred	Decreasing	Increase in size is preferred	Decreasing

Practical Suggestion

Regarding the obtained results, it is suggested that policy makers and managers of public health organization apply the following suggestions in order to promote the performance of the public health organization. The proposed models of this study can be used to determine efficient DMUs, inefficient DMUs, and the associated return to scale/DTS of departments of public health organization. The scale efficiency results can be used to regulate hospital results such as the average hospitalization of a patient, or regulating the laboratory's schedule with routine examinations. The public health organization produce health cards in order to reduce patients' release time and settlement. The process of sending physicians' prescriptions to laboratory and receiving their reply should be reduced and summarized. The facilities and equipment's for surgery operations must be maintained using more accurate procedures. There is a need to establish a managerial system in order to operation room schedule efficiently. It is mandatory to handle the patients with a non-treatment medicinal disease as soon as possible in emergency unit. No hospitalizing if suggested for such patients.

CONCLUSION REMARKS AND FUTURE RESEARCH DIRECTIONS

In this paper, several models were proposed to measure the efficiency scores of systems in the presence of undesirable outputs. In this regards, two new assumptions on production possibility sets (PPS) were proposed. In the first assumption, the WND was proposed. Under the WND situation, a DMU's interest is to reduce its inputs in order to increase its good outputs while producing a constant value of bad outputs. In the second assumption, the WMD was proposed. Under the WMD situation, a DMU's interest is to increase its inputs in order to increase its good outputs while producing a constant value of bad outputs. On the other hand, the WND perspective seeks to meet the environmental issues while improving efficiency, and the WMD perspective seeks to develop the production rate in order to improve the efficiency. Based on these two PPSs and using radial and non-radial models, several models based on DEA were developed in order to measure the efficiency scores of DMUs, to determine the return to scale, and to determine the DTS. The proposed models and

AD-minister Nº. 30 january-june 2017 pp. 237 - 265 · ISSN 1692-0279 · eISSN 2256-4322

procedures were applied in 33 public health organization in Iran and the results were discussed for the case study. The main contribution of this study were as follows:
1) Introducing two new PPSs as WND and WMD;
2) Development of new models based on these PPSs to calculate the efficiency scores using both radial and non-radial approaches;
3) Proposing some procedure in order to determine the RTS of DMUs under WND situations;
4) Proposing some procedure in order to determine the DTS of DMUs under WMD situations;
5) Handling a real case study including 33 public health organization in Tehran, Iran.
The preceding points are proposed to researchers and scholars working in the field of performance measurement. The procedure of this paper can be conducted using strong natural disposability and strong managerial disposability. The results can be compared with the results of this study. The variable RTS was used in this study, other return to scales can be considered.

As there are several qualitative criteria in performance assessment, development of the procedure of this paper in presence of qualitative and uncertain criteria can be interesting. Other applications such as baking, energy sector, production and services can be handled using proposed models of this study.

REFERENCES

Al-Shammari, M. (1999). A multi-criteria data envelopment analysis model for measuring the productive efficiency of Hospitals. *International Journal of Operations & Production Management*, *19*(9): 879-90.

Butler, T.W., and Li, L. (2005). The utility of returns to scale in DEA programming: An analysis of Michigan rural hospitals. *European Journal of Operational Research, 161*,469-477.

Blank, J.L.T., and Valdmanis,V.G. (2010). Environmental factors and productivity on Dutch hospitals: a semi-parametric approach. *Health Care Manag Sci, 13*, 27–34.

Charnes, A., Cooper W.W., and Rhodes, E. (1979). Short communication: measuring efficiency of decision making units. *European Journal of Operations Research. 3*, 339-340.

Cooper, W.W., Park, K.S., and Pastor, J.T., (2000). RAM: A range adjusted measure of efficiency. *Journal of Productivity Analysis, 11*, 5–42.

Cooper W.W., Seiford, L.M., and Zhu, J. (2011). *Handbook on Data Envelopment Analysis*, Second Edition, Spiringer.

Ding, D. X. (2014). The effect of experience, ownership and focus on productive efficiency: A longitudinal study of U.S. hospitals. *Journal of Operations Management, 32*, 1–14.

Goldstein, S.M., Ward, P.T., Leong, G.K., and Butlerd, T.W.(2002). The effect of location, strategy, and operations technology on hospital performance. *Journal of Operations Management, 20*, 63–75.

Garcia-Lacalle, J., and Martin, E. (2010). Rural vs urban hospital performance in a 'competitive' public health service. *Social Science & Medicine, 71*, 1131-1140.

Grosskopf, S., Margaritis, D., and Valdmanis, V. (2004). Competitive effects on teaching hospitals. *European Journal of Operational Research, 154*, 515–525.

Gok, M.S., and Sezen, B. (2013). Analyzing the ambiguous relationship between efficiency, quality and patient satisfaction in healthcare services: The case of public hospitals in Turkey. *Health Policy, 111*, 290– 300.

Hollingsworth, B. (2003). Non-parametric and parametric applications measuring efficiency in health care. *Health Care Management Science, 6*(4), 203–218.

Joses, M. K., Emrouznejad, A., Vaz, R. G., Bastiene, H., and Padayachy, J.(2008). Efficiency measurement of hospital. *International Journal of Productivity and Performance Management, 57*(1), 72-92.

Kawaguchi, H., Tone, K., and Tsutsui, M. (2014). Estimation of the efficiency of Japanese hospitals using a dynamic and network data envelopment analysis model. *Health Care Management Science,* 17, 101-112

Lee, K., Chun, K., and Lee J. (2008). Reforming the hospital service structure to improve efficiency: Urban hospital specialization. *Health Policy. 87*,41–49.

Nayar, P., and Ozcan, Y.A. (2008). Data envelopment analysis comparison of hospital efficiency and quality. *Journal of Medical Systems, 32*(3), 193-199.

Nunamaker, T. (1983). Measuring routine nursing service efficiency: a comparison of cost per day and data envelopment analysis models. *Health Service Research, 18*(2 Pt 1), 183–205.

O'Neill, L., Rauner, M., Heidenberger, K., and Karus, M. (2008). A cross-national comparison and taxonomy of DEA-based hospital efficiency studies. *Socio-Economic Planning Sciences, 42*(3),158–189.

Rosko, M. D., and Mutter, R. L. (2011). What have we learned from the application of stochastic frontier analysis to U.S. hospitals?. *Medical Care Research and Review, 68*(1), 75S–100S.

Sherman, H. (1984). Hospital efficiency measurement and evaluation Empirical test of a new technique. *Medical Care*, *22*(10).

Sueyoshi, T., and Goto, M., (2011a). Measurement of returns to scale and damages to scale for operational and environmental assessment: how to manage desirable (good) DEA-based and undesirable (bad) outputs. *European Journal of Operational Research, 211*, 76–89.

AD-minister Nº. 30 january-june 2017 pp. 237 - 265 · ISSN 1692-0279 · eISSN 2256-4322

Sueyoshi, T., and Goto, M. (2011b). DEA approach for unified efficiency measurement: assessment of Japanese fossil fuel power generation. *Energy Economics, 33*, 292–303.

Sueyoshi, T., and Goto, M. (2012a). Efficiency-based rank assessment for electric power industry: a combined use of Data Envelopment Analysis (DEA) and DEA Discriminant Analysis (DA). *Energy Economics 34,* 634–644.

Sueyoshi, T., and Goto, M. (2012b). Returns to scale and damages to scale under natural and managerial disposability: strategy, efficiency and competitiveness of petroleum firms. *Energy Economics, 34*, 645–662.

Sueyoshi, T., and Goto, M. (2012c). Returns to scale and damages to scale with strong complementary slackness conditions in DEA assessment: Japanese corporate effort on environment protection. *Energy Economics, 34,* 1422–1434.

Sueyoshi, T., and Goto, M. (2012d). Environmental assessment by DEA radial measurement: US coal-fired power plants in ISO (Independent System Operator) and RTO (Regional Transmission Organization). *Energy Economics, 34*, 663–676.

Sueyoshi, T., and Goto, M. (2012e). Weak and strong disposability vs. natural and managerial disposability in DEA environmental assessment: comparison between Japanese electric power industry and manufacturing industries. *Energy Economics, 34,* 686–699.

Sueyoshi, T., and Goto, M., (2012f). DEA radial and non-radial models for unified efficiency under natural and managerial disposability: theoretical extension by strong complementary slackness conditions. *Energy Economics, 34*, 700–713.

Thanassoulisa, E., Portelab, M. S., and Graveneyc, M.(2016). Identifying the scope for savings at inpatient episode level: An illustration applying DEA to chronic obstructive pulmonary disease. *European Journal of Operational Research, 255*(2), 570–582

Tsai, P.F., and Mar, M.C. (2002). A variable returns to scale data envelopment analysis model for the joint determination of efficiencies with an example of the UK health service. *European Journal of Operational Research, 141*, 21-38.

Tsekouras, K., Papathanassopoulos, F., Kounetas, K., Pappous, G. (2010). Does the adoption of new technology boost productive efficiency in the public sector? The case of ICUs system. *International Journal of Production Economics*, 128, 427–433.