Exercise 3: Probability of Default and Corporate Analysis

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Economics of Financial Intermediation

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Outline

1. Altman's Z-score
2. Probability of default
3. Risk analysis
4. Corporate analysis

Altman's Z-score

- Altman's Z-score appears frequently in practitioners publications, and it is used as an instrument to estimate the distance to default of a firm.
- The Z-score is a linear equation composed of five ratios and legitimately controls may of the risks discussed in the corporate finance literature.
- This exercises are aimed at allowing the student to compute Z-scores for firms.

Altman's Z-score

- For any firm, the Z-score is defined as follows:

$$Z_i = 1.2X_1 + 1.4X_2 + 3.3X_3 + 0.6X_4 + 1.0X_6$$

where

- X_1: working capital / total assets
- X_2: retained earnings / total assets
- X_3: EBITDA / total liabilities
- X_4: (stock price × outstanding shares) / total liabilities
- X_5: sales / total assets

Altman's Z-score

1. Using a modified discriminant function similar to Altman's, Burger Bank estimates the following coefficients for its portfolio of loans:

$$Z = 1.4X_1 + 1.09X_2 + 1.5X_3$$

where

X_1 = debt to asset ratio

X_2 = net income

X_3 = dividend payout ratio

Altman's Z-score

a) What is the Z-score if the debt to asset ratio is 40 percent, net income is 12 percent, and the dividend payout ratio is 60 percent?

b) Using Z = 1.682 as the cut-off rate, what should be the debt to asset ratio of the firm in order for the bank to approve the loan?

Altman's Z-score

a) What is the Z-score if the debt to asset ratio is 40 percent, net income is 12 percent, and the dividend payout ratio is 60 percent?

$$Z = 1.4(0.40) + 1.09(0.12) + 1.50(0.60)$$

$$= 0.56 + 0.1308 + 0.90 = 1.59$$

Altman's Z-score

b) Using Z = 1.682 as the cut-off rate, what should be the debt to asset ratio of the firm in order for the bank to approve the loan?

$$Z = 1.4X_1 + 1.09X_2 + 1.5X_3$$
$$1.682 = 1.4X_1 + 1.09(0.12) + 1.5(0.60)$$
$$1.682 = 1.031 + 1.4X_1$$
$$(1.682 - 1.031) \div 1.4 = X_1 = 0.465$$

Altman's Z-score

2. Suppose that the financial ratios of a potential borrowing firm took the following values:

$X_1 = 0.30$
$X_2 = 0$
$X_3 = -0.30$
$X_4 = 0.15$
$X_5 = 2.1$

Altman's discriminant function takes the form:

$$Z = 1.2\ X_1 + 1.4\ X_2 + 3.3\ X_3 + 0.6\ X_4 + 1.0\ X_5$$

a) Calculate the Z-score.

b) Suppose $X_3 = 0.2$ instead of -0.30. According to Altman's credit scoring model, the firm would fall under which default risk classification?

Altman's Z-score

a) Calculate the Z-score.

$Z = 1.2(0.30) + 1.4(0.00) + 3.3(-0.30) + 0.6(0.15) + 1.0(2.1)$

$Z = 0.36 + 0.00 - 0.99 + 0.09 + 2.1 = 1.56$

Altman's Z-score

b) Suppose X_3 = 0.2 instead of -0.30. According to Altman's credit scoring model, the firm would fall under which default risk classification?

$$Z = 1.2(0.30) + 1.4(0.00) + 3.3(0.20) + 0.6(0.15) + 1.0(2.1)$$
$$Z = 0.36 + 0.00 + 0.66 + 0.09 + 2.1 = 3.21$$

- A low default risk.

Probability of default

3. The following represents two yield curves:

Maturity	Pure Discount Treasury Yields	B-rated Corporate Bond Yields (Pure Discount Bonds)
1 year	3 percent	6 percent
2 year	6 percent	10 percent
20 year	12 percent	17 percent

a) What is the implied probability of repayment on one-year B-rated debt?

b) What interest rate is expected on a one-year B-rated corporate bond in one year? (Hint: Use the implied forward rate.)

Probability of default

a) What is the implied probability of repayment on one-year B-rated debt?

- $p(1 + k) = (1 + i)$ $p = (1 + i) \div (1 + k)$
- where p = probability of full repayment
- k = interest rate on corporate debt
- i = interest rate on Treasury
- $p_1 = (1.03) \div (1.06) = 0.9717$
- probability of default: $(1 - p_1) = 1 - 0.9717 = 0.0283$

Probability of default

b) What interest rate is expected on a one-year B-rated corporate bond in one year? (Hint: Use the implied forward rate.)

$$(1 + {}_{1}f_{1}R) = \frac{(1 + {}_{2}fR)^2}{(1 + {}_{1}fR)} = \frac{1.10^2}{1.06} = 1.1415$$

$$\approx 14.15\%$$

Risk analysis

4.

Borrower Rating	Loan Amount	Expected Return	Standard Deviation
A	$40 million	12 percent	1 percent
B	$20 million	15 percent	2 percent
C	$30 million	18 percent	3 percent

a) What is the FI's expected return on its loan portfolio?

b) What is the risk (standard deviation of returns) on the bank's loan portfolio if loan returns are uncorrelated ($\rho = 0$)?

Risk analysis

a) What is the FI's expected return on its loan portfolio?

Rating	Loan Amount	Loan Weight (X_i)	Expected Return [$E(R_i)$]	LW × $E(R_i)$
A	40	40 ÷ 90 = 0.4444	12	5.3333
B	20	20 ÷ 90 = 0.2222	15	3.3333
C	30	30 ÷ 90 = 0.3333	18	6.0000
Totals	90	**Portfolio expected return**		**14.67**

Risk analysis

b) What is the risk (standard deviation of returns) on the bank's loan portfolio if loan returns are uncorrelated (ρ = 0)?

$$\sigma_p^2 = X_A^2\sigma_A^2 + X_B^2\sigma_B^2 + X_C^2\sigma_C^2$$

$$\sigma_p^2 = (0.4444)^2(1) + (0.2222)^2(4) + (0.3333)^2(9)$$

$$\sigma_p^2 = (0.1975) + (0.1975) + (0.9998) = 1.3948$$

Therefore, standard deviation of portfolio is

$$\sigma_p = \sqrt{1.3948} = 1.181$$

Risk analysis

5. What is Bank A and Bank B's standard deviation of its asset allocation proportions relative to the national banks average? Use the formula in the textbook.

	National Banks	Bank A	Bank B
Real Estate Loans	60 percent	30 percent	56 percent
Consumer Loans	20 percent	30 percent	28 percent
Commercial Loans	20 percent	10 percent	16 percent

Standard deviation:

$$\sigma_j = \sqrt{\frac{\sum_{i=1}^{N}(X_{ij} - X_j)^2}{N}}$$

Risk analysis

Bank A:

$\Sigma = (0.30 - 0.60)^2 + (0.30 - 0.20)^2 + (0.10 - 0.20)^2 = 0.09 + 0.01 + 0.01 = 0.11$

$\Sigma \div N = 0.11 \div 3 = 0.036666$

$\sigma_A = \sqrt{0.036666} = 0.1915$

Bank B:

$\Sigma = (0.56 - 0.60)^2 + (0.28 - 0.20)^2 + (0.16 - 0.20)^2 = 0.0016 + 0.0064 + 0.0016 = 0.0096$

$\Sigma \div N = 0.0096 \div 3 = 0.0032$

$\sigma_B = \sqrt{0.0032} = 0.0566$

Risk analysis

6. From the Treasury strip yield curve, the current required yields on one- and two-year Treasuries are i_1 = 4.65 percent and i_2 = 5.50 percent, respectively. Further, the current yield curve indicates that appropriate one-year discount bonds are yielding k_1 = 8.5 percent, and two-year bonds are yielding k_2 = 10.25 percent.

a) Calculate the one-year forward rate on the Treasuries and the corporate bond.

b) Using the current and forward one-year rates, calculate the marginal probability of repayment on the corporate bond in years 1 and 2, respectively.

Risk analysis

a) Calculate the one-year forward rate on the Treasuries and the corporate bond.

The one-year forward rate, f_1, on the Treasury is:

$$f_1 = (1.0550)^2 / 1.0465 = 1.06357 - 1, \text{ or } \quad f_1 = 6.357\%$$

The one-year forward rate, c_1, on the corporate bond is:

$$c_1 = (1.1025)^2 / 1.0850 = 1.12028 - 1, \text{ or } \quad c_1 = 12.028\%$$

Risk analysis

b) Using the current and forward one-year rates, calculate the marginal probability of repayment on the corporate bond in years 1 and 2, respectively.

The probability of repayment in year 1 is:

$$p_1 = (1.0465) / 1.085 = 0.9645 ,$$

or probability of default = 1 – 0.9645 = 5.35%

The marginal probability of repayment in year 2 is:

$$p_2 = (1.06357) / 1.12028 = 0.9494 ,$$

or probability of default = 1 – 0.9494 = 5.06%

Corporate analysis

7. Consider the following company balance sheet and income statement.

Assets		**Liabilities and Equity**	
Cash	$4,000	Accounts payable	$30,000
Accounts receivable	52,000	Notes payable	12,000
Inventory	40,000	Total current liabilities	42,000
Total current assets	96,000	Long-term debt	36,000
Fixed assets	44,000	Equity	62,000
Total assets	$140,000	Total liabilities and equity	$140,000

Income Statement

Sales (all on credit)	$200,000
Cost of goods sold	130,000
Gross margin	70,000
Selling and administrative expenses	20,000
Depreciation	8,000
EBIT	42,000
Interest expense	4,800
Earning before tax	37,200
Taxes	11,160
Net income	$26,040

Corporate analysis

For this company, calculate the following:

a. Current ratio.

b. Number of days' sales in receivables.

c. Sales to total assets.

d. Number of days in inventory.

e. Debt to assets ratio.

f. Cash flow to debt ratio.

g. Return on assets.

h. Return on equity.

Corporate analysis

a. Current ratio: 96,000/42,000 = 2.2857

b. Number of days' sales in receivables: 52,000 x 365/200,000 = 94.90 days

c. Sales to total assets: 200,000/140,000 = 1.4286

d. Number of days in inventory.

40,000 x 365/130,000 = 112.31 days

e. Debt to assets ratio.

(42,000 + 36,000)/140,000 = .5571 = 55.71%

f. Cash flow to debt ratio.

(42,000 + 8,000)/(42,000 + 36,000) = .6410 = 64.10%

g. Return on assets.

26,040/140,000 = 0.1860 = 18.60%

h. Return on equity.

26,040/62,000 = 0.4200 = 42.00%

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