Università di Bologna – Campus di Rimini – Corso di laurea in Farmacia e CQPS

Esame MATEMATICA 1/07/2015 – Docente: Stefano Bordoni

STUDENTE: __________________________; CORSO di LAUREA: ____________

MATRICOLA: ________; N° documento: _____________; FIRMA: ____________

1. Calcolare: $\log_{\sqrt{x}}(1/x^3)$, $\dfrac{(n+1)!-n!}{n!}$ [2]

2. Risolvere: $2\sqrt[3]{x} \le 0$; $2\sqrt[4]{x} \le 0$; $2|x| \le 0$; $5/x^3 \le 0$; $5x^3 \le 0$; $5x^4 \le 0$ [6]

3. Risolvere $\left(\dfrac{1}{4}\right)^{1-x} > 2$; $\log_{1/3}(10-x^2) \ge -2$ [3]

4. Determinare dominio, grafico e codominio della funzione $y = f(x) = -e^{-x} + 1$, mostrando esplicitamente la sua costruzione a partire da $y = g(x) = e^x$. [4]

Calcolare la funzione derivata $f'(x)$ della funzione $f(x)$, e $\Phi(k) = \int_0^k e^{-x}\, dx$. [3]

∘ ∘ ∘ ∘ ∘ ∘

5. Eseguire lo studio analitico della funzione $y = h(x) = \dfrac{\ln(x)}{x}$,

e determinare una funzione primitiva di *h(x)*. [9]

6. Dopo aver determinato un intervallo sul quale la funzione $y = f(x) = x^2 - 1$ è invertibile, determinare la funzione inversa, inclusi dominio, grafico e codominio. [3]

<u>PER LA LODE</u>: Controllare la convergenza di $\Phi = \int_1^{+\infty} \dfrac{1}{2x \cdot \sqrt[2]{x}} \cdot dx$, e in caso positivo calcolarlo.

VOTO: _______

Esame Matematica 01/07/2015

1. • $\left(\frac{1}{x^3}\right) = \left(\sqrt{x}\right)^y$ $\quad x^{-3} = \left(x^{\frac{1}{2}}\right)^y$ $\quad x^{-3} = x^{\frac{1}{2}y}$ $\quad \frac{1}{2}y = -3$ $\quad \underline{y = -6}$

• $\frac{(n+1)! - n!}{n!} = \frac{(n+1)\cdot n! - n!}{n!} = \frac{\cancel{n!}(n+1-1)}{\cancel{n!}} = n$

2. • $\sqrt[3]{x} \leq \phi \rightarrow \underline{x \leq \phi}$

• $\sqrt[4]{x} \leq \phi \rightarrow \sqrt[4]{x} = \phi \rightarrow \underline{x = \phi}$

• $|x| \leq \phi \rightarrow |x| = \phi \rightarrow \underline{x = \phi}$

• $\frac{5}{x^3} \leq \phi \rightarrow x^3 < \phi \rightarrow \underline{x < \phi}$

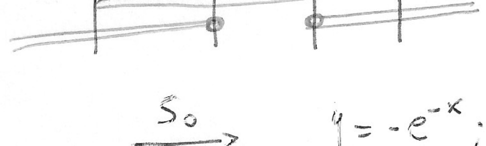
• $x^3 \leq \phi \rightarrow \underline{x \leq \phi}$

• $x^4 \leq \phi \rightarrow x^4 = \phi \rightarrow \underline{x = \phi}$

3. • $\left(2^{-2}\right)^{1-x} > 2^1$ $\quad 2^{-2+2x} > 2^1$ $\quad -2+2x > 1$ $\quad 2x > 3$ $\quad \underline{x > 3/2}$

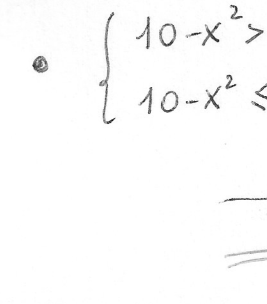
OPPURE $\left[\left(\frac{1}{2}\right)^{+2}\right]^{1-x} > \left(\frac{1}{2}\right)^{-1}$ $\quad \left(\frac{1}{2}\right)^{2-2x} > \left(\frac{1}{2}\right)^{-1}$ $\quad 2-2x < -1$ $\quad -2x < -3$ $\quad x > 3/2$

• $\begin{cases} 10 - x^2 > \phi \\ 10 - x^2 \leq \left(\frac{1}{3}\right)^{-2} \end{cases}$ $\begin{cases} x^2 - 10 < \phi \\ 10 - x^2 \leq 9 \end{cases}$ $\begin{cases} x \in \left]-\sqrt{10}; +\sqrt{10}\right[\\ 1 - x^2 \leq \phi \rightarrow x^2 - 1 \geq \phi \end{cases}$ $\begin{cases} / \\ x \in \left]-\infty; -1\right] \cup \left[+1; +\infty\right[\end{cases}$



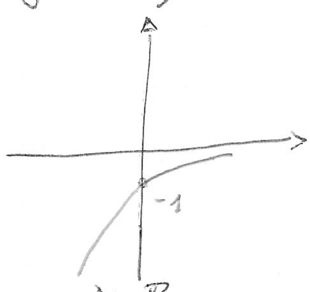
$\underline{x \in \left]-\sqrt{10}; -1\right] \cup \left[+1; +\sqrt{10}\right[}$

4. • $y = e^x$ $\quad \xrightarrow{S_0}$ $\quad y = -e^{-x}; \; -y = e^{-x}$ $\quad \xrightarrow{T_y}$ $\quad y = -e^{-x} + 1; \; y - 1 = -e^{-x}$



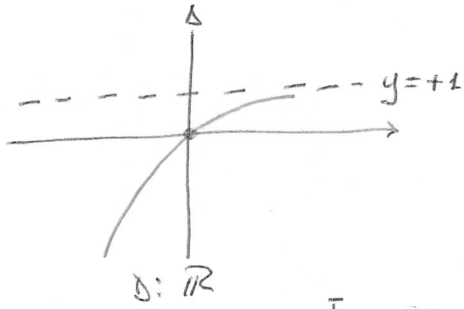
D: $\mathbb{R}$

Cd: $\mathbb{R}^+$



D: $\mathbb{R}$

Cd: $\mathbb{R}^-$



D: $\mathbb{R}$

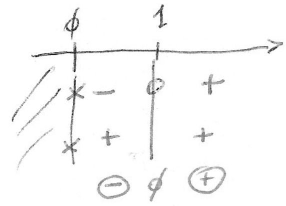
Cd: $\left]-\infty; +1\right[$

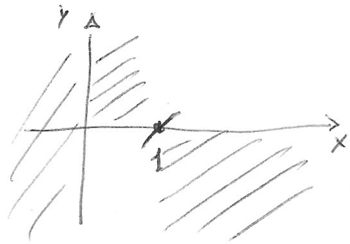
- $y' = f'(x) = -e^{-x} \cdot (-1) = e^{-x}$

- $\Phi(k) = \left[-e^{-x}\right]_0^k = -e^{-k} - (-e^{0}) = -e^{-k} + 1$

5. - $y = \dfrac{\ln(x)}{x}$ $\qquad D: \mathbb{R}^+ \qquad$ NO INVARIANZE per simmetria

- $? x: \dfrac{\ln(x)}{x} \geq 0$ $\qquad$ N: $\ln(x) \geq 0 \quad x \geq 1$ $\qquad$ D: $x \geq 0$





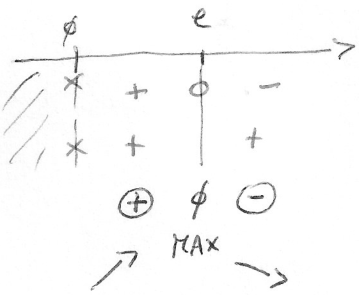
- $\lim\limits_{x \to 0^+} \dfrac{\ln(x)}{x} = \dfrac{-\infty}{0^+} = -\infty \cdot (+\infty) = -\infty$

$\lim\limits_{x \to +\infty} \dfrac{\ln(x)}{x} = \dfrac{+\infty}{+\infty} \to \lim\limits_{x \to +\infty} \dfrac{1/x}{1} = \lim\limits_{x \to +\infty} \left(\dfrac{1}{x}\right) = 0^+$

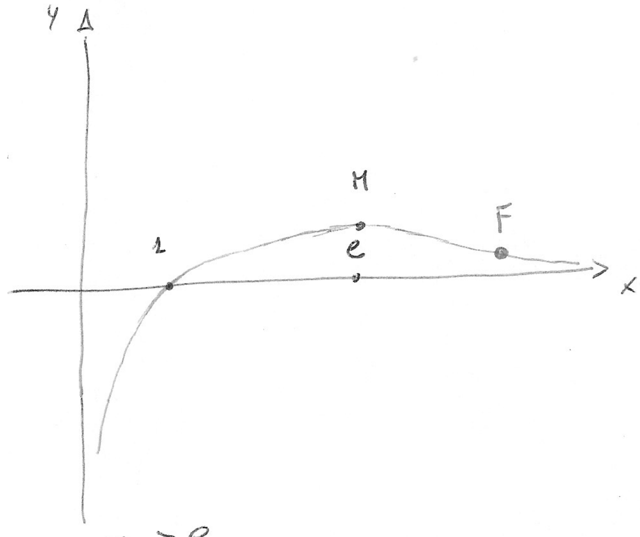
- $y'(x) = \dfrac{\frac{1}{x} \cdot x - 1 \cdot \ln(x)}{x^2} = \dfrac{1 - \ln(x)}{x^2}$

$? x: y'(x) \geq 0$ $\qquad$ N: $1 - \ln(x) \geq 0 \quad \ln(x) - 1 \leq 0 \quad \ln(x) \leq 1 \qquad x \leq e$

D: $x^2 \geq 0 \quad \forall x \in$ dominio



Massimo locale $M \equiv \left(e; \dfrac{1}{e}\right)$

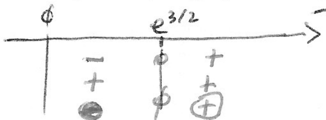


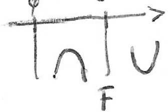
Deve esserci un punto di flesso per $x_F > e$

$y''(x) = \dfrac{-\frac{1}{x} \cdot x^2 - 2x(1 - \ln(x))}{x^4} = \dfrac{-x - 2x + 2x\ln(x)}{x^4} = \dfrac{-3x + 2x \cdot \ln(x)}{x^4} = \dfrac{x(-3 + 2\ln(x))}{x^4} = \dfrac{-3 + 2\ln(x)}{x^3}$

$? x: y''(x) \geq 0$ $\qquad$ N: $-3 + 2\ln(x) \geq 0 \quad 2 \cdot \ln(x) - 3 \geq 0 \quad 2\ln(x) \geq 3 \quad \ln(x) \leq \frac{3}{2} \quad \begin{cases} x > 0 \\ x \geq e^{3/2} \end{cases} \quad x \geq e^{3/2}$

D: $x^3 \geq 0 \quad x \geq 0$





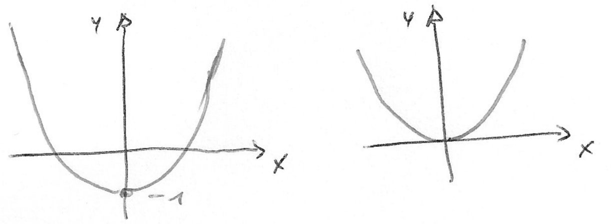
$y(e^{3/2}) = \dfrac{3/2}{e^{3/2}} = \dfrac{3}{2e\sqrt{e}} \to F \equiv \left(e\sqrt{e}; \dfrac{3}{2e\sqrt{e}}\right)$

• $P(x) = \frac{1}{2}\ln^2(x)+c$ perché $h(x) = \ln(x)\cdot D[\ln(x)] = \ln(x)\cdot\frac{1}{x}$

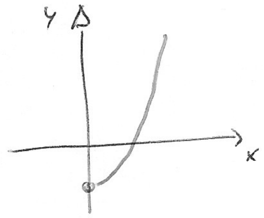
Se $\ln(x) = g(x)$ allora $h(x) = g(x)\cdot g'(x)$

La sua primitiva è quindi $\frac{g(x)^{1+1}}{1+1} = \frac{g(x)^2}{2} = \frac{\ln^2(x)}{2} \equiv \frac{[\ln(x)]^2}{2}$

6. $y = x^2-1 \leftarrow y = x^2$



È invertibile (per es.) su $x\in\mathbb{R}^+_\phi$

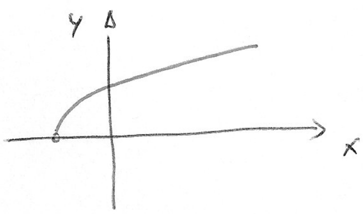


$y = x^2-1$
D: $\mathbb{R}^+_\phi$
C: $[-1;+\infty[$

Funzione inversa: $x = y^2-1$ $\quad y^2 = x+1$ $\quad y = -\sqrt{x+1}$ VEL $y = +\sqrt{x+1}$

Sapendo che il dominio sarà $x\in[-1;+\infty[$ e il codominio $y\in\mathbb{R}^+_\phi$

-> $y = +\sqrt{x+1}$



L. $\Phi = \frac{1}{2}\int_1^{+\infty}\frac{1}{x^{3/2}}\cdot dx$ è del tipo $\int_1^{+\infty}\frac{1}{x^\alpha}\,dx$

Per il criterio di Riemann, l'integrale converge se $\alpha > 1$

Poiché $\alpha = 3/2$ l'integrale è calcolabile

$$\Phi = \frac{1}{2}\left[\frac{x^{-3/2+1}}{-\frac{3}{2}+1}\right]_1^{+\infty} = \frac{1}{2}\lim_{M\to+\infty}\left[\frac{x^{-1/2}}{-1/2}\right]_1^{M} = -\frac{1}{2}\cdot\frac{2}{1}\left[M^{-1/2}-1\right] =$$

$$= \lim_{M\to+\infty}\left[-\frac{1}{\sqrt{M}}+1\right] = \phi+1 = +1$$