Fast Stochastic Non-linear Model Predictive Control for Electric Vehicle Advanced Driver Assistance Systems

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***Abstract*— Semi-autonomous driving assistance systems have a high potential to improve the safety and efficiency of the battery electric vehicles that are enduring limited cruising range. This paper presents an ecologically advanced driver assistance system to extend the functionality of the adaptive cruise control system. A real-time stochastic non-linear model predictive controller with probabilistic constraints is presented to compute on-line the safe and energy-efficient cruising velocity profile. The individual chance-constraint is reformulated into a convex second-order cone constraint which is robust for a general class of probability distributions. Finally, the performance of proposed approach in terms of states regulation, constraints fulfilment, and energy efficiency is evaluated on a battery electric vehicle.**

I. INTRODUCTION

The technological evolution of Battery Electric Vehicles (BEVs) throughout recent years turned them into more sophisticated machines [1]. However, the BEVs have limited on-board energy capacity, which limits their cruising range on a single charge. One of the well-known methods to extend the cruising range is to identify an energy-efficient velocity profile. A smarter and more energy-efficient driving is generally referred to as Ecological (Eco) driving concept. Advanced Driver Assistance Systems (ADAS) can assist human drivers to improve the trip safety and energy efficiency.

Model Predictive Control (MPC) is an attractive approach in comparison with alternative methods of multi-variable control of complex ADAS systems with hard control constraints. In MPC, an Optimal Control Problem (OCP) based on a dynamic model of the system is solved repeatedly in a receding horizon style and the first element of a finite sequence of control actions is applied to the system at each sampling time. In addition, parametric uncertainties and exogenous disturbances are pervasive features of complex dynamical systems. Stochastic MPC (SMPC) has been introduced for systems with uncertainties (see e.g. [2]). The SMPC is based on a stochastic process model and generally formulated as an expectation of the objective function with probabilistic constraints, so-called *chance-constraints* (see e.g. [3]). Non-linear MPC (NMPC) is distinguished by the use of non-linear system models in the OCP to improve performance specifications. Stochastic Nonlinear MPC (SNMPC) has been introduced to improve the shortcoming of SMPC. However, it has received relatively little attention in works of literature, which is the main concern of this paper.

The Adaptive Cruise Control (ACC), and Cooperative Adaptive Cruise Control (CACC) automate the throttle and brake control of the vehicle to maintain the pre-set longitudinal velocity while regulating a safe distance from preceding vehicles. An SMPC with driver behaviour learning capability for improving the performance of powertrain was designed in [4]. A stochastic dynamic programming based control policy with a given road grade, traffic speed was established in [5]. Energy efficient NMPC to drive a vehicle efficiently on roads containing varying traffic and signals at intersections for improved fuel economy was introduced in [6]. An energy-efficient MPC that utilise the energy consumption characteristics of a BEV was established in [7], [8]. A Stochastic NMPC (SNMPC) with the target of emission, fuel efficient driving, and infrastructure-to-vehicle (I2V) communication capability was introduced in [9].

Although the mentioned studies have considerable contributions in this field, the conventional ACC systems are not capable of dealing with curvy roads and traffic signs information where the driver intervention is required. A sophisticated Eco-ACC with extended functionalities for the BEVs still need to be explored and therefore, is the main objective of this work. This system helps to fulfil the requirements of a semi-autonomous safe and energy-efficient Eco-ADAS system in a stochastic driving environment.

This paper formulates a fast SNMPC for the BEVs specific Eco-ACC system. The proposed formulation and implementation of the SNMPC Eco-ACC system are substantially different from those of previous works of literature. First, longitudinal dynamics, energy consumption models of a BEV, road geometry and traffic speed limit zones are modelled in a high fidelity deterministic framework. Second, the main contribution is to introduce a stochastic motion of a preceding vehicle with a novel physical-statistical model based on road geometry information and 85^{th} percentile speed concept. Then, the chance-constraint is used to regulate the relative distance between the preceding and host vehicles. The chance-constraint is converted to a second-order cone constraint, that is robust for a general class of probability distributions. After that, the SNMPC with a chance-constraint is reformulated as a certainty equivalent control policy. A real-time algorithm for receding horizon implementation of the resulting non-linear OCP based on Pontryagin's Minimum Principle is adapted. Finally, the performance of the proposed

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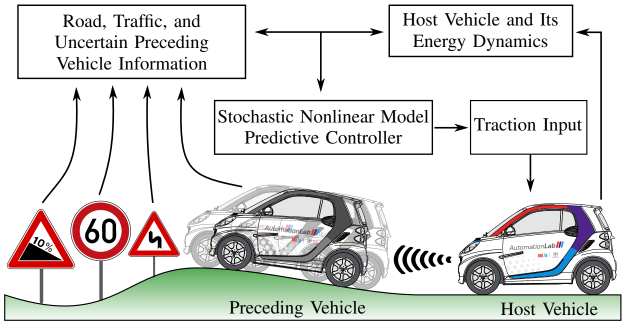

Fig. 1: Semi-autonomous Eco-ADAS system

concept in terms of system states regulation, constraints fulfilment, and energy efficiency is analysed.

The rest of this paper is organised as follows: The definitions and system models are introduced in Section II. The SNMPC based controller and problem formulation are presented in Section III. Evaluation of the proposed concept with practical experiments and numerical simulation results are presented in Section IV, followed by the conclusion and future research in Section V.

II. Definitions and System Models

The semi-autonomous Eco-ADAS concept, that extends the functionalities of an Eco-ACC system is presented in Fig. 1. Similar to the conventional ACC systems, the driver preset the desired velocity with preferred safe distance from the preceding vehicle. The Semi-autonomous Eco-ADAS system regulates the traction input with respect to the longitudinal motion, energy consumption dynamics of the BEV (host vehicle), the road geometric, traffic sign, and motion of the preceding vehicle information. While the driver handles the steering control of the vehicle, this system plans a proper safe and energy-efficient cruising velocity profile autonomously for the entire trip without requiring driver interventions.

A. Vehicle, and Energy Dynamics

The acceleration along the longitudinal direction of the BEV can be expressed by Newton's second law of motion, which it is assumed to be a point mass at the centre of gravity as follows:

$$dv_h(t)/dt = (F_{trac}(t) - F_{res}(t))/M, \tag{1}$$

where M, $F_{trac}(t)$, and $F_{res}(t)$ are equivalent mass of the vehicle, traction force, and total motion resistive forces, respectively. The traction force depends on the equivalent mass and control input as $F_{trac}(t) := Mu(t)$. The control input is bounded ($u_{min}(v_h) \leq u(t) \leq u_{max}(v_h)$) by the physical limits of the traction force that the wheel-road contact can support without slip [10]. The total resistive force including aerodynamic drag, gradient, and rolling resistance forces can be represented by:

$$F_{res} = \frac{1}{2}\rho A_f C_D(d) v_h^2 + Mgsin(\theta(s_h)) + C_{rr}(v_h)Mgcos(\theta(s_h)), \tag{2}$$

where ρ, A_f, g, $\theta(s_h)$, and $C_{rr}(v_h)$, are the air density, the vehicle frontal area, the gravitational acceleration, the road slope angle as a function of the host vehicle position, and the velocity dependent rolling resistance coefficient, subsequently. The rolling resistance coefficient for passenger vehicles on a concrete road can be written as $C_{rr}(v_h) = 0.01(1 + v/576)$ (for more details, see [1]). Note that $C_D(d)$ is the aerodynamic drag coefficient that depends on nominal aerodynamic drag coefficient and relative distance between the preceding and host vehicles, $d := s_p - s_h$. Vehicle drag reductions arising from close spacing with the preceding vehicle (for more details, see e.g. [11]).

For a given velocity at a given traction force, the operating point of the electric machine and the related power consumption/regeneration could be determined [10]. The energy consumption during cruising at constant speed is equal to the resistive power. This can be estimated by a polynomial of velocity as $f_{cruise} = b_3 v_h^3 + b_2 v_h^2 + b_1 v_h + b_0$ [6]. The acceleration and deceleration, a, of the vehicle considering only the regenerative energy zone in the hybrid (regenerative and friction) brake system, can be approximated using a polynomial of the control input as $f_a = a_2 u^2 + a_1 u + a_0$. Therefore, at any given velocity and control input, a linear relation of the traction power-to-mass ratio can describe the energy consumption of the vehicle as:

$$\dot{e}_h = f_a\ (p_{trac}/M) + f_{cruise}, \tag{3}$$

where p_{trac}, denotes the traction power (for more details, see [10]).

B. Road Geometry and Traffic Model

Road geometries and traffic information have favourable advantages for the Eco-ADAS safety and energy management applications [12]. The road slopes, road curves, and traffic speed limit zone data are modelled as continuous and differentiable functions in [10]. The road slope profile is proposed to be the sum of quadratic functions of the vehicle position representing each road segments slope data as follows:

$$f_{slp}(\theta(s)) := \sum_{n=1}^{N_{sgm}} H_n^{(s-s_{n-1})}(a_n s^2 + b_n s + c_n) H_n^{(s-s_n)}, \tag{4}$$

where N_{sgm} is the number of road segments, $H_n^{(s-s_{n-1})}$ and $H_n^{(s-s_n)}$ are hyper-functions of the *nth* road segment at the boundary position values, s_{n-1} and s_n. The road curves and traffic speed limits profiles are modelled in a similar way. The simple curve is used to express the total absolute curve profile, which is be defined as:

$$f_{crv}(\delta(s)) := \sum_{n=1}^{N_{crv}} H_n^{(s-s_{ent})} \left| \frac{1}{R_{crv_n}(s)} \right| H_n^{(s-s_{ext})}, \tag{5}$$

where N_{crv} is the number of road curves, and R_{crv_n} is the radius of a circle valid for the curve's arc length with two position points, s_{ent} and s_{ext}, at the respective entrance and exit position of the *nth* curve. Furthermore, The traffic speed limit places can be modelled as:

$$f_{lmt}(s) := \sum_{n=1}^{N_{lmt}} H_n^{(s-s_{str})}(v_{lmt} - v_{max}) H_n^{(s-s_{end})} + v_{max}, \tag{6}$$

where N_{lmt} is the number of speed limit zones, and v_{lmt} is the specified speed limit value at positions starts from s_{str} up to the end of the zone s_{end}. The v_{max} is the maximum speed value of the host vehicle. This method can also improve the trade-off challenge between the high and low-fidelity models for ADAS road models (for more details, see [10]).

C. Physical-Statistical Motion Model

Knowledge representation of traffic including a prediction model of the plausible future motion of vehicles improves the performance of decision-making processes in Eco-ADAS applications. However, high entropy in traffic system leads to a challenging task to derive a computationally efficient and tractable model to predict the motion flow. Research related to anticipating the possible trajectory of the preceding vehicle into the near/far-term future has a long track in the ADAS applications. For instance, a Markov chain model with the driver behaviour learning algorithm was proposed in [4]. A sigmoid-based function to estimate states of the preceding vehicle within the prediction horizon was introduced in [6]. A stochastic prediction method using Bayesian networks utilised for near-term future prediction was presented in [9].

Although the proposed methods mentioned in literature are effective for near-term prediction, rapid divergence can be experienced in far-term prediction. A physical-statistical motion model of the preceding vehicle robust to far-term future prediction is introduced in this paper. This model is based on 85^{th} percentile speed concept and road geometry information. The 85^{th} percentile speed is referred to as *spot speed study*, defined as the speed at or below which 85^{th} percent of vehicles travel a given location based on free-flowing conditions over a time period (for more details, see e.g. [13]). In addition to the 85^{th} percentile speed at road curves, other factors such as road slope profile and traffic speed limit zones information can be considered to estimate more appropriate trajectory. Therefore, the proposed dynamic model to propagate the velocity of preceding vehicle, v_p, at time t can be estimated as follows:

$$dv_p(t)/dt := X_{85^{th}}\left(1-\left(\frac{v_p}{f_{85^{th}}}\right)^4 - \frac{sin(f_{slp}(\theta(s_p)))}{sin(\frac{\pi}{4})}\right), \quad (7)$$

$$f_{85^{th}} := min\{\omega_{85^{th}} v_{85^{th}}(f_{crv}(\delta(s_p))), f_{lmt}(s_p)\}, \quad (8)$$

$$v_{85^{th}}(\delta(\hat{s}_p)) := m_1 exp^{(-m_2\delta(\hat{s}_p))} + m_3 exp^{(-m_4\delta(\hat{s}_p))} \quad (9)$$

where $X_{85^{th}}$ is the 85^{th} percentile acceleration of the preceding vehicle assumed to lie in a normal distribution i.i.d. $X \sim \mathcal{N}(\mu_p, \sigma_p)$ with the mean, μ_p, and variance σ_p^2. The $\omega_{85^{th}}$ is a tunable constant and $v_{85^{th}}(\cdot)$ is the position based function represents the 85^{th} percentile curve speed of the vehicles along the trip curves. The curve speed data is adapted from [13], where approximated by (9). The proposed model is continuous and differentiable, which is capable of propagating a plausible trajectory for the preceding vehicle motion profile.

III. Stochastic Nonlinear Model Predictive Control

The non-linear system to be controlled with disturbance is usually described by:

$$\dot{x} = f(x,u,\omega), \quad (10)$$

$$z = h(x), \quad (11)$$

$$\omega = \Delta(z_t(\cdot)), \quad (12)$$

where $x \in \mathbb{R}^{n_x}$ denotes the state vector, $u \in \mathbb{R}^{n_u}$ represent the input vector, $z \in \mathbb{R}^{n_s}$ refers to the output vector, and $\omega \in \mathbb{R}^{n_\omega}$ is random variable vector mapped by the causal output vector upto time, t [2]. The random variable vector ω is composed of i.i.d. random variables $\omega_i, \forall i \in \{1,..,n_\omega\}$, with probability triple sample space Ω, a set of events (σ-algebra) $\mathscr{F}$, and allocations of probabilities to the events (exogenous information), $\mathscr{P}$ on $(\Omega, \mathscr{F})$. The $\Delta(\cdot)$ is an operator standing for unmodelled dynamics that maps the output sequence over the interval $(-\infty, t]$ into ω. Assuming system states are measurable, a discrete multi-stage Stochastic OCP (S-OCP) with chance-constraints is stated as follows:

$$\underset{\pi}{\text{minimize}}\ \mathbf{E}_{x_t}\Big[\sum_{i=0}^{N-1} \mathscr{L}_c(x_i^*(t), \mu_i^*(t))\Delta\tau(t) + \mathscr{L}_t(x_N^*(t))\Big] \quad (13a)$$

$$\text{subject to} \quad (13b)$$

$$x_{i+1}^*(t) = x_i^*(t) + f(x_i^*(t), \mu_i^*(t), \omega_i(t))\Delta\tau(t), \quad (13c)$$

$$\mathbf{Pr}\{h_j(x_i^*(t)) \le 0\} \ge \beta, \quad j = 1,\ldots,q, \quad (13d)$$

$$x_0^*(t) = x(t), \quad x_i^*(t) \in \mathscr{C}, \quad x_N^*(t) \in \mathscr{C}_N \quad (13e)$$

$$\pi = \mu_i^*(t) \in \Pi, \quad \omega_i(t) = (\Omega, \mathscr{F}, \mathscr{P}), \quad (13f)$$

where $x_i^*(t)$ denotes the state vector trajectory along the τ axis, and $\mu_i^*(t) := \mathbb{R}^{n_x} \to \mathbb{R}^{n_m}$ represent the control policy that is determined on the τ axis [2]. The $\mathscr{L}_c(x_i^*(t), \mu_i^*(t))$, and $\mathscr{L}_t(x_N^*(t))$ are cost-per-stage function and terminal function, respectively. The $\mathbf{E}[\cdot]$ is expected value function, so-called *first-order moment*, of the random variable on a PDF f_{ω_i}. The $\mathscr{C}$ is the states constraints set, and $\mathscr{C}_N$ is the terminal constraint set. The h_j is joint probabilistic constraint functions (for more details, see e.g. [3]). The $\beta \in (0,1) \subset \mathbb{R}$ is the preferred confidence level lower bound that constraints should be fulfilled under uncertainty. The prediction horizon, T, is divided into N steps where $\Delta\tau(t) := T(t)/N$. Given the initial state, $x_0^*(t) = x(t)$, the finite sequence of control policy, $\{\mu_i^*(t)\}_{i=0}^{N-1}$, is optimized at each sampling interval and the first element of control action, $\mu_0(t)$, is applied to the system.

A. Problem Formulation

Solution of the S-OCP (13) became more challenging for real-time safety-critical nonlinear Eco-ADAS system. The main idea for solving approximately the S-OCP is based on suboptimal control policy so-called *certainty equivalence* principle with rolling disturbance estimation. In this method we interpret ω_i as the prediction of expected disturbance values, $\hat{\omega}_i = \mathbf{E}[\omega_i]$, for the uncertainty propagation. Hence, the proposed SNMPC emphasize on early reduction of large recourse, rather than the compensation of non-optimal decisions. Considering the preceding vehicle velocity (7) and its position prediction as disturbance of the system. Thus, the states of the SNMPC control for the proposed semi-autonomous Eco-ADAS system can be written as host vehicle position, its velocity, and energy consumption of the host vehicle, as well as the expected preceding vehicle's position and related velocity as follows $x = [s_h, v_h, e_h, s_p, v_p]^T \in \mathbb{R}^5$ (for more details see [14], [10]).

The spacing policy for regulation of the safe reference relative distance to the preceding vehicle is based on the most commonly used method so-called *time headway* defined as:

$$d_{ref} := d_0 + t_{hw} v_h, \tag{14}$$

where d_0 is a constant minimum safe distance, and t_{hw} is the desired time headway (see e.g. [12], [15]). The statistics of the stochastic position of the preceding vehicle can be estimated by:

$$\mathbf{E}[d] := \mu_{s_p} - s_h = s_p^* - s_h^*, \tag{15}$$

$$\mathbf{Var}(d) := \mathbf{E}[(d - \mathbf{E}[d])^2] \approx \dot{d}^2 \sigma_{s_p}^2, \tag{16}$$

where the $\mathbf{Var}[\cdot]$ is variance, so-called *second-order moment*, of the random variable, which can be approximated by closely related moment concept in physics. The deviation from the desired relative distance is formulated in a chance-constraint of the form:

$$\Pr\{d_{ref} \le d\} \ge 1 - \varepsilon, \tag{17}$$

where d is a random quantity and $\varepsilon = 1 - \beta$ is the risk. A distributionally robust chance-constraint for a wide class of probability distributions can be formulated to a certainty equivalent second-order cone constraint as follows (for more details, see [16], [3]):

$$\kappa_\beta \mathbf{Var}[d_{ref} - d] + \mathbf{E}[d_{ref} - d] \le 0, \quad \kappa_\beta := \sqrt{\frac{1-\varepsilon}{\varepsilon}}. \tag{18}$$

The performance index in order to achieve the ecological driving can be formulated by linearly penalising the energy consumption of the host vehicle at the end of prediction horizon as follows:

$$\mathscr{L}_t(x_N^*(t)) := q_f e_h, \tag{19}$$

where q_f is the corresponding weight. This definition provides a flexible velocity profile planning in the integral performance index that can be formulated as follows:

$$\begin{aligned} &\mathscr{L}_c(x_i^*(t), \mu_i^*(t)) := \\ &\frac{1}{2} q_c (v_h - v_{ref})^2 + \frac{1}{2}(r_u (u - u_{ref})^2) \\ &+ q_{crv,lmt}(v_h, f_{crv}(\delta(s_h)), f_{lmt}(s_h)) v_h^2 \\ &+ q_a c(v_h, v_p, d)((\mathbf{E}[d] - d_{ref})^2 + ln(1 + \kappa_\beta \mathbf{Var}(d))), \end{aligned} \tag{20}$$

where v_{ref}, u_{ref} are desired cruising velocity, and reference input respectively with relative weightings q_c, and r_u. A safe and comfortable ride during the road curve and traffic speed limit zone variations can be achieved by penalising the host vehicle velocity with relative adaptive weight (similar to the barrier methods) based on the lateral acceleration ($a_{lat} = v_h^2 f_{crv}(\delta(s_h))$) and maximum allowed lateral acceleration ($a_{lat.max}$) as follows:

$$\begin{aligned} q_{crv,lmt}(v_h, f_{crv}(\delta(s_h)), f_{lmt}(s_h)) &:= exp^{(q_{crv}(a_{lat} - a_{lat.max}))} \\ &+ exp^{(q_{lmt}(v_h - f_{lmt}(s_h)))}, \end{aligned} \tag{21}$$

where q_{crv}, and q_{lmt} are relative weights. The $q_{ac}(v_h, v_p, d)$ is an equivalent to a soft barrier function that supplies enough weight to dominate the other objectives during close approaching to the boundary value of reference relative distance defined as follows:

$$\begin{aligned} q_{ac}(v_h, v_p, d) &:= q_{acc}(q_{rv} exp(\frac{-(v_p - v_h)}{q_{rv}}) \\ &+ q_{rd} exp(\frac{\mathbf{E}[d]}{q_{rd}})) H(d_{ref} - \mathbf{E}[d]), \end{aligned} \tag{22}$$

where q_{acc}, q_{rv}, and q_{rd} are constants, while the $H(d_{ref} - d)$ is a Heaviside's sigmoid function. Note that the uncertain variation position of the preceding vehicle is taken into account during decision making that allows allocation of the trade-off between risk and return of reference relative distance tracking.

The SNMPC with the probabilistic constraint is reformulated in a computationally efficient certainty equivalent OCP problem. In this paper, the resulting problem is solved based on the Continuation and Generalized Minimal RESidual (C/GMRES) method (for more details see [14]).

IV. SYSTEM EVALUATION

The proposed Eco-ADAS system has been evaluated with practical experiments on the test track, and numerical simulations using realistic values of the parameters. A suitable prediction horizon $T = 15\ s$ is chosen to cover upcoming road geometry, traffic speed limit zone and the preceding vehicle motion prediction. This prediction horizon is discretized into $N = 30$ steps of size $\Delta t = 0.5\ s$. The constants in performance index function is set as $q_f = 2$, $v_{ref} = 20\ m/s$, $q_c = 2$, $u_{ref} = F_{res} - Mgsin(f_{slp}(s_h))$, $r_u = 60$, $q_{slk} = 1$, $q_{crv,lmt} = 1$, $a_{lat.max} = 3.7\ m/s^2$, $d_0 = 3\ m$, $t_{hw} = 1.5\ s$, $\beta = 0.95$. The parameters for the physical-statistical model are set as $\mu_p = 0\ m/s^2$, $\delta_p = 1.5$, $\omega_{85^{th}} = 0.67$, $m_1 = 20.41$, $m_2 = 13.68$, $m_3 = 13.23$, $m_4 = 151.2$.

A. Experimental Results

A *Smart Electric Drive third generation* commercial BEV, which is available for practical experiments, is chosen here to model the dynamics of a BEV and its energy consumption. The parameters of the *Smart ED* dynamic model are derived from data sheets and empirical measurements as $m = 975\ kg$, $\delta_1 = 0.04$, $\delta_2 = 0.0025$, $i_g = 9.922 : 1$, $\rho = 1.2041\ kg/m^3$, $A_f = 2.057\ m^2$, $C_{D0} = 0.35$, and $g = 9.81\ m/s^2$ (for more details, see [7], [8], [10]). In order to have a proper identification of the *Smart ED* energy consumption model, dynamometer tests have been conducted.

The parameters of the proposed model for the energy consumption, (3), is identified as $a_2 = 0.01622$, $a_1 = 0.244$, $a_0 = 1.129$, $b_3 = 0$, $b_2 = 0.02925$, $b_1 = 0.257$, and $b_0 = 1.821$ with 98.46% coefficient of determination (R-squared). The limit of control input $u_{max}(v)$ can also be identified as $u_{max}(v) = c_1 - c_2 tanh(c_3(v - c_4))$, where the constants are $c_1 = 1.523$, $c_2 = 1.491$, $c_3 = 0.08751$, and $c_4 = 15.6$ with 99.74% coefficient of determination. The maximum hybrid brake system control input is chosen to be constant, $u_{min}(v) = -5 + d_1 v\ (N/Kg)\ (d_1 = 0)$ [10].

A closed test track located at Colmar-Berg, Luxembourg, is chosen to model the road geometry with traffic information (Fig. 2) [17]. The test track has a total length of 1.255 km and

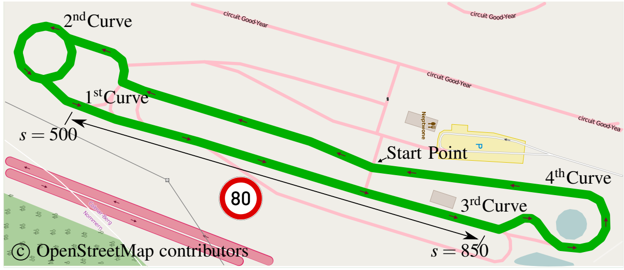

Fig. 2: Test track, Centre de Formation pour Conducteurs

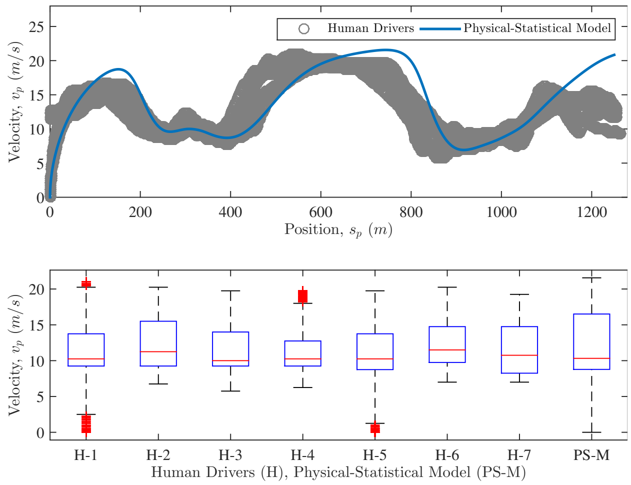

Fig. 3: Far-term future prediction without feedback update, and relative statistics of the experiments vs. proposed model

include curves, speed limit zone, and relative slope profile. This track has four main curves with 20 m, 25 m, 15 m, and 27 m radius. A speed limit zone ($v_{lmt} = 22.22\ m/s$) is assumed between positions $500_{(m)} \leq s \leq 850_{(m)}$.

The preceding vehicle motion prediction based on 85^{th} percentile speed concept with test track geometry and speed limit zone information is shown in Fig. 3. The measured data include seven different rounds of human drivers velocity profiles on the test track. It can be shown that the physical-statistical motion model is capable of foreseen an expected velocity profile. The average velocity of all human drivers is 11.68 m/s, and the average predicted velocity of physical-statistical motion model is 12.26 m/s. It is noteworthy that the prediction of the preceding vehicle in the Fig. 3 is capable of performing far-term future prediction (105 seconds) of the plausible velocity without feedback measurement updates. Significant statistical accuracy can be shown in term of the median and the related variations from the practical experiments obtained by the human drivers (H-#), and the proposed physical-statistical motion model (PS-M) on the test track.

B. Simulation Results

For the sake of comparison, the proposed SNMPC with chance-constraint for the Eco-ADAS system is compared with a conventional Deterministic NMPC (DNMPC), where the velocity of the preceding vehicle is assumed to be constant during prediction. Furthermore, these two approach is compared with the case that the motion of the preceding vehicle is known in advance namely Perfect NMPC (PNMPC). A sinusoid speed profile is considered as the simulation scenario to demonstrate the capabilities of the controllers for the unexpected behaviour of the preceding vehicle and their treatments to the state regulations, constraint fulfilment, and energy efficiency.

Fig.4a shows the velocity profile of the host and preceding vehicle with DNMPC, SNMPC, and PNMPC setting. It can be observed that the velocity profile generated by the SNMPC is closer to the PNMPC rather than the DNMPC. Fig. 4b shows the relative distance regulations between the host and the preceding vehicles. Particularly, the SNMPC fulfils the relative distance constraint with less violation rather than conventional DNMPC with relatively large deviation from reference relative distance. Moreover, an accident can be observed at time 116 s, while the SNMPC managing the situation properly. Fig. 4c shows the control input profile.

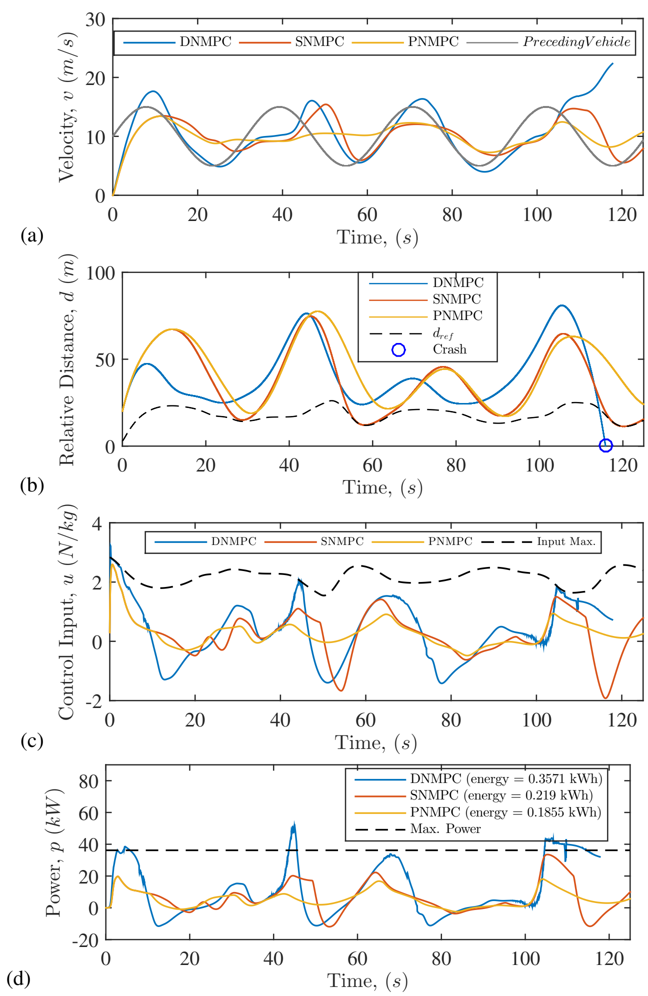

Fig. 4: Performance of the DNMPC, SNMPC, and PNMPC in terms of (a) velocity, (b) relative safe distance, (c) control input, and (d) power, with total energy consumption.

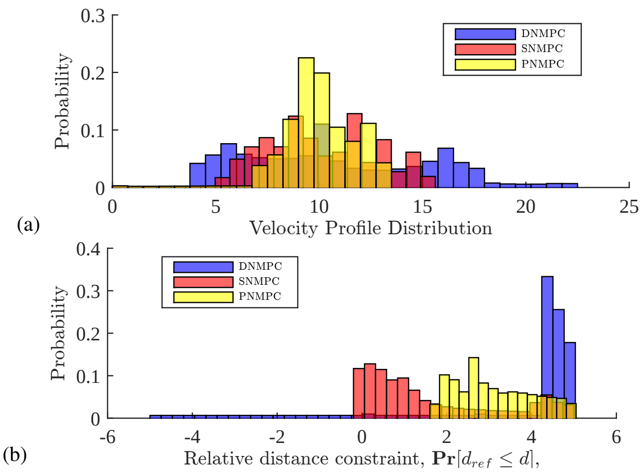

Fig. 5: Performance of the DNMPC, SNMPC, and PNMPC in terms of (a) velocity distribution, (b) probability of chance constraint around boundary region.

The DNMPC can be sensitive to unpredicted events that lead to non-smooth control behaviour with input constraint violation. On the other hand, the SNMPC not only demonstrates a robust behaviour against the uncertainties but also is capable of capturing similar behaviour to the PNMPC. It is shown that the SNMPC generate better velocity profile and reference relative distance tracking than the DNMPC, which lead a proper energy consumption profile. This can also be observed in Fig. 4d. The DNMPC can lead to a violation of maximum power constraint, and higher energy consumption than the SNMPC with relatively close to the PNMPC performance. Fig. 5a demonstrates the velocity distribution of various controllers. While PNMPC has tight variation around the average velocity of the preceding vehicle ($10 m/s$), the SNMPC could regulate the velocity distribution with lower variation compared to DNMPC. Due to (13), the SNMPC enables forming the distribution of performance in terms of first- and second- moments. Fig. 5b shows the probability of relative distance chance constraint around the boundary region. The DNMPC failed to regulated relative safe distance while the SNMPC could fulfils the chance constraint lower bound requirement.

The OCP calculation time for the proposed DNMPC is $2.9ms$, and SNMPC is $3.2ms$ in average on an Intel® Core™ i7 with memory of 7.7 GiB. The computation time of OCP might be compared with similar N/MPC controllers proposed in [4] with $1s$, [6] with $6.43ms$, and the [9] with $23.47ms$. Hence, the proposed SNMPC could be a real-time capable controller for the proposed Eco-ACC system.

V. CONCLUSION AND FUTURE RESEARCH

A semi-autonomous advanced driver assistance system to improve the safety and efficiency of the battery electric vehicle is presented. This system determines proper ecological velocity profile to improve the cruising range challenge based on the road geometry, traffic speed limit zones, and the preceding vehicle motion information. Stochastic nonlinear model predictive control with a probabilistic constraint was formulated to regulate cruising velocity autonomously. A certainty equivalent optimal control problem is obtained by rolling disturbance estimation and robust reformulation of probabilistic constraint. Achieved results show the efficiency of stochastic controller overall performance compared to a deterministic predictive controller, while the computational time of the stochastic controller was founded to be real-time capable. Further practical experiments will be conducted as the future research part.

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