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From transition magnetic moments to majorana neutrino masses

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Abstract

It is well known that a majorana mass induces a (small) transition magnetic moment. The converse is also true; in this paper we estimate the loop contribution of transition magnetic moments $[\mu]_{\alpha\beta}$ to the neutrino mass matrix $[m]_{\alpha\beta}$. We show that for hierarchical neutrino masses, the contribution of $[\mu]_{e\tau}$ to $[m]_{e\tau}$ can exceed the experimental value of $[m]_{e\tau}$.

1 Introduction

If neutrino masses are majorana, they are generated by some New Physics(NP) from beyond the Standard Model (SM). There is a plethora of models that fit the low energy neutrino mass matrix, but that differ in interactions and particle content at higher energies. This motivates the question "can we identify this new physics from data?"—preferably from experiments in our lifetime, which suggests that they should be performed at energy scales within an order of magnitude or so of m_W.

If the scale of the new physics is experimentally accessible (eg R-parity violating supersymmetry), then the answer is "yes". But if the scale of new physics is *above* that accessible to accelerators, we are lead to ask "to what degree can we reconstruct the new physics of the lepton sector by measuring the coefficients of non-renormalizable operators"?

This is an complex question. This paper considers a more manageable toy model, restricted to lepton number violating operators: the neutrino masses are majorana, and we suppose that neutrino transition magnetic moments [1] are of order their current upper bound. What can be learned from the effective theory[2] comprising these non-renormalizable operators and the Standard Model? We find constraints on transition magnetic moment operators, from their contribution to majorana masses. The effective Lagrangian point of view will allow us to derive these bounds in a model independent way.

In the remainder of this section, we discuss diagrams that could generate a majorana mass from a magnetic moment. In section 2, we review our notation, current bounds on neutrino masses and magnetic moments, the sense in which magnetic moments near their current bound are "large" but masses are "small", and models built to address such an unexpected occurrence. The transition magnetic moment operator is of dimension 7 in the electroweak sector of the SM. Above the scale m_W, and below Λ_{NP} (the scale of the new physics which generates the masses and/or magnetic moments), there should be a range in energy where the operator evolves according to the renormalization group of the SM. So in section 3, we compute the anomalous dimensions of the relevant lepton number violating operators, which gives the leading contribution of the magnetic moment operator to the neutrino masses. For some parameter choices, this contribution is large. In section 4, we discuss the implications of this calculation.

1.1 diagrams

If the light neutrinos are majorana, they can have transition magnetic moments. Like the neutrino majorana mass, the magnetic moment $[\mu]_{\alpha\beta}$ violates lepton number by two units. So a diagram connecting the photon line back to either of the neutrino lines would appear to contribute to the majorana mass—except that it is negligibly small ($O(\mu^2)$), because the only $\nu_\alpha\nu_\beta\gamma$ interaction in a low energy ($\ll m_W$) $U(1)_{em}$-invariant Lagrangian is $[\mu]_{\alpha\beta}$. Furthermore, this $O(\mu^2)$ diagram would be L conserving, so cannot contribute to majorana masses.

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However, the magnetic moment operator must be the $E \ll \langle H_u \rangle$ realisation of an $SU(2) \times U(1)$ invariant operator, so there should be a related $\nu\nu Z$ interaction, and/or possibly a $\nu e W^+$ interaction. See section 2 for a discussion of $SU(2) \times U(1)$ invariant operators which reduce to a neutrino transition magnetic moment. Replacing the photon with a Z, and connecting the Z back to either of the neutrino lines, as in fig. 1, gives a pair of one-loop diagrams contributing to $[m_\nu]_{\alpha\beta}$—which sum to zero. This is expected: $[m_\nu]_{\alpha\beta}$ is symmetric on generation indices, and $[\mu]_{\alpha\beta}$ is anti-symmetric, so $[\mu]_{\alpha\beta}$ must be multiplied by some flavour-antisymmetric matrix to contribute to $[m_\nu]_{\alpha\beta}$. This feature was pointed out in [4], and used in models that produce small majorana masses and large magnetic moments[5, 6].

The required antisymmetric matrix could be $(m^e_\beta)^2 - (m^e_\alpha)^2$, where m^e_α is a charged lepton mass. In the one loop diagram that includes the neutral Z, the remaining loop particles must also be neutral, so m^e_α cannot appear. However, if the magnetic moment operator induces a $\nu e W$ interaction,then the diagram on the left of fig. 2 can give a one-loop contribution to $[m_\nu]_{\alpha\beta}$ of order

$$\frac{g\mu_{\alpha\beta}|m^{e2}_\alpha - m^{e2}_\beta|}{16\pi^2} \log \frac{\Lambda^2_{NP}}{m^2_W} \tag{1}$$

which is $\gtrsim 10^{-2}$ eV for $[\mu]_{\tau\alpha} \simeq 10^{-12}\mu_B$. Notice that by power counting, this diagram is logarithmically divergent, *i.e.* there are no quadratic divergences. This is because the magnetic moment is antisymmetric in flavour, while neutrino masses are symmetric, so there are two charged lepton mass insertions.

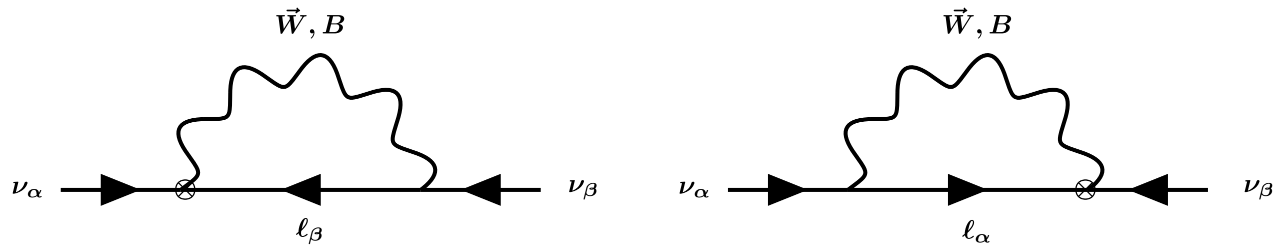

Figure 1: One-loop diagrams mediated by SU(2) gauge bosons, whose combined contribution to $[m_\nu]_{\alpha\beta}$ sums to zero. The crossed vertex is the magnetic moment $[\mu]_{\alpha\beta}$, and ℓ is the lepton doublet.

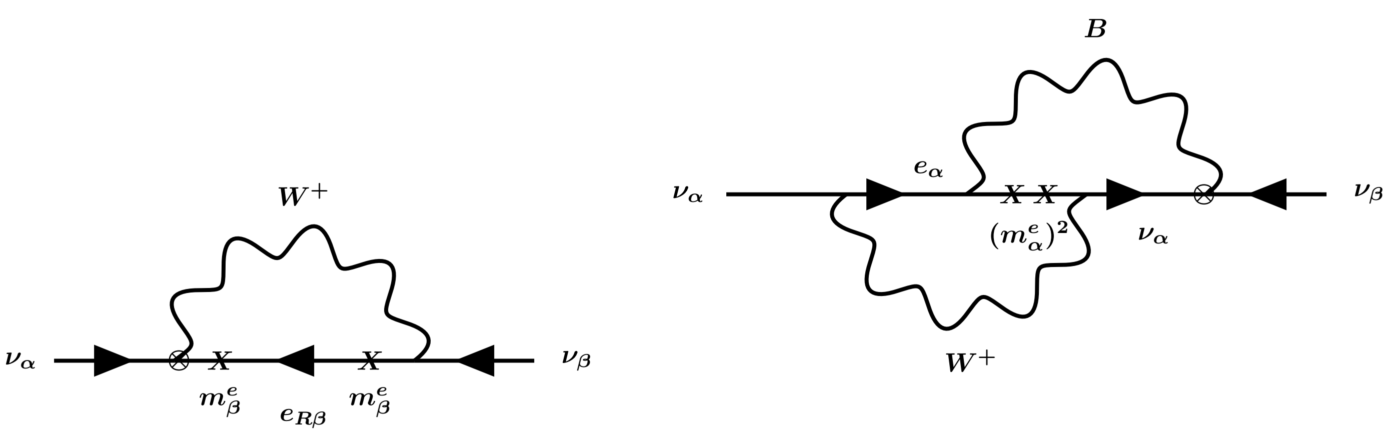

Figure 2: On the left, a one-loop contribution to $[m_\nu]_{\alpha\beta}$, where the crossed vertex is a magnetic moment $[\mu]_{\alpha\beta}$ involving the SU(2) gauge bosons $\vec{W}$, and X is a mass insertion. On the right, a two-loop diagram, the lowest order contribution when the magnetic moment $[\mu]_{\alpha\beta}$ involves the hypercharge gauge boson B.

Alternatively, if $[\mu]$ only involves the γ and Z, then two loops are required to have charged particles in the diagram. A two-loop contribution to $[m_\nu]_{\alpha\beta}$ is illustrated on the right in fig. 2. The m^e_α must appear twice on the charged lepton line, because the W only interacts with left-handed fermions. This diagram is log divergent, and should be of order

$$\frac{g^3 \mu_{\alpha\beta} |m_\alpha^{e2} - m_\beta^{e2}|}{(16\pi^2)^2} \log^2 \frac{\Lambda_{NP}^2}{m_W^2} \qquad (2-\text{loop}). \tag{2}$$

The one-loop W diagram will give the largest contribution to the mass matrix—if the $We\nu$ interaction exists. This is theoretically "reasonable": inside the crossed effective vertex of figures 1 and 2 are loops that generate $[\mu]_{\alpha\beta}$. Some of the particles in these loops must carry SU(2) quantum numbers, because there is no renormalizable vertex involving $\nu\nu$ and an SU(2) singlet. So if the effective vertex generating $[\mu]_{\alpha\beta}$ is opened up and one looks at the constituent diagrams, naively it seems that a W magnetic moment operator could arise by attaching W to an internal line. In section 3, we will calculate the W-magnetic moment contribution to the neutrino mass matrix, and estimate the two-loop Z contribution.

2 notation, bounds and expectations

Suppose the light neutrinos are majorana. Then it is known [3] that they can have transition magnetic moments, but not flavour diagonal ones. This is because the magnetic moment interaction, which we normalize in the Lagrangian as:

$$\frac{\mu_{\alpha\beta}}{2} \overline{\psi}_\alpha \sigma^{\mu\nu} \psi_\beta F_{\mu\nu} \quad \rightarrow \quad \frac{\mu_{\alpha\beta}}{2} \ \overline{\nu^c}_\alpha \sigma^{\mu\nu} P_L \nu_\beta (F_{\mu\nu}) + h.c. \tag{3}$$

flips the chirality of the fermion passing through. Greek indices from the beginning of the alphabet, *e.g.* α, β, are flavour indices, ψ is a four-component fermion, $\overline{\nu^c} = (-i\gamma_2(\nu^\dagger)^T)^\dagger \gamma_0$, and the magnetic moment $[\mu]_{\alpha\beta}$ has dimensions of 1/mass. Matrices in flavour space will often be written in square brackets, *e.g.* $[\mu]$ and $[m_\nu]$. The "right-handed" component of a light majorana neutrino is the antineutrino, so in chiral four-component notation for ν, the left hand side of eqn (3) can be rewritten as the right hand side, which vanishes for $\alpha = \beta$ ($[\mu]_{\alpha\beta} = -[\mu]_{\beta\alpha}$) by antisymmetry of fermion interchange.

The operator (3) is of mass dimension five, and consistent with electromagnetic gauge invariance. However it carries hypercharge $Y = -2$, so the SU(2)× U(1) invariant operator must be of higher dimension involving two Higgses. There are two possible dimension seven operators which give a neutrino magnetic moment interaction after spontaneous symmetry breaking:

$$[O_B]_{\alpha\beta} = g'(\overline{\ell^c}_\alpha \epsilon H) \sigma^{\mu\nu} (H \epsilon P_L \ell_\beta) B_{\mu\nu}, \qquad [O_W]_{\alpha\beta} = ig\varepsilon_{abd} (\overline{\ell^c}_\alpha \epsilon \tau^a P_L \ell_\beta)(H\epsilon\tau^b H) W^d_{\mu\nu}. \tag{4}$$

where the lepton flavour [1] indices $\alpha, \beta \in \{e, \mu, \tau\}$ are explicit, $\{\tau_i\}$ are the SU(2) Pauli matrices, the SU(2) contractions are implicit in the parentheses ($\epsilon = -i\tau_2$, $(v\epsilon u) = v_2 u_1 - v_1 u_2$), $\varepsilon_{abd} \neq \epsilon$ is the totally antisymmetric tensor, and $W_{\mu\nu}, B_{\mu\nu}$ are the gauge field strength tensors for SU(2) and U(1)$_Y$. We define the operators without hermitian conjugates; in the Lagrangian, they will appear multiplied by Wilson coefficients and $+h.c.$ —see, *e.g.* eqn (11).

We will compute the magnetic moment contribution to the masses by renormalization group mixing, for which we need the dimension seven mass term that arises with a single Higgs doublet [8]:

$$[O_M]_{\alpha\beta} = (\overline{\ell^c}_\alpha \epsilon H)(H \epsilon P_L \ell_\beta)(H^\dagger H) \quad . \tag{5}$$

In the presence of more than one Higgs doublet, there are other interesting operators which lead to neutrino masses [9, 10].

2.1 Phenomenological bounds

A $\Delta L = 2$ coupling between a gauge boson and a pair of leptons, of different flavour, could have various observable effects. We did not find significant bounds from rare decays (*e.g.* $W\nu e$ couplings), or precision lepton number conserving processes like $g-2$. The $We\nu_{\mu,\tau}$ interaction could appear at one of the vertices in neutrinoless double beta decay, but is not significantly constrained because of the flavour antisymmetry. The strongest bounds are on the magnetic moment interaction between a photon and a pair of neutrinos. This allows radiative decays $\nu_j \rightarrow \bar{\nu}_i \gamma$, contributes to the $\nu - e$ scattering cross-section, and induces the "decay" of photons in a plasma into ν pairs. Lower

[1] we can work in the flavour basis for the neutrinos, because the neutrino masses are small compared to any relevant energy scale, so will induce a negligible correction. The MNS matrix therefore will not appear in our calculation

bounds on the neutrino lifetime do not set interesting constraints on the magnetic moments, because the decay rates are already suppressed by powers of m_ν. Bounds on $[\mu]_{\ell\beta}$ from ν scattering experiments, are [11]

$$2\mu_{e\beta} \leq 0.9 \times 10^{-10}\mu_B, \quad 2\mu_{\mu\beta} \leq 6.8 \times 10^{-10}\mu_B, \quad 2\mu_{\tau\beta} \leq 3.9 \times 10^{-7}\mu_B \qquad \text{expt} \tag{6}$$

where $\mu_B = e/(2m_e)$, and the 2 is because our neutrinos are majorana [1]. For transition magnetic moments, $[\mu]_{\tau\beta}$ must satisfy the bound on $[\mu]_{e\tau}$ or $[\mu]_{\mu\tau}$, so is $\leq 6.8 \times 10^{-10}\mu_B$. The most restrictive constraint on $[\mu]_{\alpha\beta}$ comes from astrophysics. If photons in a stellar plasma can "decay" $\gamma \to \nu_\alpha\nu_\beta$, their energy escapes the star immediately. The observed cooling rate of globular cluster stars therefore sets a bound [19]

$$2[\mu]_{\alpha\beta} \lesssim 3 \times 10^{-12}\mu_B \qquad \text{astro} . \tag{7}$$

The solar neutrino flux is explained by large mixing angle (LMA) oscillations, but a subdominant effect due to neutrino magnetic moments remains possible. In the presence of the solar magnetic field, a non-zero $[\mu]_{e\alpha}$ could precess[2] the ν_e into $\bar{\nu}_\alpha$s . However, the solar bound on $[\mu]_{e\alpha}$ [18] and the sensitivity of future experiments are somewhat unclear, both theoretically and experimentally. Spin-flavour precession is usually assumed to take place in the outer regions of the sun, but a recent analysis [12] suggests that this is not the case for ^8Be neutrinos with LMA parameters. In addition, the effect depends on the solar magnetic field. In the accumulated solar neutrino data, there is some evidence for time-dependence. However, the longterm anti-correlation found between the Homestake solar ν data and the solar cycle [13] is not found by the SK collaboration [14] (but see [15]), which also does not find evidence for anti-neutrinos [16]. The sensitivity of future solar neutrino data to neutrino magnetic moments was studied in [17] (see also [12]); we cavalierly extract that $[\mu]_{e\alpha}$ in the range $10^{-10} \to 10^{-13}\mu_B$ could be interesting, and for all our numerical estimates, we will take

$$[\mu]_{\alpha\beta} \simeq 10^{-12}\mu_B \quad . \tag{8}$$

2.2 Dimensional analysis

Dimensional analysis suggests that $m_\nu \sim .1eV$ is "small", whereas $\mu \sim 10^{-12}\mu_B$ is "large". That is, new lepton number non-conserving physics at some scale M could induce transition magnetic moments and/or majorana masses; M estimated from m_ν is significantly higher than that obtained from μ.

It is well known, that if the dimension five majorana mass operator $(H\ell)(H\ell)$ induces neutrino masses $m_\nu \sim .1eV$ then the New Physics scale where this operator is generated should be $\lesssim v^2/(.1eV) \sim 10^{14}$ GeV. To repeat this argument for the dimension seven transition magnetic moment operators of section 2, requires estimating a lower bound on their coefficients C_J:

$$\mu \sim C_J v^2 \sim \frac{\mathcal{B}}{M^3}v^2 \sim 10^{-12}\mu_B \tag{9}$$

where $\mathcal{B}$ is some combination of coupling constants and $1/(16\pi^2)$ for loops, and M is the mass scale of the diagram. The magnetic moment must be suppressed by a loop factor $\mathcal{B} \lesssim g^2/(16\pi^2)$, because all its external legs are neutral, but nonetheless the photon should couple. Setting $\mathcal{B}v^2$ as large as is "reasonable" $\sim m_W^2/(8\pi^2)$, gives

$$M^3 \lesssim 5 \times 10^{11} GeV^3 \left(\frac{10^{-12}\mu_B}{\mu}\right) \tag{10}$$

or $M \lesssim 10$ TeV, if it is the same mass scale cubed. Taking $M^3 \sim m_W^2 M_{max}$, to maximise the New Physics scale, gives $M_{max} \lesssim 10^8$ GeV. If new ($\Delta L = 2$) physics arises between 10^4 and 10^7 GeV, then the $(H\epsilon\ell)(H\epsilon\ell)$ operator could give m_ν as large as GeV to MeV. The coefficient of this dimension five operator, which is determined by the New Physics, must therefore be strongly suppressed.

Notice that the magnetic moment is measured in the units used for the electron magnetic moment, $\mu_B = e/(2m_e)$. This is the relevant dimension for the electron, because the momentum in its loops (contributing, for instance, to $g-2$) is $1/p^2 \sim 1/m_e^2$, and m_e must appear upstairs to flip chirality. However, for the weakly interacting neutrino, one might expect $1/p^2 \sim 1/m_W^2$, suppressing $\mu \sim (m_e^2/m_W^2)\mu_B$. So the numerical value of eqn (8) suggests lepton number violation near the weak scale.

Various models have been constructed, which "naturally" generate a large $[\mu]$ with small $[m_\nu]_{\alpha\beta}$. From a top-down perspective, the difficulty is "inside" the magnetic moment vertex of figures 1 and 2, where the new physics generates μ. If the photon is removed from these internal diagrams, it would naively seem that the dimension five neutrino mass operator is obtained, with a "natural" coefficient of order the inverse new physics scale. This is too large. Voloshin

[2] see, *e.g.* [1] for an introduction to spin-flavour precession, and [12] for up-to-date references.

[4] addressed this in a model, by observing that $[\mu]_{\alpha\beta}$ was flavour *antisymmetric*, and arranging cancellations among the diagrams contributing to the flavour *symmetric* mass matrix. This approach has been followed by many people [5], who exploit the flavour antisymmetry of μ, and impose additional symmetries on the New Physics, to suppress contributions to the neutrino masses. Another interesting model [6], forbids by angular momentum conservation the magnetic moment diagram with its photon removed.

3 leading logarithmic contributions

In this section, we estimate the leading logarithmic contribution of the magnetic moment operators to $[m_\nu]_{\alpha\beta}$. We first study the scenario where the O_W operator gives the dominant contribution to the mixing into the dimension seven neutrino mass operator, while in the second scenario the New Physics induces the O_B operator, but not O_W. We work at one loop in unbroken $SU(2) \times U_Y(1)$, so the propagating particles in our diagrams are massless, and the charged fermion mass insertions are replaced by Yukawa couplings. Then, the dimension seven magnetic moment operator cannot mix to the dimension 5 neutrino mass operator.

Let us suppose that new (lepton number non-conserving) physics, above the scale Λ_{NP}, can be matched onto an effective theory

$$H_{\rm eff}(\Lambda_{\rm NP}) = C^W_{\alpha\beta}(\Lambda_{\rm NP})O^W_{\alpha\beta} + C^B_{\alpha\beta}(\Lambda_{\rm NP})O^B_{\alpha\beta} + C^M_{\alpha\beta}(\Lambda_{\rm NP})O^M_{\alpha\beta} + \ldots + h.c. = \vec{Q}^T\vec{C}(\Lambda_{\rm NP}) + h.c. \tag{11}$$

where the operators are defined in eqns (4) and (5). Below the scale Λ_{NP} our theory contains the interactions and particles of the electroweak standard model, the lepton number violating non-renormalizable operators of Eq. (11), and the dimension five neutrino mass operator, with small coefficient as discussed after eqn (10). This allows us to calculate the contribution of the neutrino magnetic moment to the neutrino mass in a way independent of the new physics scenario considered. To do this we solve the renormalization group equation

$$\mu\frac{d}{d\mu}\vec{C}(\mu) = \hat{\gamma}^T(g)\vec{C}(\mu) \tag{12}$$

(μ without indices or brackets being the renormalization scale) by expanding the anomalous dimension

$$\hat{\gamma}(g) = \sum_{i=0}^{\infty}\left(\frac{g^2}{16\pi^2}\right)^{i+1}\hat{\gamma}^{(i)} \tag{13}$$

and the Wilson coefficients

$$\vec{C}(\mu) = \sum_{i=0}^{\infty}\left(\frac{g^2}{16\pi^2}\right)^{i}\vec{C}^{(i)}(\mu) \tag{14}$$

in terms of the weak coupling constant. If we expand up to the second logarithmic enhanced order, the terms that will be relevant for us are

$$\vec{C}(\mu) = \vec{C}^{(0)}(\Lambda_{\rm NP}) + \frac{1}{2}\hat{\gamma}^{(0)^T}\vec{C}^{(0)}(\Lambda_{\rm NP})\frac{g^2}{16\pi^2}\log\frac{\mu^2}{\Lambda^2_{\rm NP}} + \frac{1}{8}\hat{\gamma}^{(0)^T}\hat{\gamma}^{(0)^T}\vec{C}^{(0)}(\Lambda_{\rm NP})\left(\frac{g^2}{16\pi^2}\log\frac{\mu^2}{\Lambda^2_{\rm NP}}\right)^2. \tag{15}$$

Suppose first that the New Physics generates the operator O_W (so we neglect O_B). Then only the first term from eqn (15) is required, with $g^2(\mu) = g^2$, and the $\hat{\gamma}^{(0)}$ matrix element mixing O_W to O_M. The self mixing of the magnetic moment operators can be neglected, so the Wilson coefficient (at Λ_{NP}) is matched onto the neutrino magnetic moment

$$C^W_{\alpha\beta}(\Lambda_{\rm NP}) = -\frac{[\mu]_{\alpha\beta}}{4v^2 g\sin\theta_W} \tag{16}$$

where $v = \langle H\rangle$. Using the $W^+e\nu$ interaction

$$-gC^W_{\alpha\beta}\sqrt{2}v^2(\bar{e^c}_\alpha\sigma^{\mu\nu}P_L\nu_\beta + \bar{\nu^c}_\alpha\sigma^{\mu\nu}P_Le_\beta)[2\partial_\mu W^+_\nu] \tag{17}$$

in the loop diagram on the LHS of figure (2), gives

$$C^M_{\alpha\beta}(m_W) = C^{M(0)}_{\alpha\beta} + \frac{6g^2|m^{e2}_\alpha - m^{e2}_\beta|}{16\pi^2}\frac{[\mu]_{\alpha\beta}}{4v^4 g\sin\theta_W}\log\left(\frac{\Lambda^2_{NP}}{m^2_W}\right) \tag{18}$$

where there is no sum on α, β. Taking vacuum expectation values, this contributes

$$\frac{1}{2}[\delta m]_{\alpha\beta} = \langle C_M O_M \rangle_{\alpha\beta} \simeq C^{M(0)}_{\alpha\beta} v^4 + \left| \frac{m_\alpha^{e2} - m_\beta^{e2}}{m_\tau^2} \frac{[\mu]_{\alpha\beta}}{10^{-12}\mu_B} \right| \log\left(\frac{\Lambda_{NP}^2}{m_W^2}\right) \times .014 \text{ eV} \tag{19}$$

to the neutrino mass matrix.

Consider now the second scenario, where the New Physics only produces a non-negligible Wilson coefficient for O_B. Matching O_B onto the neutrino magnetic moment gives

$$C^B_{\alpha\beta}(\Lambda_{\rm NP}) = \frac{[\mu]_{\alpha\beta}}{2g' \cos\theta_W} \tag{20}$$

In general O_B can mix into all possible dimension 7 $\Delta L = 2$ operators. For the purpose of studying the mixing into the neutrino mass operator O_M, we should calculate those divergent Green's functions of O_B which will mix into O_M. We find that O_B mixes into O_W, but not into some other possibilities. We estimate the O_B contribution to O_M from its second order mixing through $\hat{\gamma}^{(0)}_{WM}\hat{\gamma}^{(0)}_{BW}$:

$$C^M_{\alpha\beta}(m_W) \sim C^{M(0)}_{\alpha\beta} + \frac{3\tan^2\theta_W [\mu]_{\alpha\beta} |m_\alpha^{e2} - m_\beta^{e2}|}{4v^4 e} \left(\frac{g^2}{16\pi^2} \log\left(\frac{\Lambda_{NP}^2}{m_W^2}\right)\right)^2 \tag{21}$$

If the magnetic moment is generated by the O_B operator, then its contribution to neutrino masses is suppressed by $\sim (\frac{\alpha}{4\pi}\log)^2$. This will not give interesting constraints on $[C^B]$.

4 Discussion

In this section, we consider the phenomenological implications of the magnetic moment contribution to the neutrino mass matrix. We first review what is known about $[m_\nu]_{\alpha\beta}$, then discuss eqns (19) and (21).

4.1 observed parameters of the light ν sector

The ν mass matrix in flavour space is

$$[m_\nu]_{\alpha\beta} = U_{\alpha k} U_{\beta k} m_k \tag{22}$$

where $\{m_k\}$ are the neutrino masses, and U is the MNS matrix, parametrized as

$$U = \hat{U} \cdot \text{diag}(1, e^{i\alpha}, e^{i\beta}) \quad . \tag{23}$$

α and β are "Majorana" phases, and $\hat{U}$ has the form of the CKM matrix

$$\hat{U} = \begin{bmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \quad . \tag{24}$$

Current data [7] gives $\theta_{23} \simeq \pi/4$, $\sin^2\theta_{12} \simeq .29$, $\sin^2\theta_{13} \lesssim 0.035$, an atmospheric mass difference $\Delta_@ m^2 \simeq (0.049eV)^2$, and a solar difference $\Delta_\odot m^2 \simeq (0.0089eV)^2$. The absolute value of the mass scale in undetermined, as is the ordering of the eigenvalues. It is convenient to label the mass pattern as degenerate ($m_i \gg \sqrt{\Delta m^2_{atm}}$), hierarchical ($m_3^2 \simeq \Delta m^2_{atm}, > m_2^2, m_1^2$) or inverted ($m_3^2 \ll m_{1,2}^2, \quad m_{1,2}^2 \simeq \Delta m^2_{atm}$).

For comparison with the magnetic moment contributions to the mass matrix, it will be useful to have an idea of the numerical values of $[m_\nu]_{\alpha\beta}$. Taking $\theta_{12} \simeq 0.55$, $\theta_{23} \simeq \pi/4$ $m_1 = 0$, $|m_3| = .049$ eV and $k_2 = e^{2i(\alpha-\beta)} m_2/m_3$ ($|k_2| = 0.18$), in agreement with the hierarchical interpretation of current data, gives a mass matrix

$$[m_\nu] \simeq \begin{bmatrix} .30k_2 & .32k_2 + .35s & -.32k_2 + .35s \\ .32k_2 + .35s & .35k_2 + .25 & -.35k_2 + .25 \\ -.32k_2 + .35s & -.35k_2 + .25 & .35k_2 + .25 \end{bmatrix} \times .1\text{eV} \quad . \tag{25}$$

where the phase δ has been absorbed into the unknown angle $s = \sin\theta_{13} e^{-i\delta}$. From data, $|s| \leq 0.2$, so we took $\cos\theta_{13} = 1$, and dropped terms of order sk_2.

4.2 the magnetic moment contribution

The magnetic moment contribution to the neutrino mass matrix, from eqn (19), reads

$$[\delta m_\nu] \simeq b \left[\begin{array}{ccc} 0 & 0.004\tilde{\mu}_{e\mu} & \tilde{\mu}_{e\tau} \\ 0.004\tilde{\mu}_{e\mu} & 0 & \tilde{\mu}_{\mu\tau} \\ \tilde{\mu}_{e\tau} & \tilde{\mu}_{\mu\tau} & 0 \end{array} \right] \times .1\text{eV} \quad , \tag{26}$$

where the $\tilde{\mu}_{\alpha\beta}$ are the magnetic moments measured in units of $10^{-12}\mu_B$, and the log was conservatively estimated by taking $\Lambda_{NP} \sim 1$ TeV. If the magnetic moment is generated by O_W, then $b = 1$. However, if O_B is the magnetic moment operator, then $b \sim \alpha_{em}/\pi$, and the contribution to the mass matrix is reduced.

The mass matrix (26) by itself is not phenomenologically viable, because it imposes $\sum_i m_i = 0$, and predicts relations between the mixing angles and mass differences (it has only three free parameters). However, it naturally gives large mixing angles, so could contribute to $[m_\nu]$ in conjunction with some other (flavour diagonal?) source of neutrino masses [9, 10]. For the case of degenerate neutrinos, there are no bounds on $[\mu]$, in general. However, if the main source of neutrino masses is flavour diagonal, then the limits discussed below also apply.

The phenomenological consequences of eqn (26) can be seen by considering a series of cases:

1. suppose that the magnetic moment operator is O_W, so $[\mu] = 4v^2e[C^W]$ and $b = 1$ in eqn (26).
   - If $[\mu]_{\mu\tau} \gtrsim 10^{-12}\mu_B$, then for hierarchical or inverted neutrino masses (but not for degenerate), we find [3] $[\delta m_\nu]_{\mu\tau} \gtrsim [m_\nu]_{\mu\tau}$. So either $[\mu]_{\mu\tau} \leq 3 \times 10^{-13}\mu_B$, or there is some mild cancellation between $[\delta m_\nu]$ and other sources to $[m_\nu]$. The numerical "bound" is fuzzy because it depends logarithmically on Λ_{NP}, and slightly on the mass pattern and phases of $[m_\nu]$.
   - now suppose that $[\mu]_{e\tau} \sim 10^{-12}\mu_B$. If the neutrino masses are inverted, it is similar to the previous case. However, for hierarchical neutrino masses,
   $$\frac{[\delta m_\nu]_{e\tau}}{[m_\nu]_{e\tau}} \sim 10$$
   So $[\mu]_{e\tau} < 10^{-13}\mu_B$, which is too small to have an effect in solar physics, or there is a significant cancellation between $[\delta m_\nu]_{e\tau}$ and $[m_\nu]_{e\tau}$. It is usual, in quoting bounds on the coefficients of non-renormalizable operators, to suppose that only one operator at a time is present and neglect the possibility of cancellations. In this approach, the bound on the coefficient $[C_W]_{e\tau}$ corresponds to $[\mu]_{e\tau} < 10^{-13}\mu_B$.
   - the contribution of $[\mu]_{\mu e}$ to the neutrino mass matrix is always small (suppressed by m_μ^2), so if there is time-dependence in the solar neutrino signal, it is more likely due to $[\mu]_{\mu e}$.
2. Now consider the case where the neutrino magnetic moment is due to the operator O_B. The contribution to $[m_\nu]$ is of order $(\frac{\alpha}{4\pi}\log)^2$, so in eqn (26), $b \sim \alpha_{em}/\pi$, and $[\delta m_\nu]_{\alpha\beta} < [m_\nu]_{\alpha\beta}$ for magnetic moments of order $[\mu]_{\alpha\beta} \sim 10^{-12}\mu_B$. So the coefficients $[C^B]$ are more tightly constrained by the upper bounds on magnetic moments, than they are by their loop contributions to $[m_\nu]$.

   Notice,however, that [4] $[\delta m_\nu]_{e\tau} \simeq 3 \times 10^{-4} \left(\frac{[\mu]_{e\tau}}{10^{-12}\mu_B}\right)$ eV, and for a hierarchical neutrino mass pattern: $m_{e\tau} \simeq 5 \times 10^{-3}$ eV. So a magnetic moment of order $[\mu]_{e\tau} \sim 10^{-10}\mu_B$ (less than the experimental bound, but exceeding the astrophysical one) contributes a larger loop correction to $[m_\nu]_{e\tau}$ than its measured value.

The original hope was to "reconstruct" New Physics in the lepton sector. This is straightforward if the new particles and the form of their interactions are known; one can then attempt to extract the numerical value of the new coupling constants from the coefficients of operators involving SM fields [5]. However, the question is whether one can also learn about the New Physics "mechanism", that is, the particle content and type of interactions. We have not attempted to do this, but the result of this paper emphasizes the confusion: if the new physics scale is low enough to generate observable magnetic moments, then neutrino masses could arise from the usual dimension five operator, or the dimension seven operator O_M, and these are indistinguishable.

[3] For simplicity, we present bounds on the matrix elements of $[\mu]$, however in reality they apply to $[C^W]$.

[4] This relies on the rough estimate of b from eqn (21).

[5] For instance, in the SUSY type-1 seesaw, one can in principle extract the heavy RH neutrino masses and the neutrino Yukawa matrix, from the mass matrices of the sneutrinos and the light neutrinos [20].

4.3 summary

Transition (flavour changing) magnetic moments among Standard Model neutrinos are lepton number violating, so they contribute, via SM loop effects, to majorana masses. This is similar to the remark that New Physics inducing neutrinoless double beta decay necessarily contributes to majorana masses [21]. The transition magnetic moment matrix $[\mu]$ is also flavour *antisymmetric*, so must be multiplied by charged lepton masses in the SM loops contributing to the flavour *symmetric* majorana mass matrix $[m_\nu]$. The largest contribution of $[\mu]$ to $[m_\nu]$ is therefore in the third generation.

Transition magnetic moment operators in the SM have mass dimension ≥ 7, where there are two possible operators O_B and O_W (eqn 4). O_W contributes at one-loop to $[m_\nu]$, and O_B at two-loop (see *e.g.* fig. 2). We can set upper bounds on the coefficient matrices $[C^W]$ and $[C^B]$ of these operators, from requiring that their loop contributions to $[m_\nu]$ be small enough; these bounds then constrain the magnitude of the magnetic moment that the operator can generate. The resulting bounds on $[C^B]$ are weaker than current experimental limits on neutrino transition magnetic moments ($\sim 10^{-10} \to 3 \times 10^{-12} \mu_B$, see eqns 6 and 7); that is, the non-observation of neutrino transition magnetic moments sets tighter bounds on the coefficients of this operator than their loop contributions to neutrino masses. However, the case of O_W is somewhat different.

If the light neutrino masses are non-degenerate, then the bound on $[C^W]_{\alpha\tau}$ from its contribution to $[m_\nu]$, translates into the limit $[\mu]_{\alpha\tau} \lesssim 3 \times 10^{-13} \mu_B$. If the light neutrino masses are hierarchical, then we obtain $[\mu]_{e\tau} \lesssim 10^{-13} \mu_B$. Notice, however, that these bounds apply to the coefficients of the operator O_W, although for simplicity we quote them as limits on the neutrino transition magnetic moment. A larger $[\mu]_{\alpha\tau}$ could be consistent with the observed masses, if, for instance, it was generated by O_B.

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note added

In a recent paper [22], Bell et al perform a similar calculation for Dirac neutrinos. They calculate bounds

$$\mu_B \lesssim 10^{-14} \mu_B$$

on Dirac magnetic moments, from their contributions to Dirac neutrino masses.

The Dirac magnetic moment is not required to be flavour antisymmetric, so its contribution to neutrino masses is not suppressed by Yukawa couplings. This allows a stronger bound, which applies to all flavours. This suggests that only the majorana magnetic moment $\mu_{e\mu}$ can be large enough to affect solar physics.

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