Continuous matrix product states for quantum fields

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Zaragoza - November 03, 2015

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References

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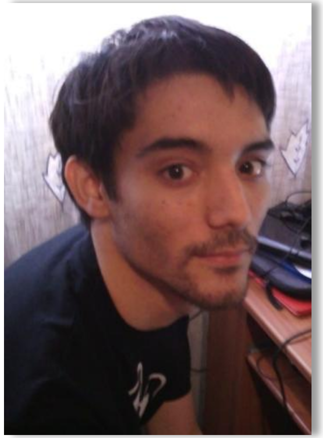
People



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Definition

$$|\chi\rangle = \langle\omega_L|\mathcal{T}\exp\left[-i\int_0^L \mathrm{d}x\,\left(K(x)\otimes I + iR(x)\otimes\hat{\psi}^\dagger(x) - iR(x)^\dagger\otimes\hat{\psi}(x)\right)\right]|\omega_R\rangle|\Omega\rangle$$

$$[\hat{\psi}(x),\hat{\psi}^\dagger(x')] = \delta(x-x')$$

$$K = K^\dagger,\ R\in\mathcal{C}^{D\times D}$$

$$\hat{\psi}(x)|\Omega\rangle = 0$$

Definition

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- Any 1D quantum field admits a cMPS representation

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- Any 1D quantum field admits a cMPS representation
  - **Coherent states**: $D = 1,\ K(x) = 0,\ |\omega_L\rangle = |\omega_R\rangle = 1$

$$|\chi\rangle = \exp\left[\int_0^L \mathrm{d}x\,\left(R(x)\hat{\psi}^\dagger(x) - R(x)^*\hat{\psi}(x)\right)\right]|\Omega\rangle$$

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  - **cMPS + cMPS = cMPS**

$$|\chi\rangle = c_1|\chi_1\rangle + c_2|\chi_2\rangle$$

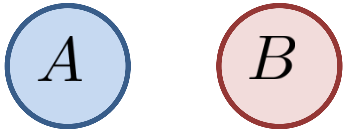
$$K = K_1\oplus K_2 = \begin{pmatrix} K_1 & 0 \\ 0 & K_2 \end{pmatrix} \qquad R = R_1\oplus R_2 = \begin{pmatrix} R_1 & 0 \\ 0 & R_2 \end{pmatrix}$$

U. Schollwoeck, Annals of Physics **326**, 96 (2011)

Continuous limit of a MPS

- What is a MPS?
  - Bipartite system

$$|\Psi\rangle = \sum_{i,j} A_{ij} |i\rangle_A \otimes |j\rangle_B$$



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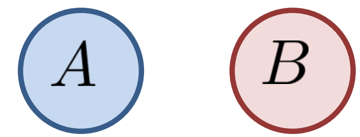

  - Schmidt decomposition

$$|\Psi\rangle = \sum_{k=1}^{r} \sqrt{\lambda_k} |k\rangle_A \otimes |k\rangle_B \qquad \sum_k \lambda_k = 1$$

Schmidt coefficients: λ_k

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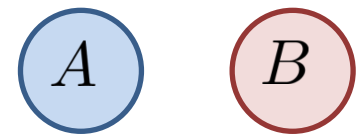

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Schmidt coefficients: λ_k

  - Entanglement: r > 1

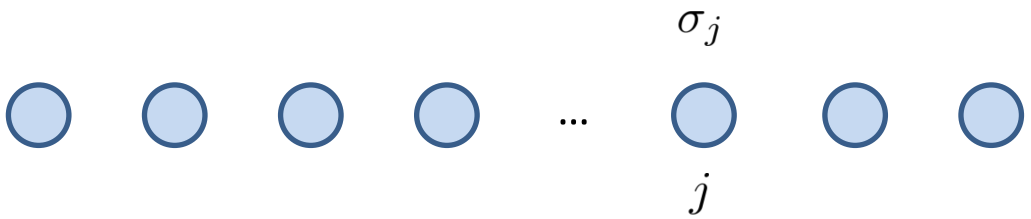
$$\rho_A = \mathrm{Tr}_B |\Psi\rangle\langle\Psi|$$

$$S(\rho_A) = -\sum_{k=1}^{r} \lambda_k \log \lambda_k \leq \log r$$

Continuous limit of a MPS

- What is a MPS?

$$|\Psi\rangle = \sum_{\sigma_1,\ldots,\sigma_L=0}^{p-1} C_{\sigma_1,\ldots,\sigma_L} |\sigma_1 \ldots \sigma_L\rangle$$



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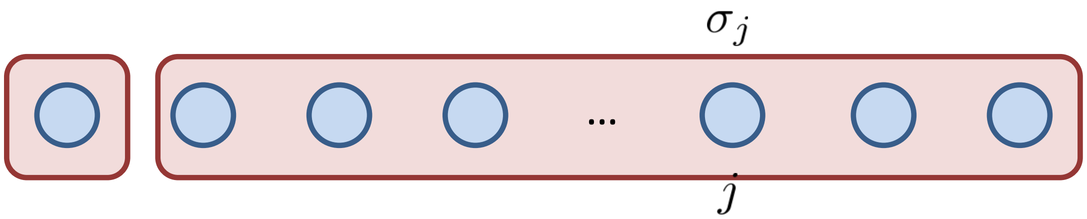

$$C_{\sigma_1,\ldots,\sigma_L} = \sum_{k_1} A^{\sigma_1}_{k_1} C_{k_1,\sigma_2,\ldots\sigma_L}$$

Continuous limit of a MPS

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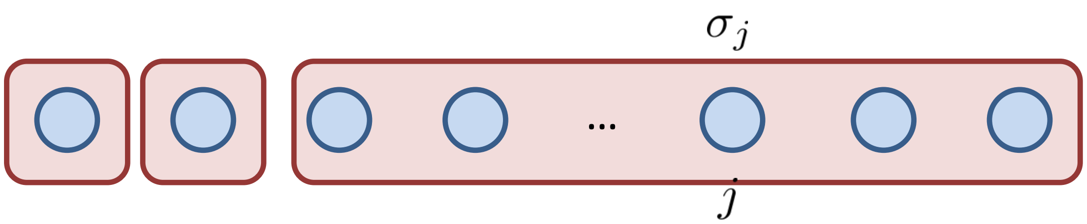

$$C_{\sigma_1,\ldots,\sigma_L} = \sum_{k_1,k_2} A^{\sigma_1}_{k_1} A^{\sigma_2}_{k_2,k_3} C_{k_3,\sigma_2,\ldots\sigma_L}$$

Continuous limit of a MPS

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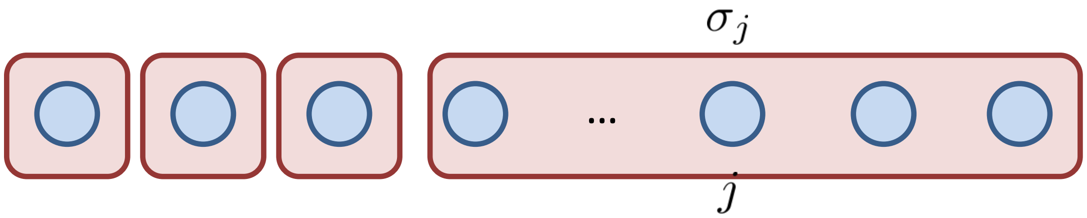
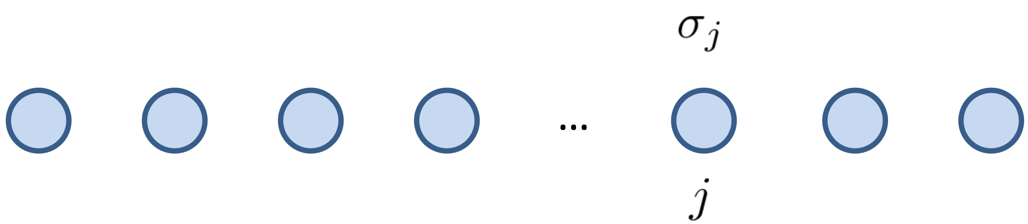


Continuous limit of a MPS

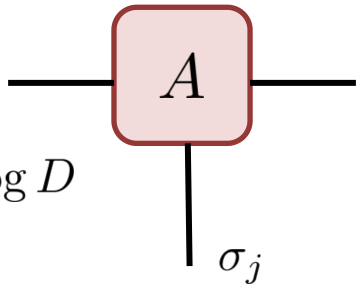
- Matrix product state



$$|\Psi\rangle = \sum_{\sigma_1,\ldots,\sigma_L=0}^{p-1} \langle \omega_L | A^{\sigma_1} A^{\sigma_2} \ldots A^{\sigma_L} | \omega_R \rangle | \sigma_1 \sigma_2 \ldots \sigma_L \rangle$$

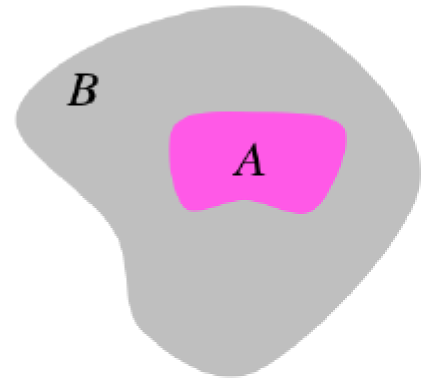
  - Tensor product
  - Bond dimension: D (Schmidt rank)

$S \leq \log D$



Continuous limit of a MPS

- Low-energy + locality = **area law**



$S \sim \partial A$

Continuous limit of a MPS

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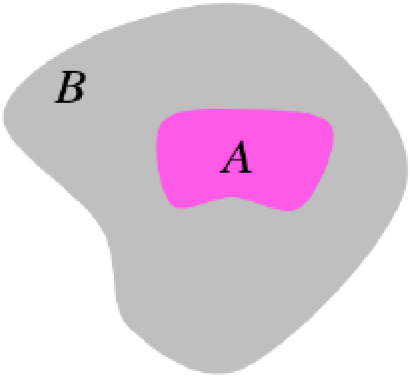


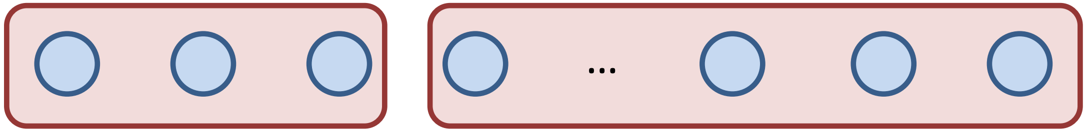
$S \sim \partial A$

- MPS: **efficient** representation of low-energy states

$$|\Psi\rangle = \sum_{\sigma_1,\ldots,\sigma_L=0}^{p-1} \langle \omega_L | A^{\sigma_1} A^{\sigma_2} \ldots A^{\sigma_L} | \omega_R \rangle | \sigma_1 \sigma_2 \ldots \sigma_L \rangle$$

$$S \leq \log D$$

$$L \times p \times D^2$$



Continuous limit of a MPS

- Quantum field

Continuous limit of a MPS

- Quantum field

$$N = L/\epsilon$$

$$|\Psi\rangle = \sum_{n_1,\ldots,n_N=0}^{p-1} \langle \omega_L | A^{n_1} A^{n_2} \ldots A^{n_N} | \omega_R \rangle (\hat{a}_1^\dagger)^{n_1} (\hat{a}_2^\dagger)^{n_2} \ldots (\hat{a}_N^\dagger)^{n_N} |\Omega\rangle$$

$$[\hat{a}_i, \hat{a}_j^\dagger] = \delta_{ij}$$

Continuous limit of a MPS

- Quantum field

$$N = L/\epsilon$$

$$|\Psi\rangle = \sum_{n_1,\dots,n_N=0}^{p-1} \langle\omega_L|A^{n_1}A^{n_2}\dots A^{n_N}|\omega_R\rangle(\hat{a}_1^\dagger)^{n_1}(\hat{a}_2^\dagger)^{n_2}\dots(\hat{a}_N^\dagger)^{n_N}|\Omega\rangle$$

$$[\hat{a}_i, \hat{a}_j^\dagger] = \delta_{ij}$$

- As $\epsilon \to 0$

$$\langle\hat{N}\rangle = \sum_n \langle\hat{a}_n^\dagger\hat{a}_n\rangle = \text{const.}$$

$$A^0 > A^1 > \dots$$

Continuous limit of a MPS

- Quantum field

$$N = L/\epsilon$$

$$|\Psi\rangle = \sum_{n_1,\ldots,n_N=0}^{p-1} \langle\omega_L|A^{n_1}A^{n_2}\ldots A^{n_N}|\omega_R\rangle(\hat{a}_1^\dagger)^{n_1}(\hat{a}_2^\dagger)^{n_2}\ldots(\hat{a}_N^\dagger)^{n_N}|\Omega\rangle$$

$$A^0 = I + \epsilon Q$$

$$A^n = \frac{(\epsilon R)^n}{\sqrt{n!}}$$

Continuous limit of a MPS

- Quantum field

$$N = L/\epsilon$$

$$|\Psi\rangle = \sum_{n_1,\ldots,n_N=0}^{p-1} \langle\omega_L|A^{n_1}A^{n_2}\ldots A^{n_N}|\omega_R\rangle(\hat{a}_1^\dagger)^{n_1}(\hat{a}_2^\dagger)^{n_2}\ldots(\hat{a}_N^\dagger)^{n_N}|\Omega\rangle$$

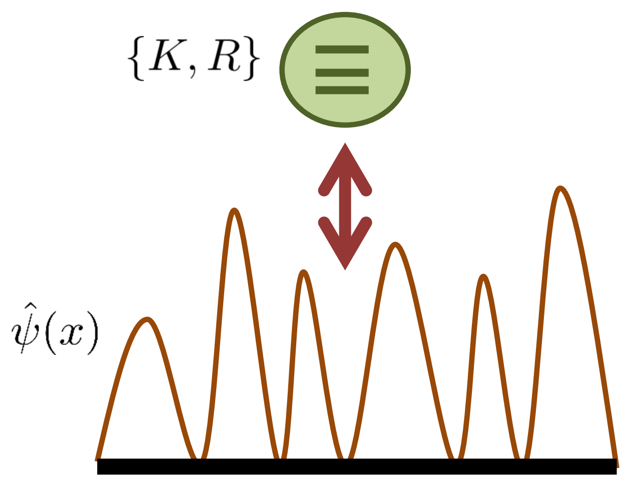
$$|\chi\rangle = \langle\omega_L|\mathcal{T}\exp\left[-i\int_0^L \mathrm{d}x \left(K(x)\otimes I + iR(x)\otimes\hat{\psi}^\dagger(x) - iR(x)^\dagger\otimes\hat{\psi}(x)\right)\right]|\omega_R\rangle|\Omega\rangle$$

Physical interpretation

$$|\chi\rangle = \langle\omega_L|\mathcal{T}\exp\left[-i\int_0^L \mathrm{d}x\,\left(K(x)\otimes I + iR(x)\otimes\hat{\psi}^\dagger(x) - iR(x)^\dagger\otimes\hat{\psi}(x)\right)\right]|\omega_R\rangle|\Omega\rangle$$

$$|\chi\rangle = \langle\omega_L|\hat{U}(0,L)|\omega_R\rangle|\Omega\rangle$$

$$\hat{H}_{\mathrm{cMPS}} = K(x)\otimes I + iR(x)\otimes\hat{\psi}^\dagger(x) - iR(x)^\dagger\otimes\hat{\psi}(x)$$



Calculus with cMPS

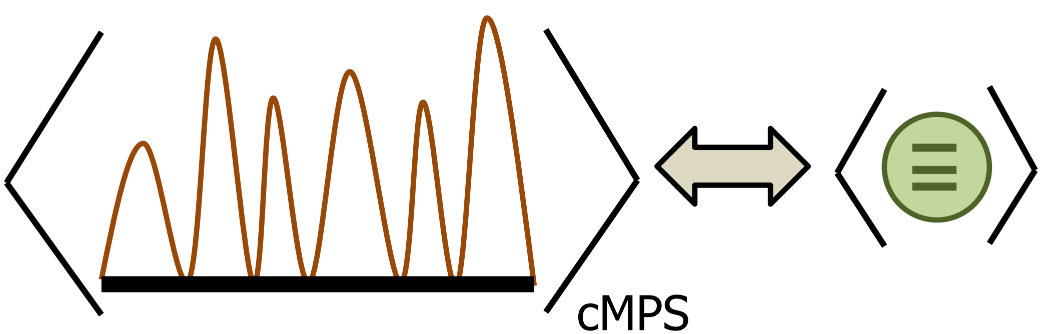
- Variational method – **ground states**
- Second quantization

$$\langle\chi|\chi\rangle = \mathrm{Tr}\left[\mathrm{e}^{TL}\right]$$

$$\langle\hat{\psi}^\dagger(x)\hat{\psi}(x)\rangle = \mathrm{Tr}\left[\mathrm{e}^{TL}(R\otimes R^*)\right]$$

$$\langle\hat{\psi}^\dagger(x)\hat{\psi}^\dagger(y)\hat{\psi}(y)\hat{\psi}(x)\rangle = \mathrm{Tr}\left[\mathrm{e}^{Ty}(R\otimes R^*)\mathrm{e}^{T(x-y)}(R\otimes R^*)\mathrm{e}^{T(L-x)}\right]$$

$$T = Q\otimes I + I\otimes Q^* + R\otimes R^*$$



Lindblad equation

- Kossakowski-Lindblad-Gorini-Sudarshan equation

$$T \to \mathcal{T}$$

$$\mathrm{d}_t \varrho = \mathcal{T}[\varrho]$$

$$\mathrm{d}_t \varrho = -i[K, \varrho] + \frac{1}{2}\left(2R\varrho R^\dagger - \{R^\dagger R, \varrho\}\right)$$

- Thermodynamic limit $L \to \infty$

$$\mathrm{d}_t \varrho_{\mathrm{eq}} = 0$$

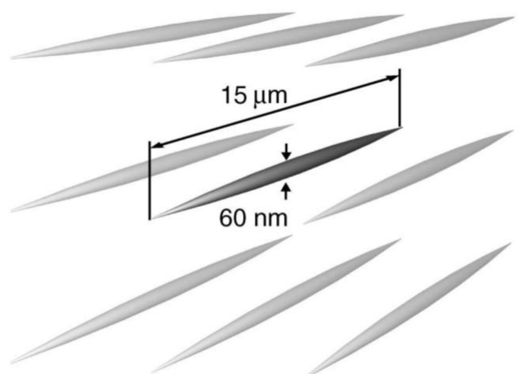
Lieb-Liniger model

$$\hat{H} = \int \mathrm{d}x \, \partial_x \hat{\psi}^\dagger(x) \partial_x \hat{\psi}(x) + c \, \hat{\psi}^\dagger(x) \hat{\psi}^\dagger(x) \hat{\psi}(x) \hat{\psi}(x)$$

$$[\hat{\psi}(x), \hat{\psi}^\dagger(x')] = \delta(x - x')$$

$$\left[\frac{\hbar}{2m}\right] = 1$$

- Interacting (repulsive) bosons in 1D (**two-body delta interaction**)
- Exactly solvable
- Experimentally accessible



Variational ground state

- **Energy density** with cMPS

$$\begin{aligned}\langle\chi|\hat{\mathcal{H}}|\chi\rangle &= \langle\partial_x\hat{\psi}^\dagger(x)\partial_x\hat{\psi}(x)\rangle + c\,\langle\hat{\psi}^\dagger(x)\hat{\psi}^\dagger(x)\hat{\psi}(x)\hat{\psi}(x)\rangle \\ &= \mathrm{Tr}\left[\varrho_{\mathrm{eq}}[Q,R]^\dagger[Q,R]\right] + c\mathrm{Tr}\left[\varrho_{\mathrm{eq}}R^\dagger R^\dagger RR\right]\end{aligned}$$

  Variational parameters:

$$\{K,R\} \to 2D^2$$

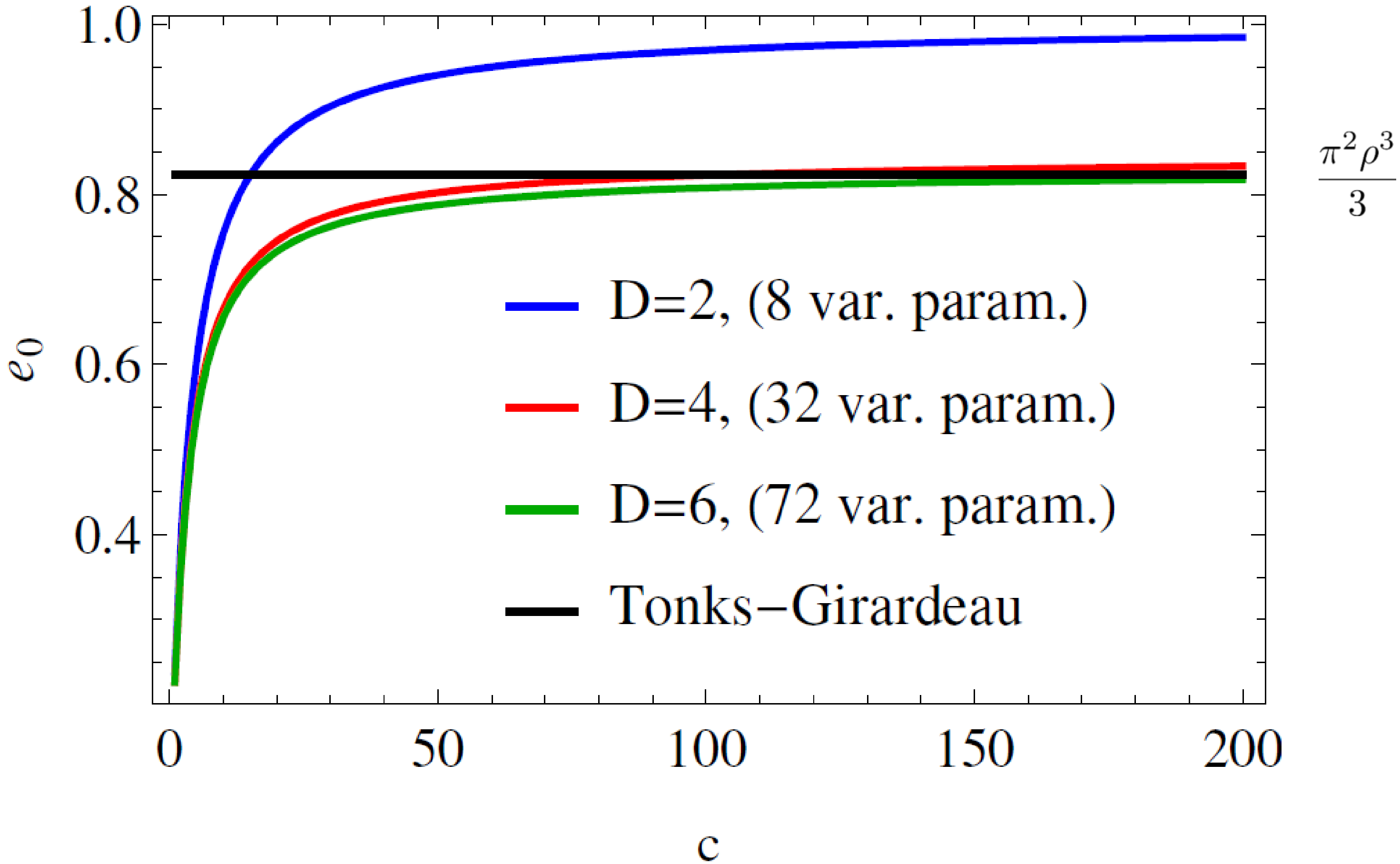
- **Variational principle**

$$e(\{K,R\}) = \langle\chi(\{K,R\})|\hat{\mathcal{H}}|\chi(\{K,R\})\rangle \geq e_0 = \langle\psi_0|\hat{\mathcal{H}}|\psi_0\rangle$$

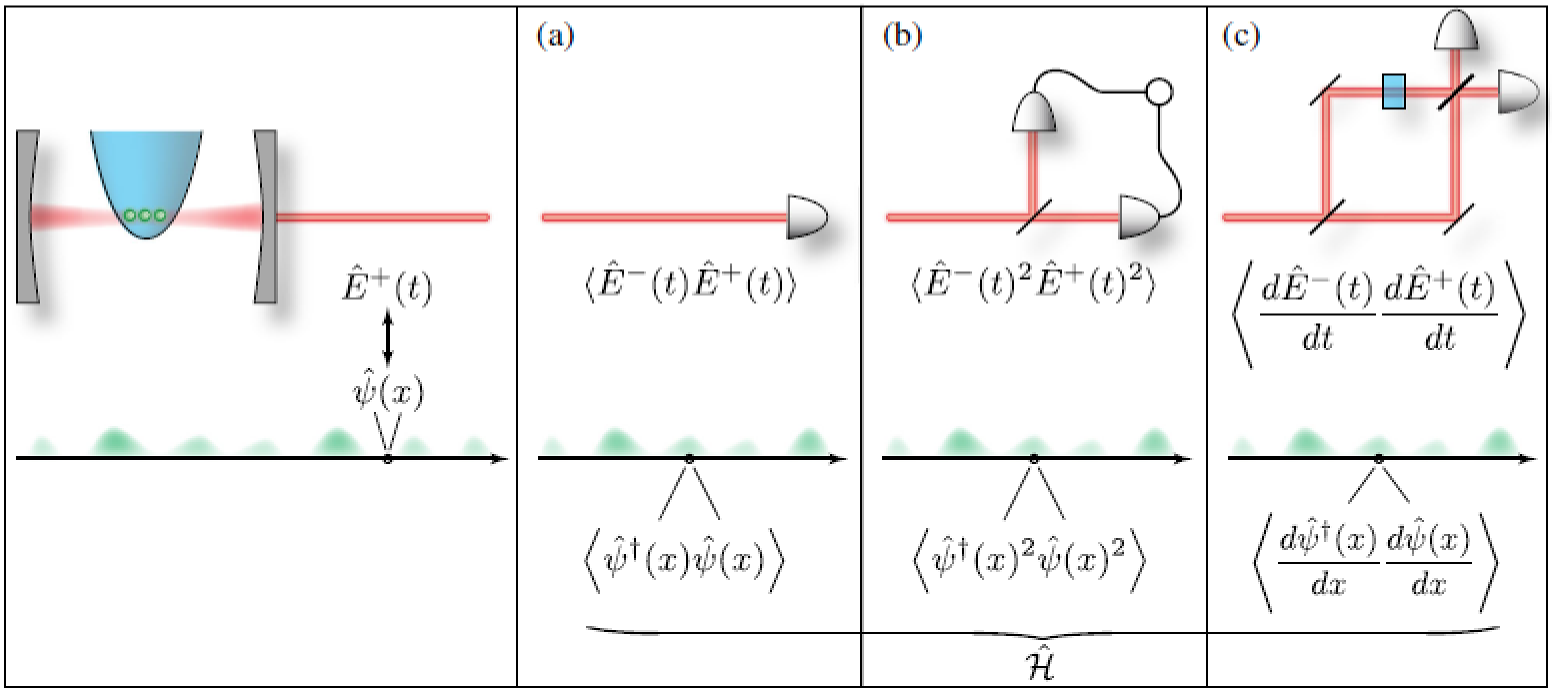
- Find the optimum set $\{K^*,R^*\}$ which minimizes the energy density → **ground state** approximation !!!

Variational ground state

$$\hat{\mathcal{H}} = \hat{\psi}^\dagger(x)\partial_x\hat{\psi}(x) + c\,\hat{\psi}^\dagger(x)\hat{\psi}^\dagger(x)\hat{\psi}(x)\hat{\psi}(x)$$

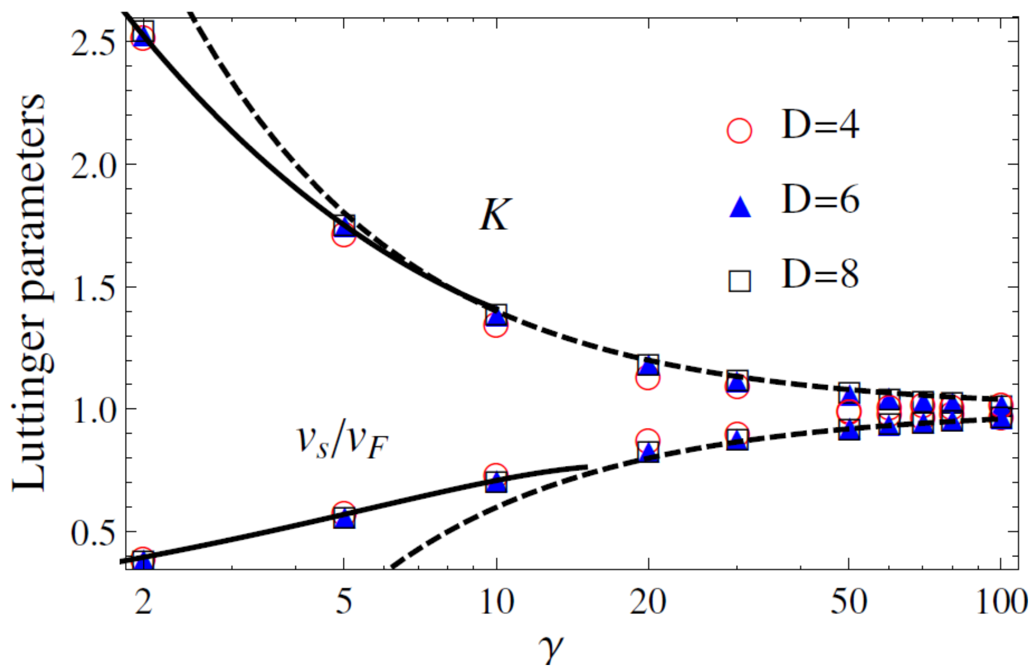


Simulation of cMPS



Low-E characterization

- Bosonization

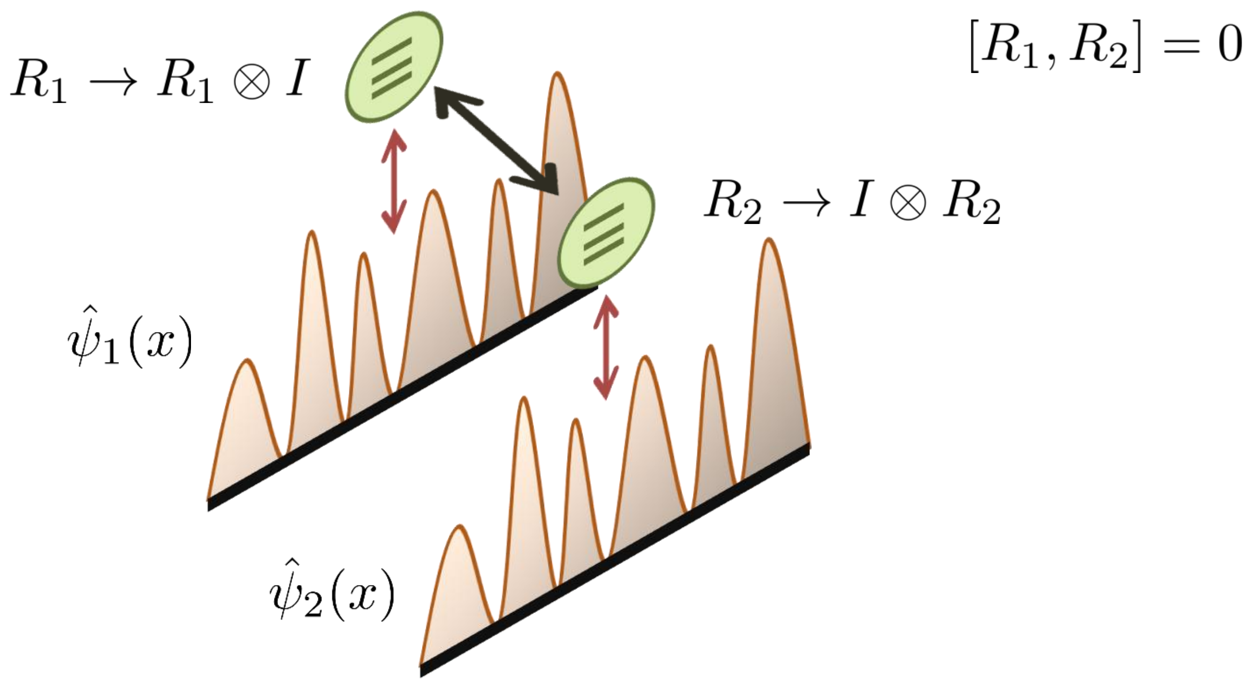


$$\hat{H} = \frac{v_s}{2}\int \mathrm{d}x \left[\frac{K}{\pi}(\partial_x\hat{\phi})^2 + \frac{\pi}{K}(\delta\hat{\rho})^2\right]$$

Quijandría, García-Ripoll and Zueco. PRB **90**, 235142 (2014)

Continuous Matrix Product States (cMPS)

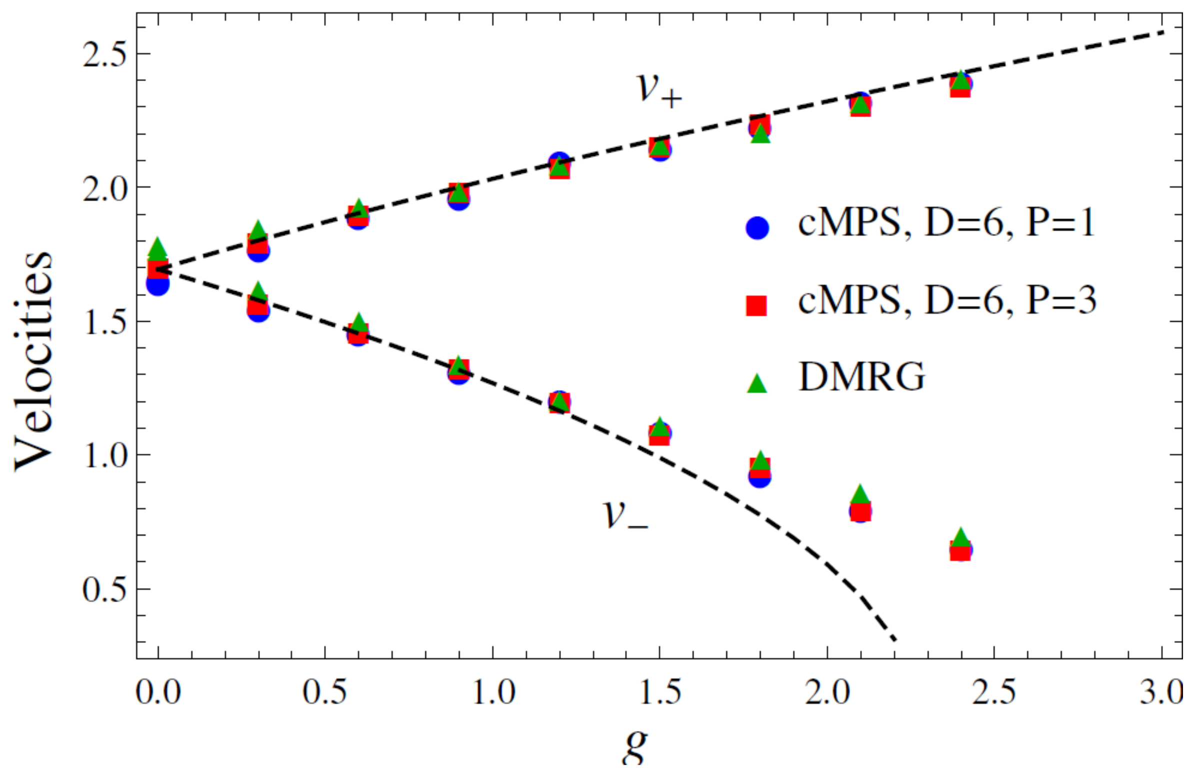
- Our work:
  - Generalization to study coupled bosonic fields



Quijandría, García-Ripoll and Zueco. PRB **90**, 235142 (2014)

Continuous Matrix Product States (cMPS)

- Our work:
  - Check: low-E (again!)

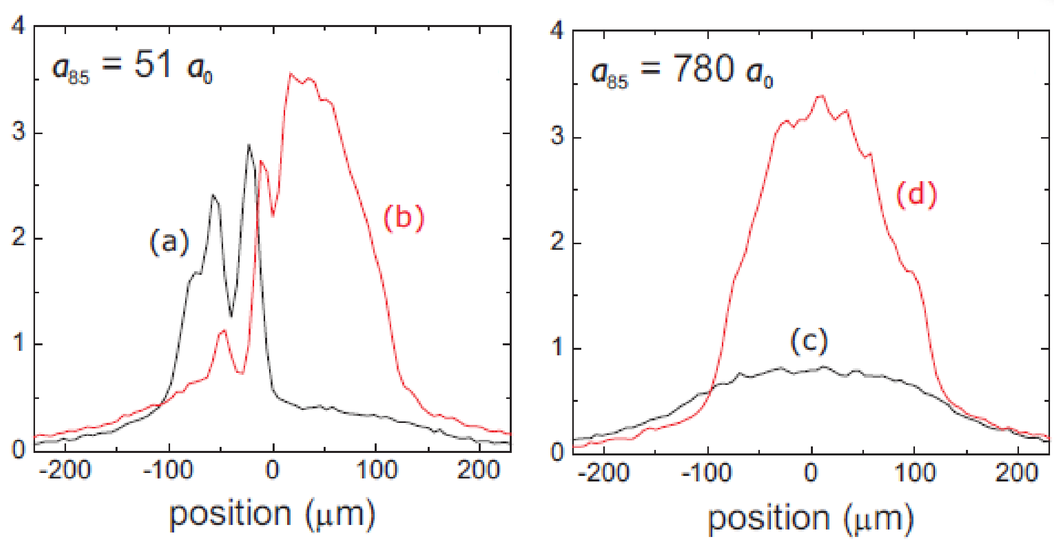


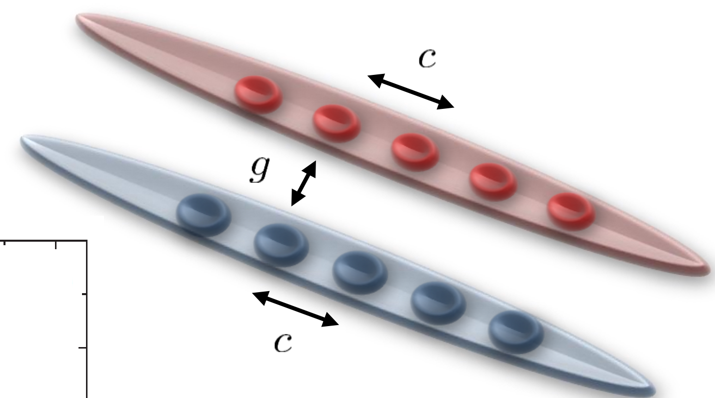
Quijandría, García-Ripoll and Zueco. PRB **90**, 235142 (2014)

DMRG: Kleine et al. New J. Phys. **10** 045025 (2008)

Continuous Matrix Product States (cMPS)

- Our work:
  - Characterization of a phase transition

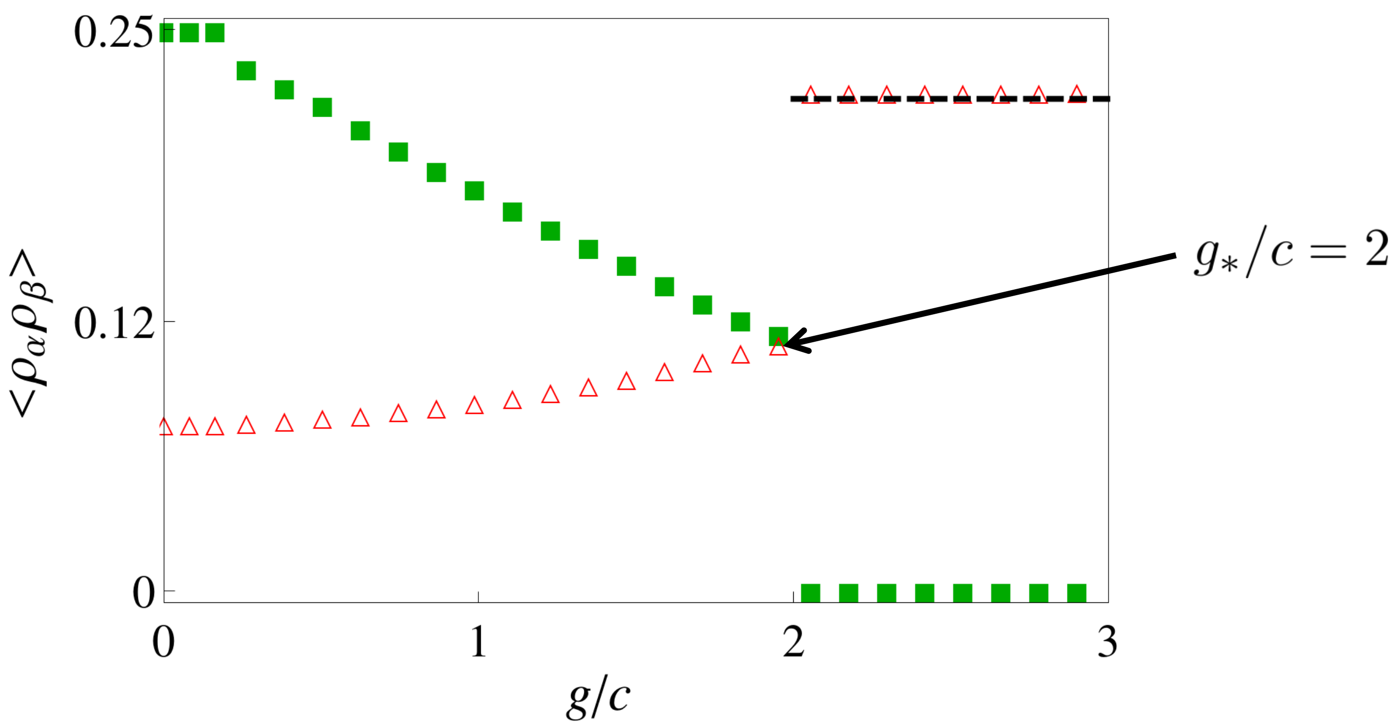




Continuous Matrix Product States (cMPS)

- Our work:
  - Full characterization of a phase transition: fluctuations

$$\langle \rho_\alpha \rho_\beta \rangle = \langle \psi_\alpha^\dagger \psi_\alpha \psi_\beta^\dagger \psi_\beta \rangle$$



Quijandría and Zueco (in press)

Conclusions

- Complete class for one-dimensional quantum field states.
- Continuous limit of matrix product states
- Open system interpretation
- Variational search of ground states
- Low-E: cMPS + bosonization
- Afford for an experimental simulation
- Extension to study interacting fields – phase transitions


That's all Folks!