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# On the production of the positive antihydrogen ion $\bar{\text{H}}^+$ via radiative attachment

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**Abstract.** We provide an estimate of the cross section for the radiative attachment of a second positron into the  $(1s^2 1S^e)$  state of the  $\bar{\text{H}}^+$  ion using Ohmura and Ohmura's (1960 Phys. Rev. **118** 154) effective range theory and the principle of detailed balance. The  $\bar{\text{H}}^+$  ion can potentially be created using interactions of positrons with trapped antihydrogen, and our analysis includes a discussion in which estimates of production rates are given. Motivations to produce  $\bar{\text{H}}^+$  include its potential use as an intermediary to cool antihydrogen to ultra-cold (sub-mK) temperatures for a variety of studies, including spectroscopy and probing the gravitational interaction of the anti-atom.

*Keywords:* antihydrogen, radiative attachment, photodetachment, antihydrogen ion

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## 1. Introduction

Over the last decade or so it has become routine to produce antihydrogen ( $\bar{\text{H}}$ ) atoms in specialised experiments (e.g., [2, 3, 4]) located at the unique Antiproton Decelerator (AD) facility at CERN [5]. More recently the ALPHA and ATRAP collaborations have succeeded in holding onto small numbers of the anti-atoms in purpose-built magnetic minimum neutral atom traps [6, 7, 8], and ALPHA has achieved confinement times of

many minutes [9]. This has allowed first experiments to be performed on the properties of antihydrogen, including the observation of a resonant quantum transition [10], setting a crude limit on the gravitational interaction of the anti-atom [11] and providing an experimental limit on its charge [12].

Though antihydrogen experimentation is still in its infancy, it has long been anticipated that trapping and cooling of anti-atomic species would play a large part in precision comparisons of the properties of antimatter with matter (see, e.g., [13] and references therein). Indeed, cooling of positrons ( $e^+$ ) and antiprotons ( $\bar{p}$ ) is key to producing antihydrogen, and at temperatures such that a small fraction of the yield can be held in sub-K deep neutral atom traps. Cooling of the antiparticles has been achieved in a number of ways: positrons are routinely captured and cooled using buffer gas accumulators (see e.g. [14]) and can then be transferred to a high vacuum apparatus for antihydrogen formation [15]. In the latter type of system the strong (of order Tesla) magnetic fields present result in positron self-cooling via the emission of synchrotron radiation. Antiprotons are routinely cooled in Penning-type traps using clouds of electrons [16] and more recently evaporative [17] and adiabatic [18] techniques have been developed to reach cryogenic temperatures.

The realisation of antihydrogen trapping allows consideration of means to manipulate the anti-atom (for instance, laser cooling [13, 19, 20]) and to allow its interactions with other trapped species, and in particular positrons. In the present work we consider the radiative attachment of a positron to antihydrogen to form the antihydrogen positive ion,  $\bar{H}^+$ , via the reaction,



Interest in  $\bar{H}^+$  stems mainly from its potential to be used as an intermediary for the creation of ultra-cold (sub-mK) antihydrogen for studies of the gravitational interaction of the anti-atom, as first suggested by Walz and Hänsch [21], and to be implemented by the GBAR collaboration [22, 23]. This is possible since  $\bar{H}^+$  is amenable to sympathetic cooling using laser-cooled positively charged ions of ordinary matter (e.g.,  $\text{Be}^+$ ). The biggest obstacle to implementing such cooling schemes is the efficiency with which the antihydrogen ion can be produced. In most antihydrogen experiments conducted to date the positron plasmas are cold and dense enough that the antihydrogen is produced in very weakly bound states (see e.g. [24, 25]) via the three-body reaction  $e^+ + e^+ + \bar{p} \rightarrow e^+ + \bar{H}$ . The length scale governing the rate of reaction is the Thomson radius,  $r_e = e^2/4\pi\epsilon_0 k_B T_e$ , where the constants have their usual meaning and  $T_e$  is the positron plasma temperature. For typical values of  $T_e$  this is of the order of  $\mu\text{m}$ , so the probability of a further positron interacting to produce  $\bar{H}^+$  (which has only a single bound state with dimensions close to the Bohr radius,  $a_0$ ) is vanishingly small. In their gravity measurement experiment, GBAR envisage producing small quantities of  $\bar{H}^+$  using a double charge exchange scheme involving successive positron capture from a cloud of positronium (Ps) atoms as,  $\bar{p} + \text{Ps} \rightarrow \bar{H} + e^-$  followed by  $\bar{H} + \text{Ps} \rightarrow \bar{H}^+ + e^-$ . The yield from this reaction scheme is less than one  $\bar{H}^+$ , per 60 million  $\bar{p}\text{s}$ , for a Ps

cloud of density of around  $7 \times 10^{11} \text{ cm}^{-3}$ , with the latter produced using an intense ( $3 \times 10^8 \text{ s}^{-1}$ ) positron beam accumulated for just over 100 s. The low yield of this Ps route to  $\bar{\text{H}}^+$  has motivated the present study. Now that  $\bar{\text{H}}$  can be trapped for extended periods, it may be feasible to exploit  $e^+ - \bar{\text{H}}$  reactions to produce the ion. In this work we will deduce cross sections for reaction (1) by invoking charge conjugation and time reversal symmetries and using data from the photodetachment of the negative hydrogen ion,  $\text{H}^-$ .

## 2. Photodetachment of $\text{H}^-$

The photodetachment of one electron from the ( $1s^2 \ ^1S^e$ ) state of the hydrogen ion  $\text{H}^-$  has long been of interest, in particular because it is known to be responsible for the opacity of the sun [26]. As a result, this process was studied extensively in the 1940s-80s [27]-[45] and more recently is experiencing a resurgence [46]-[55]. Ward, McDowell, and Humberston [56] (in the parallel  $\text{Ps}^-$  case) describe this as calculating an allowed dipole transition to the continuum of the two-electron Hamiltonian

$$\hbar\omega + \text{H}^- (1s^2 \ ^1S^e) \rightarrow \text{H}^- (1skp \ ^1P^o). \quad (2)$$

The cross section for the photodetachment of one electron from the ( $1s^2 \ ^1S^e$ ) state of  $\text{H}^-$  has been evaluated for electric-dipole allowed transitions by a number of authors utilising both the so-called velocity and length gauges. Since the cross section differences between these formulations (which are due to the approximate nature of the two-electron wave functions used) are small, we will make no distinction in this work. Here we recapitulate the effective range theory [57] formalism of Ohmura and Ohmura [1] to find the cross section (in atomic units) for photodetachment (or photoionization),  $\sigma_{PI}$ , in terms of  $k$ , the magnitude of the momentum  $\mathbf{k}$  of the detached electron as,

$$\sigma_{PI} = \frac{4\alpha a_0^2}{3} \frac{k}{k^2 + \gamma^2} \left| \int e^{-i\mathbf{k}\cdot\mathbf{r}} \left( -i\frac{1}{k} \mathbf{k} \cdot \nabla \right) C \frac{e^{-\gamma r}}{r} d^3r \right|^2, \quad (3)$$

where  $\alpha$  is the fine structure constant. This reduces to

$$\begin{aligned} \sigma_{PI} &= 0.429304 \times 10^{-17} \text{cm}^2 \frac{k^3}{(k^2 + \gamma^2)^3} \\ &= 0.429304 \times 10^{-17} \text{cm}^2 \frac{1}{\gamma^3} \left( \frac{\lambda}{\lambda_0} \right)^{3/2} \left( 1 - \frac{\lambda}{\lambda_0} \right)^{3/2}. \end{aligned} \quad (4)$$

Here  $k$  and the photon wavelength,  $\lambda$ , are related to the  $\text{H}^-$  electron affinity,  $I$ , given by  $hc/\lambda = k^2/2 + I$ , where  $h$  and  $c$  have their usual meanings. The electron affinity, namely the energy difference between the initial and final bound states, is give by  $I = \gamma^2/2$ , with  $\gamma = 0.2355883$  a.u. [58]. The constant  $C$  has its origin in the  $\text{H}^-$  wave function and is 0.315878 [49]. Furthermore, the threshold photon wavelength,  $\lambda_0$ , may be converted from atomic units to  $\text{\AA}$  via

$$\lambda_0[\text{\AA}] = \frac{0.5291772\text{\AA}}{a_0} 2 \frac{2\pi\hbar c}{k^2 + 2I} \Big|_{k \rightarrow 0} = \frac{911.2671\text{\AA}}{\gamma^2} = 16439.02\text{\AA}, \quad (5)$$

(as can the photon wavelength corresponding to any electron momentum  $k$ ).

The photoionization cross section using equation (4) is shown in Figure 1 (solid line), along with the experimental results of Smith and Burch [59]. The experimental results were originally given relative to the value at 5280 Å, but Geltman [30] put these on an absolute scale by comparing this point to the absolute integrated measurements of Branscomb and Smith [60],  $3.28 \times 10^{-17} \text{ cm}^2 \pm 10\%$ . Most theorists shift this point up or down to match their value at 5280 Å, since they are within this 10% buffer. Here we have averaged the values for the 11 theories given in Saha [45] (which do not include Ohmura and Ohmura), or  $3.04 \times 10^{-17} \text{ cm}^2$ , and have adjusted the experimental results accordingly.

One sees excellent agreement between the theory of Ohmura and Ohmura [1] and the peak position of the experiment, though the values are 3.5% higher near the peak at 7640 Å, where the average of the other theories shown in Figure 1(b) match the normalized experimental value of  $3.92 \times 10^{-17} \text{ cm}^2$  within 0.5%. Though ideally one should use a more sophisticated theory to predict the details of the cross section for radiative attachment of a second positron onto the  $\bar{\text{H}}^+$  ion, a result only a few percent high in magnitude is a small price to pay for a more rapid guide to experiment that we wish to provide in this paper. Note that the photoionization cross sections used here only include the so-called non-resonant photo-electric component. Thus, we ignore the effect of prominent autodetaching resonances close to 11 eV photon energy ( $\lambda \approx 1130 \text{ Å}$ , see e.g., Miyake *et al.* [61] for a discussion) which are at energies too high to be of interest.

Recently, Baltentkov [62] has presented calculations of cross sections for the photoionization of  $\text{H}^-$  via the magnetic-dipole mediated mechanism: these were found, except in a narrow energy region very close to the threshold (at a photon energy of  $hc/\lambda_0$ ), to be much smaller than their electric-dipole equivalents. For instance, the maximum cross section for the photo-magnetic process was found to be around  $5 \times 10^{-22} \text{ cm}^2$  at  $h\nu = 1.1 \text{ eV}$  ( $\lambda \approx 11.3 \times 10^3 \text{ Å}$ ), or around five orders of magnitude lower than the photo-electric cross sections illustrated in Figure 1. The energy region where the magnetic process dominates is restricted to below  $10^{-7} \text{ eV}$  above threshold, with a corresponding positron temperature in the sub-mK range that, as will be discussed in Section 4, is currently beyond experimental capabilities.

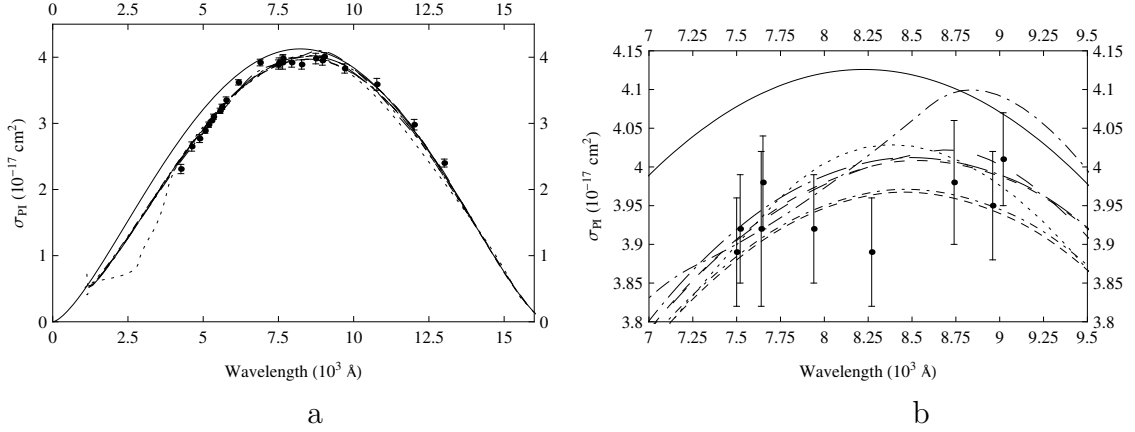


Figure 1 (a and b). The cross section for photoionizing an electron from the  $(1s^2 \ ^1S^e)$  state of the negative hydrogen ion  $H^-$ , (a) with the region near the maximum highlighted in (b). The dots are the experimental results of Smith and Burch [59] normalized at  $5280 \text{ \AA}$  to the average of the values for the 11 theories given in Saha [45],  $3.04 \times 10^{-17} \text{ cm}^2$ . The solid line gives the present recapitulation of Ohmura and Ohmura’s [1] effective range theory. Also shown are a representative sample of length gauge theories: Venuti and Decleva [46] (small dashed); Saha [45] (medium dashed); Wishart [42] (large dashed); Stewart [40] (large dot dashed); Broad and Reinhardt [37] (medium dot dashed); Ajmera and Chung [36] (small dot dashed); and Bhatia [55] (dotted).

### 3. $\bar{H}^+$ by radiative attachment

The cross section for the radiative attachment of a second positron to  $\bar{H}(1s)$  to create the  $(1s^2 \ ^1S^e)$  state of the  $\bar{H}^+$  ion, via reaction (1), can be obtained following the lead of Jacobs, Bhatia, and Temkin [63] who applied the principle of detailed balance (see e.g., Landau and Lifshitz [64]) to the photodetachment cross section to obtain the radiative attachment coefficient (for an electron) to form the  $(2p^2 \ ^3P^e)$  metastable  $H^-$  state from  $H(2s, 2p)$ . Thus [65],

$$\sigma_{RA} = \sigma_{2 \rightarrow 1} = \frac{g_1 p_1^2}{g_2 p_2^2} \sigma_{1 \rightarrow 2} = \frac{g_1 p_\omega^2}{g_2 p_e^2} \sigma_{PI}, \quad (6)$$

where  $g_j$  is the relevant statistical weight, with a ratio for the final (1) and initial (2) states of eq. (1) [65, 66] being  $g_1/g_2 = 6/12$ ; see also the appendix. Here  $p_1 = p_\omega$  is the photon momentum relative to the ion given by  $p_\omega = \hbar\omega/c = (k^2 + \gamma^2)/2c$  and  $p_2 = p_e$  is the positron momentum  $k$ . Recalling that  $c$  in atomic units is the inverse of the fine structure constant  $\alpha$  leads to

$$\sigma_{RA}(k) = \frac{6\alpha^2 (k^2 + \gamma^2)^2}{12 \cdot 2^2 k^2} \sigma_{PI} = \frac{0.429304 \times 10^{-17} \text{ cm}^2}{2^3 (137.036)^2} \frac{k}{(k^2 + \gamma^2)}, \quad (7)$$

or in terms of incoming positron energy,  $E_e = k^2/2$

$$\sigma_{RA}(E_e) = \frac{0.429304 \times 10^{-17} \text{cm}^2}{2^3 (137.036)^2} \frac{\sqrt{2E_e}}{(2E_e + \gamma^2)}. \quad (8)$$

This cross section is shown in Figure 2 as a function of  $E_e$  up to around  $E_e = 5.4$  eV.

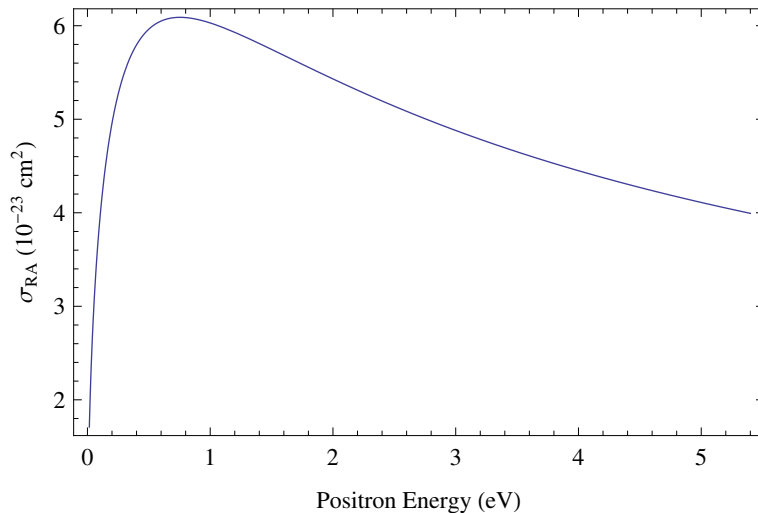


Figure 2. The cross section for radiatively attaching a second positron to  $\bar{\text{H}}(1s)$  to create the  $(1s^2\ ^1S^e)$  state of  $\bar{\text{H}}^+$  as a function of positron energy using Ohmura and Ohmura's [1] effective range theory.

Smirnov calculated the peak value (where  $k = \gamma$ ) for radiative attachment into the  $(1s^2\ ^1S^e)$   $\text{H}^-$  state, [67] and set  $g_j = 1$  but compensated by dropping a factor of two in deriving his equation (7.16) from the unnumbered equation that precedes it, giving the same value  $\sigma_{RA}(k \rightarrow \gamma) = 6 \times 10^{-23} \text{ cm}^2$  as in Figure 2. Given that the effective range theory [1] for the photoionization cross section is roughly 4% higher than the experimental results of Smith and Burch [59] at a point near the peak at 7640 Å where it matches the average of the more sophisticated theories, we expect most such theories would give a peak radiative attachment cross section to create the  $(1s^2\ ^1S^e)$  state of the positive antihydrogen ion  $\bar{\text{H}}^+$  that is about 4% lower than the  $6.07 \times 10^{-23} \text{ cm}^2$  shown in the graph above, or around  $5.8 \times 10^{-23} \text{ cm}^2$ .

To further probe the radiative attachment of a positron to antihydrogen to form  $\bar{\text{H}}^+$  as a function of temperature we can (as is common in astrophysical applications with electron attachment to H) calculate the rate coefficient  $\alpha_{RA}$  formed as the expectation value of  $v\sigma_{RA}$  with the normalized Maxwell-Boltzmann distribution  $f(v)$  as,

$$\langle v\sigma_{RA} \rangle = 4\pi \int_0^\infty v\sigma_{RA}(k(v)) \left( \frac{m}{2\pi k_B T_e} \right)^{3/2} v^2 \exp[-mv^2/(2k_B T_e)] dv, \quad (9)$$

which Bhatia [55] has calculated for temperatures in the range 1000-40,000 K using a 364-term Hylleraas wave function with both short-range and long-range correlations.

For the effective range theory result, one may analytically integrate equation (9) in terms of the incomplete gamma function to obtain

$$\begin{aligned} \langle v\sigma_{RA} \rangle \equiv \alpha_{RA} = & \sqrt{\frac{8k_B T_e}{m_e}} \frac{3}{2^4 (137.036)^2} \frac{g_1}{g_2} \left( \frac{0.429304 \times 10^{-17} \text{cm}^2}{\sqrt{2I}} \right) \\ & \times \left( \frac{I}{k_B T_e} \right)^2 \exp \left[ \frac{I}{k_B T_e} \right] \Gamma \left( -\frac{3}{2}, \frac{I}{k_B T_e} \right). \end{aligned} \quad (10)$$

Figure 3(a) shows that at high temperatures the effective range theory approaches a constant value whereas Bhatia's benchmark result peaks and experiences a slow fall-off. This high temperature behaviour of the former may be due to the significant high-energy (small-wavelength) overestimate of the effective range theory in Figure 1, a feature shared by most of the more sophisticated theories displayed in that figure. However, at low temperatures the two curves have similar behaviour down to Bhatias's lowest tabled temperature, 1000 K. Figure 3(b) shows the near linear  $T_e$ -dependence of the rate coefficient (solid curve), whilst a series expansion of equation (10) yields a term linear in  $T_e$  in lowest-order, shown as the dot-dashed line. Bhatia does not give results in this region. For  $T_e \lesssim 6K$  the rate coefficient may be fit by  $\alpha_{RA} = 0.001071 \times 10^{-15} \text{cm}^3 \text{s}^{-1} T_e \text{K}^{-1}$ .

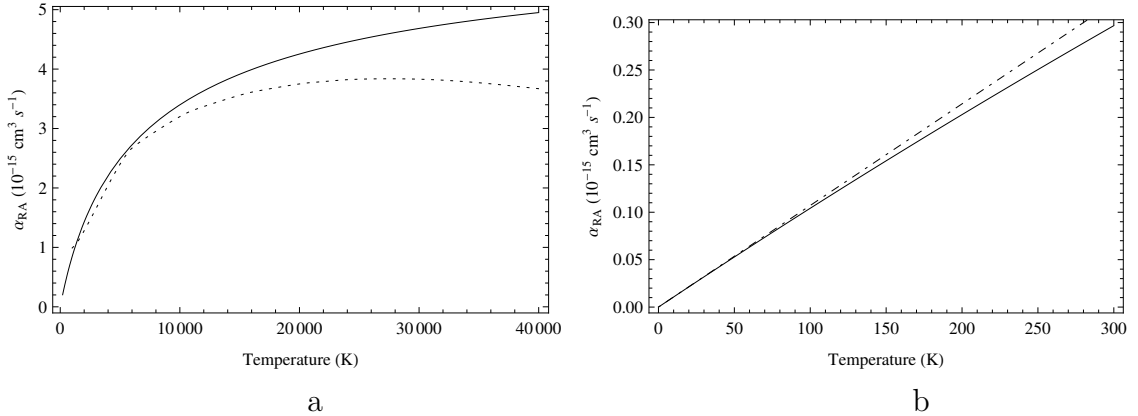


Figure 3. The rate coefficient  $\alpha_{RA}$  for attaching a positron to antihydrogen to form  $\bar{\text{H}}^+$ . (a) The solid curve gives the present effective range theory and the dotted curve is interpolated from Bhatia's table IX [55]. For low temperatures (b), the rate coefficient increases nearly linearly with  $T_e$ , with a series expansion of equation (10) yielding a term linear in  $T_e$  in lowest-order, and shown as the dot-dashed line.

We also present for information the analogous expectation value,  $\langle \sigma_{RA} \rangle$ , of equation (9) as a measure of the radiative attachment cross section for a Maxwell-Boltzmann ensemble. This is given by

$$\begin{aligned} \langle \sigma_{RA} \rangle = & \frac{3}{2^4 (137.036)^2} \frac{g_1}{g_2} \left( \frac{0.429304 \times 10^{-17} \text{cm}^2}{\sqrt{2I}} \right) \\ & \times \left( \frac{I}{k_B T_e} \right)^{3/2} \exp \left[ \frac{I}{k_B T_e} \right] \frac{2^3}{3\sqrt{\pi}} \Gamma \left( -1, \frac{I}{k_B T_e} \right), \end{aligned} \quad (11)$$



the curve shown in Figure 4. We can now proceed to use these cross sections and rates to estimate rates of production of  $\bar{\text{H}}^+$  in experiments with trapped antihydrogen.

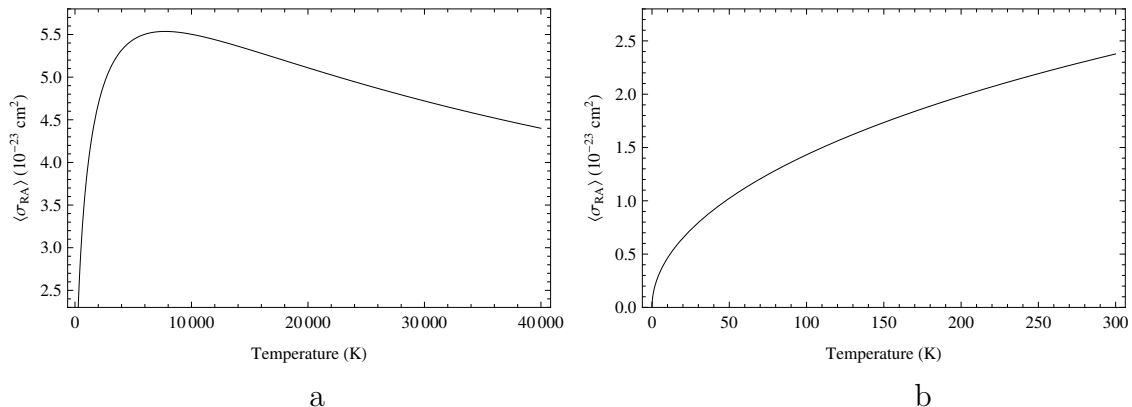


Figure 4. The Maxwell-Boltzmann-averaged cross section as  $\langle\sigma_{RA}\rangle$  for radiatively attaching a second positron to  $\bar{\text{H}}(1s)$  to create the  $(1s^2\ ^1S^e)$  state of the positive antihydrogen ion  $\bar{\text{H}}^+$  versus temperature of the positron cloud for (a)  $T_e$  up to 40000 K and (b) below 300 K.

#### 4. Discussion and concluding remarks

We have determined cross sections,  $\sigma_{RA}$ , for the radiative attachment of a positron to an antihydrogen atom to create the  $(1s^2\ ^1S^e)$  state of the positive antihydrogen ion according to reaction (1) for electric-dipole allowed transitions. Values for  $\sigma_{RA}$  have been presented in the positron kinetic energy range from zero to around 5.4 eV. The cross section peaks at just below  $6 \times 10^{-23}$  cm $^2$  for positron energies in the range 0.6-0.7 eV, or a temperature ( $T_e$ ) equivalent in the region of 6000 K.

Our results for  $\alpha_{RA}$  can be used to estimate formation rates of  $\bar{\text{H}}^+$ . We presume that the interactions between  $e^+$  and  $\bar{\text{H}}$  occur with a positron plasma held in a Penning-type trap such as those used to form antihydrogen; see section 1 and [13] for a review. We assume, as an example, a positron density of  $n_e = 10^{16}$  m $^{-3}$  can be achieved in a magnetic field of 1 T at a sub-mm plasma radius in which the positron speeds due to rotation of the plasma can be neglected. Furthermore, we assume that the interaction takes place with trapped antihydrogen, which has been held for long enough to ensure that if, as likely, it is formed in an excited state, then it will have decayed to the ground state [8, 9] to effect the reaction.

Assuming unit overlap between the positron plasma and the antihydrogen, the reaction rate is the product  $n_e\alpha_{RA}$ , which at the maximum in  $\alpha_{RA}$  yields a rate of around  $4 \times 10^{-5}$  s $^{-1}$  per antihydrogen atom. However, the value of  $T_e$  at this maximum is much above those currently used to form and trap antihydrogen, which are in the range of 10's of K or lower. Here, from Figure 3,  $\alpha_{RA}$  is below  $5 \times 10^{-17}$  cm $^3$ s $^{-1}$ , falling linearly with  $T_e$ . Thus, reaction rates will be below  $5 \times 10^{-7}$  s $^{-1}$  per antihydrogen atom.

The magnetic wells used to capture antihydrogen are around 0.5 K deep or less

[6, 7, 8] and, for instance, ALPHA is currently only able to hold around  $1 \bar{\text{H}}$  per  $10^4$  antiprotons in their mixing experiments. Increasing the trapped yield can, to the best of current trap capabilities, probably only be effected by decreasing the positron plasma temperature, such that colder  $\bar{\text{H}}$  is formed. If all ALPHA's antiprotons could be converted into trapped  $\bar{\text{H}}$ s, whilst still allowing the anti-atoms to interact with warm positron clouds, the  $\bar{\text{H}}^+$  rates would still be lower than  $5 \times 10^{-3} \text{ s}^{-1}$ , which might just be observable, given the long antihydrogen storage times achieved by ALPHA [9]. However, an important caveat to this is that the very cold antihydrogen atoms would likely heat rapidly via elastic  $e^+ - \bar{\text{H}}$  collisions, resulting in them leaving the shallow neutral trap. Thus, the trapped antihydrogen would have to be actively cooled, for instance using lasers [13, 19, 20], although it is not straightforward to judge whether this can be effective in this case. Though a great technical challenge, laser cooling of trapped antihydrogen is currently under development since colder anti-atoms will bring gains in precision to investigations of the spectroscopic and gravitational properties of antimatter.

Within the next 5 years CERN's AD facility will be enhanced by the addition of a further storage ring, ELENA (see e.g., [68]) which will allow delivery of antiprotons to experiments at an energy of around 100 keV (rather than the 5 MeV currently available). It is expected that this will enhance cold  $\bar{p}$  numbers, and hopefully by extension those of trapped  $\bar{\text{H}}$ , by around a factor of  $10^2$ . Translating these gains into observable yields of  $\bar{\text{H}}^+$  will require further technical progress trapping and manipulating the anti-atom, as alluded to above.

In conclusion, we find that rates of  $\bar{\text{H}}^+$  formation using the radiative attachment approach will be very low, even with optimistic projections on trapped antihydrogen yields and cooling technique development. Inclusion of magnetic-dipole transitions [62] below  $3 \times 10^{-4} \text{ K}$  will at most double observed rates. Thus, observation of reaction (1) will be difficult unless some way can be found of stimulating the process.

## Acknowledgements

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## Appendix A. Statistical weights used to derive $\sigma_{RA}$

Here we give details on the derivation of the statistical weights used in the application of the detailed balance argument to derive the radiative attachment cross sections from those for photoionization. We have found that such details are frequently missing, incomplete, or in error in many previous works. Following from equation (6), the final state (1) of eq. (1) has [65, 66]

$$\begin{aligned} g_1 &= g_{Photon} g_{Target_1} g_{Relative} = Polarizations \times (2S_{Target_1} + 1)(2L + 1) \\ &= 2(2 \times 0 + 1)(2 \times 1 + 1) = 6. \end{aligned} \quad (\text{A.1})$$

Likewise, the initial state (2) of eq. (1) has

$$\begin{aligned} g_2 &= g_{\text{Electron}}g_{\text{Target}_2}g_{\text{Relative}} = (2S_{\text{Electron}} + 1)(2S_{\text{Target}_2} + 1)(2L + 1) \\ &= (2 \times \frac{1}{2} + 1)(2 \times \frac{1}{2} + 1)(2 \times 1 + 1) = 12, \end{aligned} \quad (\text{A.2})$$

leading directly to equations (7) and (8) in the main text.

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