Embrapa

Brazilian Corporation for Agricultural Research

Technical Efficiency of Production in Agricultiural Research: A Case Study

Geraldo da Silva e Souza
Eliseu Alves
Antonio Flavio Dias Avila

Brasília, May 1998

Technical Efficiency of Production in Agricultural Research: A Case Study

Geraldo da Silva e Souza[1]

Eliseu Alves

Antônio Flávio Dias Ávila

Embrapa - Empresa Brasileira de Pesquisa Agropecuária

Brasília, DF, Brazil, 70770-901

[1]Research with support of Conselho Nacional de Desenvolvimento Cient´ıfico e Tecnológico - CnPq, Brazil

SUMMARY

We define and model research production at Embrapa, the major Brazilian institution responsible for applied agricultural research in the country. The main theoretical framework we use is data envelopment analysis. We explore the economic interpretation of these models to assess cost and technical efficiencies for the production of agricultural research in Brazil. Efficiency results are then compared with alternative measures defined via a stochastic frontier.

1. INTRODUCTION

It is of importance to the administrators of research institutions to have at their disposal measures and procedures that make feasible an evaluation of the quantum of productivity as well as the technical efficiency of the production process of their institutions. In times of competition and budget constraints a research institution needs to know how much it may increase its production with quality without absorbing additional resources. The quantitative monitoring of the production process allows for an effective administration of the resources available and the observation of predefined research patterns and goals. In this context we developed for Embrapa a production model based on the input-output data of its research units. The model serves the purpose of quantitative productivity evaluations at relative and absolute levels. The theoretical framework of this model is the analysis of production frontiers. We make intensive use of the DEA (Data Envelopment Analysis) models described in Seiford and Thrall (1990), Färe, Grosskpof and Lovell (1994), Charnes, Lewin and Seiford (1995), Sengupta (1995), and Färe and Grosskopf (1996). The DEA models are linear programming models that essentially generalize the notion of productivity. The dual problems of these models provide a rich economic framework relative to which it is possible to assess scale of production and input congestion. Our discussion of the subject is as follows. In Section 2 we detail the data envelopment models exploring the approach of Färe, Grosskpof and Lovell (1994). We use the notion of radial measure of technical efficiency to define production frontier and the concept of dominance to define efficient production frontier for a set of decision making units. The complementary slackness theorem has a crucial role in the discussion of these two concepts. In Section 3 we introduce the input and output measures of Embrapa's production process . In Section 4 we present our empirical findings.

The period covered in the analysis is 1996. The analysis is carried out for cost and quantity data. In Section 5 we compare our results with the econometric fit of stochastic frontiers. Finally in Section 6 we conclude our discussion and indicate directions for further studies.

2. DATA ENVELOPMENT PRODUCTION MODELS

Consider a production process composed of n decision making units (DMUs). Each DMU uses varying quantities of m different inputs to produce varying quantities of s different outputs. Denote by

$$Y = (y_1, y_2, \ldots, y_n)$$

the sxn production matrix of the n DMUs. The rth column of Y is the output vector of DMU r. Denote by

$$X = (x_1, x_2, \ldots, x_n)$$

the mxn input matrix. The rth column of X is the input vector of DMU r. The matrices $Y=(y_{ij})$ and $X=(x_{ij})$ must satisfy: $p_{ij} \geq 0$, $\sum_i p_{ij} > 0$ and $\sum_j p_{ij} > 0$ where p is x or y.

Definition 2.1 *The measure of technical efficiency of production (under constant returns to scale) for DMU* $o \in \{1, 2, \ldots, n\}$*, denoted* $\mathrm{E}^{\mathrm{CR}}(o)$*, is the solution of the linear programming problem*

$$\mathrm{E}^{\mathrm{CR}}(o) = \max_{u,v} \frac{y_o' u}{x_o' v}$$

subject to i) $x_o' v = 1$*, ii)* $y_j' u - x_j' v \leq 0,\ j = 1, 2, \ldots, n$ *e iii)* $u \geq 0,\ v \geq 0$.

If we look at the coefficients u and v as input and output prices, we see that the measure of technical efficiency of production is very close to the notion of productivity (output income /input expenditure). Technical efficiency, in this context, basically, is looking for the price system (u,v) for which DMU o achieves the best relative productivity ratio.

An interesting motivation for the concept of technical efficiency obtains from the case s=m=1. In this instance condition (ii) implies that

$$v = \frac{1}{x_o}$$

Let

$$R = \max_{j=1,\dots,n} \frac{y_j}{x_j}$$

be the largest output to input ratio (largest productivity) in the set of the n DMUs. Constraints (ii) e (iii) imply that

$$0 \leq u \leq \frac{1}{x_o R}$$

Hence,

$$E^{CR}(o) = \frac{y_o}{x_o R}$$

and the maximum is achieved when

$$u = \frac{1}{x_o R}$$

Thus we see that in the simple case of one input and one output the measure of technical efficiency is simply a normalization procedure. In other words, the DMU with best productivity ratio has unit technical efficiency. Any other DMU has its efficiency evaluated dividing its productivity ratio by the best productivity ratio. It is interesting to observe that the quantity $E^{CR}(o)$, in this simple context, represents the proportional reduction one should apply to input quantity x_o in order to induce o to achieve the best productivity ratio R. Equivalently the reciprocal of technical efficiency define the proportional increase in output production necessary to obtain R. This is the essence of DEA models.

The dual problem of the linear programming problem of Definition 2.1 has an important economic interpretation which we will explore. The features of the case s=m=1 will be more evident in the context of the dual problem. Before introducing this interpretation we find convenient to present some theoretical aspects of linear programming problems.

Table 1 shows the non symmetric formulations of the primal and dual problems which will be of concern in our subsequent discussions. The following theorem establishes the relationship existing between the solutions of the two problems. See Mas-Collel, Whinston and Green (1995) and Gass (1969) for more details.

Theorem 2.1 *(Dual Theorem) There is an optimum solution for the primal if and only if there is an optimum solution for the dual problem. The optimum values of both problems when they exist coincide.*

An equivalent formulation of the dual problem of importance for DEA models is Theorem 2.2.

Theorem 2.2 *(Complementary Slackness Theorem) In regard to the optimum solutions of the pair primal-dual we may say the following. If strict inequality occurs in the jth constraint of one of the dual problems the value of the jth variable in the optimum solution of the corresponding primal problem will be zero. If the value of the jth variable in the optimum solution of one of the primal problems is positive then the jth restriction of the corresponding dual problem will be an equality.*

Proof Consider the first pair of problems in Table 1. The result is analogous for the second pair. Let $A=(a_{ij})$ be mxn, c nx1, x nx1, b mx1 and w is mx1. Denote by f(x) and g(w) the objective functions of the primal and dual respectively. Let w_{m+j} be nonnegative slack variables such that

$$a_{1j}w_1 + a_{2j}w_2 + \ldots + a_{mj}w_m - w_{m+j} = c_j \;\; j = 1, \ldots, n.$$

Multiply this equation by x_j, sum in j, and subtract g(w) from the result to obtain

$$\begin{aligned} f(x) - g(w) &= (b_1 - \sum_{j=1}^{n} a_{1j}x_j)w_1 + \ldots + (b_m - \sum_{j=1}^{n} a_{1j}x_j)w_m + \sum_{j=1}^{n} x_j w_{m+j} \\ &= \sum_{j=1}^{n} x_j w_{m+j} \end{aligned}$$

Then if $\hat{x}$ and $\hat{w}$ are the optimal solutions of the primal and dual, respectively, we have $\sum_{j=1}^{n} \hat{x}_j \hat{w}_{m+j} = 0$. Since variables x_j e w_{m+j} are restricted to be nonnegative, $\hat{x}_j \hat{w}_{m+j} = 0$ for every j. Result then follows.

□

In matrix terms we may write the linear programming problem of Definition 2.1 as

$$\max_{u,v,\delta}(y_o', 0, 0) \begin{pmatrix} u \\ v \\ \delta \end{pmatrix}$$

subject to the constraints

$$\begin{pmatrix} 0 & x_o' & 0 \\ Y' & -X' & I \end{pmatrix} \begin{pmatrix} u \\ v \\ \delta \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

where δ is a vector of slack variables and I is the identity of order n.

The corresponding dual problem is $\min_{\theta,\lambda} \theta$ subject to

$$\begin{pmatrix} 0 & Y \\ x_o & -X \\ 0 & I \end{pmatrix} \begin{pmatrix} \theta \\ \lambda \end{pmatrix} \geq \begin{pmatrix} y_o \\ 0 \\ 0 \end{pmatrix}$$

or, equivalently, $\min_{\theta,\lambda} \theta$ subject to i) $Y\lambda \geq y_o$, ii) $X\lambda \leq \theta x_o$ and iii)$\lambda \geq 0$; θ free.

The matrix products Yλ and Xλ with $\lambda \geq 0$ represent linear combinations of the columns of Y and X respectively. A sort of weighted averages of output and input vectors. In this way, for each λ, we can generate a new production relation (a new pseudo producer). Trivially the set of DMUs $1, 2, \ldots, n$ are included among those new producers. Making allowance for these newly defined production relationships the question that the dual intends to answer is: What proportional reduction of inputs θx_o it is possible to achieve for DMU o and still produce at least output vector y_o? The solution $\theta^*(x_o, y_o)$ is the smallest θ with this property. In this context the quantity $\theta^*(x_o, y_o)$ is known as a radial measure of technical efficiency. It is radial in the sense that the proportional reduction is applied uniformly to the entire input vector. The analogy with the case s=m=1 is perfect.

The two relevant notions in the study of the nonparametric measure of technical efficiency are the concepts of envelope and dominance within the envelope. The idea of envelope is inherited from the constraints of the dual problem. Formally the envelope is the set

$$\mathrm{E} = \{(x, y); \exists\lambda \geq 0, X\lambda \leq x, Y\lambda \geq y\}$$

It is clear that the envelope defines the kind of producers we allow to participate in the optimization process. We notice that the component x of a point (x,y) of E represents an input vector and the component y represents an output vector.

If (z,w) e (x,y) are distinct points of E we say that (z,w) dominates (x,y) when and only when z≤x and w≥y. In other words, when the producer (z,w) is able to produce more than (x,y) spending less.

The frontier (isoquant) for the input (reduction) oriented linear programming problem of Definition 2.1 is defined by the set

$$F = \{(x_o, y_o); \theta^*(x_o, y_o) = 1\}$$

The efficient frontier is [1]

$$EF = \{(x_o, y_o); (x_o, y_o) \text{ can not be dominated in E}\}$$

Proposition 2.1 *The efficient frontier EF is a subset of F.*

Proof Suppose EF not empty and let (x_o, y_o) be a point in EF. Consider the dual problem of Definition 2.1. The optimum $\theta^* = \theta^*(x_o, y_o)$ occurs when $\lambda = \lambda^*$. Suppose $0 < \theta^* < 1$ and let z=Xλ^* and w=Yλ^*. Clearly (z,w) ∈ E and (z,w) is distinct from (x_o, y_o). Thus (z,w) dominates (x_o, y_o). Hence (x_o, y_o) cannot be a point in EF, a contradiction.

□

Proposition 2.2 *Let the DMU o be such that $E^{CR}(o) = 1$. The necessary and sufficient condition for o to be a point in EF is that the optimum multipliers (shadow prices) u^* and v^* are strictly positive.*

Proof The condition is sufficient. Indeed, suppose the condition satisfied and that (x_o, y_o) does not belong to EF. There exists (z,w) in E dominating (x_o, y_o). Thus there exists $\bar{\lambda} \geq 0$ such that $X\bar{\lambda} \leq x_o$ and $Y\bar{\lambda} \geq y_o$. Thus $(1, \bar{\lambda})$ is feasible and therefore optimal for the dual problem. Since $X\bar{\lambda} \neq x_o$ or $Y\bar{\lambda} \neq y_o$ we have a contradiction by the complementary slackness theorem. Thus $(x_o, y_o) \in$ EF. The condition is also necessary. Indeed, suppose that (x_o, y_o) is a point in EF and that some component of the optimum price system (u^* , v^* is zero. Then there exists a pair $(\bar{x}, \bar{y})$ distinct of (x_o, y_o) such that $\bar{x} \leq x_o$, $\bar{x}'v^* = 1$, $\bar{y} \geq y_o$ and

[1] Notice that (0,0) is a point in E that cannot be dominated. Our definition of EF however does not include the zero vector. The definitions of F and EF in the present context are restricted to the DMUs being evaluated.

$\bar{y}'u^* = 1$. Consider the linear programming problem $\max_{u,v} \bar{y}'u$ subject to the constraints i) $\bar{x}'v = 1$ e ii) $y_j'u - x_j'v \leq 0$ $j = 1, \ldots, n$. This problem reaches its optimum solution in $u = u^*$ and $v = v^*$. By Theorem 2.1 its dual problem has an optimum solution. Thus we may find $\lambda^* \geq 0$ such that $X\lambda^* \leq \bar{x} \leq x_o$ and $Y\lambda^* \geq \bar{y} \geq y_o$. It follows that $(\mathrm{x}_o, \mathrm{y}_o)$ is dominated in E, a contradiction.

The dual version of Proposition 2.2 requires $\mathrm{Y}\lambda^* = y_o$ e $\mathrm{X}\lambda^* = x_o$ for the optimum solution $(1, \lambda^*)$ of the dual problem.

An inefficient DMU can be made more efficient by projection onto the isoquant. This projection is defined by the mapping $(x_o, y_o) \longrightarrow (\theta^* x_o, y_o)$.The projection will be a point in EF when $X\lambda^* = \theta^* x_o$ and $\mathrm{Y}\lambda^* = y_o$.

We can define the concept of technical efficiency of production in a context of fixed inputs instead of fixed outputs, i.e., in a program of output augmentation. In this environment the measure of technical efficiency of production of DMU o, under constant returns to scale, is defined by $\phi^*(x_o, y_o)) = \max_{\phi,\lambda} \phi$ subject to i) $\mathrm{Y}\lambda \geq \phi y_o$, ii) $\mathrm{X}\lambda \leq x_o$ e iii) $\lambda \geq 0$, ϕ free.

In the output augmentation program the question we ask is what proportional rate ϕ can be uniformly applied to augment the output vector y_o without increasing the input vector x_o. The solution ϕ^* is the largest ϕ with this property. Projection onto the frontier with fixed inputs is achieved with the mapping $(x_o, y_o) \longrightarrow (x_o, \phi^* y_o)$. We have $\phi^* = 1/\theta^*$. Again the analogy with the case s=m=1 is perfect.

Our aim now is to define a couple of DEA models that will allow us to define a new measure of technical efficiency, namely the scale measure of technical efficiency. This measure will be denoted by θ^*_{sca}. It will also varies in the interval (0,1] with values less than one meaning inefficiencies. We want to know why a production pair $(\mathrm{x}_o, \mathrm{y}_o)$ is inefficient according to Definition 2.1 (technical efficiency less than one). When this happens the DMU belongs to a region of increasing returns to scale or to a region of decreasing returns to scale in the space xy. In the former case y_o is too small for $(\mathrm{x}_o, \mathrm{y}_o)$ to be efficient. In the latter case x_o is too large. This kind of information is extremely relevant to the implementation of production policies. Inefficiencies in the region of increasing returns requires, possibly, projection onto the frontier via output augmentation. Inefficiencies in the region of decreasing returns requires, possibly, projections via input reduction.

The notion of scale of production can be made precise with the use of production sets. Färe, Grosskpof e Lovell (1994) explain in detail these sets. As before let y_o be the output vector of the DMU being evaluated.

. Production set under constant returns:

$$L(y_o, \mathrm{CR}, \mathrm{S}) = \{x; (x, y_o) \in \mathrm{E}\}$$

. Production set under decreasing returns:

$$L(y_o, \mathrm{DR}, \mathrm{S}) = \{x; (x, y_o) \in \mathrm{E}_1\}$$

. Production set under variable returns:

$$L(y_o, \mathrm{VR}, \mathrm{S}) = \{x; (x, y_o) \in \mathrm{E}_2\}$$

The sets E_1 and E_2 are derived from the envelope E imposing the constraints $\sum_i \lambda_i \leq 1$ and $\sum_i \lambda_i = 1$ respectively. We may also define the production set under increasing returns imposing in E the restriction $\sum_i \lambda_i \geq 1$. We will not need this definition. The three production sets show strong disposability (S) in the sense that if $x \in L$ then if $z \geq x$, $z \in L$. In other words, strong disposability occurs when with more input one can produce at least the same amount of output.

The production set $L(y_o, \mathrm{CR}, S)$ shows constant returns to scale in the sense that for any $\alpha > 0$

$$L(\alpha y_o, \mathrm{CR}, \mathrm{S}) = \alpha L(y_o, \mathrm{CR}, \mathrm{S})$$

Note that

$$\begin{aligned} \mathrm{E}^{\mathrm{CR}}(o) &= \mathrm{E}^{\mathrm{CR,S}}(o) \\ &= \theta^*_{\mathrm{CR,S}}(x_o, y_o) \\ &= \min_{\theta \in (0,1]} \{\theta; \theta x_o \in L(y_o, \mathrm{CR}, \mathrm{S})\} \end{aligned}$$

The production set $L(y_o, \mathrm{DR}, S)$ shows decreasing returns to scale in the sense that

$$L(\alpha y_o, \mathrm{DR}, S) \subseteq \alpha L(y_o, \mathrm{DR}, S)$$

for every $\alpha > 0$.

Let $\theta^*_{\mathrm{DR,S}}(x_o, yo)$ be the optimal solution to $\min_{\theta,\lambda} \theta$ subject to i)$\mathrm{Y}\lambda \geq \mathrm{y}_o$, ii) $\mathrm{X}\lambda \leq \theta \mathrm{x}_o$ and iii) $\sum_i \lambda_i \leq 1$, $\lambda_i \geq 0$, θ free. We have

$$\begin{aligned} \mathrm{E}^{\mathrm{DR,S}}(o) &= \theta^*_{\mathrm{DR,S}}(x_o, y_o) \\ &= \min_{\theta \in (0,1]} \{\theta; \theta x_o \in L(y_o, \mathrm{DR}, \mathrm{S})\} \end{aligned}$$

We notice that $\mathrm{E}^{\mathrm{DR}}(o)$ is the measure of technical efficiency of DMU o under the assumption of decreasing returns. In an analogous manner we define the measure of technical efficiency under the assumption of variable returns to scale.

$$\mathrm{E}^{\mathrm{VR,S}}(o) = \theta^*_{\mathrm{VR,S}}(x_o, y_o) = \min_{\theta \in (0,1]} \{\theta; \theta x_o \in L(y_o, \mathrm{VR}, \mathrm{S})\}$$

We see that $\theta^*_{\mathrm{VR,S}}(x_o, y_o)$ is the optimum of $\min_{\theta,\lambda} \theta$ subject to i) $Y\lambda \geq y_o$, ii) $X\lambda \leq \theta x_o$ and iii) $\sum_i \lambda_i = 1$, $\lambda_i \geq 0$, θ free.

Clearly,

$$\mathrm{E}^{\mathrm{CR,S}}(o) \leq \mathrm{E}^{\mathrm{DR,S}}(o) \leq \mathrm{E}^{\mathrm{VR,S}}(o)$$

The measure of scale technical efficiency is defined by the ratio of the technical efficiency under constant returns to the technical efficiency under variable returns.

$$\theta^*_{\mathrm{sca}}(x_o, y_o) = \frac{\theta^*_{\mathrm{CR,S}}(x_o, y_o)}{\theta^*_{\mathrm{VR,S}}(x_o, y_o)}$$

Suppose $\theta^*_{\mathrm{sca}}(x_o, y_o) < 1$. If $\theta^*_{\mathrm{CR,S}}(x_o, y_o) = \theta^*_{\mathrm{DR,S}}(x_o, y_o)$ DMU o operates in a region of increasing returns. If $\theta^*_{\mathrm{CR,S}}(x_o, y_o) < \theta^*_{\mathrm{DR,S}}(x_o, y_o)$ the DMU operates in a region of decreasing returns.

Now we are going to define a measure of technical efficiency that will make it possible the investigation of weather or not there exists an input component that is congestive. Congestion of the input variables means that increasing the quantity of resources used actually implies in reduction of the output level. The presence of congestive inputs destroys the property of strong disposability. The new measure of technical efficiency will be named congestion measure of technical efficiency and denoted by θ^*_{cong}. Its definition involves the comparison of the solutions of two linear programming problems. One under the assumption of strong disposability and the other under weak disposability. We use the following production set to handle weak disposability

$$L(y_o, \text{VR}, \text{W}) = \left\{x; \exists\lambda \geq 0 \text{ and } 0 < \sigma \leq 1 \text{ st } Y\lambda \geq y_o; X\lambda = \sigma x_o; \sum_i \lambda_i = 1\right\}$$

The measure of technical efficiency under the assumption of variable returns and weak disposability is

$$E^{\text{VR,W}}(o) = \theta^*_{\text{VR,W}}(x_o, y_o) = \min_{\theta\in(0,1]} \{\theta; \theta x_o \in L(y_o, \text{VR}, \text{W}\}$$

Clearly

$$\text{E}^{\text{CR,S}}(o) \leq \text{E}^{\text{DR,S}}(o) \leq \text{E}^{\text{VR,S}}(o) \leq \text{E}^{\text{VR,W}}(o)$$

Equivalently we may compute $\text{E}^{\text{VR,W}}(o)$ as the solution of the linear programming problem $\min_{\theta,\lambda} \theta$ subject to i) $Y\lambda \geq y_o$, ii) $X\lambda = \theta x_o$ e iii) $\sum_i \lambda_i = 1$; $\lambda_i \geq 0$; θ free.

We define,

$$\theta^*_{\text{cong}}(x_o, y_o) = \frac{\theta^*_{\text{VR,S}}(x_o, y_o)}{\theta^*_{\text{VR,W}}(x_o, y_o)}$$

When $\theta^*_{\text{cong}}(x_o, y_o) < 1$ it is of interest to pinpoint which inputs, or combination of inputs, are responsible for the observed congestion. This is accomplished with the use of partial measures of technical efficiency. Let B be a subset of $\{1, 2, \ldots, m\}$ with at least one element and B^c its complement. Suppose we want to investigate if the input set B^c causes congestion. Partition X e x_o according to the partition induced by B. In other words, write

$$X = \begin{pmatrix} X^B \\ X^{B^c} \end{pmatrix} \text{ e } x_o = \begin{pmatrix} x_o^B \\ x_o^{B^c} \end{pmatrix}$$

Find the solution $\theta^*_{\text{cong,B}}(x_o, y_o)$ of the linear programming problem $\min_{\theta,\lambda} \theta$ subject to i) $Y\lambda \geq y_o$, ii) $X^B\lambda \leq \theta x_o^B$, iii) $X^{B^c}\lambda = \theta x_o^{B^c}$ and iv) $\sum_i \lambda_i = 1$, $\lambda_i \geq 0$; θ livre. If $\theta^*_{\text{cong,B}}(x_o, y_o) = \theta^*_{\text{VR,S}}(x_o, y_o)$ the subvector of inputs B^c congests production. Note that there is not uniqueness in the notion of congestion. The analysis has to be carried out for all possible subsets of the input list.

We thus have the following decomposition

$$\mathrm{E}^{\mathrm{CR,S}}(o) = \theta^{*}_{\mathrm{sca}}(x_o, y_o)\theta^{*}_{\mathrm{cong}}(x_o, y_o)\mathrm{E}^{\mathrm{VR,W}}(o)$$

It follows that a DMU is inefficient either due to scale problems, congestion or because it does not belong to the frontier of the production problem under the assumption of variable returns and weak disposability.

To summarize we present the four main linear programming problems involved in the decomposition of the technical efficiency under constant returns to scale in primal form. These problems are known as multipliers problems and are handy for computational purposes. In general we are looking for

$$\max_{u,v,u^*} y_o'u + u^*$$

subject to $x_o'v = 1$ and $Y'u - X'v + u^*1 \leq 0$. Imposing additional restrictions on the variables u, v and u^* we can generate all four linear programming problems:

1. constant returns, strong disposability: $u, v \geq 0$ e $u^* = 0$.
2. decreasing returns, strong disposability: $u, v \geq 0$ e $u^* \leq 0$.
3. variable returns, strong disposability: $u, v \geq 0$ e u^* free.
4. variable returns, weak disposability: $u \geq 0$ e u^*, v free.

If in addition to the quantity matrices Y and X a vector p of input prices is available for each DMU we may also compute cost measures of efficiency. Our discussion will assume constant returns to scale but obvious modifications may lead to more general cost measures. Let p_o and y_o denote prices and outputs for DMU o and let $C(p_o, y_o)$ be the solution of $\min_{\lambda,x} p_o'x$ subject to the conditions $Y\lambda \geq y_o$ and $X\lambda \leq x$, where x and λ are nonnegative. The measure of cost efficiency for DMU o is

$$\theta^{*}_{cost}(o) = \frac{C(p_o, y_o)}{p_o'x_o}$$

We see that the cost efficiency is given by the ratio of the minimum cost attainable to observed cost. Whenever $\theta^*_{cost}(o) < 1$ DMU o is spending more on inputs than is necessary to produce y_o. As in Färe, Grosskopf and Lovell (1994) the excess is due to either or both of two factors (i) using too much of all inputs, and (ii) using inputs in the wrong mix. The first factor is measured by $\theta^*_{\text{CR,S}}(o)$ and the second is measured by the allocative measure of efficiency. This is simply the the ratio $A(o)$ of $\theta^*_{cost}(o)$ to $\theta^*_{\text{CR,S}}(o)$. It follows that

$$\theta^*_{cost}(o) = \theta^*_{\text{CR,S}}(o) \times A_o$$

If only total input costs and output quantity data it is still possible to define a measure of technical efficiency. Let Q be the cost n vector. We now look for the minimum, in λ and x, both nonnegative, of $Q'\lambda$ subject to the conditions $X\lambda \leq x$ and $Y\lambda \geq y_o$. We will not make use of this measure in this paper.

3. EMBRAPA'S PRODUCTION SYSTEM

Embrapa's research system comprises 37 units (DMUs) or research centers. Input and output actions have been defined from a set of performance indicators known to the company since 1991. The company uses routinely some of these indicators to monitor performance through annual work plans. The system of performance indicators is detailed in Embrapa (1996a). With the active participation of the board of directors of Embrapa as well as the administration of each of its research units we selected 28 output and 3 input indicators as representative of production actions in the company. A full explanation of these items is given in Embrapa (1996b).

We begin our discussion of EMRAPA's production system with the output. The output indicators were classified into four categories. Scientific production, production of technical publications, development of technologies, products and processes and diffusion of technologies and image. By scientific production we mean the publication of articles and book chapters aimed mainly to the academic world. We require that each item be specified with complete bibliographical reference. Specifically the category of scientific production includes the following items.

1. Scientific articles published in refereed journals and book chapters - domestic publications.

2. Scientific articles published in refereed journals and book chapters - foreign publications.

3. Articles and summaries published in proceedings of congresses and technical meetings.

The category of technical publications groups publications produced by research centers aiming primarily agricultural businesses and agricultural production. Specifically,

1. Technical Circulars. Serial publications, written in technical language, listing recommendations and information based on experimental studies. The intended coverage may be the local, regional or national agriculture.

2. Research bulletins. Serial publications reporting research results.

3. Technical communiqués. Serial publications, succinct and written in technical language, intended to report recommendations and opinions of researchers in regard to matters of interest to the local, regional or national agriculture.

4. Periodicals (document series). Serial publication containing research reports, observations, technological information or other matters not classified in the previous categories. Examples are proceedings of technical meetings, reports of scientific expeditions, reports of research programs, etc.

5. Technical recommendations/instructions. Publication written in simplified language, aimed at extensionists and farmers in general, and containing technical recommendations in regard to agricultural production systems.

6. Ongoing research. Serial publication written in technical language and approaching aspects of a research problem, research methodologies or research objectives. It may convey scientific information in objective and succinct form.

The category of development of technologies, products and processes groups indicators related to the effort made by a research unit to make its production available to society in the form of a final product. We include here only new technologies, products and processes. These must be already tested at the client's level in the form of prototypes or through demonstration units or be already patented. Specifically,

1. Cultivars. Plant varieties, hybrids or clones.
2. Agricultural and livestock processes and practices.
3. Agricultural and livestock inputs. All raw material that may be used or transformed to obtain agricultural and livestock products, including stirps.
4. Agro-industrial processes. Operations carried out at commercial or industrial level envisaging economic optimization in the phases of harvest, post harvest and transformation and preservation of agricultural products.
5. Machinery (equipment). Machine or equipment developed by a research unit.
6. Scientific methodologies.
7. Software.
8. Monitoring, zoning (agroecologic or socioeconomic) and mapping.

Finally, the category of diffusion of technologies and image encompasses production actions related with Embrapa's effort to make its products known to the public and to market its image. Here we consider the following indicators.

1. Field days. These event are organized by research units aiming the diffusion of knowledge, technologies and innovations. The target public is primarily composed of farmers, extensionists, organized associations of farmers (cooperatives), and undergraduate students. The field day must involve at least 40 persons and last at least 4 hours.
2. Organization of congresses and seminars. Only events with at least 3 days of duration time are considered.
3. Seminar presentations (conferences and talks). Presentation of a scientific or technical theme within or outside the research unit. Only talks and conferences with a registered attendance of at least 20 persons and duration time of at least one hour are considered.
4. Participation in expositions and fairs. Participation is considered only in the following cases.

(a) with the construction of a stand with the purpose of showing the center's research activities by audiovisuals and distributing publications uniquely related to the event's theme.

(b) co-sponsorship of the event.

5. Courses. Courses offered by a research center. Internal registration is required specifying the course load and content. The course load should be at least 8 hours. Disciplines offered as part of university courses are not considered.

6. Trainees. Concession of college level training programs to technicians and students. Each trainee must be involved in training activities for at least 80 hours to be counted in this item.

7. Fellowship holders. Orientation of students (the fellowship holders). The fellowship duration should be at least six months and the work load at least 240 hours.

8. Folders . Only folders inspired by research results are considered. Reimpressions of the same folder and institutional folders are not counted.

9. Videos . Videos should address research results of use for Embrapa's clients. The item includes only videos of products, services and processes with a minimum duration time of 12 minutes.

10. Demonstration units. Events organized to demonstrate research results - technologies, products and processes, already in the form of a final product, in general with the co-participation of a private or government agent of technical assistance.

11. Observation units. Events organized to validate research results, in space and time, in commercial scale, before the object of research has reached its final form. Observations units are organized in cooperation with producers, cooperatives, other agencies of research or private institutions. The events may be organized within or outside the research unit.

The input side of Embrapa's production process is composed of three factors. Personnel, operational costs (consumption materials, travel and services less income from production projects), and capital measured by depreciation.

3.1 Input and ouput indexes

As indicators (inputs and outputs) of the production process we consider a system of dimensionless relative indices. These are all quantity indexes. The idea, from the output point of view, is to define a combined measure of output as a weighted average of the relative indicators (indices) in the system. The relative indices are computed for each production variable and for each research unit within a year dividing the observed production quantity by the mean per research unit. Only research units that can potentially exercise the production activity related to the production variable in question are included in the computation of the mean. We see that, within a given year, the base of our system of production indices is defined by the set of means per unit defined by the production variables. In case of inputs the means use all 37 cases. DEA assumes quantity data. We use the number or employees to represent the factor personnel. Division of money expenses by their respective means will produce a quantity index under the assumption of a common price to all research units. This is a reasonable assumption for operational and capital expenses considering the interest rate as the relevant price. The input indices are indicated by x_i^o, $i = 1, 2, 3$. These quantities represent relative indices of personnel, operational expenditures, and capital expenditures, respectively. A combined measure of inputs x_o is defined as the simple average of the three quantities x_i^o.

Output measures per category are defined as follows. The output component y_i, $i = 1, 2, 3, 4$ of each production category is a weighted average of the relative indices composing the category. If o is the DMU (research unit) being evaluated then

$$y_i^o = \sum_{j=1}^{k_i} a_{ji}^o y_{ji}^o; \quad 0 \leq a_{ji}^o; \quad \sum_{j=1}^{k_i} a_{ji}^o = 1$$

where a_{ji}^o, $j = 1, \ldots, k_i$ is the weight system for DMU o in the category of production i, k_i is the number of production indicators comprising i and y_{ji}^o is the relative index of production j. The weights in principle are supposed to be user defined and should reflect the administration perception of the relative importance of each variable to each DMU. Defining weights is a hard and questionable task. In our application in Embrapa we followed an approach based on law of categorical judgment of Thurston. See Torgerson (1958) or Souza (1988). The model is competitive with the AHP method of Saaty (1990) and is well suited when several judges are involved in the evaluation process. Basically we sent out about 500 questionnaires to researchers and administrators (on a per research center basis) and asked them to rank in importance - scale from 1 to 5, each production category and each production variable within the corresponding production category. We assume that the psycological continuum of the responses projects to a lognormal distribution. Based on the analysis of the inquiry, final weights were set interacting with the board of directors of Embrapa. Minor adjustments to Thurston's analysis were then made to better reflect the administration policies for each research unit.

DEA models implicitly assume that the DMUs are comparable. This is not strictly the case in Embrapa. To make them comparable it is necessary an effort to define an output measure adjusted for differences in operation and perceptions. At the level of the partial production categories we induced this measure allowing a distinct set of weights for each DMU. In principle one could go ahead and use DEA with multiple outputs. This would minimize the effort of defining weights leaving to DEA the task of finding these coefficients. The problem with such approach is that there is a kind of dimensionality curse in DEA models. As the number of factors (inputs and outputs) increases, the ability to discriminate between DMUs decreases, i.e., as Seifford and Thrall (1990) put it "given enough factors, all (or most) of the DMUs are rated efficient. This is not a flaw of the methodology, but rather a direct result of the dimensionality of the input/output space relative to the number of DMUs". In our case with 4 separate measures of output we found that more then 60% of the DMUs were efficient. In this context we found convenient to extend the weight system to produce a single measure of output y_o. This further established a common basis to compare research units and avoided the incidence of zero output (shadow) prices, another common occurrence in multiple output models (and also a disturbing fact for management interpretation!). A single output also allows a simple comparison of DEA results with efficiency measures generated by the fit of stochastic frontiers, as we show later.

The (combined) measure of productivity for DMU o is given by the ratio $\text{Prod}(o) = y_o/x_o$.

We call a research unit productive when its productivity measure is greater than or equal to one.

4. DATA ANALYSIS I (Envelope Problems)

We performed a DEA analysis with 34 of the 37 research centers of Embrapa for the year 1996. Three research centers were eliminated from the analysis due to the particular nature and size of their operation. These are coded as UD-07, UD-19, and UD-37. The coding in use for research centers follows the actual convention used in Embrapa to designate its units. UD-19 deals mainly with the production of software, UD-07 with agricultural machinery, and UD-37 with environmental monitoring. The research units of Embrapa's system are classified into 3 types according to their missions and research objectives. Ecoregional research units (E, total of 13 units), product oriented (simply referred as product) research centers (P, total de 15 units) and thematic research centers (T, total of 9 units). As described in Section 3 the production system comprises 28 output items and 3 inputs. The output variables are reduced to a single output measure with the use of a weight system variable per research unit. For the 4 broad categories of output weights were defined by type. Within each of this categories we allowed variation among research units only for variables classified as development of technologies, products and processes. This is the production category where one can observe the major differences in perception, among research units, of the relative importance of each individual production variable. We carried out the analysis of technical efficiency with the use of three macros SAS: (1) EFIC computes the measures of technical efficiency under the assumptions of constant returns - strong disposability, decreasing returns - strong disposability, variable returns - strong disposability, and variable returns - weak disposability, (2) CONGEST analyzes partial congestion, and (3) COSTEFIC which analyzes cost efficiency for a given set of prices[2]. All macros assume the presence of a data set with data on input and output indexes. The variables should be output (Y), inputs (X_1, X_2 and X_3) and the identification of the DMUs (ID). In COSTEFIC quantity data are represented

[2]The macros EFIC, CONGEST, and COSTEFIC are available via anonymous ftp in ftp.sede.Embrapa.br in the directory /pub/dea/paper/. In the directory the data sets with the 1996 data are DADOS.DAT and PRICES.DAT. The SAS code that generates input and output indices to be used with EFIC and CONGEST is in BASIC.SAS and includes the weights being used.

by QY, QX_1, QX_2 and QX_3, respectively. We note that the macros are crude but can be easily generalized to a greater number of inputs and outputs.

Table 2 shows the results of DEA on quantity data. Shadow prices are shown in Table 3 and partial congestion measures on Table 4. On the average thematic units are more efficient than ecoregional and product research centers. Averages for these units are 0.57, 0.66 and 0.82 respectively. Figure 1 sheds some light on the distribution of efficiencies. The evidence is for a density with two modes indicating the presence of two subpopulations. A close look at Table 3 shows that units are more efficient in the use of operational expenses than personnel and capital. The last four units in Table 3 are technical efficient but only UD-01 belongs to the efficient frontier EF. The location of operation relative to the efficient frontier is as follows. Research units UDs 06, 10, 18, 20, 22, and 23 show decreasing returns to scale. The others, with the exception of the four technical efficient, show increasing returns. Congestion measures are particularly low for UDs 10, 22, 28, 32, and 33. In all these research units the congestive component is operational expenses. UD-32 also shows capital congestive. See Table 4.

Table 5 shows cost efficiencies. Prices for capital and operational expenses factors were considered constant for all units and the price for personnel is an index computed from the average year salary of each unit. The basis is the company average salary. We see that inefficiencies come much more from spending too much on all inputs than due to a poor allocation of resources. It is interesting to note that of the four units technical efficient only one is fully cost efficient.

5. DATA ANALYSIS II (Stochastic Frontier)

A single equation stochastic frontier model, Bauer (1990), has the form

$$\log y_t = \alpha + \beta_1 \log x_{1t} + \beta_2 \log x_{2t} + \beta_3 \log x_{3t} + v_t - u_t$$

where we choose the response (true stochastic frontier) in the Cobb-Douglas family, the residuals v_t are normally distributed with mean zero and variance σ_v^2, the residuals u_t are nonnegative and distributed as a half normal, truncated normal or exponential distribution with variance σ_u^2. The errors $\epsilon_t = v_t - u_t$ are assumed independent across research units.

Let $\sigma^2 = \sigma_u^2 + \sigma_v^2$ and $\lambda = \sigma_u/\sigma_v$. Assuming a half normal distribution for u_t a measure of production inefficiency is given by

$$E(u/\epsilon) = \frac{\sigma\lambda}{1+\lambda^2}\left[\frac{\phi(\epsilon\lambda/\sigma)}{1-\Phi(\epsilon\lambda/\sigma)} - \left(\frac{\epsilon\lambda}{\sigma}\right)\right]$$

Here $\phi(.)$ and $\Phi(.)$ are the density and distribution function of the standard normal, respectively. See Greene (1995) for the other forms of this quantity under the assumptions of truncated normal and exponential distributions for the component u_t. We used LIMDEP to fit the Cobb-Douglas function via maximum likelihood assuming, in turn, each of the 3 distributions above. Ordinary least squares produced a fit with $R^2 = 0,47291$ and a significant F statistic. Ordinary least squares residuals for the Cobb-Douglas fit are negatively skewed, an important property for mle estimation of stochastic production function frontiers. We tried more general forms than the Cobb-Douglas. Those alternatives did not pass the skewness condition. The parametric estimates of technical efficiencies above cannot be shown to be consistent for cross section data, but we used them anyway to access the nonparametric efficiency measures. To make the measurements comparable we inverted the stochastic frontiers estimates and normalized dividing by the maximum. Final results are shown in Table 6. The hypothesis of constant returns is not rejected in any of the 3 fits. Although individual efficiencies may differ, Spearman and Pearson correlation coefficients with CR are on the order of 90%. Between stochastic frontier fits the correlations are on the order of 99%. On the average inefficiencies are lower in the nonparametric case but in many cases we have a reasonable agreement between the two methods. It is worth to mention that, independently of the residual distributional assumption, the important variable in the stochastic frontier fit is operational expenses which has an elasticity estimate of about 0,69 with a standard error of 0.25.

6. CONCLUSION AND FUTURE PERSPECTIVES

A nonparametric approach to the analysis of production frontiers is in use in Embrapa to assist management. An important contribution in this context was the definition of input and output measures that allow the company to identify the strengths and weaknesses of its research centers inducing a more effective management of resources. A further exercise is now under way relating management practices to efficiencies in an effort to identify relevant factors for near optimum administration. An important by-product of Embrapa's study is the possibility of the establishment of production goals easier to monitor with the help of other quantitative management techniques. A typical example is the balanced scorecard. See Kaplan and Norton (1996). Embrapa is successfully implementing a pilot project with this approach. Of particular interest for managers of agricultural research institutions like Embrapa is the potential use of the production frontier approach in external comparisons. In this context we are already in touch (and gathering data) with other comparable institutions (as INTA of Argentina, INIA of Chile, and the group of research institutions under the administrative coordination of ISNAR in Holland). The international setting poses challenging problems to the definition of output and input measures.

In the near future more data will be collected and other econometric techniques can be evaluated. Of particular concern is the possibility of panel data analysis from both points of view - parametric and nonparametric. Stochastic frontiers in case of panel data will generate consistent estimates of efficiencies.

7. REFERENCES

Bauer, P. W. (1990), Recent developments in the econometric estimation of frontiers, *Journal of Econometrics* 46, 39-56.

Charnes, A., Lewin A. L. and Seiford L. M. (1995), *Data Envelopment Analysis: Theory, Methodology and Applications.* Kluwer, Boston.

Embrapa (1996a), Sistema de informação gerencial dos planos anuais de trabalho - SISPAT: manual do usuário. Sec. de Admin. Estratégica. Presidência, Embrapa sede, Brasília.

Embrapa (1996b), Sistema de avaliação de unidades: glossário de termos técnicos e indicadores de desempenho utilizados no modelo de análise de produtividade e eficiência relativa das unidades descentralizadas da Embrapa. Sec. de Admin. Estratégica. Presidência, Embrapa sede, Brasília.

Färe, R., Grosskopf, S. and Lovel C. A. K. (1994), *Production Frontiers.* Cambridge, New York.

Färe R. and Grosskopf S. (1996), *Intertemporal Production Frontiers: With Dynamic DEA*. Kluwer, Boston.

Gass, S. I. (1969), *Linear Programming*. McGraw-Hill, New York.

Greene W. H. (1995), *LIMDEP version 7.0*. Econsoft, New York.

Kaplan R. S. and Norton D. P. (1996), *The Balanced Scorecard, Translating Strategy into Action*, Havard Business School Press, Boston.

Mas-Colell, Whinston D. M. and Green, J. G. (1995), *Microeconomic Theory*. Oxford, New York.

Saaty T. L. (1990), *The Analytic Hierarchy Process*. RWS, Pittsburgh.

Seiford, L. M. and Thrall R. M. (1990), Recent developments in DEA , the mathematical programming approach to frontier analysis, *Journal of Econometrics* 46, 7-38

Sengupta J. K. (1995), *Dynamics of Data Envelopment Analysis, Theory of Systems Efficiency*. Kluwer, Boston.

Souza J. (1988), *Métodos de Escalagem Psicossocial*. Thesaurus. Brasília.

Torgenson W. S. (1958), *Theory and Methods of Scaling*, Wiley, New York.

Figure 1. Box plot and density estimation of CR.

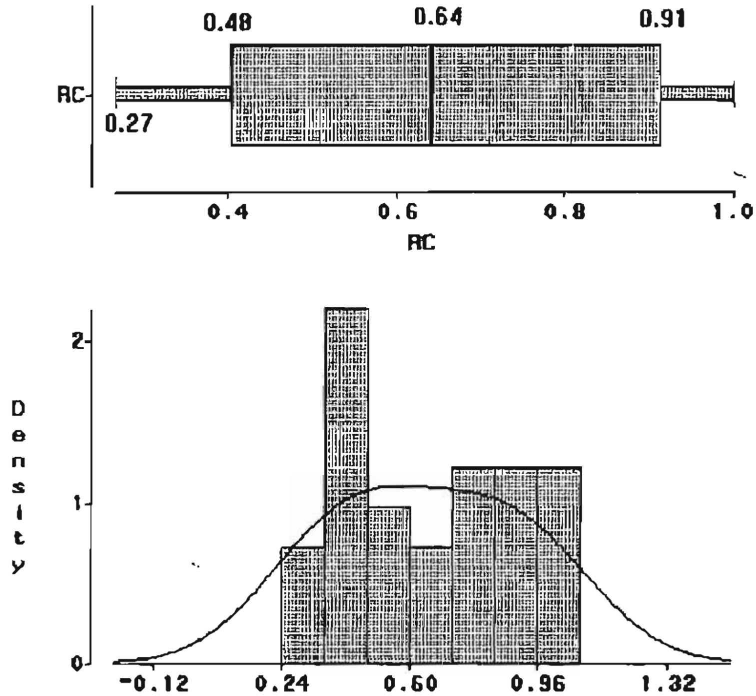

Table 1. Unsymmetric primal-dual problems.

Primal problem	Constraints (primal)	Dual problem	Constraints (dual)
$\max_x c'x$	$Ax = b,\ x \geq 0$	$\min_w b'w$	$A'w \geq c$
$\min_x c'x$	$Ax = b,\ x \geq 0$	$\max_w b'w$	$A'w \leq c$

Table 2. Productivity (Prod). Efficiencies CR(S), DR(S), VR(S), VR(W), Sca (Scale), and Cong (Congestion).

UDs	Type	Prod	CR	DR	VR	VR(W)	Sca	Cong
28	E	0.3965	0.2663	0.2663	0.4441	0.7990	0.5997	0.5558
21	E	0.4405	0.2772	0.2772	0.3867	0.4309	0.7168	0.8973
33	E	0.6724	0.3673	0.3673	0.4018	1.0000	0.9140	0.4018
25	E	0.6639	0.3936	0.3936	1.0000	1.0000	0.3936	1.0000
31	E	0.6914	0.3964	0.3964	0.4914	0.4925	0.8067	0.9978
26	E	0.7412	0.4029	0.4029	0.5901	0.6342	0.6828	0.9305
22	E	0.6560	0.5089	0.5385	0.5385	1.0000	0.9451	0.5385
32	E	0.9839	0.5823	0.6520	0.6520	1.0000	0.8930	0.6520
27	E	1.1322	0.6944	0.6944	1.0000	1.0000	0.6944	1.0000
29	E	1.2841	0.7844	0.7844	0.9832	0.9858	0.7978	0.9975
24	E	1.3931	0.8450	0.8450	0.9215	0.9219	0.9169	0.9996
23	E	1.2449	0.9130	1.0000	1.0000	1.0000	0.9130	1.0000
30	E	1.3072	0.9706	0.9706	1.0000	1.0000	0.9706	1.0000
09	P	0.5934	0.3317	0.3317	0.4228	0.4389	0.7845	0.9632
02	P	0.7122	0.3879	0.3879	0.5099	0.5304	0.7608	0.9612
11	P	0.5632	0.4039	0.4039	0.4869	0.5416	0.8295	0.8989
10	P	0.6134	0.4090	0.4175	0.4175	1.0000	0.9797	0.4175
16	P	0.6251	0.4388	0.4388	0.5022	0.5581	0.8738	0.8998
34	P	0.7189	0.4788	0.4788	0.6668	0.7536	0.7181	0.8848
17	P	0.8701	0.5995	0.5995	0.6010	0.6795	0.9975	0.8846
08	P	1.0310	0.6533	0.6533	0.7254	0.7272	0.9005	0.9976
14	P	1.4788	0.7394	1.0000	1.0000	1.0000	0.7394	1.0000
04	P	1.1935	0.7602	0.8446	0.8446	1.0000	0.9001	0.8446
06	P	1.3678	0.7907	0.7907	0.8639	0.8654	0.9153	0.9984
20	P	1.1444	0.9232	0.9353	0.9353	1.0000	0.9871	0.9353
18	P	1.5571	0.9320	0.9930	0.9930	1.0000	0.9386	0.9930
13	P	2.0343	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
35	P	1.7933	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
15	T	0.7593	0.5003	0.5003	0.6975	0.7151	0.7172	0.9755
05	T	0.9174	0.6295	0.6295	0.7556	1.0000	0.8331	0.7556
12	T	1.0595	0.8266	0.8266	0.8779	1.0000	0.9417	0.8779
36	T	1.1819	0.9441	0.9441	0.9659	1.0000	0.9774	0.9659
01	T	1.5123	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
03	T	1.5898	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 3. Shadow prices of production (Y), personnel (X_1), operational expenses (X_2), and capital (X_3).

UDs	Y	X_1	X_2	X_3
28	0.5935	0.0000	1.1430	0.0000
21	0.5168	0.0000	0.9953	0.0000
09	0.5068	0.3749	0.4814	0.0000
33	0.2343	0.0000	0.0000	0.7814
02	0.6391	0.4729	0.6071	0.0000
25	1.5638	1.3796	1.2879	0.0000
31	0.6914	0.5115	0.6568	0.0000
26	0.7058	0.9782	0.2393	0.0545
11	0.6506	0.0000	1.2531	0.0000
10	0.4741	0.0000	0.9132	0.0000
16	0.6167	0.4562	0.5858	0.0000
34	0.9479	0.7013	0.9004	0.0000
15	0.8256	1.2109	0.2960	0.0000
22	0.4268	0.0000	0.8219	0.0000
32	0.4067	0.0000	0.7833	0.0000
17	0.3938	0.3474	0.3243	0.0000
05	0.7272	1.2157	0.0000	0.1852
08	0.7247	0.6394	0.5969	0.0000
27	1.5138	1.1200	1.4381	0.0000
14	0.3052	0.5102	0.0000	0.0777
04	0.5115	0.0000	0.9851	0.0000
29	1.0991	1.5234	0.3726	0.0849
06	0.7490	1.0381	0.2539	0.0579
12	0.7742	1.1355	0.2775	0.0000
24	0.0217	0.7559	0.9705	0.0000
23	0.4746	0.0000	0.9140	0.0000
20	1.1299	0.0000	2.1762	0.0000
18	0.5878	0.4349	0.5584	0.0000
36	0.7828	1.1481	0.2806	0.0000
30	0.8615	1.7577	0.0000	0.0000
01	0.3790	0.5254	0.1285	0.0293
03	0.7980	0.7040	0.6572	0.0000
13	0.4979	0.3684	0.4730	0.0000
35	1.3364	0.0000	2.5739	0.0000

Table 4. Partial congestion measures: Capital (X_3), operational expenses (X_2), personnel (X_1), personnel-operational expenses (X_{12}), personnel-capital (X_{13}), and operational expenses-capital (X_{23}).

UDs	X_3	X_2	X_1	X_{12}	X_{13}	X_{23}
09	0.4228	0.4389	0.4228	0.4389	0.4228	0.4389
11	0.5383	0.4868	0.5275	0.5275	0.5416	0.5383
22	0.5516	0.5385	1.0000	1.0000	1.0000	0.5516
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
04	0.8446	0.8446	1.0000	1.0000	1.0000	0.8446
29	0.9833	0.9857	0.9833	0.9857	0.9833	0.9857
01	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
18	1.0000	0.9930	0.9930	0.9930	1.0000	1.0000
16	0.5581	0.5022	0.5022	0.5022	0.5581	0.5581
35	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
20	1.0000	0.9352	1.0000	1.0000	1.0000	1.0000
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
08	0.7273	0.7255	0.7255	0.7255	0.7273	0.7273
17	0.6795	0.6010	0.6010	0.6010	0.6795	0.6795
15	0.7150	0.7012	0.6975	0.7012	0.7150	0.7150
36	1.0000	0.9659	0.9659	0.9659	1.0000	1.0000
10	0.4206	0.4175	1.0000	1.0000	1.0000	0.4206
34	0.7536	0.6668	0.6668	0.6668	0.7536	0.7536
12	1.0000	0.8779	0.8779	0.8779	1.0000	1.0000
03	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
31	0.4914	0.4925	0.4914	0.4925	0.4914	0.4925
06	0.8639	0.8653	0.8639	0.8653	0.8639	0.8653
05	0.7556	1.0000	0.7556	1.0000	0.7556	1.0000
02	0.5098	0.5304	0.5098	0.5304	0.5098	0.5304
28	0.4605	0.4441	0.7987	0.7987	0.7987	0.4605
32	0.6520	0.6520	1.0000	1.0000	1.0000	0.6520
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
24	0.9220	0.9217	0.9217	0.9217	0.9220	0.9220
26	0.5901	0.6342	0.5901	0.6342	0.5901	0.6342
33	0.4018	0.4740	1.0000	1.0000	1.0000	0.4740
21	0.3975	0.3867	0.4309	0.4309	0.4309	0.3975
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 5. Cost efficiency (EFCOST) and allocative efficiency (ALLOC).

UDs	EFCOST	ALLOC
28	0.1968	0.7390
21	0.2164	0.7807
09	0.2915	0.8788
33	0.3300	0.8984
02	0.3501	0.9026
25	0.3249	0.8255
31	0.3396	0.8567
26	0.3635	0.9022
11	0.2758	0.6828
10	0.3028	0.7403
16	0.2993	0.6821
34	0.3520	0.7352
15	0.3857	0.7709
22	0.3225	0.6337
32	0.4896	0.8408
17	0.4255	0.7098
05	0.4643	0.7376
08	0.5151	0.7885
27	0.5553	0.7997
14	0.7268	0.9830
04	0.5902	0.7764
29	0.6317	0.8053
06	0.6709	0.8485
12	0.5277	0.6384
24	0.6852	0.8109
23	0.6124	0.6708
20	0.5577	0.6041
18	0.7671	0.8231
36	0.6015	0.6371
30	0.6560	0.6759
01	0.7758	0.7758
03	0.7729	0.7729
13	1.0000	1.0000
35	0.8839	0.8839

Table 6. Stochastic frontier efficiency: half-normal (U), truncated normal (V), and exponential (W).

UDs	V	W	
28	0.4004	0.4090	0.3822
21	0.4207	0.4289	0.4079
09	0.4531	0.4607	0.4473
33	0.5250	0.5298	0.5305
02	0.5196	0.5249	0.5244
25	0.4378	0.4459	0.4289
31	0.5182	0.5237	0.5225
26	0.4968	0.5031	0.4985
11	0.5170	0.5228	0.5202
10	0.5618	0.5656	0.5695
16	0.5154	0.5219	0.5172
34	0.5262	0.5323	0.5297
15	0.5142	0.5209	0.5160
22	0.6341	0.6369	0.6416
32	0.7491	0.7495	0.7529
17	0.6581	0.6621	0.6628
05	0.5275	0.5334	0.5322
08	0.6630	0.6665	0.6686
27	0.6734	0.6762	0.6803
14	0.8548	0.8550	0.8522
04	0.8565	0.8562	0.8541
29	0.7153	0.7180	0.7192
06	0.7617	0.7636	0.7630
12	0.7175	0.7213	0.7187
24	0.7930	0.7946	0.7925
23	0.9285	0.9287	0.9228
20	0.8396	0.8406	0.8359
18	0.9255	0.9262	0.9200
36	0.7622	0.7657	0.7603
30	0.7120	0.7157	0.7142
01	0.8550	0.8575	0.8482
03	0.8625	0.8644	0.8568
13	1.0000	1.0000	1.0000
35	0.9463	0.9459	0.9439