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Uncertainty in Electric Bus Mass and its Influence in Energy Consumption

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Abstract

Throughout recent years, a public awareness of climate change and a social trend for preserving the environment have emerged. Transport sector is the principal contributor to greenhouse gas emissions, consequently electric buses are a great opportunity to reduce these emissions and fossil fuel dependence. To increase the competitiveness of electric buses, batteries with an accurate size are needed in order to optimize the charging infrastructure and reduce the total costs. Therefore, it is necessary to analyse the influence of certain parameters on electrical consumption.

This thesis evaluates the impact of passenger loads on the electrical consumption of an electric city bus and provides a reliable energy consumption forecast. An electrical consumption sensitivity analysis was created with the number of passengers in the bus as uncertainty. This uncertainty is stochastically modelled for each stop in the bus route based on actual data and it is evaluated with the Monte Carlo method. In addition, the uncertainty in the number of stops is also considered.

An algorithm for passenger load calculation was created in Matlab, based on driving cycles generated randomly (with a random number of stops and different speed profiles). Passenger data for each bus stop were represented by a normal probability distribution and they were related to each other using a multivariate normal distribution. These are the uncertain inputs of the model, as well as the number of stops which was modelled previously by another normal distribution. A validated electric bus model created in Simulink was simulated by means of the Monte Carlo sampling method, varying in each iteration the driving cycle and passenger flow introduced.

The results obtained for a particular bus route, described as a probability distribution, define an electrical consumption with an average of 0.549 kWh/km. It is also possible to assure with an 80% of probability that the electrical consumption in this route will be between 0.485 kWh/km and 0.613 kWh/km. These results represent an electrical consumption forecast for the route, including all the possible outcomes taking into account the uncertainties of the model. Moreover, the analysis of the results indicates that the passenger load has a clear influence on the bus electrical consumption that increases with the number of passengers. In addition, the results show a clear influence of driving cycle average speed and number of stops on the consumption. Electrical consumption increases as the number of stops increases and as the average speed decreases. The results also confirm that Monte Carlo method provides an efficient tool for estimating the consumption of an electric city bus since it enables to obtain results for the different possible scenarios and cover all the variations.

Keywords Battery electric city bus, passenger load, Monte Carlo method, electrical consumption, simulation model

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Symbols and Definitions

A_d	[m^2]	Vehicle's frontal cross-sectional area
C_d	[-]	Drag coefficient
E	[J]	Battery electrical energy
F_{acc}	[N]	Acceleration resistance
F_{aero}	[N]	Aerodynamic drag
F_g	[N]	Gradient resistance
F_I	[N]	Inertial resistance
F_R	[N]	Rolling resistance
F_T	[N]	Vehicle tractive force
$F_{X_i}{}^{-1}$	[-]	Inverse cumulative distribution function of random variable X$_i$
I	[A]	Battery current
J	[m^4]	Total moment of inertia at the driven axle
K	[-]	Kernel smoothing function
M	[kg]	Mass
M_T	[kg]	Vehicle total mass
N	[N]	Normal force
P	[W]	Battery electric power
P_{loss}	[W]	Power loss
T	[K]	Battery internal temperature
T_a	[K]	Ambient temperature
T_b	[N·m]	Brake torque
T_{ref}	[N·m]	Reference motor torque
T_x	[N·m]	Motor torque
V	[V]	Battery voltage
V_{OC}	[V]	Battery open circuit voltage
V_{ref}	[V]	Reference voltage
a	[m/s^2]	Real vehicle acceleration
f_{rr}	[-]	Rolling coefficient
g	[m/s^2]	Gravitational acceleration
h	[-]	Bandwidth in Kernel estimation
i_{ref}	[A]	Reference current
m	[kg]	Curb weight of the vehicle
n	[-]	Number of samples in Kernel estimation
r	[m]	Dynamic radius of the tire
v	[m/s]	Real vehicle velocity
Σ	[-]	Covariance matrix for n-dimensional normal distribution
α	[rad]	Angle forming the surface road with the horizontal
λ	[-]	Poisson distribution parameter
μ	[-]	Mean of normal distribution
ρ	[kg/m^3]	Air density
σ	[-]	Standard deviation of normal distribution
ω	[rad/s]	Motor angular velocity
ω_{ref}	[rad/s]	Reference angular velocity

Abbreviations

AC	Alternating current
BEB	Battery electric bus
CNG	Compressed natural gas
DC	Direct current
EV	Electric vehicle
FCEB	Fuel cell electric bus
GPS	Global positioning system
HEB	Hybrid electric bus
HEV	Hybrid electric vehicle
ICE	Internal combustion engine
ME	Method of moments
ML	Maximum likelihood
PDF	Probability density function
PID	Proportional-integral-derivative
RBS	Regenerative braking system
SOC	State of charge
SOD	State of discharge

1 Introduction

Nowadays, society must cope with the continuous growth of the fuel price [1] and its future exhaustion, as it is not a renewable resource. Not less important are the pollution problems associated with the use of fossil fuels such as the greenhouse effect, acid rain or deforestation [2]. Due to these concerns and to the growth of the energy consumption a change is necessary, not only in the way to obtain enough energy in order to satisfy the demand, but also in its efficient and intelligent use.

Within this issue, the role of transport has a significant importance as the global number of vehicles is increasing mainly due to emerging countries such as China and India [3]. This represents a huge increase in CO_2 emissions and a decrease in oil reserves. Due to this increase in pollutant emissions, urban mobility will be reduced within urban cities, as it is already happening in cities such as London [4] and in the future, more cities will promote the mobility reduction of high emission cars from their urban areas.

To solve the problems that arise with conventional internal combustion engine vehicles, ecological vehicles are being developed. These are hybrid vehicles (internal combustion engine and package of batteries) and pure electric vehicles. Hybrid vehicles are a short-term solution, although in the medium term the best solution is electric vehicles due to their zero emissions. For a long-term solution and definitive implementation of these vehicles, high investments are required in order to adapt and increase the electric network. Therefore, the electric vehicle presents itself as a future alternative as far as urban transport is concerned, since it brings with it a considerable reduction of the environmental pollution.

For more than ten years, electric vehicles have been available as passenger cars and heavy vehicles. However, in most cases, their life cycle costs are still higher than those of conventional vehicles [5]. Regarding the electrical buses, even though the capital costs are high, their lower energy consumption significantly reduces the operating costs converting them in a potential alternative for conventional fossil fuel city buses. To increase the competitiveness of the electric buses, the total costs need to be reduced. This can be done by optimization of the route as well as the design of the charging infrastructure, since the number of charging points has a significant influence on total costs [6]. To carry this out, accurate models of electric buses are needed in order to perform reliable simulations.

1.1 Background

Since 1970´s, when the interest in hybrid vehicle simulation grew in order to predict the performance before prototyping phase [7], modelling and simulation has become an important tool for electric vehicle design. Since then, several computer programs have arisen to describe the operation of electric vehicles, including: SIMPLEV [8], MARVEL [9], CarSim [10], JANUS [11], ADVISOR [12], Autonomie [13] and Simulink [14]. These are MATLAB-based software developed for automotive control-system design, simulation and analysis.

A large number of analyses and simulations have been performed using driving cycles based on actual data, noting the great influence of the driving cycles on the consumption and the traction performance of vehicles [15]. Although consumption and efficiency of a city bus are highly dependent on engine and vehicle characteristics, other external factors can modify the consumption such as travel speed, number of stops, traffic state, road grade and passenger

load [16]. In order to develop methods for consumption prediction and optimization, it is necessary to make a quantitative exploration of the effects of these factors. Following this trend, Joel Anttila in his Master´s Thesis [17] developed a validated energy flow model of a battery electric bus by means of Autonomie program, run in MATLAB. Moreover, a driving cycle sensitivity analysis was done indicating the influence of the uncertainty in the number of stops in energy consumption.

In addition to the number of stops, one of the most important sources of uncertainty that affect the electric consumption is the passenger volume [18]. However, prediction about the passengers´ arrival to each stop has not been deeply studied before [19]. Studies about passenger arrival can be found only to predict the number of vacancies of a bus or the transit demand [20], but not to analyse their influence in the vehicle consumption. For example, in the study realized in [20], Kalman-filter algorithms are applied to make predictions about travel times and arrival times. There are also several studies about the impact that boarding and alighting activities have on the bus dwell time and therefore on driving time. There is a study where the analysis is carried out at a microscopic user-by-user level with an electronic smart card system [21]. Furthermore, origin-destination matrices have become a requirement in transport planning [22]. These dynamic passenger matrices describe the demand between different transportation zones and they are helpful to predict the passenger arrival at each stop. According to the available literature, most of the studies carried out on passenger loads are aimed at transport planning [23], [24].

On the other hand, there is little published work about the influence of passenger loads on bus fuel consumption [25], although a high correlation between fuel consumption and number of passengers has been found [26]. However, generally in consumption modelling, passenger load is neglected and a fixed vehicle mass is used [25]. Therefore, there is a gap in this issue and that is the motivation of this research.

1.2 Research Problem

Knowing the variation of the electric consumption, size accurate batteries could be designed and charging stations could be better settled [18]. This optimization leads to improvement in cost and efficiency of the electric bus, as well as reduction in environmental pollution.

As the objective is to assess the vehicle performance, one of the main factors involved (apart from the design aspects) is the environment where the electric vehicle is moving through and some characteristics of its movement such as speed, accelerations and decelerations. This can be analysed by using the driving cycles which represent the speeds to which a vehicle would circulate under certain traffic conditions or environment. Moreover, as it has been explained above, it is important to analyse as well the influence of the mass fluctuations in the bus, caused by the passengers, as a factor with great effect in consumption. The quantity of passengers and their weight affect the driver´s behaviour as the vehicle´s speed changes.

Taking into account the importance of the passenger factor, the research problem addressed in this thesis is the energy consumption quantification of an electric city bus depending on the mass of the bus, through the variation of the number of passengers inside the bus.

1.3 Objective and Scope

Hence, the aim of this thesis is to evaluate the impact of passenger loads on energy consumption of an electric city bus and provide a reliable energy consumption forecast. To achieve that goal, an energy consumption sensitivity analysis is created, with the number of passengers in the bus as uncertainty. This uncertainty is stochastically modelled for each stop in the bus route and it is evaluated with Monte Carlo Method. In addition, the uncertainty in the number of stops is also considered in this study.

The scope of research includes the statistical analysis of passenger fluctuations in each stop of the bus route, followed by the creation of the passengers flow for each driving cycle. In addition, a mathematical algorithm to calculate the mass for the energy flow simulation model is developed, and the consequent simulation and validation of the model are done. Finally, electric consumption forecast is estimated by means of Monte Carlo method and the results are analysed.

This study has some limitations, as only the uncertainties in the number of stops and passenger load are taken into account. Other influential factors such as traffic or road grade are left out of the study.

On the other hand, only one type of electric city bus configuration and one specific route fragment are studied, hence the forecast obtained with this analysis is only valid for these particular bus and route. Accordingly, passenger load sensitivity analysis must be carried out for each bus and route combination individually. Thereby, more size accurate batteries will be designed, depending on the bus and the route that it follows.

Furthermore, the data acquired for the study is slightly limited, and only the number of passengers inside the bus after each stop was registered. Thus the exact number of passengers boarding and alighting in each bus stop is not known, just the difference between them.

Another limitation is that the electric bus model used to perform the simulations was computed with some simplifications, and only longitudinal dynamics were taken into account, excluding all lateral effects.

Moreover, only the mass influence on electric bus energy consumption is analysed, thus the number of boarding and alighting passengers in a bus stop is only used to determine the flow of passengers inside the bus and to be able to calculate the mass during the driving cycle. However, the influence of boarding and alighting of passengers on the dwell time and route time is not taken into account.

1.4 Methods

The route analysed in this thesis is an urban bus route operated by line 11 in Espoo, Finland. Only eight bus stops of the route and their corresponding passenger fluctuations are considered in this study.

First of all, the data are statistically analysed with Matlab, creating a probability distribution for boarding and alighting passengers in each bus stop. Subsequently, these distributions are used to build an algorithm that defines the passengers flow according to the driving cycles generated in reference [17].

On the other hand, the bus mass is defined for each driving cycle, taking into account the passengers flow, and it is included in the energy flow electric bus simulation model. This electric bus model was done in Simulink and it is run with Matlab.

Finally, Monte Carlo method is used to analyse the sensitivity to passenger flow in the driving cycles, and thus avoid measuring the passengers in hundreds of driving cycles. In this way, the number of passengers is sampled with this method to quantify its influence on the electrical consumption.

2 Literature Review

In this chapter, a general overview of the drivetrains incorporated into transit electric buses is presented, focusing on battery electric buses, hybrid electric buses and fuel cell electric buses. In addition, Monte Carlo method is described, as well as the three steps needed to perform the Monte Carlo simulation.

2.1 *Electric and Hybrid Electric Buses*

Electric vehicles are those that use an electric motor to propel the vehicle, regardless of the power source used. They can be classified into two large groups: purely electric vehicles (EV) and hybrid electric vehicles (HEV). In both cases, several energy storage systems can be used such as battery, ultracapacitors, fuel cell, or solar cells, among others. The model of bus used in this thesis is a battery electric bus.

The development of electric buses has somehow followed the development of electric passenger cars, which began receiving greater attention in the 1990s [27]. During that decade, most of the research on electric buses was focused on pure battery-powered systems. However, from the late 1990s, hybrid systems began to attract more attention for both heavy-duty and light-duty vehicles [27]. Hybrid systems with diesel or gasoline engines combined with an electric motor arose as a viable alternative to solve the limitations of battery technology available in those days.

Nowadays, hybrid buses have some of the benefits of electric vehicles, such as the higher efficiency and the reduction of emissions and noise, while maintaining the performance and vehicle utility of diesel fuel based buses. However, emerging battery storage systems, such as ultracapacitors, flywheels, hydraulic systems and lithium batteries, are attracting continued research and investment to improve their characteristics [27]. Also, investigators have considered fuel cells development as the next phase of electric power source, and transit buses have been used as a test bed to advance this technology toward commercialization [27].

This section will present a general overview of the drive technology incorporated into transit electric buses. Furthermore, the main characteristics of each technology and the advantages and disadvantages of electric and hybrid vehicles will be stablished. In this section only electric buses with on board power source are considered, leaving the electric trolleybuses outside the scope of this review, as they draw electricity from overhead catenary lines [27].

Therefore, the three types of electric drive systems that are here considered are battery electric buses (BEB), hybrid electric buses (HEB) and fuel cell electric buses (FCEB), since they are the ones that are currently being implemented in transit buses [27].

2.1.1 Battery Electric Buses

Battery electric buses are often called pure electric buses because they are powered only by the energy stored in the battery. The battery pack may be rechargeable or may be changed when the batteries are depleted.

The drive system for a BEB consists of a battery pack that provides energy storage, an electric motor that is powered by that energy and a control system that governs the operation

of the vehicle [27]. EV are, from the point of view of the mechanical components layout, simpler than the current internal combustion vehicles. The internal combustion vehicle has many more components and these are subjected to higher wear and tear. The BEB powertrain configuration is shown in figure 1.
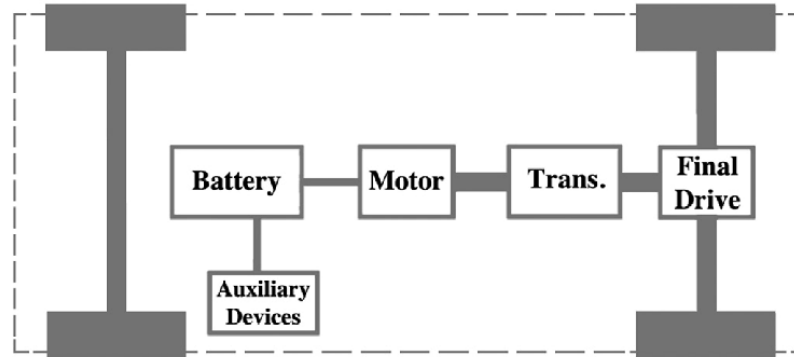

Figure 1. Battery electric bus powertrain configuration [28]

Electric buses offer a number of advantages over conventional buses. First, the efficiency of the electric motor is higher, and produces less noise than an internal combustion engine (ICE). Electric motors provide the highest torque at low speeds, which leads to better accelerations from a stop. The motors used in EVs cover the different existing technologies, both DC and AC. However, asynchronous motors have been consolidated as the most suitable for electric mobility due to their robustness and simple maintenance. Electric vehicles also improve energy efficiency with the addition of regenerative braking, which is one of the key benefits of electric drive systems [27].

Regenerative braking systems (RBS) are braking mechanisms that allow the generation of electric energy from the kinetic energy of the vehicle to recharge the batteries [29]. The idea is to recover part of the energy involved in braking. In friction brakes of conventional vehicles this energy is released as heat, since there is no mechanism to recover it [27]. In the case of electric powertrains, the electric motor itself assists in braking, acting as generator during this braking process. An electric motor is basically a generator, to which an electric current is applied instead of generating it. In electricity generation, the current flow is opposite in comparison to what happens when the generator acts as a motor, producing now a torque applied in the opposite direction of travel. It is important to notice that the RBS does not completely replace the conventional friction brake, since at low speeds it loses effectiveness. In addition, the system may fail and it requires a brake backup. Furthermore, an emergency may require too much braking above the system capacity that can only be achieved through conventional braking [29].

Battery electric propulsion systems are a relatively new and promising technology. In contrast, there is still concern about their energy capacity to feed large city bus throughout the day [30]. Until recent years, electric buses have been limited to small capacities (from 7 to 9 meters long) and with maximum speeds of 40 to 65 km/h because they were conditioned by the volume and weight of the on-board battery. Nevertheless, some technological innovations have been carried out to reduce the weight of the buses, e.g. the modification of the chassis material by a lighter one [30]. On the other hand, the growing development in battery manufacturing technology is significantly increasing the range and power of batteries

and it is expected to continue to do so in the future. Despite this, battery-powered buses have not been able to reach enough scope at a commercially competitive cost. As a result, battery electric buses nowadays are mainly found in the medium-duty bus market [27]. In contrast, the main advantage of battery electric buses is their independence from fossil fuels and the reduction of pollutant emissions.

2.1.1.1 Battery Technology

As mentioned above, battery technology improves year by year and can be considered the most critical component in the battery electric propulsion system. The batteries contribute to a significant part of the weight of the bus and their performance can vary depending on several factors [30]:

• Specific energy: the ratio between the battery's energy output and its mass, typically measured in watt-hour per kilogram.

• Specific power: the ratio between the battery's power and its mass, expressed in watts per kilogram.

• Cycle life: the number of complete discharge-recharge cycles a battery can go through before its capacity declines to 80 per cent of its original capacity.

• Calendar life: the amount of time that a battery can provide power and capacity for its uses.

Table 1 shows the characteristics of existing battery technologies for BEBs.

Table 1. Battery technology for battery electric buses (table is based on [30])

Chemical Composition	Specific Energy (Wh/kg)	Specific Power (W/kg)	Cycle Life (number of cycles)	Relative Cost
Lead Acid	35	200	500-800	$
Nickel-Cadmium	30	260	1000	$$$
Nickel Metal Hybrid	45-75	850	900	N/A
Sodium Nickel Chloride	995	170	1000	$$$
Lithium-ion	100-180	700-1300	1000-4000	$$$$
USABC Minimum Goals	150	300	1000	$
USABC Long-Term Goals	200	400	1000	$

Figure 2 represents a roadmap with the evolution of battery technologies for electric buses and their current state, differentiating between mature technologies, new technologies

already introduced and promising technologies in development. The strengths and weaknesses of each technology are also shown in the figure.
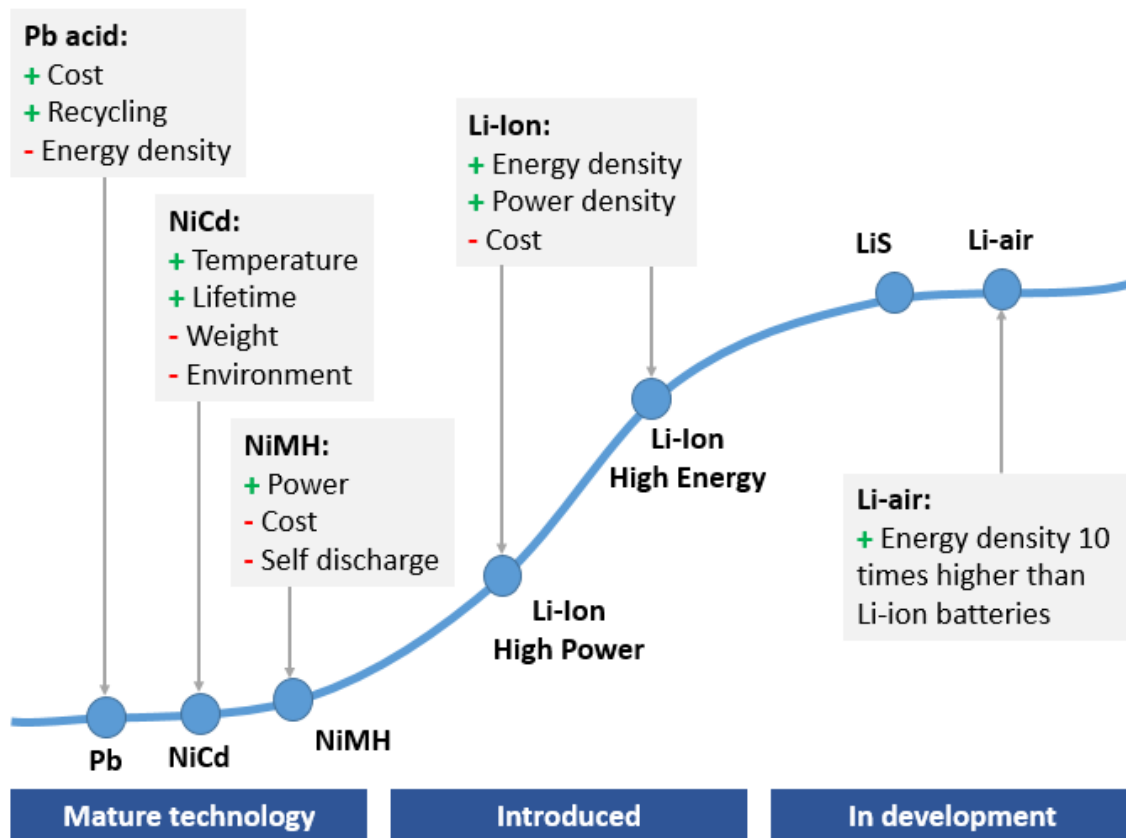

Figure 2. Evolution of battery technology for battery electric buses (figure based on [30])

Usually, BEBs have a large amount of batteries to reach a sufficient driving range. This leads to the increase of bus weight and space required in it. To solve this problem, the manufacturers produce electric buses with high-power ultra-fast battery charging systems [31].

2.1.2 Hybrid Electric Buses

As noted above, HEB seeks to combine the advantages of electric traction such as quiet operation, higher torque and lower emission levels; with the operating range of fuel sources. A hybrid system combines propulsion of an electric motor with another source of energy such as an ICE engine, a turbine or a fuel cell stack [27]. In the case of hybrid electric transit buses, electricity and an ICE are typically the two sources of power [30], and therefore they contain two different engines: an ICE engine and an electric motor.

In HEVs the most used fuel for the ICE engine is low-sulphur diesel, although other sources of energy can also be used such as gasoline, compressed natural gas (CNG), biofuels or biogas [27]. Hybrid transit buses commonly use a diesel engine as well. Gasoline engine and CNG have been also introduced in transit buses [30], but they are less frequently used. The majority of hybrid transit buses combine these engines with an electric motor and batteries, although in some cases hybrid buses replace the battery packs with an ultracapacitor. This is

an advanced energy storage system and, unlike a conventional battery, ultracapacitors have a longer life and provide a higher power density (while batteries provide a higher energy density) but only for a short period of time [27], [31].

The combination of conventional vehicles and electric vehicles gives hybrid bus operators the advantages and disadvantages of both types of vehicle. On the one hand, by having an internal combustion engine the operational range of the vehicle is no longer a problem since it presents the same refuelling facility as conventional vehicles. Furthermore, the hill-climbing power is greater than in pure battery electric buses [27]. On the other hand, being equipped with an electric motor implies a higher energy efficiency, better acceleration from a stop, a significant fuel saving and quieter operation [27].

The most notable feature of plug-in hybrid electric buses (PHEBs) compared to conventional hybrid buses is the possibility of recharging the batteries not only by the combustion engine (standard hybrids) but also by the connection to the electrical network (plug-in hybrids), causing so further increasing of the energy efficiency. The key of PHEBs is the possibility to run in electric mode a certain amount of time. The time run in electric mode depends on the route characteristics, bus configuration and charging frequency [31]. The majority of plug-in hybrids run with batteries while most of the standard hybrids run with supercapacitors. This is because the capacitors are electrostatics and they can release charge rapidly. Otherwise, batteries are based on a chemical process and the discharge is slower [31].

Therefore, hybrid-electric buses configuration combines the elements of battery electric bus described in section 2.1.1 (electric motor, battery packs and control system) with an ICE engine coupled to an electric generator [27]. They also include the regenerative braking technology that allows them to recover and store part of the energy involved in braking [30]. Generally, hybrid buses' drivetrains can be configured in series or parallel, each one with a different mode of operation according to the direction of the energy flow [27].

2.1.2.1   Series Hybrid System

In series hybrids the combustion engine is connected to a generator that transforms the mechanical energy produced by the fuel combustion into electric energy. The generator is connected to an electric motor and to a battery system being able to supply power to either of them. In this configuration the electric motor is in charge of the traction of the vehicle as it is connected to the vehicle´s transmission system, while the combustion engine is in charge of providing it with energy and recharging the batteries with the surplus, as it is completely mechanically decoupled from the drive axle [27]. In figure 3 series configuration is illustrated.
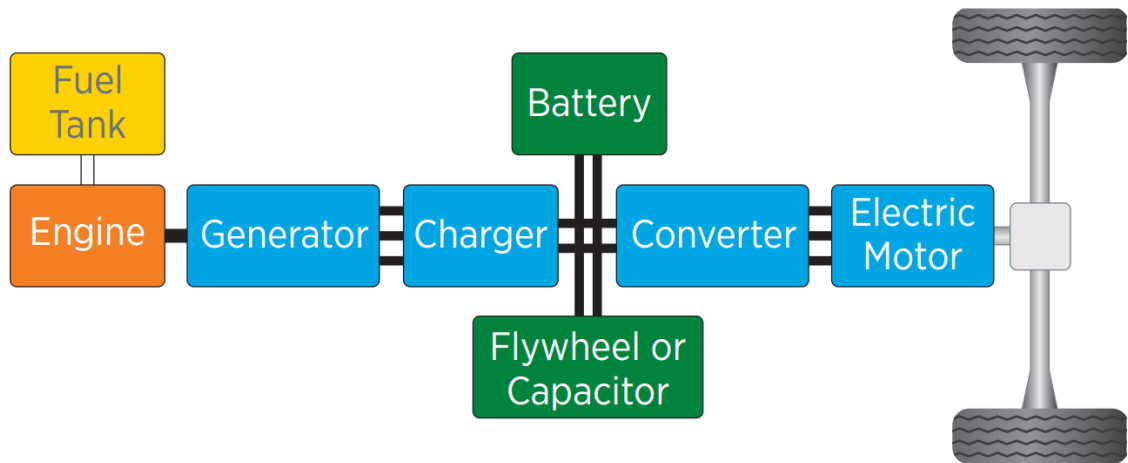
Figure 3. Series hybrid bus powertrain configuration [32]

PHEBs can operate in the following ways:

• Single energy source: The PHEB's electric motor is supplied either by the battery or by the generator. In the event that the generator produces more energy than necessary, the excess is derived to the batteries for later use.

• Mixed energy source: The electric motor is fed simultaneously from both the battery and the generator. It takes place in situations of high power demand.

• Regenerative braking: In case of deceleration or braking the resulting energy is sent by the electric motor to the batteries for storage and subsequent use.

The main advantage of this configuration is that the thermal combustion engine is not directly connected to the transmission, which allows the operation at constant speed and at the optimum operating point, which is not the case in conventional vehicles [31]. The ICE engine can be switched off for a period of time, allowing an all-electric operation [27]. As a drawback, it should be noted that in order to provide competitive performance the dimensioning of both engines could result in high dimensions machines, which implies an increase in the overall cost of the vehicle [30]. Another disadvantage are the losses produced by the energy transformations taking place throughout the process.

2.1.2.2 Parallel Hybrid System

Parallel hybrid vehicles have two parallel drive systems. According to this configuration, both the ICE engine and the electric motor are connected to the transmission system of the vehicle. The connections are direct and independent and each of them can provide mechanical power to the wheels allowing the movement of the vehicle [27], [30]. The two systems can be used independently or simultaneously to obtain maximum power. Parallel drivetrain configuration is illustrated in figure 4.
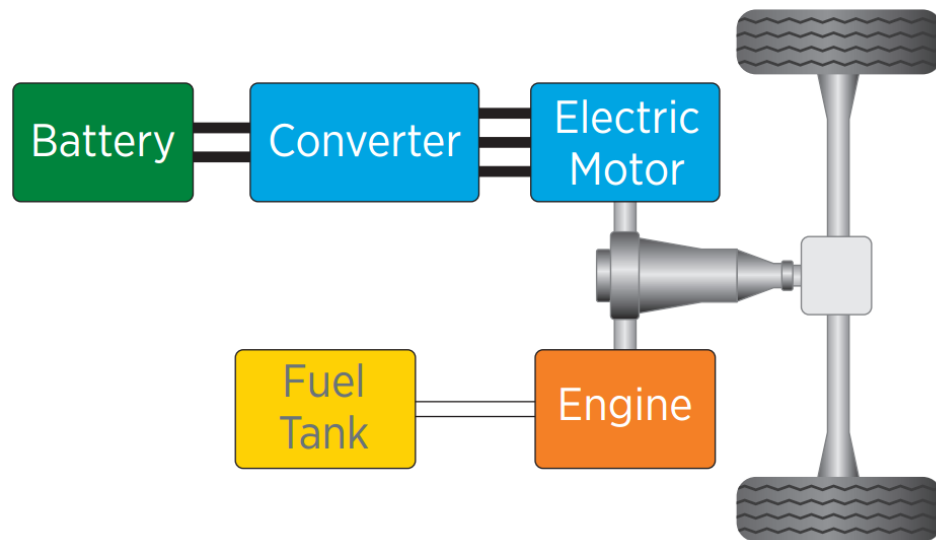
Figure 4. Parallel hybrid bus powertrain configuration [32]

Although parallel hybrids are mechanically more complex, this configuration avoids the losses inherent to the conversion of mechanical energy into electric energy that takes place in series configuration.

Parallel hybrid systems are designed to power the bus using the ICE engine at high, constant speeds, and using the electric motor at low speeds and during stop and go traffic [31]. Both power sources can operate together during accelerations [27], or in some cases, the electric motor only provides supplemental power [30].

When circulating in city, much power is not needed and the electric motor allows a remarkable saving of fuel and zero emissions to the atmosphere as it can be fed with the energy stored in the batteries. These batteries can be recharged by connecting the vehicle to the electrical network or by the generator coupled to the internal combustion engine. In this configuration the regenerative braking is also implemented to recover some of the energy lost during braking and charge the battery [31]. The parallel hybrid can operate in the following ways:

• Fully electric mode: The vehicle only moves thanks to the action of the electric motor, whose power is provided by the batteries. This mode of operation is reserved for situations of low power demand.

• Combined mode: Both engines operate simultaneously by pulling the drive wheels, being able to operate in situations of high demand of power due to the combined action of both motors.

• Regenerative braking: In case of deceleration or braking the resulting energy is sent by the electric motor (acting as generator) to the batteries for storage and subsequent use.

Thanks to the possibility that both engines work simultaneously, both the power and the dimensions can be lower, being that a significant advantage over the hybrid vehicles in series.

2.1.3 Fuel Cell Electric Buses

During the latest years, fuel cell technology has garnered great attention as it offers a clean, efficient and fossil fuel independent transportation system [30]. This technology is interesting for bus applications as the storage space for hydrogen is not a problem in larger vehicles, and they do not need to use such a high level of compression.

Fuel cell technology generates electrical energy by combination of hydrogen and oxygen (from air) in an electrochemical process. The transformation also generates water (evacuated as a vapour) and some nitrogen, which are not polluting emissions [30]. Fuel cell technology can be used as the prime power source in an electric drive system (replacing the batteries) or as the prime power source in a hybrid drive system (replacing the ICE engine and generator) [27]. The electricity produced by the hydrogen stored on the vehicle can be generated continuously since fuel cells do not need to be recharged, unlike batteries. The electricity generated is then transferred to an electric motor that is connected to the drive axle [30]. The drivetrain configuration is shown in figure 5.
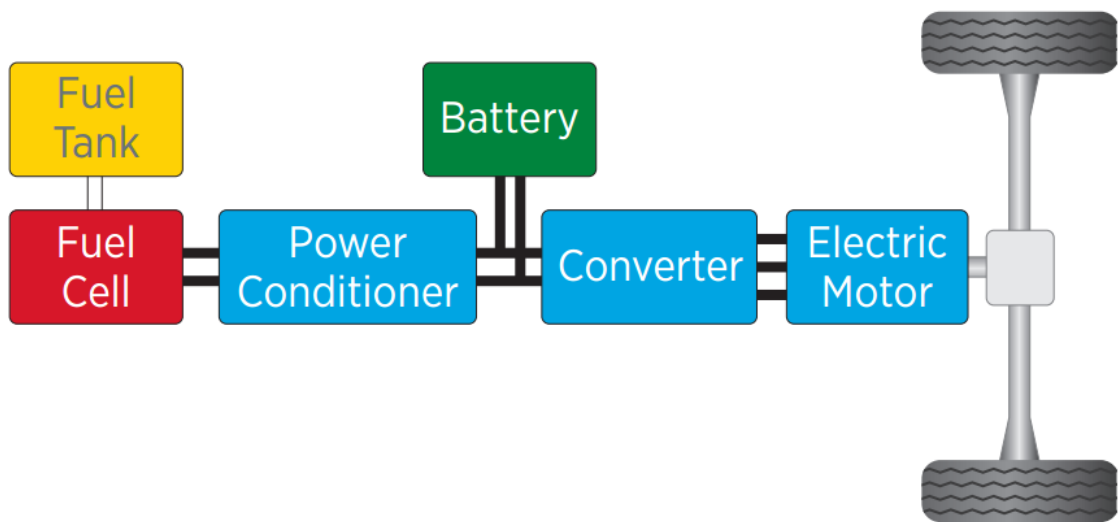
Figure 5. Fuel cell hybrid electric bus powertrain configuration [32]

One of the major problems of this technology is obtaining the hydrogen. On the one hand, there is the environmental problem: hydrogen is mainly obtained by the natural gas reforming, so the production itself generates emissions. However, it can also be produced from clean sources such as wind or solar technologies. On the other hand, there is the economic problem: hydrogen cost cannot compete with electricity cost. Furthermore, fuel cells' electrical efficiency cannot compete with the efficiency of a battery [32], and the fuel storage, robustness and durability are still many years away from those of other technologies [27]. Thus, this technology has substantial room for improvement if it is to compete with current transportation technologies.

The US National Renewable Energy Laboratory has compared in a recent report [33] the energy efficiency of different buses in real-life applications. The results are the following:

- Diesel buses have a consumption of 56 litres of diesel per 100 kilometres.

- Hydrogen fuel cell buses have a consumption of 39.2 litres of diesel equivalent every 100 kilometres.
- Natural gas buses have a consumption of 71.28 litres of diesel equivalent every 100 kilometres.
- Battery electric buses have a consumption of 13.84 litres of diesel equivalent every 100 kilometres.

As it can be seen, battery electric bus has no equal in terms of energy consumption. Fuel cell bus has an energy consumption 30% lower than a diesel bus and 45% lower than natural gas bus. An improvement of 30% compared to a diesel bus is not bad, but there is no comparison possible with the battery electric bus, which consumes 75% less energy than the diesel bus.

The components of a fuel cell bus are shown in figure 6.
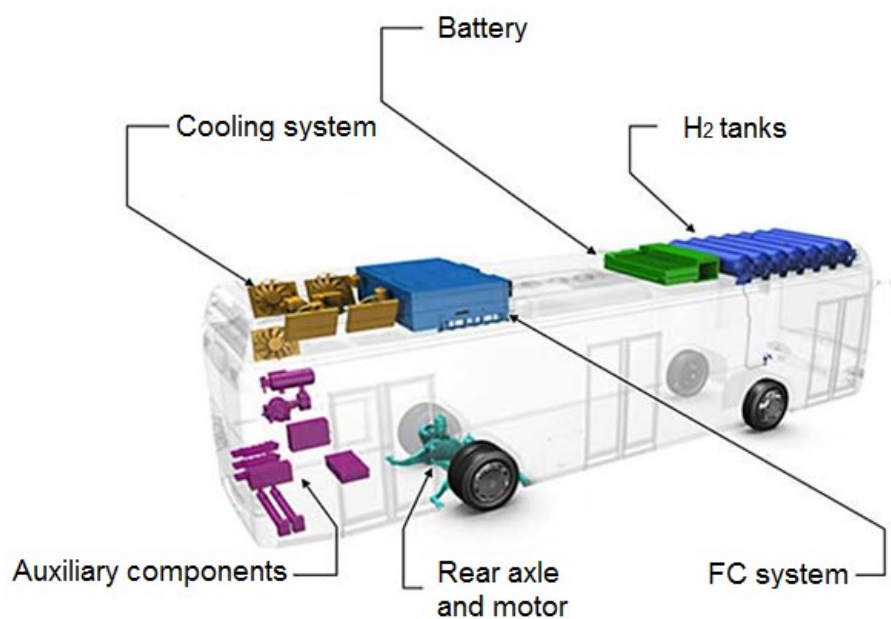
Figure 6. Fuel cell electric bus components [30]

Although one of the biggest challenges of this technology is the on-bus hydrogen storage, as it requires high pressure, other problems such as infrastructure and maintenance costs make it difficult to place fuel cell buses in the market. In spite of this, it is an attractive technology for transit bus applications since they have greater efficiency and quieter, smoother and zero emission operation [30]. At present, there are no long-term commercial hydrogen fuel cell buses in operation: however there are several projects going on and different companies are developing prototypes [30].

2.2 Monte Carlo Method

Monte Carlo method is a powerful technique that allows the solving of physical and mathematical problems by means of repeating random sampling to compute the results. It is one of the most used computational methods as it helps to solve complex problems which are infeasible to compute an exact result with a deterministic algorithm [34]. This method was so named because of its clear analogy with the roulette games of casinos, the most famous of which is the Monte Carlo one, in Monaco.

Monte Carlo methods are mainly used in three types of applications: optimization problems, numerical integration and generating samples for a probability distribution [35]. However, due to its stochastic nature, it is used in a wide range of study fields, such as finance [36], physics [37], [38] and biomedicine [39], [40]. In addition, Monte Carlo method has also been used in engineering, including the electric buses field, mainly for the analysis of energy storage systems [41] and electric power systems planning [42]. Thanks to the advance in computer design, Monte Carlo simulations that once would have been inconceivable, today they are available for the resolution of this kind of problems.

Monte Carlo simulation can also be described as a method for propagating uncertainties in model inputs into uncertainties in model outputs. Typically, the model of study depend on a number of different input parameters, which are processed through mathematical expressions in the model, and the results, which can be one or several outputs, are obtained. Taking into account that the outputs of a model with uncertain inputs have to be uncertain as well, the model performance can be predicted by representing the inputs as probability distributions. Hence, the solution of the model is going to be another probability distribution [34].

Therefore, a Monte Carlo simulation is carried out by associating a statistical distribution to each one of the input parameters. Then, random samples from each distribution are generated and they are taken as the values of the input variables. Subsequently, the inputs are introduced in the model and results are obtained, getting a set of output parameters for each set of input parameters, which belong to a particular scenario. The results are collected from a large number of simulations and a statistical analysis of the output parameters is performed and so the output variation can be characterized [43].

Following the process explained above, three steps are required to perform a Monte Carlo simulation [34]. First, the uncertain inputs have to be chosen and modelled. Second, the sampling method and the number of iterations that are going to be simulated are selected. This has to be determined in order to acquire as accurate results as possible, depending on the problem of study and its complexity. The third step is the simulation of the model and the collection and statistical analysis of the outputs. The Monte Carlo process is shown in the schematic diagram represented in figure 7 and, in the following sections, these three steps are going to be more deeply explained.
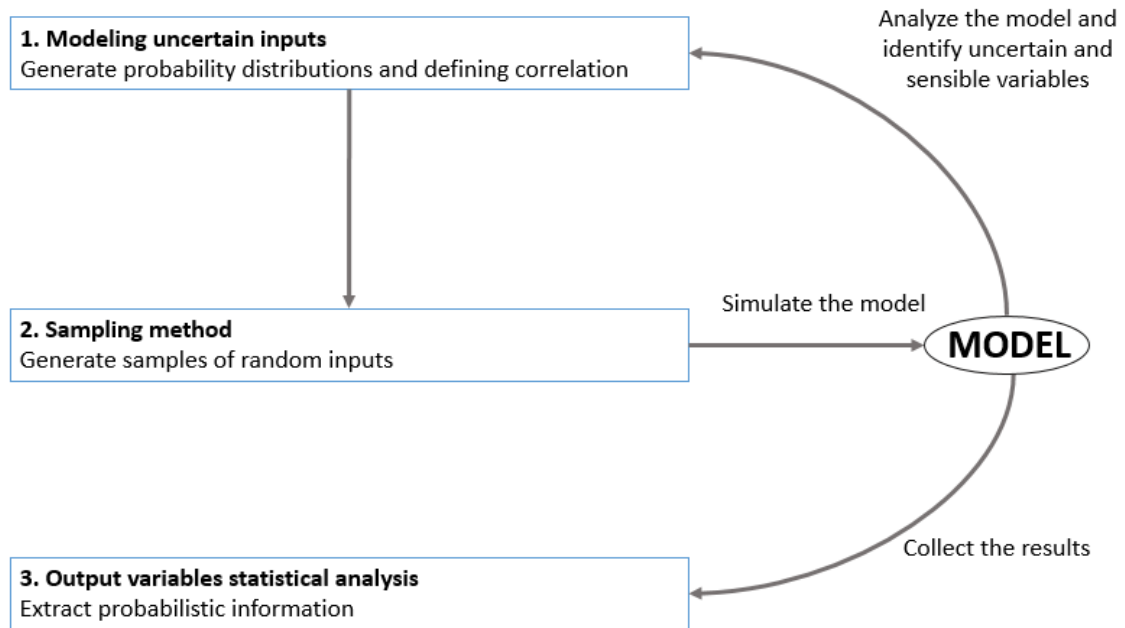
Figure 7. Monte Carlo simulation process with three main steps: modelling the uncertain inputs, sampling the model and output analysis.

2.2.1 Modelling Uncertain Inputs

The uncertain variables are modelled by probability density functions (PDF), basing their characteristics on expert judgements, theoretical knowledge or real data. This first step will have a deep impact on the simulation results as it determines the samples that are going to be simulated. Thus the PDFs have to represent the full range of possible values of the uncertain input, but they also have to highlight the high probability areas as well as the low ones [34]. In this way, the risk associated with each input variable is taken into account, including the best possible scenario and the worst one.

According to the literature [43], there are different standard statistical procedures to identify the most suitable input distributions, often called distribution fitting. Numerical methods are used to fit historical data to one theoretical discrete or continuous distribution. The objective is to find the probability distribution most suitable for a given set of data. A few standard procedures exist, but the most commonly used are Method of Maximum Likelihood (ML), Method of Moments (ME) and Nonlinear Optimization. However, the Maximum likelihood method is by far the most used for estimating the unknown parameters of a distribution. The fundamental idea of this method is to take as an estimate of the studied parameter the value that maximizes the probability of obtaining a particular result from a sample [44].

On the other hand, several probability distributions can be used to fit a dataset, both discrete and continuous. Within the discrete distributions, the most common are Binomial, Geometric and Poisson. In the case of continuous distributions, the most common are Normal, Lognormal, Uniform or Exponential.

For example, as it is shown in figure 8, the historical data can be represented as a histogram which visually reveals the primary shape of the distribution. Then, the best-fitting probability

distribution is chosen by means of maximum likelihood method which provides an algorithm that chooses the parameters that maximize the likelihood function for the data. In this example, four normal curves characterized with different parameters are represented and the maximum likelihood algorithm will choose the best-fitting one.
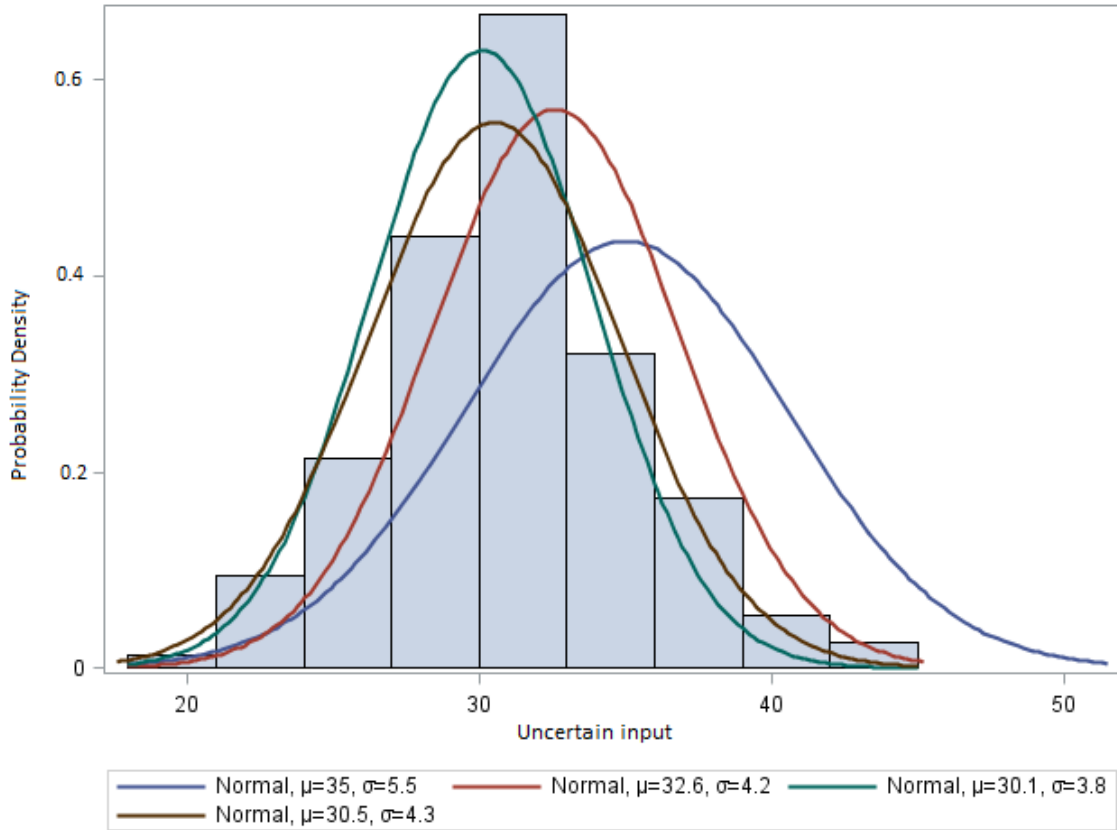
Figure 8. Maximum likelihood method. Four normal PDFs with different parameters try to fit the data. Maximum likelihood method will determine the set of parameters that best fit the data.

Furthermore, in this first step the correlation or dependencies between input variables must be defined as they may have a significant influence in the model outputs. Large-cross correlation reduces the "degrees of freedom" by limiting the value range that the variable can acquire [45]. Hence, it is important to include correlations in Monte Carlo simulation rather than assuming independent variables. In order to generate correlations among stochastically dependent random variables, they can be linked to each other via linear combination.

2.2.2 Sampling Method

There are different sampling methods, such as hypercube sampling and quasi-Monte Carlo sampling, but the most popular of them is Monte Carlo sampling [34]. This method uses pseudo-random numbers, between 0 and 1, to approximate a uniform distribution.

The goal of Monte Carlo sampling is to generate samples of the input variables X= (X1, X2,…Xn) according to their distributions. For this purpose, two steps are involved [46]:

- Step 1: Generation of random variables uniformly distributed between 0 and 1.

The key of this step is that uniform numbers within the range [0, 1] can be turned into real values that follow any distribution of interest. Nowadays, any computer has the capability to generate random variables uniformly distributed between 0 and 1. Random variables generated this way are called pseudorandom numbers. Although this sequence of numbers is not truly random sequence, they satisfactorily simulate the behaviour of true randomness. Sources of truly random numbers are rare in practice, as they do not produce the number of values required for practical purposes, so they have to be replaced by pseudorandom numbers.

- Step 2: Transformation of the uniform variable values, obtained previously in step 1, to the values of random variables that follow the probability distribution of each input variable.

The samples of uniform variable, $z = (z_1, z_2, \ldots, z_N)$, where N is the number of samples, will give N random variables through a transformation method. There are several methods to carry out this transformation but the most simple and direct is the inverse transformation method. By this method, the random variable in given by:

$$x_i = F_{X_i}^{-1}(z_i), \quad i = 1, 2, \ldots, N \qquad (1)$$

Where $F_{X_i}^{-1}$ is the inverse of the cumulative distribution function of the random variable X_i.

After the samples generation, they are introduced in the model and the output variables are calculated through the performance function $Y = g(X)$, corresponding to the model algorithm. In this way a large number of experiments will lead to a set of output samples and the variable Y will be available for statistical analysis. If N samples of each random variable are generated, which means N input sets to the model, N output sets of samples will be obtained.

In Monte Carlo methods the error is approximately $1 / \sqrt{N}$, and therefore gaining a decimal figure in precision implies increasing N by 100 times. These methods have an absolute error that decreases according to Central Limit Theorem. This theorem practically ensures that if the number of iterations (N) is large enough, the output stochastic variable tends toward a Gaussian distribution. Furthermore, if the number of iterations approaches infinity, the output variable will converge to the true solution [46].

Therefore, a number of iterations must be performed taking into account the trade-off between convergence in the solution and simulation time. The different output sets are collected forming a probability distribution, which is the core of Monte Carlo simulation.

2.2.3 Statistical Analysis of Output Variables

Finally, the third step of the simulation is tracking the results. The outputs generated are stochastic so they can be studied as a statistical distribution [34]. As mentioned before, the process is repeated until it has a statistically representative sample and the output values are aggregated into groups by size, displaying the values as a frequency histogram [46]. The

histogram reveals the approximate shape of the probability density function of the output variable.

The histogram data can be used to calculate an empirical distribution itself by means of calculating the percentiles and other statistics, or they can be fitted to a theoretical probability distribution, as it was explained in section 2.2.1 [43]. With this second possibility, the theoretical statistics can be calculated and can be used for determining the mean, variance, mode and confidence interval [34]. The mean is the average of all the observations and it can be interpreted as the desired result [39]. The mode is the most likely value to occur, i.e. the most repeated value in the output set. There may be more than one mode, even though it may also not exist if no value is repeated. The confidence interval gives an estimated range of values which is likely to contain the parameter of interest [47]. Confidence intervals are constructed at a selected confidence level, such as 95 %.

The knowledge of all these characteristics helps with the understanding of simulation results and facilitates the decision making about the model studied. The distribution represents the forecast of the problem studied and it includes all the possible outcomes taking into account all the uncertainties of the model. With all this data, a sensitivity analysis can also be performed to find out the input variables that cause more variation in the output parameter.

3 Methods

The generation of the passengers flow is the first step needed in order to proceed with the electrical consumption sensitivity analysis. The process to be followed consists of the statistical study of the data collected in the bus route and the definition of the probability distributions that best describe it. Subsequently, a passenger flow model is developed with variations in the number of passengers depending on the driving cycle.

In addition, the electric bus model that is used to perform the simulations is described in this chapter. Furthermore, the sampling method, including the number of iterations needed to achieve an adequate accuracy in the results, is also presented.

3.1 Data Processing

3.1.1 Data Collection and Driving Cycles Generation

The bus route that is the object of this study is the one corresponding to line 11 in Espoo, Finland, with direction from Friisilä to Tapiola. The total length of the route is 10.1 kilometres but, due to the occurrence of constant interruptions in the GPS signal in each measurement and to simplify the data processing, only 3.3 kilometres of the route will be analysed in this study. In the figure 9 the total length of the route is shown and the part to be analysed is highlighted. It is a section with 8 bus stops and it is characterized by having a roundabout, an intersection with a turn and a speed limit of 40 km / h.

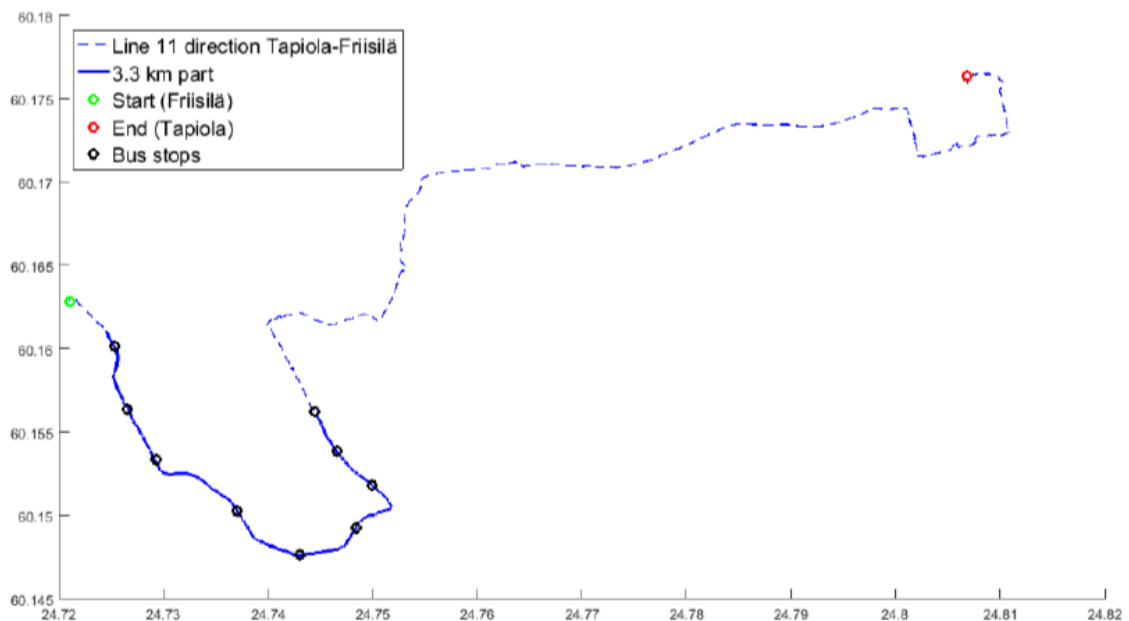**Figure 9. Line 11 bus route in Espoo. The solid blue line represents the 3.3 km section of the route to be analysed below [17].**

First of all, the data corresponding to the evolution of the bus speed during the time were collected in order to analyse the driving cycles in Master´s Thesis [17]. This data were taken on three consecutive days and at different times of the day, between noon and evening.

Moreover, the measurements were taken in a random way, including both electric buses and ICE buses operating on the route studied. The data collection was conducted on ten journeys, to characterize the different speed profiles of the route. Once the data were filtered, ten different driving cycles were obtained for the same section of the route. The driving cycles differ in speed, number of stops, acceleration and duration.

In order to study the uncertainty of the number of bus stops and their influence on the electric bus consumption, an algorithm was developed in Matlab [17]. This algorithm generates new cycles with a random number of stops and a random combination of actual measured driving cycles. Based on the ten driving cycles measured, the algorithm creates new driving cycles by randomizing the base cycle that is being used each time the bus stops at each station. Thus, the changes in the speed profile are achieved combining different parts of the actual driving cycles. The probability of stopping at each stop was set to 50%, so driving cycles with different number of stops are created. Therefore, the algorithm can alter the number of stops (adding or cutting out a stop) of the driving cycle.

Hence, it was intended to cover all the variations in the number of stops and speed profiles and thus perform a robust sensitivity analysis. As the new cycles created were based on actual measurements, they reflect situations that may occur in reality on the studied route and therefore they are valid for carrying out the study.

The knowledge of the driving cycles is highly important for this study since the passenger flows that are going to be generated are based on the characteristics of the driving cycles. On the one hand, it will be necessary to know specifically in which stop the bus stops in order to be able to associate the corresponding probability distribution. On the other hand, knowing the length of the segment between each stop will be useful to determine the time that a certain number of passengers remain on the bus. In addition, the driving cycle will be necessary when simulating the model of the electric bus in Simulink, as it is an essential input variable.

3.1.2 Passengers Data Processing

Likewise, the number of passengers in the bus was recorded in the complete route, only the 3.3 km part will be considered. However, in this case passenger's data were collected from a larger number of driving cycles. Besides the ten driving cycles of the previous study, passenger´s data from driving cycles of a different month were collected. This new data were taken during all the days of a single week and at different times of the day, with the aim of obtaining a greater range of variability and a higher number of measurements.

Table 2 shows the measurements taken, where the first ten rows correspond to the data of the first driving cycles measured and the following correspond to the new data recorded:

Table 2. Number of passengers in the bus after each stop.

Date	Departure time	Number of passengers in the bus							
		Stop 1	Stop 2	Stop 3	Stop 4	Stop 5	Stop 6	Stop 7	Stop 8
08/03/2016	14:31	1	0	0	0	0	2	2	2
08/03/2016	15:46	3	3	1	1	1	3	5	12
09/03/2016	12:37	1	1	1	1	2	2	4	5
09/03/2016	13:47	0	0	2	2	2	3	3	5
09/03/2016	16:15	1	1	2	3	3	3	3	5
09/03/2016	17:28	0	0	3	3	3	3	7	10
10/03/2016	11:52	1	2	3	3	3	3	3	3
10/03/2016	13:02	0	1	2	2	2	4	5	7
10/03/2016	14:12	1	2	2	2	2	2	3	5
10/03/2016	15:31	1	1	5	5	5	6	7	7
03/10/2016	17:52	1	1	1	2	3	3	3	7
03/10/2016	19:02	0	0	2	2	2	3	3	3
04/10/2016	14:46	2	2	4	4	4	4	4	3
04/10/2016	16:02	1	1	5	5	6	6	7	12
04/10/2016	17:27	0	0	0	1	1	2	3	3
05/10/2016	15:02	0	0	1	1	3	6	6	8
05/10/2016	16:46	0	0	1	1	1	2	3	5
05/10/2016	18:17	1	1	3	4	5	6	6	7
06/10/2016	13:47	1	0	0	0	1	2	3	4
06/10/2016	15:16	1	2	4	4	4	3	3	3
06/10/2016	16:46	2	2	4	4	4	2	4	4
07/10/2016	10:42	0	0	2	2	4	4	5	8

The measurements taken are the number of passengers inside the bus after each stop. When there is no change in this number between two consecutive stops, it could mean that the bus did not stop or that the number of passengers getting on and off the bus was the same. In order to implement the algorithm that will create the passengers flow in each driving cycle, it would be necessary for these collected data to be expressed as passengers boarding or alighting the bus at each stop. It should be noticed that to proceed with this calculation, it is necessary to know the number of passengers inside the bus before the first stop, which had also been collected. The modified data is shown in table 3 where negative values correspond to passengers alighting.

Table 3. Number of passengers boarding or alighting from the bus in each stop.

Date	Dep. time	Passengers before first stop	Number of passengers boarding or alighting (-)							
			Stop 1	Stop 2	Stop 3	Stop 4	Stop 5	Stop 6	Stop 7	Stop 8
08/03/2016	14:31	1	0	-1	0	0	0	2	0	0
08/03/2016	15:46	3	0	0	-2	0	0	2	2	7
09/03/2016	12:37	1	0	0	0	0	1	0	2	1
09/03/2016	13:47	0	0	0	2	0	0	1	0	2
09/03/2016	16:15	1	0	0	1	1	0	0	0	2
09/03/2016	17:28	0	0	0	3	0	0	0	4	3
10/03/2016	11:52	2	-1	1	1	0	0	0	0	0
10/03/2016	13:02	0	0	1	1	0	0	2	1	2

10/03/2016	14:12	1	0	1	0	0	0	0	1	2
10/03/2016	15:31	1	0	0	4	0	0	1	1	0
03/10/2016	17:52	1	0	0	0	1	1	0	0	4
03/10/2016	19:02	0	0	0	2	0	0	1	0	0
04/10/2016	14:46	2	0	0	2	0	0	0	0	-1
04/10/2016	16:02	1	0	0	4	0	1	0	1	5
04/10/2016	17:27	0	0	0	0	1	0	1	1	0
05/10/2016	15:02	0	0	0	1	0	2	3	0	2
05/10/2016	16:46	0	0	0	1	0	0	1	1	2
05/10/2016	18:17	0	1	0	2	1	1	1	0	1
06/10/2016	13:47	1	0	-1	0	0	1	1	1	1
06/10/2016	15:16	0	1	1	2	0	0	-1	0	0
06/10/2016	16:46	2	0	0	2	0	0	-2	2	0
07/10/2016	10:42	0	0	0	2	0	2	0	1	3

These data provide guidance to create the flow of passengers that could exist in this section of the bus route. Although it is an uncertain variable, the data will allow to generate probability distributions that can reflect what might happen in reality.

3.2 Mass Algorithm

The uncertain variable to be studied is the mass of the bus, which fluctuates due to the boarding and alighting of passengers. In order to perform the analysis, taking into account the actual data of the bus route, an algorithm is generated in Matlab. This algorithm is capable of generating a mass flow, depending on the driving cycle that the bus follows. However, the mass is not generated as a random variable obtained from a single probability distribution, but it is created with a more complex algorithm which can generate a value for the number of passengers boarding or alighting at each bus stop on the driving cycle.

To perform a robust electrical consumption sensitivity analysis, the algorithm must cover all possible variations in terms of number of passengers within the bus for each driving cycle. Therefore, different probability distributions will be used for each stop although they will have a correlation relationship between them that has to be modelled. Moreover, it is necessary to take into account that the flow of passengers must simulate the actual one in the route thus the probability distributions must resemble the data collected.

Instead of a single random variable that corresponds to the total mass inside the bus, nine random variables will be analysed. The first one corresponds to the number of passengers inside the bus before arriving at the first stop studied. The other eight variables refer to the number of passengers that get on or off in each stop of the route section studied. For this reason, once the probability distribution is selected, the correlation between the different variables must be modelled.

3.2.1 Selection of the Probability Distributions

First of all, the probability distributions that best represent the data collected must be chosen. To carry this out, the data referring to each bus stop are represented in histograms. The same is done with the number of passengers inside the bus before the first stop in the route. The histograms reveal the underlying shape of the probability distribution and they are traditionally used for estimating the probability density. They represent the frequency with

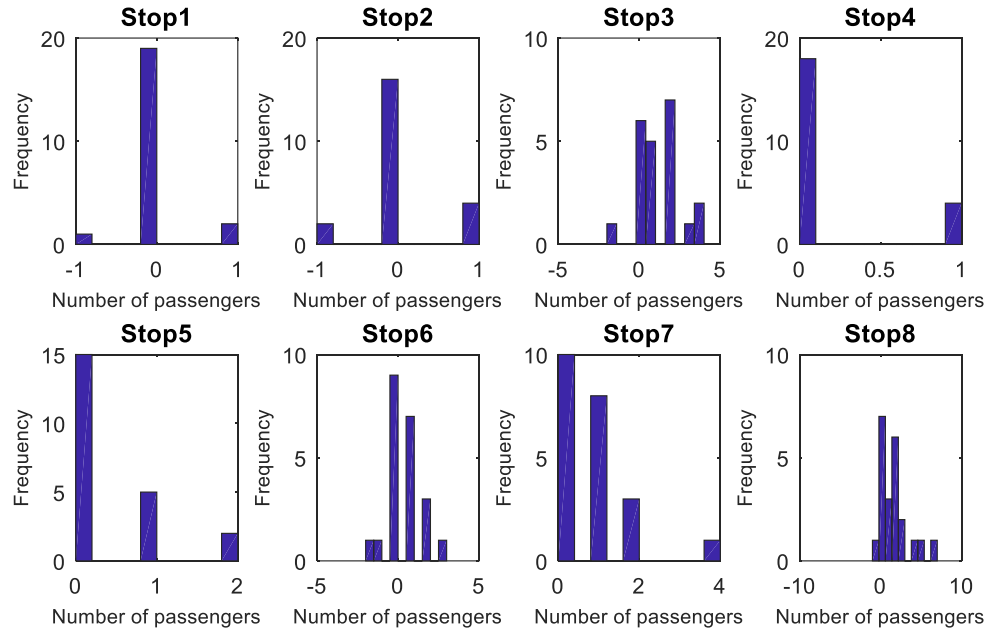which the values appear in the data collected. The figure 10 shows the histograms for the data of each stop.

Figure 10. Histograms for each stop in the measured data.

When the objective is to characterize a random variable from the available data it is assumed that the observed data is a random sample of a probability distribution that is being identified. To begin with, it must be decided whether it is a discrete or continuous variable. The data obtained in the sample belong to a discrete random variable, since it takes a finite number of values within a range, i.e. the number of people boarding or alighting the bus can only be integers (0, 1, 2 ...).

Taking the data of stop 6 as an example, since it is the one that has greater variability in values (from -2 to 3), the most suitable probability distribution that best fit the data is going to be defined. As a discrete random variable, the probability models commonly used in these cases are the Binomial and Bernoulli models. In this specific case, the variable to be studied will be fitted firstly to a Poisson model, which is usually used in queuing theory [48], [49].

The Poisson process generalizes in some way the Bernoulli process. It consists in observing the number of times that an event occurs in a certain interval (usually of time). In these processes it is assumed that there is stability and the number of events per unit of time (λ) remains constant [50]. λ ($\lambda > 0$) is the parameter of the model and its probability function is:

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!} \qquad\qquad x = 0,1,2,\dots \qquad\qquad (2)$$

Although the data collected is not adequate to obtain the passenger rate over time λ (t), an attempt to adjust them to a Poisson distribution was made. Therefore, Poisson distribution is used in Matlab to model the passengers boarding and alighting in each bus stop, according

to the particular passengers flow patterns. For both boarding and alighting of passengers, each bus stop has a different arrival rate based on the experimental data.

Poisson distribution becomes attractive for representing count data as it takes on a probability value only for nonnegative integers [51]. For this reason it is impossible to model with a single probability distribution the passengers that get on and off the bus at each stop. Therefore, the data for each stop will be studied separately with two different Poisson distributions. Figure 11 show the two distributions for stop number six.
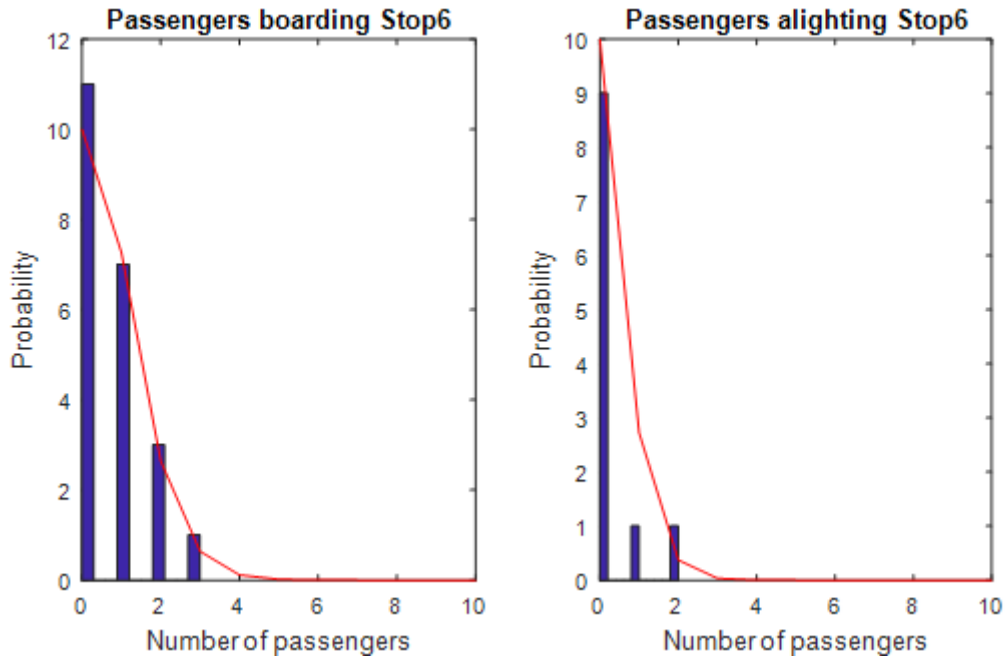

Figure 11. Poisson distributions fitted to measured data of stop 6.

On the other hand, the data collected correspond to the difference between the number of passengers boarding and alighting the bus, which makes it convenient to maintain a single probability distribution to model mass fluctuations at each stop instead of separating it in two.

While the objective of the algorithm is to cover all possible variations in terms of number of passengers that can occur in reality, the possibility to use Poisson probability distribution is ruled out. In order to group the data in a single distribution and be able to obtain samples with a greater variety of values than those obtained in reality, it was decided to use a continuous variable to model each stop.

A random variable, x, is continuous when it can take any value in an interval of the real number line with a density function f (x). A probability density function (PDF) is a mathematical relationship that describes how the probability density of a continuous random variable may vary over a permitted range of values [52].

The normal distribution is the most used of all probability distributions when modelling continuous random variables since a large number of experiments fit the distributions of this

family [50]. The density function corresponding to a normal distribution of mean μ and variance σ² is:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \qquad -\infty < x < \infty \qquad (3)$$

The normal PDF graph is a bell-shaped curve with very low probabilities in the tails. The horizontal axis of a PDF graph represents the possible values of a variable, while the vertical axis gives the probability density. Furthermore, the area under a PDF graph between any two selected values of the variable is the probability that the variable may take any value within the specified interval. Consequently, the total area under a PDF graph has a probability of one, since the variable must take some value within its permitted range [52].

By means of a Matlab function, normal PDFs were fitted to data of each stop, using maximum likelihood estimation. Each PDF is characterized by its mean and standard deviation, and the results are illustrated in figure 12.
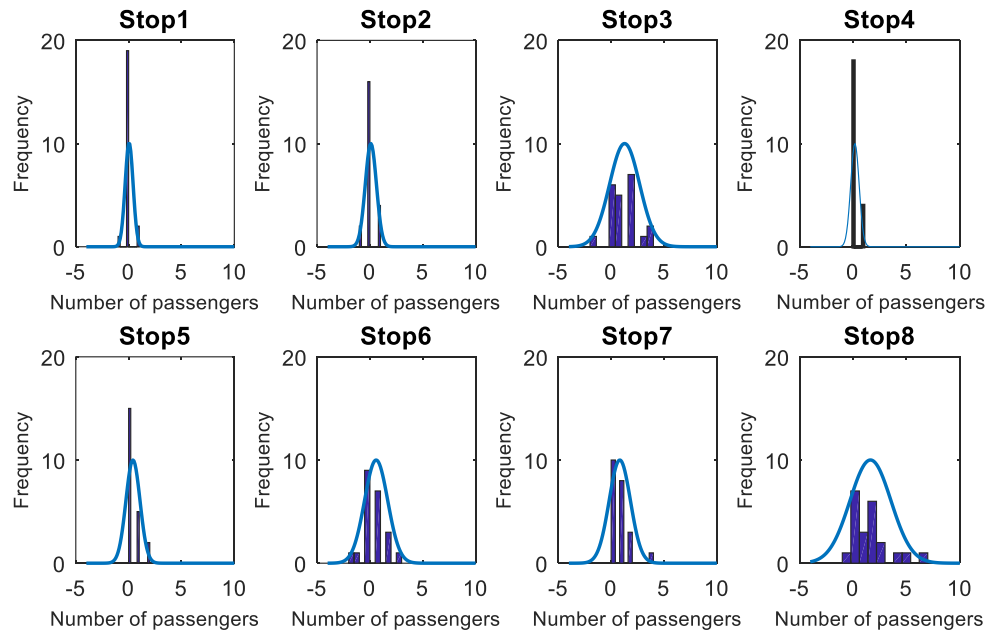

Figure 12. Normal PDFs fitted to measured data of each stop.

In Figure 13 the PDFs are shown for each of the stops and it can be seen that f(x) is bigger than 1 in some cases. This is not a problem since it is not interpreted as a probability. The probabilities in this case are the areas under the density function.
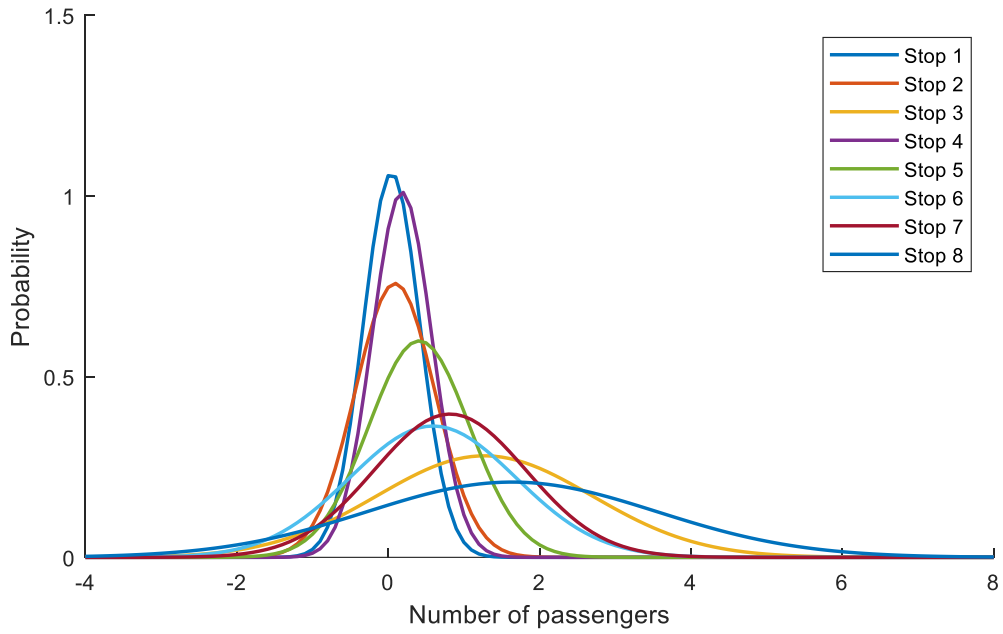
Figure 13. Normal PDFs for each stop.

In the case of the number of passengers inside the bus before the first stop, it is modelled with a normal PDF as well (figure 14). However, when generating a sample of this distribution, it must be taken into account that only positive values are valid for the correct performance of the algorithm as negative values have no sense.
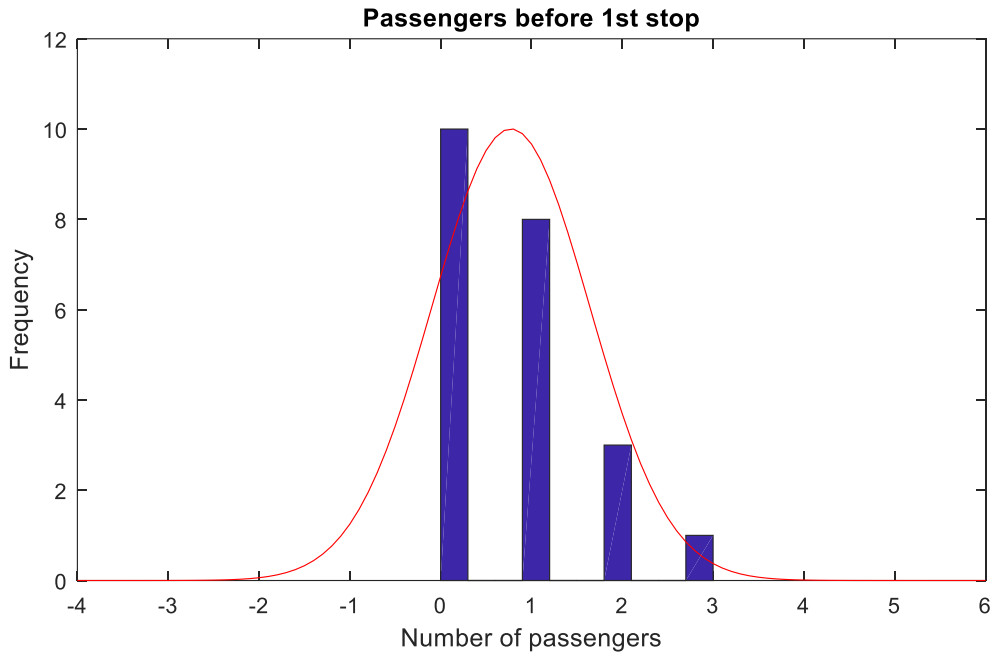

Figure 14. Normal PDF fitted to the measured data for the number of passengers inside the bus before the first stop analysed.

3.2.2 Correlation Definition

The key to a correct reproduction of the problem is the ability to predict the behaviour of the model under certain circumstances and to find appropriate values to simulate each variable of the problem. When there are multiple variables involved, the relationship between them should be introduced as well [53].

The difficulty of the passenger problem is that the demand for a given route is random and it fluctuates throughout the day. Therefore, the correlation between the data of different stops depends on the time of the day in which the measurements are taken and the day of the week. Correlation conditions restrict random selection, so in order to not distort the results, correlation coefficients between variables are defined, considering them a valid estimate.

As each variable was modelled as a normal distribution, their correlation will be studied as a multivariate normal distribution. Normal one-dimensional distribution, as it was noted in the previous section, is the most used distribution in statistics. However, its good properties are even more evident when studying multidimensional statistical models.

A random vector $X = (X_1, \ldots, X_n)$ is distributed according to an n-dimensional normal distribution with mean vector μ and variance-covariance matrix Σ if its PDF has the form [50]:

$$f(x) = \frac{1}{|\Sigma|^{\frac{1}{2}}(2\pi)^{n/2}} \cdot exp\left\{-\frac{1}{2}(x-\mu)^t \Sigma^{-1}(x-\mu)\right\} \qquad x \in \mathbb{R}^n \qquad (4)$$

In spite of its complexity, it is easily observed that it is a natural generalization of the formula for the one-dimensional case. The role of variance is now played by the variance-covariance matrix and the vector of means does what the mean did before. The distribution can be written as:

$$X \sim N_n(\mu, \Sigma) \qquad (5)$$

A property of multivariate normal distribution is that any vector Y obtained as a linear combination of X has multivariate normal law. In particular, any subset of X has multivariate normal distribution and the marginal $X_1, \ldots, X_n$ have univariate normal distributions.

The conditional distribution in variables with multivariate normal distribution can be calculated considering partitions of vector X. Then, the procedure is going to be illustrated:

Considering the partition of the vector $\mathbf{X} = (\mathbf{X_1'}, \mathbf{X_2'})'$, where $\mathbf{X_1} = (X_1, \ldots, X_{n1})'$, $\mathbf{X_2} = (X_{n1+1}, \ldots, X_n)'$ and $n_2 = n - n_1$. Accordingly, $\mu_1 = E(\mathbf{X_1})$ and $\mu_2 = E(\mathbf{X_2})$ are the mean vectors. The covariance matrix of $\mathbf{X}$ can be portioned as follows:

$$\Sigma = \left(\begin{array}{c|c} \Sigma_{11} & \Sigma_{12} \\ \hline \Sigma_{21} & \Sigma_{22} \end{array}\right) \qquad (6)$$

Where:

Σ_{11}=Var($\mathbf{X_1}$), with size n_1 x n_1 Σ_{12}=Cov($\mathbf{X_1}$, $\mathbf{X_2}$), with size n_1 x n_2

Σ_{22}=Var($\mathbf{X_2}$), with size n_2 x n_2 Σ_{21}= Σ_{12}', with size n_2 x n_1

Then, the distribution of $\mathbf{X_1}$ conditional on $\mathbf{X_2} = x_2^0$ is a multivariate normal distribution with mean:

$$\mu(X_1|X_2 = x_2^0) = \mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(x_2^0 - \mu_2) \tag{7}$$

And covariance matrix:

$$\Sigma(X_1|X_2 = x_2^0) = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21} \tag{8}$$

In order to calculate the conditional distribution in Matlab, these equations have to be implemented in the algorithm and, for that purpose, the mean vector and covariance matrix have to be calculated. The covariance matrix is calculated directly with a Matlab function.

As an example, the formulas will be described for the most complicated case, which would be to calculate the distribution for the variable that corresponds to the number of passengers boarding or alighting at stop 8 conditioned to the values that the rest of variables take. These other variables refer to the initial number of passengers inside the bus and the passengers boarding and alighting at stops 1 to 7. Therefore, there would be 8 variables with known value and one (which would be variable $\mathbf{X_9}$) whose distribution will depend on the value that the other 8 had taken.
Hence, the covariance matrix can be portioned as follows:

$$\Sigma = \begin{pmatrix} \sigma_{11} & \cdots & \sigma_{18} & \sigma_{19} \\ \vdots & \ddots & \vdots & \vdots \\ \sigma_{81} & \cdots & \sigma_{88} & \sigma_{89} \\ \sigma_{91} & \cdots & \sigma_{98} & \sigma_{99} \end{pmatrix} = \begin{pmatrix} \Sigma_{(12345678)(12345678)} & \Sigma_{(12345678)9} \\ \Sigma_{9(12345678)} & \Sigma_{99} \end{pmatrix} \tag{9}$$

And the distribution of $\mathbf{X_9}$ conditional on $\mathbf{X_1} = x_1^0, \mathbf{X_2} = x_2^0, \mathbf{X_3} = x_3^0, \mathbf{X_4} = x_4^0, \mathbf{X_5} = x_5^0, \mathbf{X_6} = x_6^0, \mathbf{X_7} = x_7^0$ and $\mathbf{X_8} = x_8^0$ is a multivariate normal distribution with mean:

$$\mu(X_9|(X_1, X_2, \ldots, X_8)' = (x_1^0, x_2^0, \ldots, x_8^0)') = \mu_9 + \Sigma_{9(1\ldots8)}\Sigma_{(1\ldots8)(1\ldots8)}^{-1}\begin{pmatrix} x_1^0 - \mu_1 \\ \vdots \\ x_8^0 - \mu_8 \end{pmatrix} \tag{10}$$

And covariance matrix:

$$\Sigma(X_9|(X_1, X_2, \ldots, X_8)' = (x_1^0, x_2^0, \ldots, x_8^0)') = \Sigma_{99} - \Sigma_{9(1\ldots8)}\Sigma_{(1\ldots8)(1\ldots8)}^{-1}\Sigma_{(1\ldots8)9} \tag{11}$$

28

Figure 15 shows the initial PDF for stop 8, represented with dashed line, and five PDFs of the same stop conditioned by the different values that the previous variables have taken. It can be observed that both the mean and the variance vary due to the influence of the other variables values.
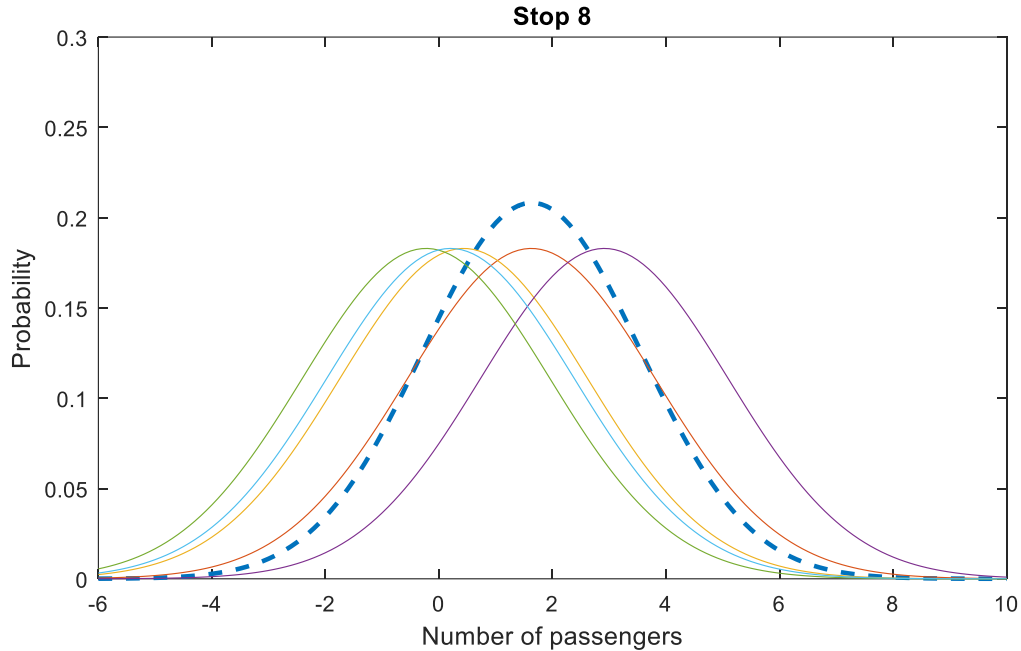

Figure 15. Initial PDF for stop 8 (represented with dashed line) and other five PDFs of the same stop conditioned by different values of the other variables.

Once the formulas are implemented in Matlab, the variables of the problem and their dependency relationships are already fully defined. In this way, any correlation will have been explicitly taken into account after the completion of the Monte Carlo simulation [52].

3.2.3 Algorithm Performance

Finally, it is going to be explained how the algorithm works. First, an initial number of passengers inside the bus is randomly generated, according to the probability distribution that defines this variable. In this case, since this number cannot be negative, and being the variable modelled by a normal distribution with a mean close to zero, random numbers must be generated until a positive number is obtained, thus negatives numbers are rejected.

Subsequently, according to the driving cycle introduced, and taking into account the sample generated for the previous variables, a value for each stop´s variable is generated following the same procedure. Since continuous distributions have been considered for variables definition, whenever a sample is calculated it should be rounded to an integer, since it is not possible to have a decimal number as the number of passengers. When the bus does not stop in a certain bus stop, the value corresponding to the variable of that stop will be directly 0.

Taking into account the initial number of passengers inside the bus and the values of each stop variable, the passenger flow inside the bus is obtained. In addition, lower and upper limits are introduced, forcing the number of passengers inside the bus to be greater or equal

to zero and lower than 45, which is the average passenger capacity for a medium sized city bus [54].

Therefore, the flow of passengers has been created following the driving cycle and the variation in the number of passengers is obtained as a function of time. Figure 16 shows a driving cycle and one of the possible passenger flows that can be generated for that specific cycle.
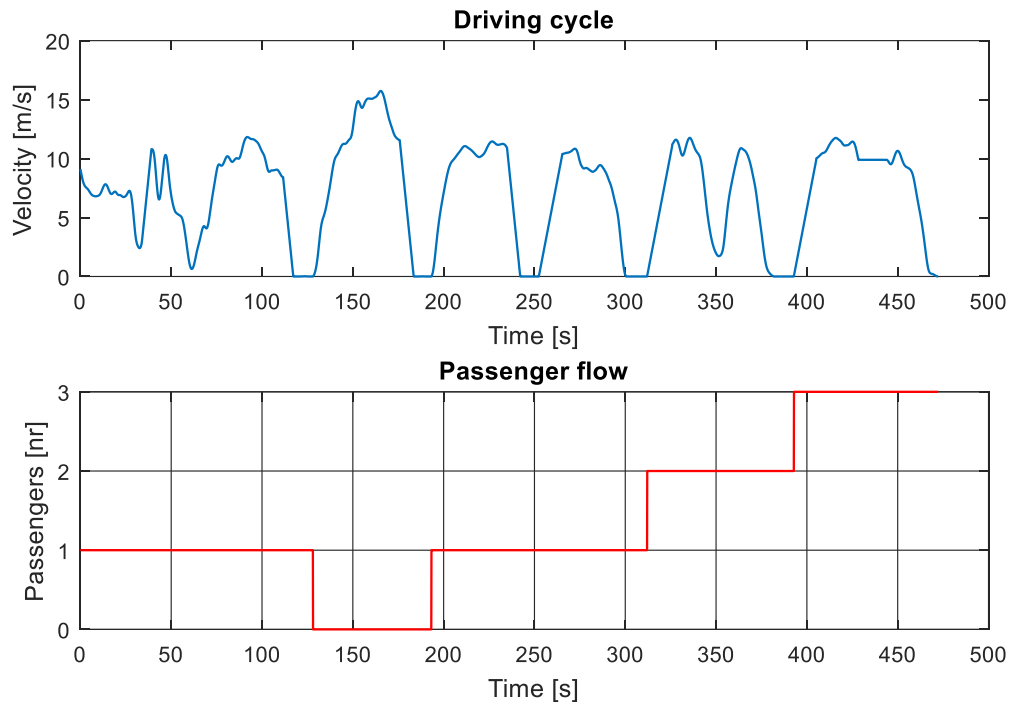

Figure 16. Passenger flow generated for a specific driving cycle.

The algorithm can generate numerous different passenger flows for each driving cycle, which allows to cover all possible situations. Figure 17 shows five iterations done with the algorithm following the same driving cycle.
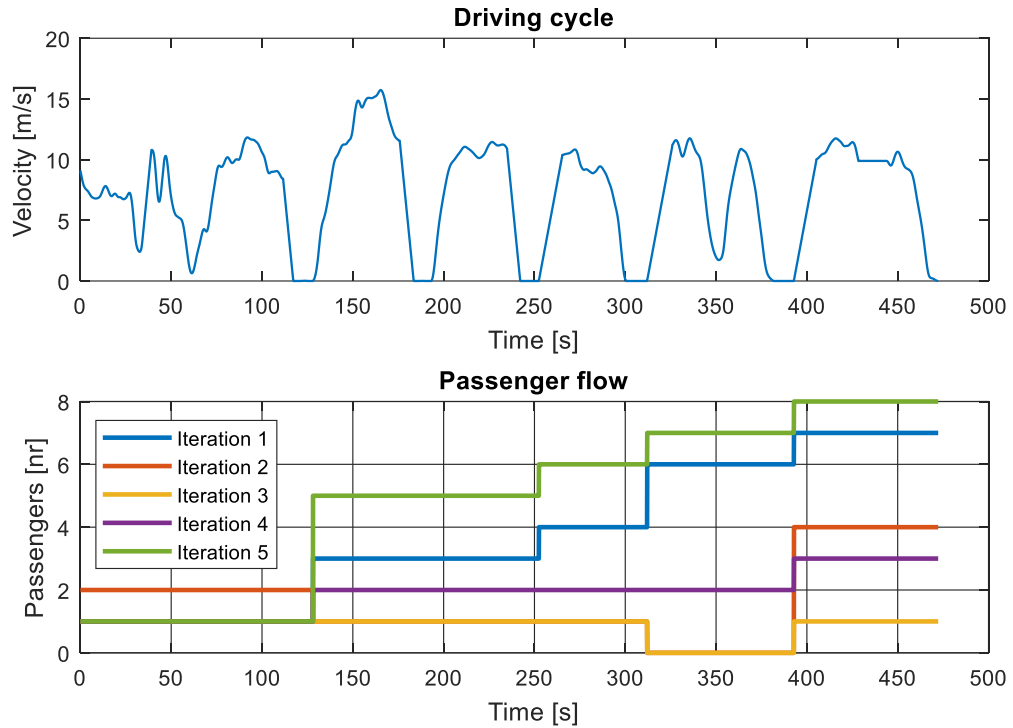
Figure 17. Five different passenger flows generated for the same driving cycle.

3.3 Electric Bus Model

In the research environment, it is very important to be able to simulate results before building a definitive model. The possibilities offered by the creation of virtual models are infinite and, through software programs such as Matlab, the electric vehicle behaviour can be modelled.

By means of Matlab and Simulink, an electric bus model was created at Aalto University by doctoral students Jari Vepsäläinen and Klaus Kivekäs through macroscopic representation of physical systems. The aim is to determine the consumption and operation range that the model can have based on the input variables for different driving conditions and driving cycles.
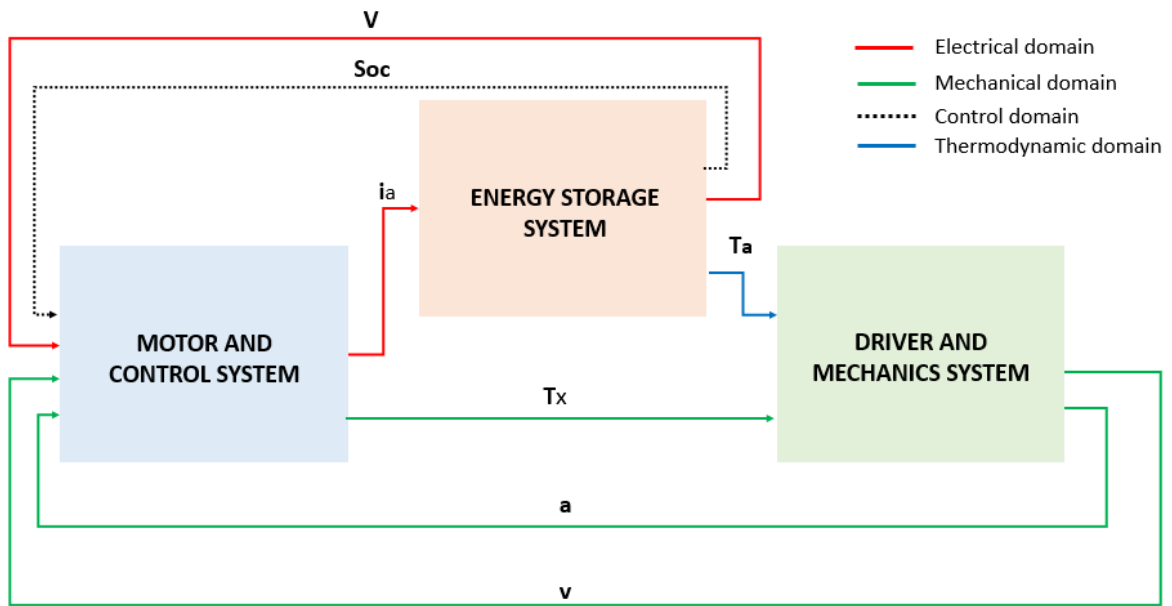
Figure 18. Electric bus energy flow model main systems.

As it can be seen in figure 18, the model is divided into three main systems: Motor and Control (where motor and regenerative braking blocks are defined as well as multiple controllers), Energy Storage (which consist on the battery model) and Driver and Mechanics (where vehicle dynamics are modelled). The three systems are interrelated via the input and output variables. Each one of these systems is explained in the following sections.

3.3.1 Motor and Control System

In this system the motor torque (T_x) required to move the bus is calculated as well as the battery current needed (i_a). The inputs in this system are the battery voltage (V), the battery SOC (state-of-charge), the real vehicle velocity (v) and real vehicle acceleration (a). It contains three main blocks: the driver, the regenerative braking and the AC-motor. Moreover, there are two control blocks: PID speed control and PID current control. In figure 19, it is shown the general view of the system.
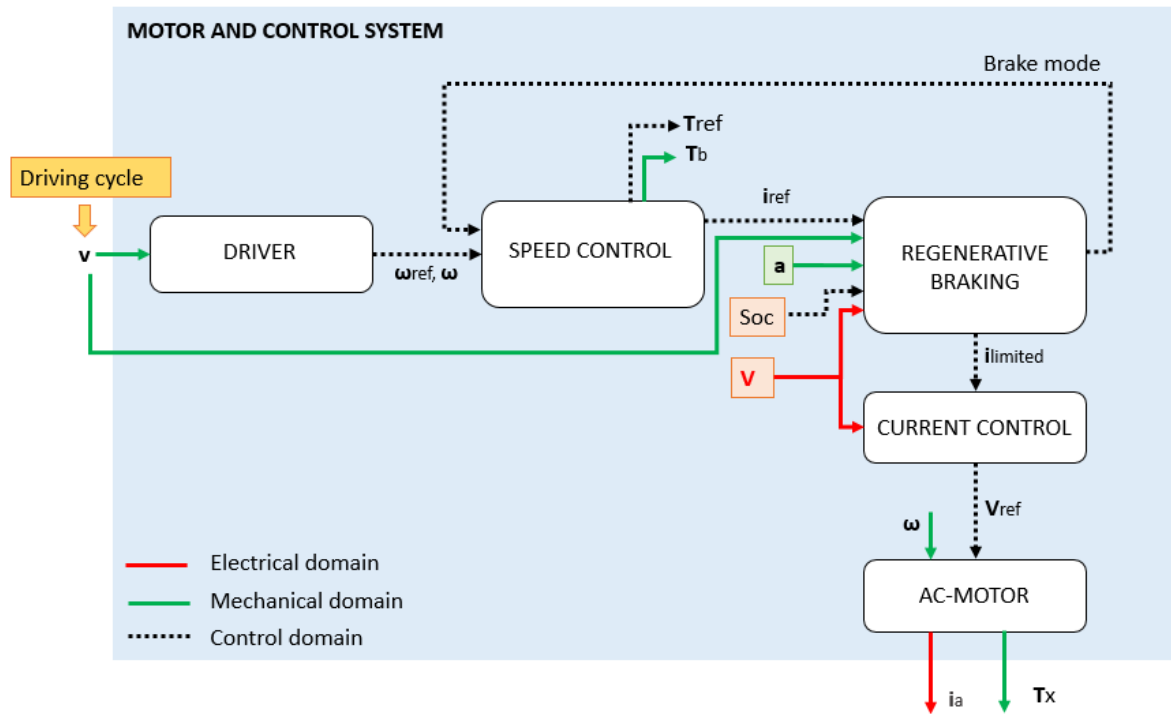
Figure 19. Motor and control system of the electric bus model.

The first inner block configured is the driver block that contains the predefined driving cycle which is considered as the reference vehicle velocity. This velocity is the one that the vehicle must follow and therefore, the electric motor has to supply a mechanical torque that allows the vehicle to acquire this desired velocity. Consequently, the reference vehicle acceleration and motor angular velocity can be calculated.

Then, the PID speed control block compares the real vehicle velocity, which is an output of driver and mechanics system, with the reference one. The control is done by calculating the reference motor torque and the brake torque needed to acquire the desired vehicle velocity at each instant.

Next block, regenerative braking, contains a Matlab function with the regenerative braking algorithm that controls when the regeneration should be done, knowing the values of the SOC, deceleration and speed. Depending on the values of these parameters, the algorithm has three types of breaking modes: no regeneration, partial regeneration and full regeneration. The three modes are explained in table 4. Also, this function limits the current in the motor, obtaining the reference current value.

Table 4. Modes of braking system.

No regeneration mode	SOC > 90 % Deceleration > 4 m/s^2 speed < 5.4 m/s
Partial regeneration mode	SOC < 90 % Deceleration < 4 m/s^2 speed > 5.4 m/s
Full regeneration mode	SOC < 90 % Deceleration < 1 m/s^2 speed > 10.8 m/s

Some parameters are needed to carry out the calculations and therefore the input parameters are shown in the table 5. These parameters were selected based on default values in paper [55].

Table 5. Regenerative braking model parameters.

Regenerative braking	
Parameter	**Value**
Max. Battery voltage (V)	750
Max. regeneration current (A)	150
Max. Discharge current (A)	260
Battery SoC regeneration limit (%)	90
Partial reg. velocity threshold (km/h)	5
Full reg. velocity threshold (km/h)	10
Partial reg. deceleration threshold (m/s^2)	4
Full reg. deceleration threshold (m/s^2)	1

After this block, another PID controller is used to ensure that the motor current follows the desired value, which is the reference value calculated in regenerative braking block. The output in this case is the reference voltage.

Finally, the Alternating Current motor block is implemented. The motor absorbs electrical energy from the battery and transforms it into mechanical torque and rotation. The battery is responsible for generating the proper voltage to power the motor. Thus, this block has as inputs the reference voltage supplied by the battery and the required angular velocity in the motor, both previously calculated. By means of the pertinent computations, the electromagnetic torque generated by the motor is calculated, and adding it to the brake torque obtained with the friction brakes, they constitute the mechanical torque that causes the vehicle movement. The AC motor model is initiated with the parameters shown in table 6.

Table 6. AC-motor model parameters.

AC-motor	
Parameter	**Value**
Internal resistance (ohm)	0.5
Stator armature inductance (H)	5e-3
Number of pole pairs	6
Flux induced by magnets (Vs)	0.96
Inverter efficiency (%)	97

3.3.2 Energy Storage System

This system implements the battery model of the electric bus and it is responsible for supplying the power to the electric motor. The system has one input, the battery current, and three outputs: the battery voltage, the battery SOC (state of charge) and the ambient temperature (T_a).

Figure 20 shows the blocks that constitute the energy storage. It is composed by a main block, the battery model, which has been performed as a Lithium-ion battery. Different variables related to battery behaviour are obtained from this block: voltage, SOC, current and temperature. With this data, the total energy consumption required by the electric bus can be calculated. Multiplying the voltage and the current generated by the battery, the total power needed to move the bus is obtained. If it is integrated over time, the total energy consumption of the electric bus is calculated. In order to obtain the electric consumption per km, it is divided by the distance travelled.
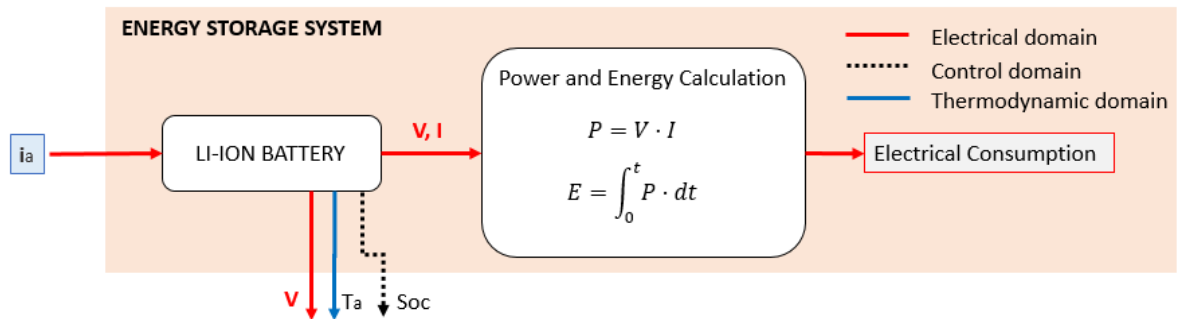

Figure 20. Energy storage system of the electric bus model.

First of all, a brief theoretical explanation of the Li-ion battery is going to be done and afterwards, the battery model in Simulink is presented.

3.3.2.1 Lithium-Ion Battery

Nowadays, Lithium-ion batteries are the most widely used in electric vehicles due to their great potential. Their main features are high energy density, high power density, environmental friendliness, and long lifecycle [56]. However, this is not the perfect technology for energy storage as it has several defects in terms of safety, uniformity, cost and durability. These problems limit the wide application of Li-ion batteries in vehicles. Not being a fully mature technology, and being in continuous development, they still have room

for improvement and they could be the great promise for energy storage that will lead to the electric vehicle revolution [57].

Li-ion batteries are composed of several cells that store electrical energy as chemical energy in two electrodes: the anode (reductant) and the cathode (oxidant). The electrodes are separated by an electrolyte which conducts the ionic component of the chemical reaction that occurs inside the cell and forces the electricity to flow outside the battery. The battery outputs are an external current I and a voltage V, which together constitute the output power. A rechargeable battery has a reversible chemical reaction at the two electrodes. [58]

3.3.2.2 System Description

Inside the Li-ion battery block, there are two main blocks: Heat losses and SOD calculation. Both of them can be seen in figure 21.
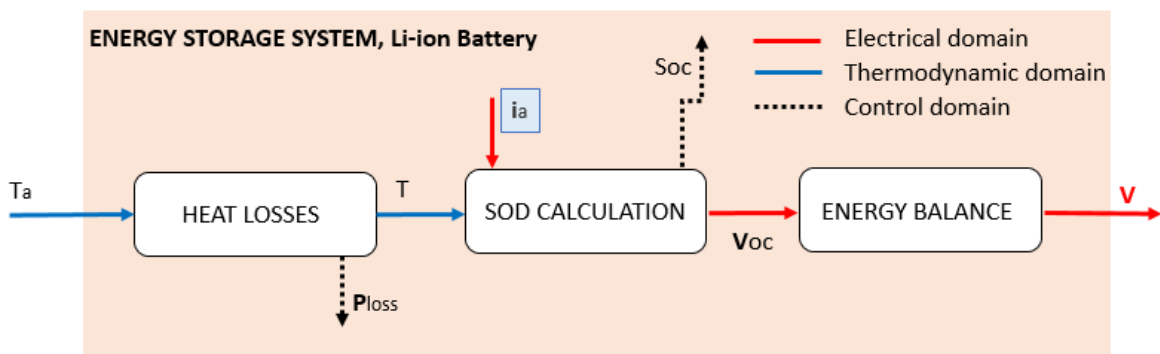

Figure 21. Energy storage system of Li.ion battery in the electric bus model.

Heat losses block consist on a thermal model that calculate the power loss by heat generation during charge and discharge process. Thus, the outputs of this block are the power loss (P_{loss}) and the internal temperature of the battery (T).

On the other hand, the SOD calculation block implements the equations to calculate the battery state of discharge (SOD) and consequently the state of charge (SOC). The battery state-of-charge is measured as a percentage: for a fully charged battery, the SOC is 100% and for an empty battery is 0%. Along with the SOC, the other output of this block is the battery open circuit voltage (V_{oc}).

With the knowledge of these new variables and implementing the corresponding equations, the battery voltage is calculated. Subsequently, the battery power and efficiency are calculated as well. Other internal parameters are needed to proceed with the calculations, thus the battery model is initiated with the parameters shown in table 7, based on the default values described in paper [55]:

Table 7. Li-ion battery model parameters.

Lithium-ion battery	
Parameter	**Value**
Number of cells in series	168
Number of cells in parallel	38
Nominal cell capacity (Ah)	2.1
Max. Voltage of a cell (V)	4.2
Internal resistance of a cell (mOhm)	150
Capacitance of a Cell (F)	4
State of charge (%)	85
Cell star temperature (ºC)	23
Ambient temperature (ºC)	25
Weight of a cell (kg)	0.0410
Surface area of a cell (m²)	0.0043
Specific heat (J/kgK)	925
Heat transfer coefficient	40
Number of cycles to N% aging	500
Capacity remained after N% aging	80
Number of cycles used	0
Self-discharge (%/month)	4

3.3.3 Driver and Mechanical System

Performance and specifications, among other characteristics of the vehicle, determine the energy requirements, thus the mechanical properties required for driving a vehicle are considered in this block.

In this system, the speed and acceleration of the vehicle are calculated. To carry out this calculation, the dynamic equations of the vehicle are implemented, so a brief theoretical explanation about vehicle dynamics is going to be done before the system description.

3.3.3.1 Linear Motion Dynamics

In order to understand the behaviour of a vehicle, whether electric or conventional, it is convenient to know the forces involved in its movement. In this case, the model is simplified considering only the action of longitudinal forces. The main acting forces can be easily interpreted through the corresponding equations based on physical laws. The equivalent forces acting on a vehicle in motion are represented in figure 22.
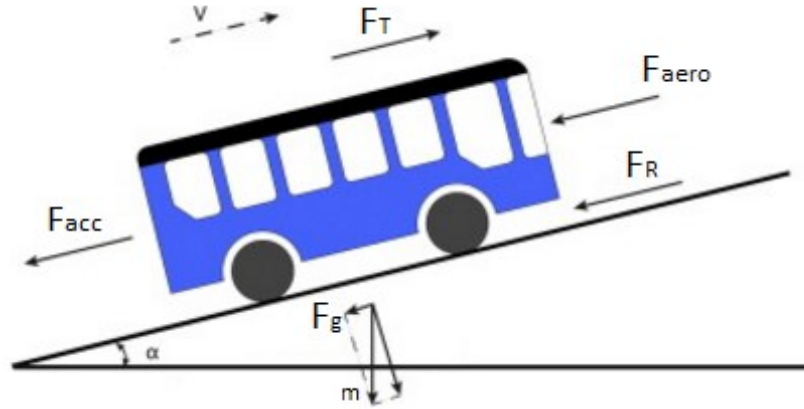
Figure 22. Longitudinal forces acting on a bus in motion (based on [25]).

The total resistance to motion effort that the vehicle must overcome is the sum of rolling resistance, gradient resistance, aerodynamic drag and acceleration resistance. In order to overcome this resistance, a sufficient quantity of traction force has to be applied.

- Rolling resistance

It is the equivalent force related to the resistance of the wheels to rotate due to a deformation processes, which occur when the tire is in contact with the road [59]. Moreover, the rolling coefficient, and subsequently the rolling resistance, vary according to type of road surface.

$$F_R = f_{rr} \cdot N \tag{12}$$

Where f_{rr} is the rolling coefficient and N is the normal force which can be expressed as:

$$N = M \cdot g \cdot \cos \alpha \tag{13}$$

Where M is mass, g the gravitational field strength and α is the angle forming the surface with the horizontal.

- Gradient resistance

It is the force due to gravity necessary to raise a body of a certain mass along an inclined plane with an angle α to the horizontal.

$$F_g = M \cdot g \cdot \sin \alpha \tag{14}$$

Where M is mass, g the gravitational field strength and α is the angle forming the surface with the horizontal.
In this study an angle of 0 degrees has been considered (horizontal profile), but it could be modelled for different driving profiles.

- Aerodynamic drag

The aerodynamic drag is the force that opposes vehicle movement when it travels through the air. Vehicles have to set aside a significant amount of air in order to advance. The

pressure of the air against the front side and the vacuum (suction) generated in the back are combined to generate a force opposite to the movement. The aerodynamic drag is calculated with the following formula:

$$F_{aero} = \frac{1}{2} \cdot \rho \cdot A_d \cdot C_d \cdot v \qquad (15)$$

Where ρ is the air density, Cd is the drag coefficient, Ad is the vehicle's frontal cross-sectional area and v is the driving speed (taking into account the headwind speed which in this study is going to be considered 0 m/s) [59]. The drag coefficient depends on crosswind effects and vehicle speed and requires detailed data of regional wind statistics [60]. In this study it is considered as a constant.

- Acceleration resistance

In addition to the driving resistance that occurs in steady state motion, inertial forces also occur during acceleration and braking, and they have to be taken into account. This resistance can be divided in two components: the acceleration resistance of the total mass of the vehicle and the inertial resistance of the rotating parts. According to Newton's second law of motion, the vehicle acceleration can be expressed as:

$$F_{acc} = M \cdot \frac{dv}{dt} \qquad (16)$$

Where v is the vehicle speed and M is the vehicle total mass.

On the other hand, inertial resistance of the rotating parts is the equivalent force necessary to have a torque that gives the angular acceleration. The rotating components need a certain force to overcome the moment of inertia and produce the necessary rotation. This force can be calculated with the following formula:

$$F_I = \frac{J}{r^2} \cdot \frac{dv}{dt} \qquad (17)$$

Where J is the total moment of inertia at the driven axle and r is the dynamic radius of the tire.

The vehicle tractive force (F_T) is a sum of all the resistive forces: rolling resistance (F_R), gradient resistance (F_g), aerodynamic drag (F_{aero}), acceleration resistance (F_{acc}) and inertial resistance (F_I) [60].

$$F_T = F_R + F_g + F_{aero} + F_{acc} + F_I \qquad (18)$$

Substituting the values of all the forces in equation 18, it leads to:

$$F_T = f_{rr} \cdot M \cdot g \cdot \cos \alpha + M \cdot g \cdot \sin \alpha + \frac{1}{2} \cdot \rho \cdot A_d \cdot C_d \cdot V + \frac{dV}{dt} \cdot (M + \frac{J}{r^2}) \qquad (19)$$

3.3.3.2 System Description

The driver and mechanical system consists of the elements that can be seen in figure 23. The system has two inputs: the ambient temperature and the motor torque. The ambient temperature is used to calculate the air density and the motor torque is needed to obtain the motor force.
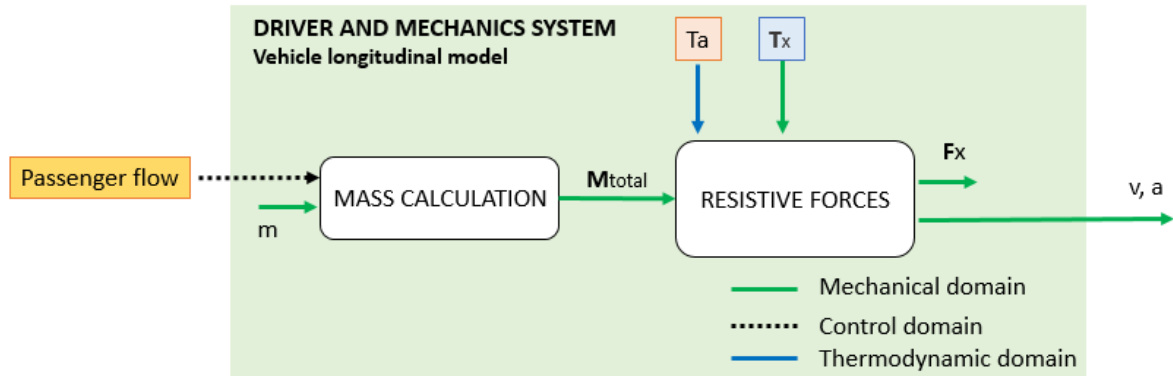

Figure 23. Driver and mechanics system of the electric bus model.

This block contains the bus longitudinal dynamics model and it calculates all resistive forces and, based on them and the force provided by the motor, the acceleration is obtained. To compute the resistive forces, the vehicle total mass is needed, and it is implemented adding the mass of the passenger flow generated in Matlab to the curb weight of the vehicle (m). A new passenger flow is generated in each simulation and it is multiplied by 70 kg which is assumed to be the standard weight of a person [61], [62], [63].

Finally, the system has two outputs: the real acceleration and real speed of the bus. The values of the block parameters are shown in the table 8.

Table 8. Driver and mechanics model parameters.

Driver and Mechanics	
Parameter	**Value**
Gear ratio	4.93
Tire diameter (m)	0.43
Vehicle mass (kg)	12345
Motor inertia (kgm^2)	0.63
Tire inertia (kgm^2)	8
Head wind (m/s)	0
Road angle (degrees)	0
Coefficient of drag	0.493
Vehicle frontal area (m^2)	6.2
Rolling resistance coefficient	0.008

3.4 Electric Bus Model Simulation and Validation

Before proceeding with the Monte Carlo simulation, it is necessary to guarantee that the Simulink model represents adequately the actual electric bus performance, thus the electric bus Simulink model has to be validated. To carry this out, the results of the model will be compared with the electrical consumption measured on an actual bus in the same route. These measurements were taken with a chassis dynamometer, and were presented in study [64].

The same driving cycle measured in the actual bus is simulated with the Simulink model. Also, the same bus mass was introduced (12345 kg) and no additional mass corresponding to the passengers was taken into account. The results obtained are presented in table 9.

Table 9. Comparison between electrical consumption in actual measurements and in model simulation.

	Electric consumption (kWh/km)
Actual bus measurement	0.581
Electric bus model simulation	0.5867
Relative error	0.98 %

The value obtained with the simulation model is quite similar to the measured one. The relative error is less than 1% and therefore, the simulation model is valid to reproduce the actual bus behaviour.

Therefore, the bus model is repeatedly simulated with Matlab (with the driving cycles and passenger flows generated), obtaining enough representative results. The number of trials has to be settled first. Figure 24 represents the simulation process.
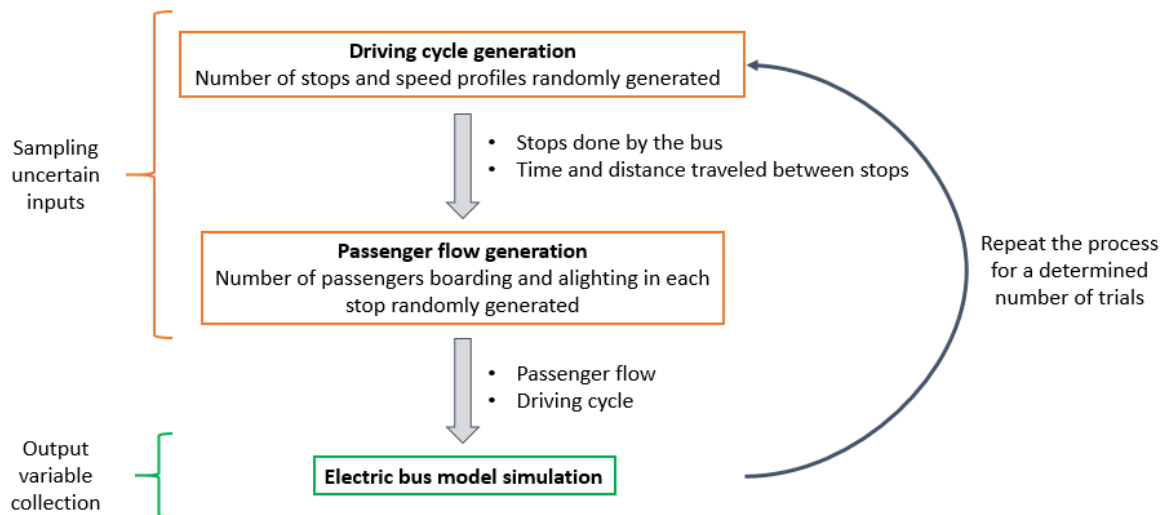

Figure 24. Simulation process diagram. First, the uncertain inputs are sampled with driving cycle generator algorithm and passenger flow generation algorithm. Second, the driving cycle and passenger flow are introduced in the electric bus model, and it is simulated. Third, the output variable is collected. This process is repeated a determined number of trials.

Every Monte Carlo simulation requires the development of a model that closely resembles the real scenario, and thereby to be able to analyse meaningful results. In this thesis the analysis focuses on one of the outputs of the model: the electrical consumption.

3.5 Monte Carlo Sampling

Monte Carlo Method is applied to ensure the reliability of the model. The Monte Carlo method procedure generates a random value for each input, and through mathematical relations, they are used to produce a probability distribution as output. However, one disadvantage of Monte Carlo method is that a single simulation (or trial) is not enough to produce a reliable result [52].

Therefore, when designing Monte Carlo simulations, two considerations must be taken into account. First, the random component must be introduced reflecting the interdependence that needs to be evaluated. And second, the number of simulations performed has to be enough to achieve an accurate estimation of the output probability distribution [45]. This second consideration is going to be analysed in Monte Carlo sampling chapter.

3.5.1 Uncertain Inputs

First, the number of inputs that are going to be sampled has to be defined, as it is a crucial factor for the number of simulations needed to get an accurate estimation. The uncertain inputs of the model are the number of stops in the driving cycle, the number of passengers inside the bus before the first stop and the number of passengers boarding or alighting in each stop. Thus, ten uncertain inputs are involved in this model.

In the case of the number of stops, the actual distribution was unknown, so the uncertainty was modelled with a normal distributed PDF, represented in figure 25, and the probability of stopping at each stop was set to 50% [17].
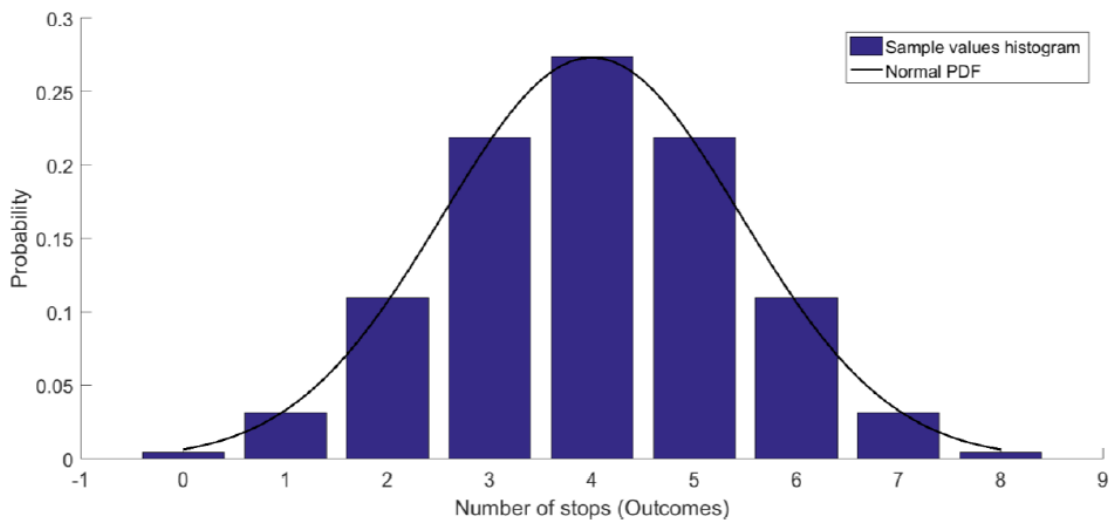

Figure 25. Normal PDF that model the number of stops in a driving cycle.

On the other hand, normal PDFs were obtained for describing the number of passengers boarding and alighting in each stop, fitting the actual data to normal distributions. Through random sample generation, the following histograms (figure 26) for each distribution can be

obtained and they can give an idea of the probability of each value. The same process is performed to model the uncertain number of passengers inside the bus before the first stop.
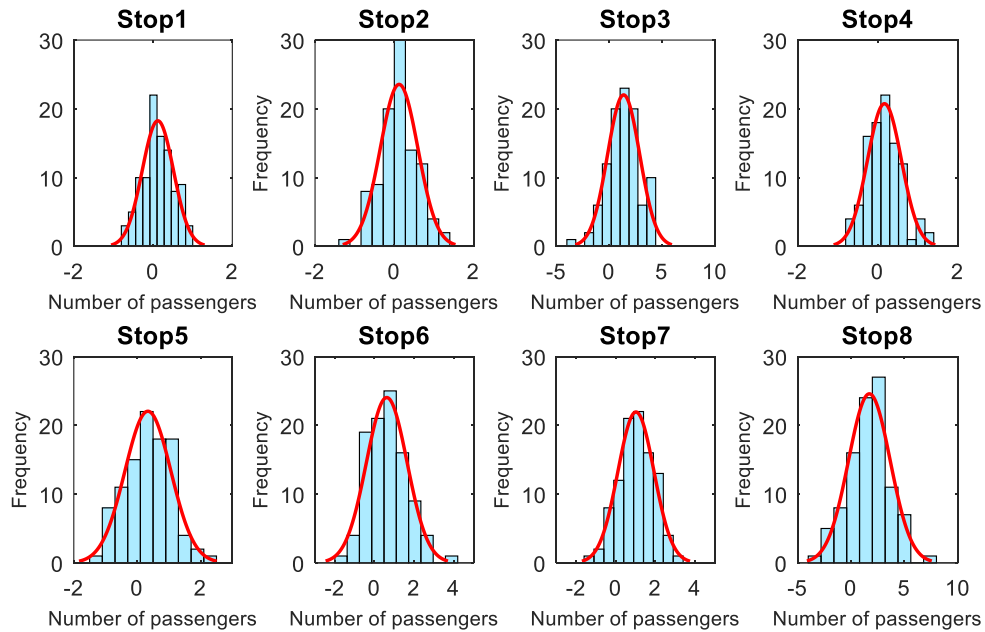

Figure 26. The number of passenger boarding or alighting at each stop is sampled from normal PDFs.

In every simulation cycle, each uncertain input is randomly sampled taking into account the dependency relationships between them, and a value for each input is obtained generating sets of inputs. This means that a passenger flow is going to be created for each set of samples and for each driving cycle. Consequently, the electric bus model is going to be simulated a specific number of times obtaining as a result an aggregate probability distribution which gives the uncertain output values.

3.5.2 Number of iterations required

Working conveniently with the random variables, a complete study of the reliability of the electric bus model can be made, as well as the sensitivity of each of the variables can be extracted. To carry this out as it was mentioned before, the number of simulations performed has to be determined.

The simulation process is repeated a large number of times in order to obtain a sufficient amount of results as output. Considering that the larger the sample size, the greater the fit between the sample distribution and the theoretical distribution on which the sample is based, it is decided to simulate the model 1000 times and verify its convergence. This is a sufficient number to get an output PDF with constant mean and variance [34]. However, if the number of iterations is not high enough, it is possible to over-sample some segments of the distributions and to sub-sample other segments. Hence, as the number of simulations increases, the error decreases.

The simulation was run for 1000 iterations and the estimated mean was computed after each iteration. The obtained results are illustrated in figure 27. This graph will converge towards

a specific value, the true mean, and will eventually reach a point at which any additional iterations will not result in a significant change to the estimated mean. The number of iterations required is the one that leads to an acceptable error interval around the simulation results.
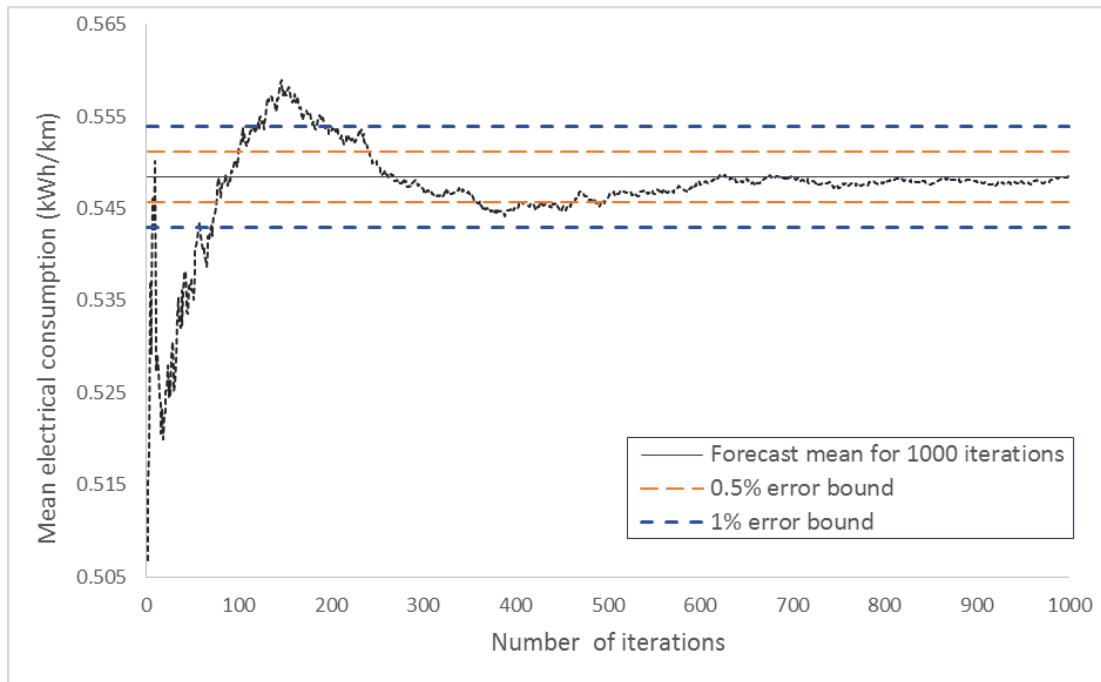

Figure 27. Convergence of electrical consumption mean depending on the number of iterations performed.

Based on the convergence graph, it can be defined the number of iterations to reach a particular accuracy. The greater the number of iterations, the more stable will be the output mean. It can be seen that the forecast mean for 1000 iterations is 0.548 kWh/km, achieving less than 1% error after 200 iterations and less than 0.5% error after 500 iterations.

Depending on the model complexity (number of uncertain variables and range of values accepted), the number of iterations and error percentage accepted can vary [34]. It is crucial to achieve a proper compromise between the estimation accuracy and the simulation time. As it is a model with 10 uncertain variables, the solution variability is higher as well as the error bound accepted. Thus, an error bound of 0.5% is considered acceptable and at least 500 simulations are required in this case to provide accurate results with Monte Carlo Method.

4 Results

Every Monte Carlo simulation requires the development of a model that closely resembles the real scenario. In this case the electric bus model done in Simulink was used. Mathematical relationships that transform the input parameters into the desired output parameters were applied. In this thesis the analysis focuses on one output of the model: the electrical consumption.

To proceed with the analysis, 500 simulations were carried out and the electrical consumption value was collected for each set of inputs. In addition, as he results depend on several inputs, the values of other parameters of interest were collected, such as the number of passengers in the bus, the speed, the number of stops, and the time in which a certain number of passengers remain inside the bus.

In this section, the samples obtained in the simulations are evaluated, presenting the electrical consumption distribution, its variation with the number of passengers and the influence of passenger load, speed and number of stops on electrical consumption.

4.1 Electrical Consumption Distribution

For estimating the electrical consumption forecast, given the uncertainties of all the inputs considered, the output results are gathered in a distribution histogram. The histogram is illustrated in figure 28 and the mean, mode and 80 % confidence interval are plotted in it.

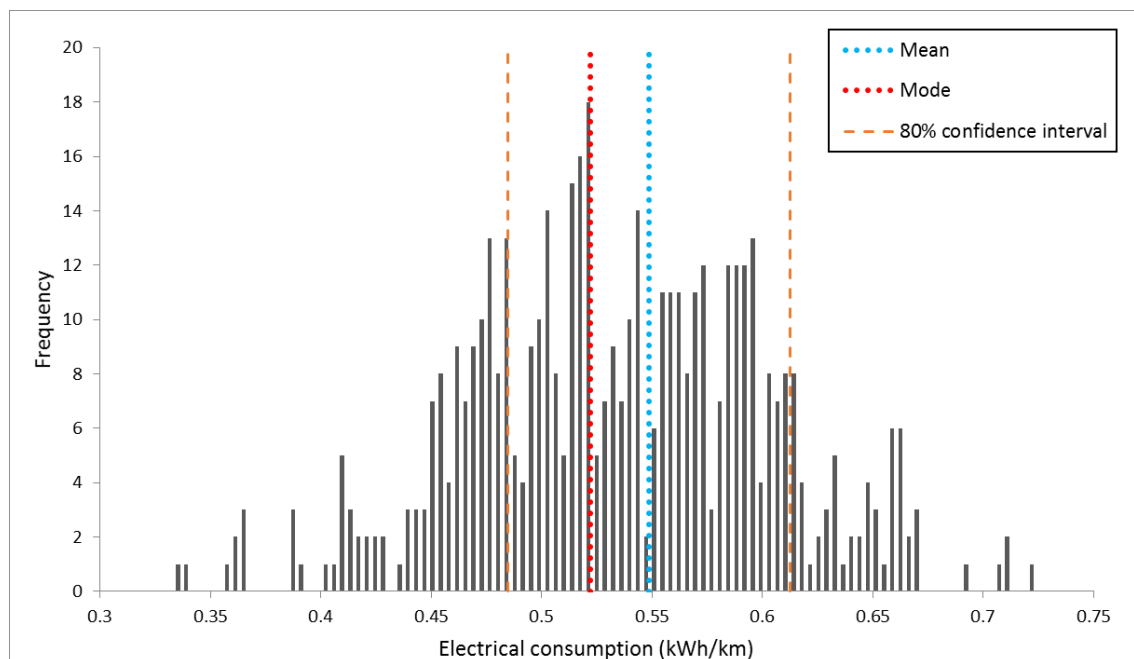**Figure 28. Electrical consumption histogram.**

The mean of the sample, illustrated in blue, is defined as the sum of the observations divided by the total number of them. While the mode, represented in red, is the most frequent value in the output results. It is a single mode forecast and it indicates that the most frequent value for electrical consumption is located in the zone of values below the mean.

45

In addition, the 80% confidence interval is also shown. It was calculated assuming that the probabilistic distribution follows the law of normal distribution. The lines that limit the interval represent the 10% and 90% distribution percentiles and they define the 80% confidence interval on the results, considering the uncertain inputs. This means that there is an 80% probability for the electrical consumption to occur within this interval.

Moreover, table 10 includes some information of the results obtained, where numerical values that describe the basic features of the output distribution can be observed.

Table 10. Numerical values of the output distribution, considering a normal PDF.

Distribution characteristics	Electrical Consumption (kWh/km)	
Mean	0.549	
Mode	0.536	
Median	0.546	
80% confidence interval	0.485-0.613	26.37% of variation
Minimum value	0.335	-39.01% of difference to the mean
Maximum value	0.749	+36.43% of difference to the mean
Absolute difference between minimum and maximum	0.414	75.44% of difference

Although the obtained histogram indicates that the distribution of the results could resemble a normal one, and the information obtained was calculated assuming a normal PDF, the validity of the normal distribution to represent the results is analysed in greater depth. In a normal distribution, mean, mode and median have the same value, and this is not the case, even though mean and median are very similar. To proceed with the analysis, a Kernel PDF estimation of the electrical consumption was done with Matlab, based on the forecast. Then, it is compared with the normal distribution fitted to the forecast as well.

The non-parametric estimation of density functions using the Kernel method is an ingenious way of estimating a PDF that does not follow a known model (Normal, Binomial, Exponential, etc.). Furthermore, it is very useful in Monte Carlo simulation [65], [34]. A kernel estimation for the outcome data was carried out by means of a Matlab function.

Kernel distribution is defined by a smoothing function and a bandwidth value that controls the smoothness of the resulting density curve [66]. The kernel algorithm is:

$$\widehat{f_h}(x) = \frac{1}{nh} \sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right); \quad -\infty < x < \infty \tag{20}$$

Where n is the number of samples, h is the bandwidth and K is the kernel smoothing function.

The shape of the curve used to generate the PDF is determined by the kernel smoothing function. The Matlab function compiles the probability distribution based on the output samples and it sums the smoothing functions component for each value to generate a smooth and continuous probability curve.

The electrical consumption kernel estimation was computed using the default bandwidth, which is theoretically optimal for estimating densities for the normal distribution [66]. Figure

46

29 illustrates the comparison between the kernel PDF estimation and the normal one. Also, the distribution histogram is represented.
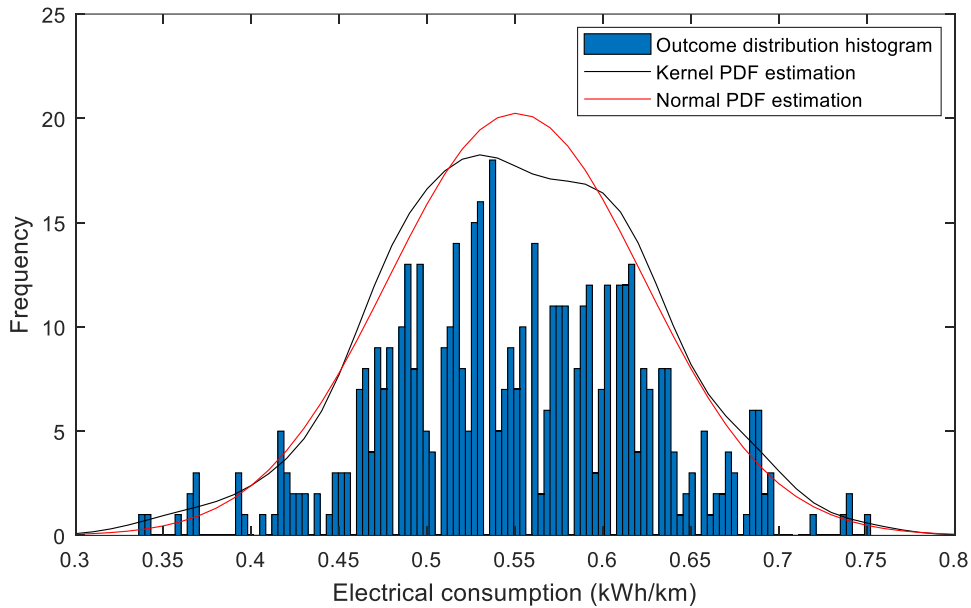

Figure 29. Kernel PDF estimation (black line) and Normal PDF (red line) both fitted to output distribution histogram.

It can be observed that the kernel estimation fits the histogram shape more closely than the normal PDF estimation, and consequently Kernel PDF provides a more accurate description of the output. The deviation of the mode towards values below the mean makes that the normal PDF peak does not fit to the one of the outcome histogram.

4.2 Electrical Consumption Mean by Number of Passengers

Knowing the values for each parameter and inputs of the model, the influence of the passenger load on electrical consumption can be analysed according to the number of passengers that are inside the bus during each driving cycle. In order to carry out this analysis, it has been necessary to calculate the average number of passengers on the bus in each driving cycle. This has been done by taking into account the time that the passengers remain inside the bus. The highest number obtained was 5.6 and the lowest 0, so the average number of passengers was divided into 6 bins, in the following way:

Table 11. Average number of passengers divided into bins.

Bin name	Range of values (number of passengers)
1	0-1
2	1-2
3	2-3
4	3-4
5	4-5
6	5-6

Then, the mean and standard deviation of the electrical consumption were calculated for each bin. Both values are illustrated in figure 30.
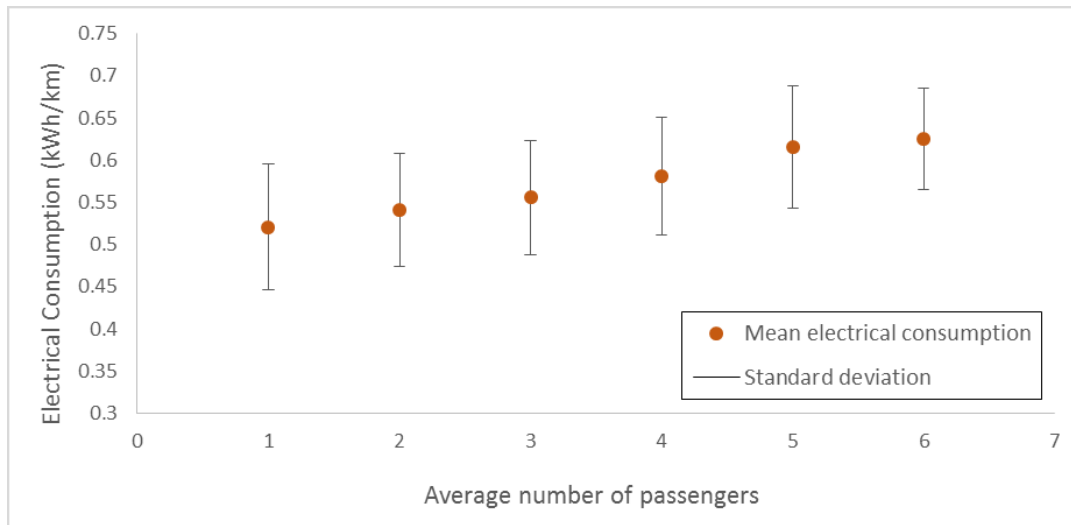

Figure 30. Electrical consumption mean and deviation by number of passengers.

As expected, the electrical consumption of the bus increases with the number of passengers, i.e. they are directly proportional. Regarding the standard deviation, the electrical consumption variation can be explained by the different speed profiles and number of stops in each driving cycle performed. Moreover, the number of samples in each bin is not the same, as high numbers of passengers (5 and 6) are not very frequent in the driving cycles simulated.

4.3 Influence of Passenger Load on Electrical Consumption

The effect of passenger load on the electrical consumption was further studied. The consumption strongly depends on two other parameters that are the bus speed and the number of stops. Then the results were analysed in a combined way: on the one hand, average number of passengers combined with number of stops and on the other hand, average number of passengers combined with average speed.

4.3.1 Electrical Consumption at Different Passenger Load and Number of Stops

With the previous divisions realized for the average number of passengers and taking into account the number of stops done by the bus in each driving cycle, a three-dimensional chart is constructed (figure 31).
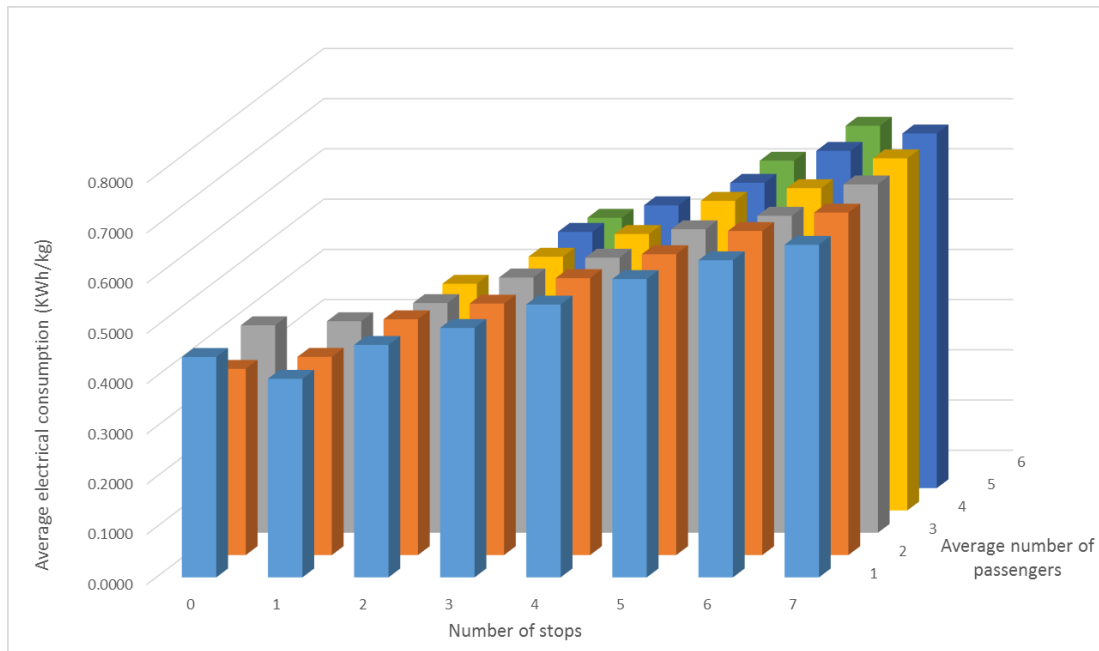
Figure 31. Electrical consumption at different passenger load and number of stops.

It can be observed in figure 31 that, with certain exceptions, electrical consumption shows a significant increasing trend when increasing the number of stops and passenger load. Hence, electrical consumption is directly proportional to both parameters. Consequently, the highest value for electric consumption will occur for the highest number of passengers and highest number of stops.

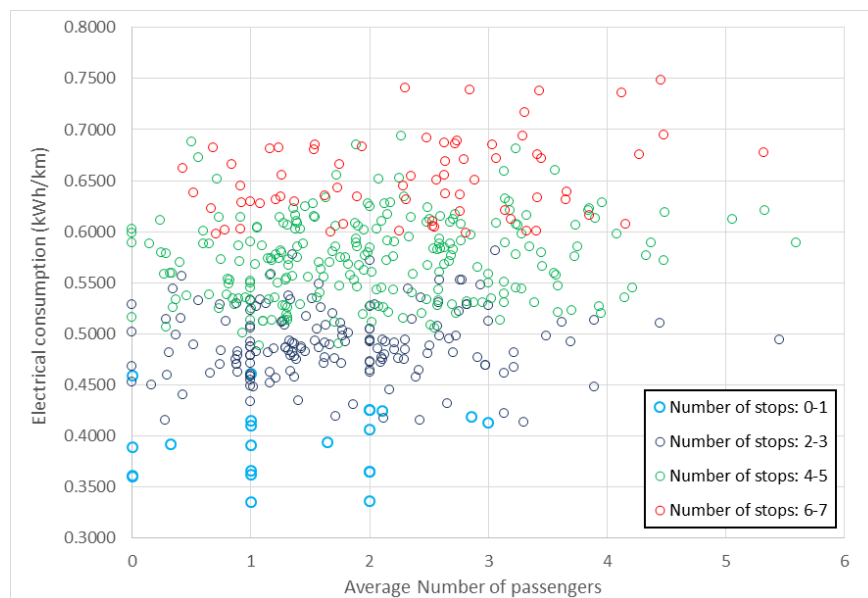**Figure 32. Number of stops influence on electrical consumption.**

Figure 32 shows how the effect of the number of stops on electrical consumption is more pronounced than the one caused by the number of passengers. The data have been divided into 4 groups, depending on the number of stops. The light blue circles represent driving cycles with 0 or 1 stops, the dark blue circles represent driving cycles with 2 or 3 stops, the

green circles represent driving cycles with 3 or 4 stops and the red circles represent driving cycles with 6 or 7 stops. It can be observed that electrical consumption increases as the number of stops increases. However, the number of passengers has influence as well, causing an increase in electrical consumption.

4.3.2 Electrical Consumption at Different Passenger Load and Average Speed

To evaluate the effect of speed on electrical consumption, the average bus speed of each driving cycle was calculated. Thus, an average value for each simulation performed was obtained, and then they were divided into six bins depending on the following intervals:

Table 12. Average speed divided into bins.

Bin name	Interval	
	m/s	km/h
25.2	6-7	21.6-25.2
28.8	7-8	25.2-28.8
32.4	8-9	28.8-32.4
36	9-10	32.4-36
39.6	10-11	36-39.6
43.2	11-12	39.6-43.2

Electrical consumption as a function of passenger load and average speed is illustrated in figure 33.
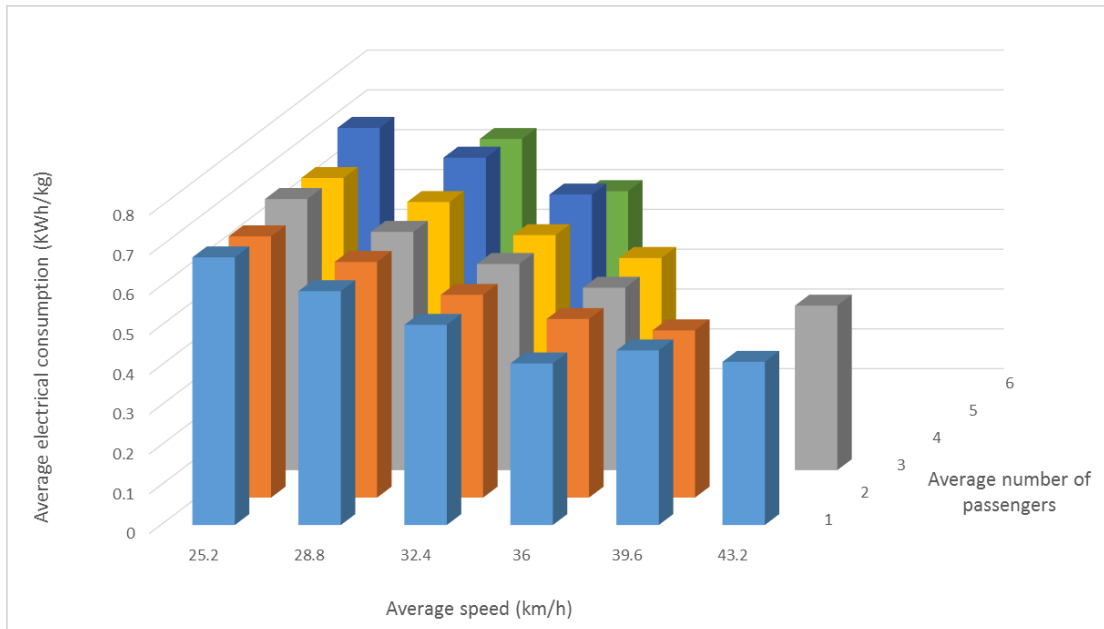
Figure 33. Electrical consumption at different passenger load and average speed.

In this case, it can be observed that the electrical consumption of the bus is higher when the average number of passengers is bigger and the average speed is lower. This is logical as the average speed is lower when more stop-go events are performed, i.e. when it does more stops. Therefore, electrical consumption is directly proportional to passengers and inversely proportional to speed.

Figure 34 shows that electrical consumption has a higher dependence on speed than in the number of passengers. This may be because there is not much variation in the number of passengers. Values with greater variability will produce greater changes in consumption.

Figure 34. Speed influence on electrical consumption.

5 Discussion

First of all, this section summarizes the steps taken for the development of the study. Furthermore, the main conclusions are formulated as well as the limitations of the study and the recommendations for further research.

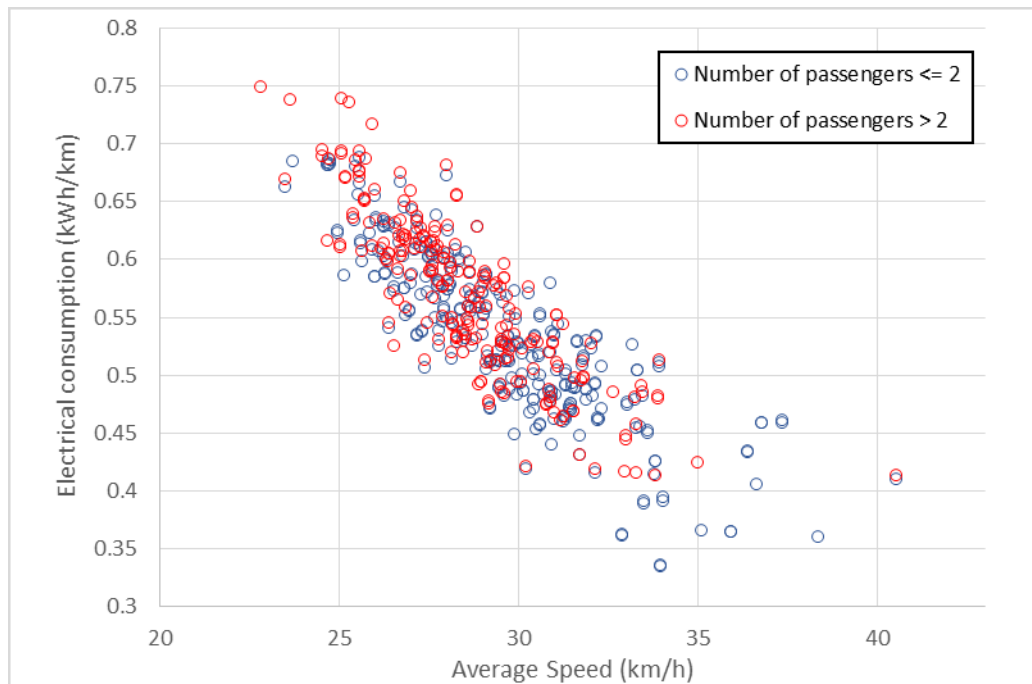## 5.1 Research Summary

In this thesis a study about the influence of passenger load on electrical consumption of an electric city bus was performed, providing a reliable energy consumption forecast. To address reliability and accuracy to the results, Monte Carlo Method was utilized.

An algorithm for the calculation of the passenger load was created in Matlab, based on driving cycles generated randomly (with a random number of stops and different speed profiles). This algorithm calculates the number of passengers inside the bus during each driving cycle introduced, creating a passenger flow. The algorithm was computed based on actual bus data of the studied route. Monte Carlo method allows the introduction of uncertainty in the algorithm inputs. Therefore, passenger data for each bus stop were represented by a normal probability distribution and they were related to each other using a multivariate normal distribution. Thus, these are the uncertain inputs of the model, as well as the number of stops which was modelled previously by another normal distribution.

An electric bus model created in Simulink was simulated with a real urban cycle and constant mass to conduct the validation of the model. Once it was validated, Monte Carlo simulation was performed by varying in each iteration the driving cycle and the introduced passenger flow.

The number of iterations performed was analysed based on the convergence of the electrical consumption mean. Finally, it was determined that 500 simulations would yield an accurate output estimation.

The results obtained were analysed with different input values, taking into account the variation in the electrical consumption caused by the passenger load impact.

5.2 Conclusions

Based on the results obtained in the different scenarios simulated, the following conclusions can be drawn.

First of all, it should be noted that the possibility of simulating with random numbers provides quality to the study. The consistent and reasonable results obtained confirm that the Monte Carlo method provides an efficient tool for estimating the consumption of an electric city bus. By means of this method, the amount of information obtained to describe the possible behaviour of the bus is greater, since it allows the introduction of uncertainty in the inputs of the model which enable to obtain results for the different possible scenarios and cover all the variations.

In order to increase the precision in the estimation, a larger number of simulations must be performed. However, more simulations involve more time so a compromise between accuracy and time must be found. Finally, 500 simulations were carried out to provide

stability and robustness, and a sufficiently reasonable estimate was obtained for electrical consumption.

On the other hand, in the analysis of the actual data used to create the mass algorithm, it was observed that the number of passengers in the bus depends on the time of the day and day of the week. Therefore, the introduction of the correlations between the different variables was of great importance to bring realism to the model.

The results described as a probability distribution define an electrical consumption with an average of 0.549 kWh/km. It is also possible to assure with an 80% of probability that the electrical consumption will be between 0.485 kWh/km and 0.613 kWh/km. These results obtained with the Monte Carlo method are an estimation of the bus electrical consumption depending on the number of stops performed and the number of passengers boarding and alighting at each stop. The results are valid for the estimation of the consumption of a battery electric bus that circulates in the 3.3 km section of the line 11 route in Espoo (Finland), which was the one studied.

Moreover, the analysis of the results indicates that the passenger load has a clear influence on the bus electrical consumption. This is because the mass of the bus plays an important role in the consumption, since it affects the necessary force that is required to accelerate the vehicle. The rolling resistance is determined by the weight of the vehicle and the pressure of the tires. The weight of the bus itself and the one of its occupants directly influences consumption, especially in the start and acceleration phases. At low speed the component of mechanical resistance is higher than aerodynamic one, so the influence of mass is also greater.

The results obtained confirm that the electrical consumption of the bus increases with the number of passengers. Furthermore, it has been observed that variations in the speed and number of bus stops affect electrical consumption more than mass variations. This may be due to the fact that there were no big differences in the average mass between the different driving cycles (there were variations from 1 to 6 passengers, which is 70 to 420 kg), which leads to a slighter variation in consumption.

The average speed of the driving cycle and the number of stops are related to each other. Since more stops lead to more stop-go events and therefore, the average speed is lower leading to a higher electrical consumption of the bus. It can be concluded that when there is a driving cycle that combines the highest number of passengers and the largest number of stops (and therefore lower average speed), the maximum electrical consumption will be obtained.

Comparing the results obtained in the present study with those obtained in previous study [17], where the uncertain input was the number of stops, it is verified that the mean of the electric consumption, which in study [17] was 0.609 kWh/km, differs slightly from the one obtained in this present study (0.549 kWh/km). This may be due to the use of different bus models with different parameters and assumptions. In addition, in the previous study [17] the difference between the minimum and maximum consumption obtained in the histogram was 49.4%, while in the present study it was 75.44%. The same occurs with the confidence interval which in the present study varies by 26.37%, and in study [17] it varies by 14.4%. This can be explained by the fact that when considering a larger number of uncertain inputs,

a higher variation in the output is obtained, since a larger number of scenarios are considered. Furthermore, a greater number of simulations are necessary to achieve accuracy in the results. Therefore, the present study is more complete and represents better the possible actual behaviour of the bus on the analysed route, as it covered more possible scenarios.

The introduction of several uncertainties adds sensitivity to the analysis. Introducing the possibility of different scenarios improves the estimation as more realistic values are obtained. Obtaining an accurate estimation of the consumption of an electric bus is important for transport planning. Estimating the possible load scenarios can solve the uncertainties related to when and where the bus should be charged. Furthermore, it can help with establishing route schedules. In addition, the battery size can be designed more accurately.

Lastly, it is evident that the influence of the passenger load is different when studying the total consumption or the consumption per passenger. When the consumption is divided by the number of passengers inside the bus, it is obtained that there is a lower consumption per capita when the number of passengers is higher. Therefore, there exists an inverse correlation between the consumption per passenger and the passenger load. This indicates that when the bus is in low load, the consumption per passenger can be very similar to that of a private vehicle, henceforth it can be concluded that a reasonable planning for transit bus routes is important to reduce the consumption. This can be achieved by adjusting the schedule to the passenger demand, and also the punctuality of the buses will help.

5.3 Limitations

However, this study has some limitations that will be listed in this section.

First, the size of the sample could be expanded. The actual data measured in the route analysed shows a bus line with a low number of passengers, with the maximum number of passengers inside the bus being 12. This fact limits the study since it makes the passenger load very low and there is not much variation between the average passenger load in different driving cycles. This causes that the fluctuations in electrical consumption due to passenger load were not very strong.

Secondly, the data measured on the route does not correspond to the number of passengers that get on or off at each bus stop, but it corresponds to the number of passengers inside the bus after each stop. This means that when performing the probability distributions for each stop in the mass algorithm, the number of passengers getting on or off at each stop was calculated as the absolute difference between the passengers inside the bus before and after each stop. Therefore, it is not known the exact number of passengers boarding and alighting at each stop, only the absolute difference.

Third, the mass algorithm used to calculate the number of passengers inside the bus is not totally accurate, since normal (continuous) distributions have been used to model the boarding and alighting of passengers on the bus at each stop, which is a discrete variable. However, the algorithm reproduces with sufficient accuracy the approximation of the passenger load fluctuation on the route.

On the other hand, although the obtained results correctly estimate what the actual consumption of the bus would be, only the influence of the passenger load and the number of stops were taken into account. However, not all the parameters that affect the electrical

consumption of the bus have been considered in the model, since the traffic and road grade have not been analysed and they are influential factors as well.

In the electric bus model only longitudinal dynamics were taken into account, excluding all lateral effects. Also, some simplifications were taken, such as the headwind speed that was considered 0 m /s and the road angle that was considered 0 degrees (horizontal profile).

Finally, the results are valid only for the specific fragment of the route studied and the bus configuration modelled. For other types of buses or routes another sensitivity analysis would have to be carried out since the characteristics of the route would vary, as well as the traffic or the passenger demand of the line. Thereby, more size accurate batteries can be designed, depending on the bus and the route that it follows.

5.4 Suggestions for Further Research

As mentioned before, this study does not include all the parameters that affect the electrical consumption of the bus, hence in future studies, other influential parameters could be incorporated. The objective would be to obtain more accurate forecasts, closer to what could happen in reality. Some of the parameters that could be included in future studies are the level of traffic congestion, the road grade, the driver behaviour (accelerations and abrupt braking) and the stops due to traffic lights or signals. Although some parameters are difficult to define, such as traffic or driver behaviour, they modify the driving cycle, which undoubtedly affects the bus electrical consumption.

This study has proved the importance of developing tools that allow you to predict possible scenarios and therefore, possible behaviour of electric buses, as they can improve the current transport planning system. In this way, transport operators would have a better knowledge of the impact of several factors in the electrical consumption and consequently in the battery life time, which allows the optimization of the charging points on the route.

Finally, another suggestion would be to include the existing relationship between the number of passengers boarding or alighting the bus with the dwell time at each station. In this way, the driving cycle would change because the time in which the bus was stopped at each station would be different, being greater when a higher number of passengers is involved in the getting on and off actions. The dwell time would therefore affect the total route time and it could also be analysed in order to establish the route schedules.

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Appendix 1: Mass Algorithm with Matlab

```matlab
%Load the data from excel datasheet
 Load ('C:\Users\ASUS\Desktop\master
thesis\simulink_model_def\passengersdata_multivariate.mat')
%load data for passengers boarding and alighting
passengers_stop=passengersdataS1;
%initial number of passengers, this is before arriving at the first stop
studied
passengers_before=passengersbefore;

%create the normal distribution acording to data
pd_normal_in = fitdist(passengers_before,'normal');
mu(1)=mean(pd_normal_in);

%generate normal distribution for passengers boarding or alighting in
each stop
for i=1:8
pd_normal=fitdist(passengers_stop(:,i),'Normal');

%record mu and sigma of each one
mu(1,i+1)=mean(pd_normal);
sigma(1,i+1)=std(pd_normal);

%Plot histogram and normal PDF for each stop
figure
hist(passengers_stop(:,i))
hold on
x_values = -4:0.1:10;
y = pdf(pd_normal,x_values);
scale = 10/max(y);
plot(x_values,y.*scale)
end

%calculate covariance matrix: C=cov(A);If A is a matrix whose columns
represent random variables and whose
%rows represent observations, C is the covariance matrix with the
corresponding column variances along the diagonal.
multinorm_data=cat(2,passengers_before,passengers_stop);%matrix with
passengers before (first column) and passengers stop data
covar=cov(multinorm_data);

%create the probability density of the multivariate normal distribution
%with mean the vector mu and covariance sigma
F = mvnpdf([multinorm_data(:,1) multinorm_data(:,2) multinorm_data(:,3)
multinorm_data(:,4) multinorm_data(:,5)...
    multinorm_data(:,6) multinorm_data(:,7) multinorm_data(:,8)
multinorm_data(:,9)],mu,covar);

%Create passenger flow for each driving cycle introduced
%simulaatiot = number of driving cycles generated
%nr_passengers = number of passengers after each stop
for zz=1:simulaatiot
load(strcat('Cycle_combined_nr',num2str(zz),'.mat')); %load the driving
cycle generated previously
```

```matlab
%initialitation
stop=sch_stops; %stops of the driving cycle
nr_passengers=zeros(1,8);
sch_passengers=zeros(size(sch_cycle));

%generate a random number from the normal distribution, this is going to
%be used as the number of passengers in  the bus when it arrives at the
first stop
%the normal distribution has negative values, and only positive values
are
%taken
initial_passengers=random(pd_normal_in);
while initial_passengers<0
initial_passengers=random(pd_normal_in);
end
initial_passengers=int16(initial_passengers) %integer

%conditional distribution. Each variable correspond with the passengers
%boarding and alighting in each bus stop. the conditional distribution of
%each variable is done knowing the values of the previous variables.

%x= vector with the value of each variable
%the first variable x1=initial_passengers
x(1)=initial_passengers;
sum_pass=0;

%knowing the first value variable, the conditional distribution of the
%following variables are created
%variables x2.....x9= number of passengers boarding or alighting in each
%bus stop
for i=2:9
sum_pass=sum_pass+x(i-1);
 if stop(i-1)==1
    for j=1:i-1
        a(j)=x(j)-mu(j);
    end
    a=double(a);

    %calculate the expectation (E) and variance (var) values of
conditional normal
    %distribution
    E=mu(i)+covar(i,1:i-1)*inv(covar(1:i-1,1:i-1))*a(1:i-1)';
    var=covar(i,i)-covar(i,1:i-1)*inv(covar(1:i-1,1:i-1))*covar(1:i-1,i);

    %create the conditional normal distribution of variable x(i), knowing
    %the values of the previous variables
    pd_condnormal=makedist('Normal','mu',E,'sigma',var);
    x(i)=int16(random(pd_condnormal));%integer

% avoid that when there are 0 passengers on the bus, they remain 0 after
a stop
while (sum_pass==0) && (x(i)<=0)
        x(i)=int16(random(pd_condnormal));
    end

 else
    x(i)=0;
 end
```

```matlab
 %Calculate number of passengers after each stop
 nr_passengers(1,1)=x(1)+x(2);
  for z=3:9
nr_passengers(1,z-1)=nr_passengers(1,z-2)+x(z);
  end

%limit the bus capacity
%minimum limit
if nr_passengers(1,i-1)<=0
    nr_passengers(1,i-1)=0;
end
%maximum limit
if nr_passengers(1,i-1)>=45;
    nr_passengers(1,i-1)=45;
end


end


%create the passengers flow following the driving cycles.
%passengers in the bus during driving cycle time
% sch_cycle(:,1)= time vector for the driving cycle
%passengers = passengers vector for the driving cycle
[m,n]=size(sch_cycle);
passengers=zeros(m,1);
passengers(:,1)=initial_passengers;

%record the positions when the bus start movement after a bus stop
ii=1;
positions_zero=find(sch_cycle(:,2)==0);  %find the positions in which the
velocity =0
siz=size(positions_zero);
sx=siz(1,1);
for i=1:(sx-1)
if (positions_zero(i+1)-positions_zero(i))>100
    start_movement(ii,1)=positions_zero(i); %vector with the positions in
which the bus start movement after each stop
    %the vector dimension is equal to the number of stops in the driving
    %cycle
ii=ii+1;
end

if sch_stops(8,1)==1
    start_movement(ii+1,1)=positions_zero(sx);
end
end

%create passengers flow
jj=1;
for k=1:8
if sch_stops(k,1)==1
        for kk=1:m
         if kk>start_movement(jj,1)
        passengers(kk,1)=nr_passengers(1,k);
        end
        end
     jj=jj+1;
end
end
```

```matlab
%plot passengers flow against driving cycle
figure
subplot(2,1,1);
plot(sch_cycle(:,1),passengers(:,1))
grid on;
hold on
xlabel('Time [s]');
ylabel('Passengers [nr]');
subplot(2,1,2);
plot(sch_cycle(:,1),sch_cycle(:,2))
hold on
xlabel('Time [s]');
ylabel('Velocity [m/s]');

%create matrix with time and passengers
sch_passengers(:,1)=sch_cycle(:,1);
sch_passengers(:,2)=passengers(:,1);

%simulate the simulink model
 stop_time=max(sch_cycle(:,1));
 loc=stop_time/0.01+1; %0.01 is the sample time. with this we can
calculate the consumption at the end of the driving cycle
 %initial condition for integrator, initial speed of the driving cycle
    v0=zeros(loc,2);
    v0(:,1)=0:0.01:stop_time;
    v0(:,2)=sch_cycle(1,2);
 %simulate electric bus model eBus_V2
 simOut =
sim('eBus_V2','StopTime','stop_time','ReturnWorkspaceOutputs','on')
  %save the consumption of each sample (it is in kwh/km)
 output = simOut.get('Batconsumption');
 consumption(zz)=output(loc);
 %save other data
 consumption_sim=consumption(zz);
 nr_stops(zz)=sum(sch_stops(:) == 1);
 average_passenger_load(zz)=mean(sch_passengers(:,2));
 average_speed(zz)=mean(sch_cycle(:,2));
 name=strcat('sim',num2str(zz));
save(name,'sch_stops','sch_cycle','sch_passengers','nr_passengers','consu
mption_sim')

end
save('results.mat', 'consumption','nr_stops',
'average_passenger_load','average_speed')
```