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Robust Optimization Framework to Operating Room Planning and Scheduling in Stochastic Environment

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Robust Optimization Framework to Operating Room Planning and Scheduling in Stochastic Environment

By

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Robust Optimization Framework to Operating Room Planning and Scheduling in Stochastic Environment

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Declaration of Co-authorship and previous publication

DECLARATION OF CO-AUTHORSHIP

I hereby declare that this dissertation incorporates material that is the result of research conducted under the supervision of my supervisors, Dr. M. Ahmadi and Dr. Fazle Baki. Results related to this research are reported in Chapters 2 through 5.

I am aware of the University of Windsor's Senate Policy on Authorship and I certify that I have properly acknowledged the contributions of the other researchers to my dissertation, and I have obtained written permission from my co-author to include the aforementioned materials in my dissertation.

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Dissertation Chapter	**Publication Title/full Citation**	**Publication Status**
Chapter 2	M. Lalmazloumian, F. Baki , M. Ahmadi, "A novel robust optimization transformation framework to operating room capacity allocation problem under uncertainty at a publically funded hospital" submitted to *IEEE Transaction on Engineering Management*, pp. 42, 2017.	Submitted
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Abstract

Arrangement of surgical activities can be classified as a three-level process that directly impacts the overall performance of a healthcare system. The goal of this dissertation is to study hierarchical planning and scheduling problems of operating room (OR) departments that arise in a publicly funded hospital. Uncertainty in surgery durations and patient arrivals, the existence of multiple resources and competing performance measures are among the important aspect of OR problems in practice. While planning can be viewed as the compromise of supply and demand within the strategic and tactical stages, scheduling is referred to the development of a detailed timetable that determines operational daily assignment of individual cases. Therefore, it is worthwhile to put effort in optimization of OR planning and surgical scheduling. We have considered several extensions of previous models and described several real-world applications.

Firstly, we have developed a novel transformation framework for the robust optimization (RO) method to be used as a generalized approach to overcome the drawback of conventional RO approach owing to its difficulty in obtaining information regarding numerous control variable terms as well as added extra variables and constraints into the

model in transforming deterministic models into the robust form. We have determined an optimal case mix planning for a given set of specialties for a single operating room department using the proposed standard RO framework. In this case-mix planning problem, demands for elective and emergency surgery are considered to be random variables realized over a set of probabilistic scenarios. A deterministic and a two-stage stochastic recourse programming model is also developed for the uncertain surgery case mix planning to demonstrate the applicability of the proposed RO models. The objective is to minimize the expected total loss incurred due to postponed and unmet demand as well as the underutilization costs. We have shown that the optimum solution can be found in polynomial time.

Secondly, the tactical and operational level decision of OR block scheduling and advance scheduling problems are considered simultaneously to overcome the drawback of current literature in addressing these problems in isolation. We have focused on a hybrid master surgery scheduling (MSS) and surgical case assignment (SCA) problem under the assumption that both surgery durations and emergency arrivals follows probability distributions defined over a discrete set of scenarios. We have developed an integrated robust MSS and SCA model using the proposed standard transformation framework and determined the allocation of surgical specialties to the ORs as well as the assignment of surgeries within each specialty to the corresponding ORs in a coordinated way to minimize the costs associated with patients waiting time and hospital resource utilization.

To demonstrate the usefulness and applicability of the two proposed models, a simulation study is carried utilizing data provided by Windsor Regional Hospital (WRH). The simulation results demonstrate that the two proposed models can mitigate the existing variability in parameter uncertainty. This provides a more reliable decision tool for the OR managers while limiting the negative impact of waiting time to the patients as well as welfare loss to the hospital.

Dedication

To my wife

for her unconditional support and sacrifices throughout completion of this journey.

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There are several people whom I owe my sincere appreciation for their generous help, guidance and contributions to completion of this doctoral dissertation.

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Nomenclature

h	Planned available operating rooms hours per day for surgery operation
a_i	Number of operating room of type i available in each day
k_{st}	Maximum capacity (i.e. number of surgeons) available for surgical specialty s on day t
c_{is}^1	Cost associated with allocating OR block times of type i to specialty s
c_{tz}^2	Cost of postponement of elective surgery demand from day t to day z
c^3	Penalty rate associated with the unmet elective surgery demand
c^4	Cost of underutilization of operating room hours
λ	Weighting penalty to measure trade-off between risk and expected outcome
ω	Weighting penalty for the infeasibility of the random elective demand constraint
e_{st}^{ε}	Random parameter representing elective surgery demand for specialty s on day t under scenario ε
g_{st}^{ε}	Random parameter representing emergency surgery demand for specialty s on day t under scenario ε
π_{ε}	Probability of occurrence of scenario ε
X_{ist}	Number of operating rooms of type i allocated to specialty s on day t
U_{st}	Amount of emergency surgery demand of specialty s met in a dedicated emergency room on day t
Y_{stz}^{ε}	Amount of elective surgery demand hours of specialty s postponed from day t to day z under scenario ε
ρ_{st}^{ε}	Elective surgery demand of specialty s on day t under scenario ε that is rescheduled to be met outside the normal shift operation through overtime and/or moving to another local hospital

ι_{st}^{ε}	Undersupply of operating room hours allocated to specialty s on day t under scenario ε relative to its desired level
μ	Expected value of the second stage cost being made after realization of the random variable is observed
d^{ε}	Variability cost of deviation from the mean expected value of the objective function in each scenario ε
θ^{ε}	Deviational variable for violation of the mean objective function in each scenario ε
f_{st}^{ε}	Deviation variable by which the random elective demand constraints of specialty s on day t can be violated under scenario ε
$\gamma_{st}^{\varepsilon}$	Deviational variable for infeasibility of the random elective demand constraints of specialty s on day t under scenario ε
d_i	Elapsed days since referral of patient i for surgery
ρ_i	Urgency coefficient of patient i in days
B_s	Subset of patient belong to specialty S
o_s	Number of surgeon available for specialty S in the planning horizon
k_{rt}	Available capacity (hours) for surgery in OR r on day t
K_{rt}	Maximum available capacity (hours) for surgery in OR r on day t
$V^{\max}$	Maximum daily number of operating rooms that can reserve OR hours for emergency cases
β_t	Number of available ICU beds on day t
N	Subset of OR blocks not available for specialty S if $(s,r) \in N$
η_i	1 if patient i is expected to need ICU bed after operation; 0 otherwise
α_{rt}	1 if OR r is not available on day t ; 0 otherwise
w_l	Weights of the objective function l given by the decision makers
$\psi^{+/-}$	Weighting factor for over (under) time of specialty
$\sigma^{+/-}$	Weighting factor for over (under) utilization of OR capacity
λ	Weighting scale to measure the trade-off between risk and expected outcome
ω_1, ω_2	Weighting penalty to trade-off solution for model robustness
p_i^{ε}	Stochastic elective surgery duration of patient i under scenario ε
u_t^{ε}	Stochastic emergency arrival time on day t under scenario ε
x_{irt}	$\begin{cases} 1 & \text{If patient } i \text{ is assigned to OR } r \text{ on day } t \\ 0 & \text{Otherwise} \end{cases}$
y_{srt}	$\begin{cases} 1 & \text{If specialty } s \text{ is assigned to OR } r \text{ on day } t \\ 0 & \text{Otherwise} \end{cases}$
v_{rt}	$\begin{cases} 1 & \text{If operating room time is reserved for emergency cases in OR } r \text{ on day } t \\ 0 & \text{Otherwise} \end{cases}$

g_{rt}^{ε}	Operating room hours reserved for emergency surgery in OR r on day t under scenario ε
χ_{srt}^{ε}	Surgery demand of specialty s that cannot be met in OR r on day t under scenario ε
ϕ_{srt}^{ε}	Undersupply of OR block times allocated to specialty s in OR r on day t under scenario ε relative to its desired level
$\zeta_{rt\varepsilon}^{+}$	Over-utilization hours of overall capacity of OR r on day t under scenario ε
$\zeta_{rt\varepsilon}^{-}$	Under-utilization hours of overall capacity of OR r on day t under scenario ε
μ	Expected value of the second stage cost being made after realization of the random variable is observed
d^{ε}	Variability cost of deviation from the mean expected value of the objective function in each scenario ε
θ^{ε}	Deviational variable for violation of the mean objective function in each scenario ε
$f_{srt\varepsilon}^{1}$	Deviation variable by which the allocated OR block to specialty s in room r on day t can be violated under scenario ε
$f_{rt\varepsilon}^{2}$	Deviation variable by which the OR capacity utilization in room r on day t can be violated under scenario ε
$\gamma_{srt}^{\varepsilon}$	Deviational variable for infeasibility of the random allocated OR block constraint of specialty s in room r on day t under scenario ε
$\delta_{rt}^{\varepsilon}$	Deviational variable for infeasibility of the random OR capacity utilization constraint in room r on day t under scenario ε

Abbreviations

ECS	Elective case scheduling
FCFS	First come first serve
ICU	Intensive care unit
ILP	Integer linear programming
LOS	length of stay
MILP	mixed-integer linear programming model
MIP	mixed-integer programming
MSS	master surgery schedule
MSSP	master surgery schedule problem
OR	Operating room
ORP	Operating room planning
OT	Operating theatre
PACU	post anesthesia care units
PWTRL	Prioritized wait time related loss
RO	Robust optimization
RO-MR	Robust optimization with model robustness
RO-SR	Robust optimization with solution robustness
ROM-T	Robust optimization model with tradeoff
SAA	sample average approximation
SCA	Surgical case assignment
SCAP	Surgical case assignment problem
SLP	stochastic linear programming
SP	stochastic programming
WRH	Windsor Regional Hospital
WTIS	Wait time information system

Chapter 1

Introduction to Operating Room Planning and Scheduling

1.1 Healthcare Operations Management

Healthcare expenditure as a portion of gross domestic product (GDP) has grown considerably in North America over the past few decades. The operating costs of surgery departments contain a significant portion of the total cost of a healthcare system. Many healthcare studies undertaken in Canada show a substantial growth in health spending during the past few decades. The amount was close to 10.9% of the Canada's GDP and reached to $219.1 billion in 2015 representing a 1.6% growth compared to the last year [1]. Along with the costs, demand for healthcare services is increasing which makes maintaining healthcare services a challenging and complicated process due to the limited resources. One way to overcome those challenges and improve healthcare operations without increasing available resources or compromising required demands is by improving the efficiency of the healthcare capacity [2].

Hospital managers typically face two types of variability in a health care facility that impacts the efficiency and effectiveness of the facility. One is the natural variability, which is inherent to the uncertain world of healthcare such as the variability in patient arrivals or surgery

durations. The other is artificial variability that originates from poor scheduling policy. For example a poor operating room schedule could lead to a longer waiting time and lower service level. While many studies in the literature have considered surgery operation durations and capacity of available operating rooms as known and fixed parameters [3,4], others have argued that uncertainty in surgery durations or resource availability may lead to a solution that is infeasible [5,6]. In our model, we include uncertainty of surgery durations as well as patient arrivals. Figure 1.1 and 1.2, for instance, depict the likely variability in surgery durations. Figure 1.1 shows the variation of surgery durations (in minutes) by surgical specialties and Figure 1.2 depicts the distribution of surgery operations in operating rooms (Figure 1.2 and 1.2 shows actual data taken from the operating room department of a local hospital in Southwestern Ontario).

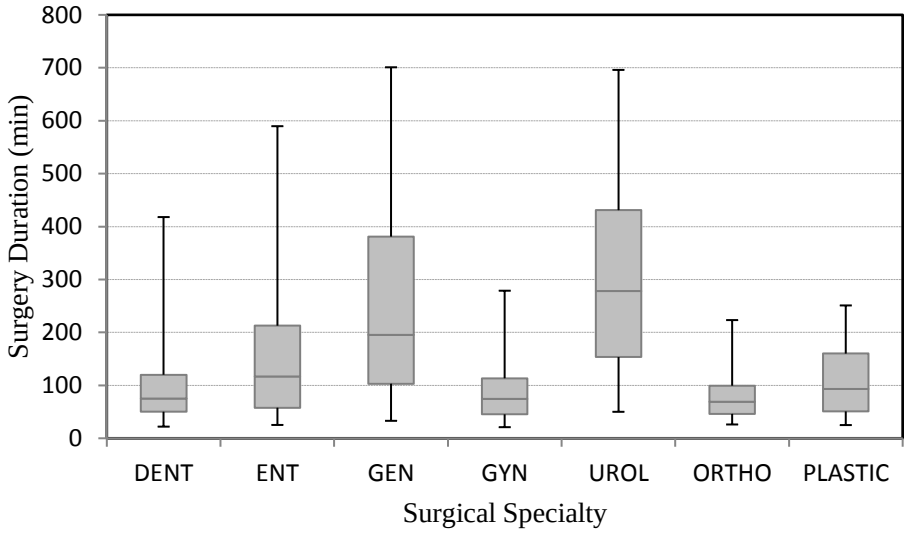

Figure 1.1: Surgery durations by surgical specialty

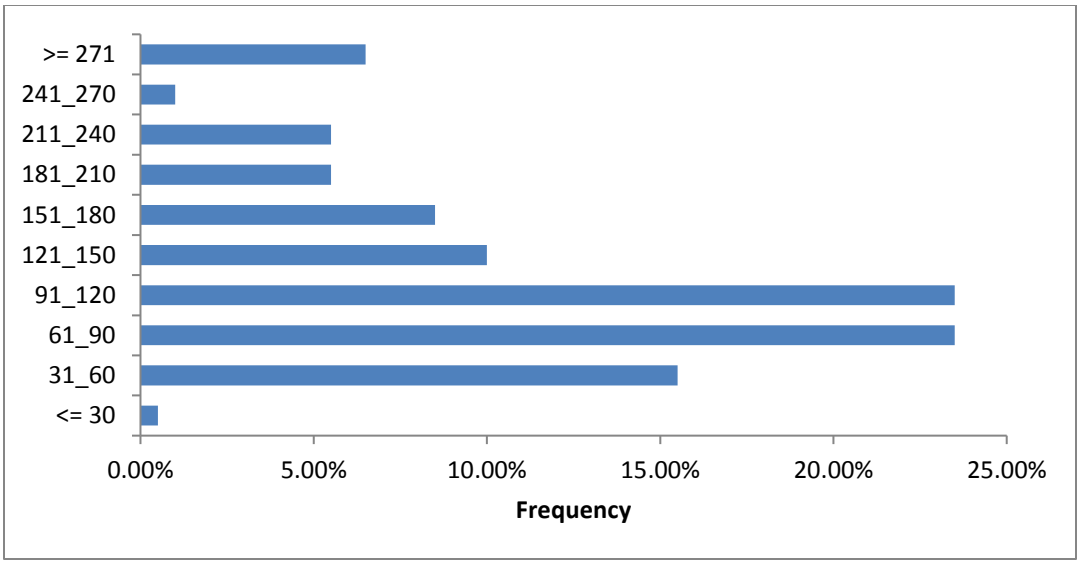

Figure 1.2: Duration distribution of surgery operations

Effective planning and scheduling of operating room (OR) operations within the surgery department of a hospital becomes critical when uncertainty is present in the system. Sources of uncertainty can be due to patient arrivals and emergency cases, resource availability, and surgery durations. This uncertainty makes the planning and scheduling decisions more complex and challenging for hospital managers. The capacity allocation decisions and the operating cost of ORs are greatly impacted by this uncertainty. The decision for OR planning and scheduling becomes even more complex when emergency patients are taken into account. Taking care of emergencies is in fact one of the most important source of uncertainty that impacts the availability of the resources as well as the performance of the healthcare services. In such an environment, effective Operation Research tools can help decision makers in finding a balance in the trade-off between capacity over-allocation and shorten patients waiting time at the cost of higher idle time for resources or under-allocation and thus lower overtime at the cost of higher waiting time for patients.

Previous studies show that ORs and post-surgical units resources are amongst the most costly bottleneck in a healthcare system [7]. The cost of running an OR with all the required staff and a surgeon is estimated to be as high as $60.50 per minute [8]. Therefore, efficient OR scheduling program is the key to success a healthcare operation. The OR planning and scheduling literature generally focused on two major characteristics of the patients in the literature, namely elective and non-elective patients. While the surgery operation is usually planned early in advance for the former class, this surgical treatment is carried out within an urgent situation, and consequently emergency, for the later one.

The surgical scheduling process in a medical facility is a complex and critical process where the choice of schedules and availability of resources directly influence on the patient throughput, the postponement or cancellation of surgeries, utilization of resources, wait times, and the overall performance of the system [9]. Therefore, a systematic approach that takes into account a variety of surgical specialties, priorities for service, post- surgical capacity, and the combination of both scheduled (elective) and unscheduled (emergency) procedures could lead to an improved capacity allocation among various specialties within the system and enhanced policy implication that results in expanded effectiveness and patient throughput and reduced wait times. Therefore, there is an increasing demand for an efficient operational research study to optimize resources in the healthcare system and bridge the gap between surgeons and hospital managers through the optimization approaches.

In general, the procedure for any scheduling of elective surgical processes is considered as a three-level process [10,11]. These stages involve activities from allocation of the OR time among surgical specialties to the actual assignment of individual cases within the allocated OR blocks in a hospital and are usually classified as strategic, tactical, and operational levels of the surgery scheduling process respectively [12]. Figure 1.3 gives an overview of the process with

respect to the decision that is being made at each decision levels. The assignment of OR time to the surgical specialties are the first decision that must be made, usually called case mix planning.

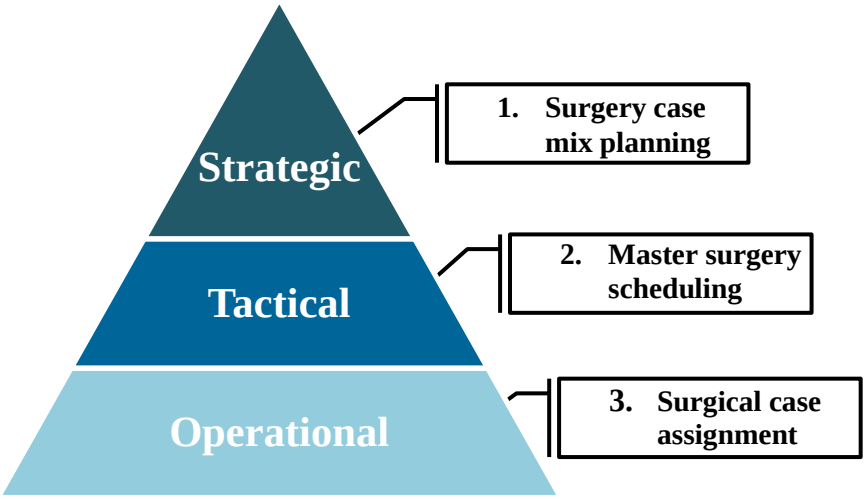

Figure 1.3: Decision hierarchy in surgery planning process

The allocation of surgical block schedules is developed at the second stage which simply determines the surgery time for each specialty in ORs on each day on a cyclic timetable, usually called master surgery schedule (MSS). Finally, the third level schedules individual cases on a daily basis, normally known as advance scheduling.

Due to the inter dependence of the decisions at these three levels of planning, any scheduling policy obtained in isolation can result in solutions that may not meet the requirements set to the decisions made at the upstream stages. Therefore, it is crucial to address the concerns of healthcare management regarding the influence of assumptions made about the surgical schedules set at higher levels on downstream operations. These assumptions can be for instance the amount of flexibility that planning decision of allocating surgical specialties to the OR blocks may provide for the decision makers in their attempt to assign individual patients from the associated waiting list of those specialties to the allocated blocks or which OR within the system has been allocated to which specialty.

This study is primarily motivated by surgical activities within the OR department of a local hospital to develop a hierarchical planning and scheduling models over all three stages as defined above when demand for both elective and emergency cases is considered stochastic and surgery durations are not known.

The surgical scheduling process for elective patients involves a range of activities from determining OR time to be allocated in a hospital to actual scheduling of individual cases. One of the major activities within this process is to define and assign the OR time to the surgical specialties. One main decision that needs to be addressed is: How many OR time in total is allocated to each surgical specialty. At this stage the budget often determines the total OR time available, also several factors such as waiting time can influence the amount of OR time required by each surgical specialty. According to [9], there are several criteria that impact the required number of OR time by each surgical specialty, including waiting times as it directly affects the throughput of patients, fairness among all the specialties, and maximization of OR efficiency.

In the first stage, we determine optimal OR hours assigned to each surgical specialty under uncertain elective and emergency surgery demands where the proportion of time allocated to surgical specialties is subject to several factors such as limited OR capacity as well as underage cost of idle resources and overage cost of surgical overtime. The solution at this hybrid stage aims at finding an optimal OR allocation planning for surgical specialties that minimizes the postponed and/or unmet surgeries resulting from variability in elective and emergency patients demand and the expected total cost of resource underutilization and overtime.

A hybrid tactical and operational framework is developed to capture the integrated allocation problem of surgical specialties with the assignment of individual surgery cases within the assigned OR blocks by specifying the appointment schedule of each specialty at which the required resource(s) such as OR, surgery teams and equipment as well as patients are available.

Scheduling of surgical cases directly affects the amount of overtime and undertime of the healthcare resources [13]. In OR department, any deviation from the staff scheduled hour can lead to a huge staff overtime as well as additional overhead costs. On the contrary, the cost of idle time is considered considerable as a result of the cost involved in underutilization of available resources.

Due to the uncertainty in emergency arrivals and surgery durations, some surgeries may take longer than planned and might go overtime or even postponed to the succeeding planning horizon and start later than its original scheduled time which could trigger controversial social issues related to the maximum allowed time for the patients in the waiting list. On the other hand, some OR blocks may be under-utilized due to the difference between actual and planned duration of surgery operations that could lead to expensive OR idle time. Therefore, there is always a trade-off between under-and-overutilization of OR time, overtime, and patient waiting times. The solution to this hybrid stage aims at finding an integrated schedule for surgical specialties and surgery cases that minimizes the postponed surgeries resulting from patients stayed in the waiting list beyond the determined durations and the expected total cost of underutilization and overtime of resources.

In most of the publicly funded hospitals, the maximum patient waiting time before receiving surgery operation is normally determined by the government. Hospital decision makers must attempt to satisfy this requirements. Limited availability of the operating rooms directly affects the number of patients admitted to a hospital within a time period and as a result can violate the regulated waiting time. The desired service level of a healthcare provider is directly influenced by that waiting factor, and hence, has to be incorporated in the decision making framework.

Effective management of surgical planning and scheduling is an area that draws considerable attention from the healthcare community to reduce costs and increase service level [14,15]. The primary goal of the OR scheduling problem is to minimize the total fixed and variable costs associated with the overall monthly or yearly schedule while maintaining the service level. The operating cost of surgery departments contains a significant portion of the total cost of a health care unit. It has been estimated that operating cost of the surgery units accounts for more than 40% of the entire expenses of a hospital, [6,7,16,17]. Thus, substantial cost savings can be achieved in surgery department. It is well-documented in the literature that operating rooms are important revenue generators in a hospital, but also the largest cost centers in a hospital [10]. Conflicting objectives of various stakeholders (e.g., patients, OR managers, surgeons, anesthetists, and nurses) that need to be reconsidered makes the process of developing a surgical plan and schedule a complex issue. Hence, an effective use of the operating rooms can lead to a huge cost reduction in hospitals which is the ultimate goal of healthcare managers along with optimal utilization of resources to deliver a surgery operation at a right time to the maximum number of patients with a minimum amount of waiting [18].

Operating rooms normally represent a form of bottleneck factor constraining the overall surgical throughput in a healthcare system. Thus, it is vital to develop an allocation program that utilizes the available resources in an optimal way. The importance of developing a smooth allocation program for ORs is not only because of its impact on the surgery operations that is performed in a surgical center, but it also determines the amount of resources that are required to be assigned to each operation along with the planning horizon. Hence, developing an effective OR scheduling plan can assist managers in reducing cost and improving the resource usage.

The OR planning and scheduling process is strongly characterized by the uncertainty. In the key variables impacting the system, some have serious impact on the patients' satisfaction, and hence, needs to be efficiently and effectively handled over the specific decision levels that are

addressed. In previous literature on OR planning and scheduling, the inclusion of uncertainty was mainly limited to either the uncertainty in arrival of the patients or the duration of the surgical procedures [16]. However, the activities inside the OR have a significant impact on many other activities within a hospital. For instance, patients waiting for surgery operations are expected to be admitted in a certain period of time to comply with governmental regulations. The consideration of other source of uncertainties, such as emergency cases is very crucial in developing an efficient planning and scheduling program. Consequently, a successful operating room schedule depends upon how various source of uncertainties are incorporated into the model.

Although the study of OR planning and scheduling problems has received extensive attention in the literature during the past few decades, the majority of this research has either considered unrealistic assumptions by overlooking the existing uncertainty or failing to incorporate the impact of inherent variability in emergency arrivals and surgery durations when dealing with elective surgery scheduling. In subsequent chapters, we survey the related work about the problem under discussion in more details. Other stochastic researches have considered isolated decision levels in their attempt to provide optimal plan for the surgery scheduling failed to incorporate integrated approach that involves activities from determining allocations of OR time blocks through to the actual assignment of individual cases.

Even though a lot of research has been done in accounting for uncertainty, application of robust optimization model has been limited as compared to other stochastic approaches. In general, a healthcare system can be called robust if the optimal acquired service level is feasible regardless of how variable parameters resulting from inherent uncertainty in the system can influence it. More specifically, according to [19], the “robust planning” approach addresses the physically efficient system. It is aimed at recognizing and exploring the uncertainty that is inherent in the system, and distilling from it planning decisions that will yield more predictable and stable results. Unlike in deterministic approaches, variability of the outcomes (e.g., patient

arrivals and surgery durations) is considered in this thesis as the replacement of each random variable by its expected value. Since costs are extremely important and lack of flexibility is always a problem in many healthcare systems, one important factor of cost containment is reduction of the number of schedule changes. However, the existing uncertainty will render obsolete any medium-term plan based on deterministic numbers, forcing a re-planning cycle. The use of risk assessment and risk planning techniques clarifies the effects of uncertainty on planning performance. This in turn allows deciding on appropriate levels of resources, yielding schedules that remain valid for a longer time period, hence the term "robust planning". In order to quantify this robustness, several approaches are possible. One approach tries to find the decision policy that yields the most stable outcome, i.e. with low variability of the key performance measures (such as postponed surgery or under-and overutilization of resources) which is called "solution robust". Another approach attempts to find a policy that reduces the number of changes to the plan, while keeping the key performance measures fixed at their target level which is named "model robust". The conceptual meaning and advantages of robust planning is visualized in Figure 1.4. As it is clearly shown, in deterministic approaches one "optimal" schedule is obtained for a deterministic value of each variable (mostly the average or a "good guess"), while the robust plan provides a "near optimal" solution, which stays valid over a range of variable values at a predictable but higher cost.

Data uncertainties may lead to quality, optimality and feasibility problems when deterministic models are used. Therefore, it is required to generate a solution which is immune to data uncertainty. In other words, the solution should be robust [20]. A large portion of the literature on OR planning and scheduling deals separately with either determining the number of patients that need to be scheduled into a surgical suite or the sequence of cases within an operating theatre on a certain day. Hierarchical OR planning and scheduling that includes all three decision levels (i.e., strategic, tactical, and operational) to systematically address the

allocation of available OR capacity among surgical specialty, development of surgical block schedule and assignment of individual cases to the OR blocks in an integrated manner has not been fully investigated in the literature. Most of the previous work on OR block allocation problems have considered unrealistic assumptions such as the consideration of one type of elective surgery demands. In this thesis, we develop a richer model by considering both elective and emergency cases in the allocation of surgical OR time among specialties [21]. Furthermore, most of the previous works have been developed based on a deterministic planning model and hence, the effect of uncertainty has not been incorporated in those model. Moreover, the few papers that considered stochastic dependence in the key variables are based on developing complicated stochastic programing model that failed to incorporate the effect of emergency uncertainty into the modeling framework or considered heuristic approaches with some unrealistic assumptions that was unable to generate optimal solutions [16]. Finally, the concept of incorporating different aspects of service level into the decision making framework through the incorporation of maximum allowed waiting time has not been considered in previous literatures.

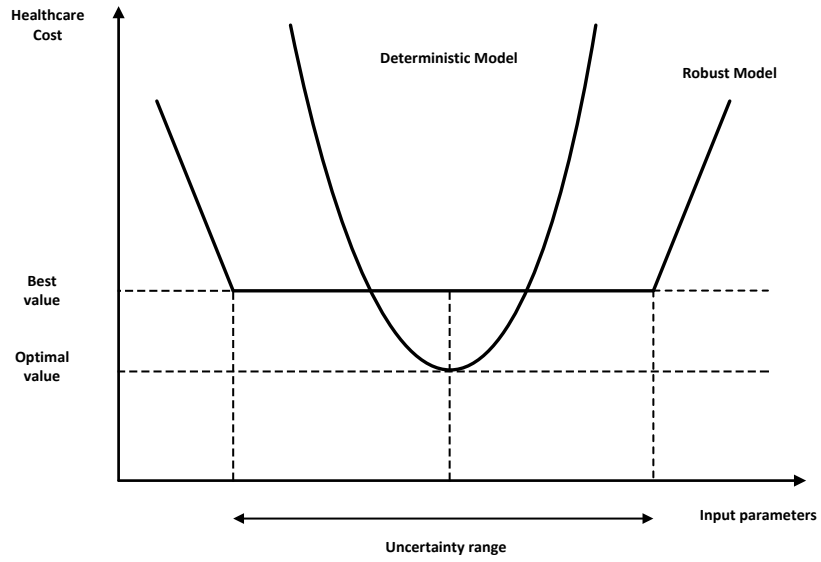

Figure 1.4: Effect of robust optimization model on total costs

The solution methods in the literature are diverse including deterministic mixed-integer programming (MIP) model to allocate block times of operating room capacity to various specialties' emergency and non-emergency surgery [4,14,22], stochastic programming (SP) approaches [6,16,23,24], Monte Carlo simulation method [24], stochastic dynamic programming [25], and robust optimization [17, 25]. We refer the readers to [27] for details. The literature can be classified by the different methodologies used and modelling aspects considered to evaluate various performance measures [28–30]. The main conclusions are that the optimal solutions provided by the existing literature are case-specific and cannot be generally applied to the case where a systematic planning and scheduling approach is required to capture the hierarchical impact of the solutions on the upstream stages. Moreover, robust optimization (RO) as a solution methodology has not been fully investigated to account for the effect of inherent uncertainty in the system.

The objective of this research is twofold: 1) To acquire the maximum service level for a healthcare provider by developing a RO programming approach to allocate the optimal OR times to surgery specialties that can meet the target number of operated patients, thereby minimizing the loss incurred to the hospital due to the underutilized capacity of allocated resources while the uncertain nature of emergency capacity is taken into consideration. The main trade-off includes meeting the target number of patients and making efficient use of resources i.e. a high utilization of resources. 2) To develop an integrated master surgery schedule with a surgical case assignment problem that address hybrid planning and scheduling problem of the hospital in a systematic hierarchical approach. The aim is to obtain an optimal surgery schedule that minimize patient waiting times as a societal factor that is crucially important in the Canadian health care system as well as the costs associated with underage and overage of OR resources. There is an important trade-off between underutilization, overtime, and patient waiting times.

Robust optimization techniques [30, 31] provide effective tools with regards to dealing efficiently with the inherent uncertainty that often characterizes OR planning programs. In Chapter 2, we introduce surgery planning and scheduling models developed in the literature with a focus on their strength and weaknesses. We also relate our work to the literature and stress the main differences. In Chapter 3, a brief description of the conventional RO methodology is presented and the novel general transformation framework is developed to remedy the drawbacks in complexity of using the conventional RO model and to reduce its computational difficulties. In Chapter 4, a typical surgery allocation procedure is developed and solved using our proposed RO transformation framework. In Chapter 5, the proposed RO transformation framework is applied on a hybrid MSS and SCA problem to develop an integrated robust OR planning and scheduling model. The model determines the optimal assignment of OR block among surgical specialties and the allocation of individual cases to the OR blocks within the planning horizon. Furthermore, the robust optimization model investigates the effects of uncertain emergency arrivals as well as surgery durations. A numerical experiment is conducted to demonstrate the RO model and to show that it captures the influence of uncertain parameters in a polynomial time. In Chapter 6, our findings are summarized and some future research directions are pointed out.

1.2 Overview of the Thesis

1.2.1 Chapter 2: Standard Robust Optimization Transformation Framework

In Chapter two, we develop a novel standard framework to transform deterministic linear programming models into the robust optimization forms. Deterministic models fail to capture all aspects of the real world problems due to the variability in the input data. Facing noisy and inaccurate data is an inevitable part of dealing with real-world optimization problems. Assuming that all the parameters are known with certainty is a highly optimistic assumption in solving optimization models [33]. In todays' world, sources of uncertainty exist in various real world problems. Therefore, failing to consider uncertain data can seriously degrade a system's performance. However, developing stochastic models that incorporate the complications of

unreliable data uncertainty into the model represents a challenge and requires sophisticated knowledge and considerable time and efforts. Therefore, presenting an effective approach to generalize transformation of deterministic model to the stochastic form that can handle the uncertainty of the real world problems would be of great value.

Robust Optimization (RO) is a modelling approach that involves uncertainty and was initially developed by [34] and [35] to handle the difficulty of unreliable data. RO is a general stochastic formulation framework that was constructed based on integration of goal programming formulation with a scenario-based description of a problem data to take the various decision makers' risk aversion into account. It is basically a proactive approach to mathematical programming for producing solutions which are less sensitive to the input data. Hence, RO can be applied in the optimization areas where the optimal solutions obtained through using other optimization approaches are highly sensitive to small changes of the input parameters. In fact, this methodology is an alternative approach to the stochastic linear programming (SLP) which applies probabilistic input data when the probability distributions governing the data are known or can be estimated.

RO is a relatively new approach to optimization under uncertainty when the uncertainty does not have a stochastic background and/or that information on the underlying distribution is not available, which is often the case in real-world optimization problems. Despite various advantages of the RO technique in generating solutions that are relatively less sensitive to the realization of noisy data and are immune to the anticipated uncertainty in the problem parameters, there is no general framework in the literature that can facilitate the transformation of a deterministic model into the RO form. We develop a generalizable RO transformation framework to remove this barrier and expand application of RO models in solving stochastic problems. Our aim is to provide a RO transformation framework for use as a tool in the context of operation research in order to generate the robust part of the deterministic models. Such a framework can

assist decision makers in solving complex optimization problem through providing an instructional guideline that makes the transformation process more effective and at the same time easier to implement. The proposed framework also reduces the formulation burden which has always been an obstacle to application of RO in solving operation research problems [32]. The proposed novel transformation framework is constructed based on the RO model developed by [32]. Three different robust models are then formulated using the proposed transformation framework to highlight the capability of the RO model in dealing with variability in stochastic environments. The proposed formulation can be generally used as a standard framework to transform any linear deterministic model into the stochastic robust form. We demonstrate the effectiveness of the proposed framework by applying it on a surgery planning and scheduling case of a healthcare problem in the following chapters. The randomness of the actual process is captured by testing the proposed formulation on a realistic model from a real case. We elaborate on the difference between our proposed framework and the SLP method to highlight the advantage of our framework. We believe our proposed framework can assist decision makers in solving complex optimization problem through providing an instructional guideline that makes the transformation process more effective and also easier to implement while it reduces the formulation burden which has always been an obstacle in applications of RO in solving operation research problems. Using actual data from a local health care system we demonstrate that our RO transformation framework is more efficient than the method presented in [5] as it works on a predefined framework that requires less information about the original deterministic problem while it is solved on a polynomial time. Our setting is quite general, thus it can be applied to various real life situations, including but not limited to health care, production planning and scheduling, and supply chain management while it is sufficiently generic to efficiently solve the problems presented in this study.

1.2.2 Chapter 3: Robust Surgery Mix Planning

In the current literature, OR planning and scheduling of healthcare systems is mostly considered in a static environment where the bulk of the key variables are known for certain. To compensate for the omission dedicated ORs to serve emergency patients or assign a fixed portion of existing OR capacity to perform only the emergency surgeries. However, this can easily be overlooked when addressing the emergency cases of patients who need to be served on the day of arrival as it happens [24]. Uncertainty is always involved in the number of emergency cases a hospital can get in a certain day, and hence, even a pre-determined portion of OR capacity may not fully absorb the impact of stochastic emergency cases in a developed model. Most of the previous work rely only on developing planning and scheduling models for elective patients.

In Chapter 3, we consider the problem of surgery capacity planning with discrete random arrivals for elective and emergency patients under the assumption that surgery demands are known only within certain bounds such that the probability distributions of the stochastic data are not known. We apply the proposed RO framework to incorporate the uncertainty that in this model. The majority of the earlier work on healthcare problems have been conducted under a deterministic environment [37]. This demonstrates the importance of developing a stochastic model that can capture the impact of the existing uncertainties of the healthcare services in order to tackle the challenges of the real-world needs of the underlying healthcare problems.

We consider the problem of operating room (OR) block allocation planning for multiple surgical specialties of a healthcare system on a given day, where possible mixtures of elective and emergency patients require simultaneously various surgery teams and OR blocks. Since patient arrivals are realized under uncertain circumstances, random characteristics in term of arrival time will be observed in surgical demands of different specialties. We first develop a deterministic surgery capacity allocation problem through a linear mixed-integer programming (MIP) approach to allocate block times of operating room capacity to various specialties' emergency and non-

emergency surgeries. We then formulate a two-stage stochastic programming model for the surgery capacity allocation problem and demonstrate its advantages over the deterministic model. We finally use the RO transformation framework proposed in previous chapters to develop an alternative approach that can efficiently handle the trade-off associated with the expected cost and its variability in the objective function. The incompleteness of the elective surgery demand data and the randomness arises in the emergency surgery demand is incorporated in the model to develop robust allocation plans that efficiently utilize the resource capacities in order to maintain the required service level. The main contribution of our work in this chapter is the proposed RO transformation framework as a modelling tool for surgery block allocation problems. We also consider emergency surgeries in allocation of surgery capacities in addition to a single class of patient (i.e. elective patients), and introduce the patients length of stay (LOS) as a function of the surgery postponements to manage the service level in the hospitals. Three RO models with different variability measures are proposed: the RO model with solution robustness, the RO model with model robustness, and the RO model with trade-off between solution robustness and model robustness to evaluate the operational performance and to analyze the enhancement of the trade-off between efficiency and health service delivery. A real case healthcare system is used to illustrate the application of the model. The resulting combinatorial programming models are conducted on AMPL optimization software and solved by CPLEX 12 in a reasonable amount of time. A framework for analysis is also proposed to select among three RO models based on the risk aversion levels and feasibility consideration of decision makers for the robustness of postponed/unmet demand size (i.e. hospital's service level) and the increased total cost. The results of the two-stage stochastic programming and the robust optimization models are evaluated to provide a comparison between the variability of output measures and infeasibility of the second stage constraints. Finally, a trade-off between the variability of the performance measures and the expected total costs is performed to acquire managerial insights on the optimal allocation plans.

1.2.3 Chapter 4: Robust integrated master surgery scheduling and surgical case assignment problem

In chapter 4 we look at a different but related problem of integration of planning and scheduling level in health care system with a focus on patient service level. We investigate the integration between OR planning and advance scheduling in a robust optimization setting, present experimental findings on OR allocation that hospitals can offer to surgical specialties and surgery cases scheduling for patients on the waiting lists that increases patient service level as well as the hospital's throughput. Since management and development of surgical activities at ORs can enormously impact the quality of surgery processes undergone by patients as well as patient waiting times, effective management efforts to increase performance are always needed. In particular, we investigate the commonly observed situation reported in the literature [27] where surgery durations were assumed a known parameter causing canceled surgery operations due to over scheduling of allotted OR block times by surgical specialties.

The efficient allocation of OR capacities to surgical specialties is a persistent problem in hospitals, especially when flat rate payments for patients based on diagnosis-related groups (DRGs) are taken into account [38]. Under the flat rate payment system, hospitals will only be reimbursed based on a pre-defined model developed by the government to establish a formal link between healthcare providers and quality. Introduction of DRG in the Canadian healthcare system forced hospitals to allocate their resources more economically.

Making plans for ORs is considered to be a very challenging task due to a number of different perspectives. The operating room department is a volatile environment where the uncertainty in emergency patient arrivals and surgery durations together with their impact on other departments in the hospital makes planning and scheduling a very complex decision [39]. According to a recent review made by [40] and [28] there are various conflicting objectives in OR planning and scheduling process due to different stakeholder criteria. The inherent variability in

various resources greatly impacts the trade-off between a hospital's desirability to reduce cost against the quality of health services that coordinates to lower patients waiting time. Therefore, there is a strong need for developing an efficient model the allocation of surgical disciplines to available OR sessions and the assignment of surgical cases to the allocated operating room. We take a systematic look at this problem and provide an integrated model that concurrently solves the master surgery schedule problem (MSSP) and the surgical case assignment problem (SCAP) to acquire optimum allocation of surgical specialty (MSS) while the assignment of patients to the OR blocks (SCA) is optimized to identify and analyze a combined tactical and operational decision for the OR department with the aim of reducing costs associated with patients waiting time, surgeons over (under) time and ORs capacity disruption. The integration of planning and scheduling levels provide some stability, in terms of repeatability of personnel schedules and predictability of bed occupancy pattern in post anesthesia care units (PACU) as well as flexibility, in terms of adaptability of weekly plans to the changing waiting lists for the decision makers. We also seek the trade-off between higher capacity, which will reduce the waiting time as well as OR productivity due to under (over) utilization, and a lower capacity that result in postponement as well as ORs overtime. We consider two types of patients: elective cases with uncertain surgery durations and emergency patients with stochastic arrivals.

We also consider a weighted multi-objective RO approach, which integrates optimization modules that take into account the number of surgeries scheduled, the waiting time and tardiness of each patient associated with patient urgency factors, and weighted resource utilization rates. The multi-criteria objective function is focused on conflicting resource perspective as well as patient perspective at the same time. A regulated waiting time target for elective cases derived from Ministry of Health and Long-Term Care in Ontario is utilized to manage patient admission that weighs the chronological waiting time with the urgency coefficient of the corresponding Urgency Related Group (URG) of each patient. The wait time targets are developed with the help

of clinical experts and serve as a method of accountability and provide a goal to achieve. These targets include urgency classifications and are incorporated in the regulated Wait Times Information System (WTIS).

A mixed-integer linear programming model is first developed where the uncertainty considerations are excluded. The deterministic model is then transformed into a two stage stochastic programming model as well as a robust optimization (RO) model to incorporate the impact of uncertainty into the decision making process. A novel transformation framework, presented in Chapter 2, is utilized to develop the robust counter part of the deterministic model. The incompleteness of the random surgery durations and the randomness arising in the emergency arrivals are considered using a discrete set of scenarios. The proposed RO framework makes use of a linear programming model and does not require the specifications of the probability density functions of the uncertain parameters. All three models are then analyzed over a set of real life based instances to evaluate their behavior in terms of computational effort and solution quality. Moreover, assuming lognormal distributions for the emergency arrivals and surgery durations, a set of randomly generated scenarios is used in order to compare the proposed solutions in terms of OR utilization rate and number of postponed patients. The compromised allocation of OR blocks as well as the assignment of patients obtained from the RO framework is able to handle the variability within the uncertain parameters through generating optimal scenario-dependent solutions. The trade-off between the allocation plan's robustness (i.e. postponed/cancelled surgery) and underutilization of OR blocks for different values of robustness is demonstrated that the proposed RO model is progressively less sensitive to the realization of the variable input parameters, while generating more feasible solutions as compared with the two-stage stochastic recourse programming model. Moreover, the impact of introducing overtime in the model formulation is evaluated and a sensitivity analysis on the choice of the key parameters is performed. Our approach is demonstrated to improve patient satisfaction through reducing

prioritized weighted waiting times and improving health care efficiency by reducing overall operation costs, and hence has more societal benefits for the hospitals.

1.3 Outline of the Thesis

The rest of this thesis, as discussed in section 1.2, is organized as a series of chapters. At the beginning of each chapter, we outline the problem to be discussed, investigate the motivations, and illustrate the significance of examining the related work. We then provide our modelling approach followed by analysis of the results. We conclude each chapter with a summary of the main findings. In addition to the chapters discussed in Section 1.2, Chapter 5 gives a summary of the thesis contributions and provides a brief discussion of future research directions.

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Chapter 2

A Transformation Framework for Robust Optimization[1]

2.1 Introduction

Facing noisy, inaccurate, or unspecific data is an inevitable part of dealing with real-world optimization problems for decision makers in their attempts to reduce variability and to show the overemphasis of feasibility of optimization models. Assuming that all the parameters are known for certainty is a highly optimistic assumption in developing optimization models [1]. Failing to consider variable and uncertain data can seriously degrade a system's performance in the real world situation, where various sources of uncertainty are present. Thus, presenting an effective approach to encompass all the uncertainty in the real world problems would be of great value. To handle the difficulty of such unreliable data, Mulvey and Vanderbei [2] and Mulvey and Ruszczynski [3] develop a general stochastic formulation framework, called Robust

[1] A version of this chapter has been submitted for publication. Lalmazloumian M. , Baki F. and Ahmadi M. A novel robust optimization transformation framework to operating room capacity allocation problem under uncertainty at a publicly-funded hospital.

Optimization (RO). The authors construct their approach based on integration of goal programming formulation with a scenario-based description of a problem data to take the various decision makers' risk aversion. They introduce RO methodology as a proactive approach to mathematical programming for producing solutions which are less sensitive to the input data. Hence, RO can be applied in the optimization areas where the optimal solutions obtained through using other optimization approaches are highly sensitive to small changes of the input parameters. In fact, their methodology is an alternative approach to the stochastic linear programming (SLP) which applies probabilistic input data.

2.2 Robust Optimization

RO is a relatively new approach to optimization under uncertainty when the uncertainty cannot be captured by specific probability distributions or that information on the underlying distribution is not available, which is often the case in real-world optimization problems. In RO, stochastic parameters are separated by a set of discrete scenarios. RO searches for solutions that are relatively immune to anticipated uncertainty in the problem parameters to produce less sensitive solutions to the realization of noisy data from each scenarios. Stochastic programming (SP) and RO are both scenario-based methods trying to proactively utilize information about uncertainty. RO uses the method of two-stage programming, in which the violation of inequality constraints will be penalized in the objective function. While both SP and RO emphasize the minimization of expected costs or maximization of expected profits, RO also focuses on reducing the variability of the expected cost. Furthermore, in RO, the number of variables approximately only doubles, whereas in SP it increases exponentially with the number of uncertain parameters.

Using the scenario-based approach in which random variables take on specified values in each scenario, RO provides solutions which are progressively less sensitive, and more flexible to the realization of stochastic variables. Properties of the optimal solutions in RO are classified as "solution robust" and "model robust" to reduce variability of the objective value and also the

infeasibility of the solution for any realization of the stochastic parameters. An optimal solution to model is defined as solution robust if it remains 'close' to optimal for all input data scenarios, while the model is called robust if it remains 'almost' feasible for all data scenarios.

The use of risk assessment and risk planning techniques clarifies the effects of uncertainty on planning performance. This in turn yields plans that remain valid for a longer time period, hence the term "robust planning". In order to quantify this robustness, several approaches are possible. One approach tries to find the decision policy that yields the most stable outcome, i.e. with low variability of the key performance measures (such as service level or total underutilization) which is called "solution robust". Another approach tries to find a policy that reduces the number of changes to the plan, while keeping the key performance measures fixed at their target level which is called "model robust".

Mulvey and Vanderbei [2] describe the notions of RO in a stochastic optimization model. Variables and constraints in RO include two distinct characters. Structural or design variables are those whose optimal values are not dependent upon the realization of uncertain input parameters. Furthermore, the design variables' values cannot be adjusted once a realization of the uncertain data is known. On the other hand, the optimal values of control variables depend upon the realization of uncertain parameters, as well as the optimal values of the design variables. Like the variables, robust modeling contains two type of constraints which are structural constrains free of noise coefficients, and control constraints with noisy coefficients. According to Leung et al. [4], the structural constraints are linear constraints whose technology coefficients are affected by randomness and its input data are free of any noise, the control constraints contain data that can be uncertain.

RO was developed to reduce variability and citing the overemphasis of feasibility in optimization models, [2] present the framework for the conventional RO model. Using a

scenario-based approach in which random variables take on specified values in each scenario, this technique seeks to measure the trade-off between solution robustness (i.e., a measure of optimality) and model robustness (i.e., a measure of feasibility). According to the authors, a robust solution is one that is almost optimal in all scenarios, while a robust model is one that remains almost feasible in all scenarios. Hence, RO extends SLP by including higher moments in the objective function (i.e., variance of total costs) and allowing for infeasibilities (i.e., model robustness). By incorporating risk into the objective function, robust optimization allows for a more passive management style than stochastic linear programming. Unlike its stochastic linear programming counterpart, a robust optimization model is not considered infeasible even when one or more infeasibilities occur. According to [5] the solutions developed by the RO model is progressively less sensitive to the realization of data in a scenario sets. However, the complexity of developing the robust counterpart of an integer linear programming model is deemed a huge barrier that restricts the implementation of the RO technique in healthcare optimizations [6]. Therefore, development of a standard framework that coordinates the transformation of deterministic models into the robust optimization forms is of a great value.

As described above, transformation of a deterministic model into the RO form can be very complicated and at the same time lengthy process that is seen as a barrier to using RO as a progressive tool to tackle the uncertainty in solving optimization problems. The aim of this chapter is to provide a generalizable RO transformation framework for use as a tool in the context of operation research in order to generate the robust part of the deterministic models. Such a framework can assist decision makers in solving complex optimization problems through providing an instructional guideline that makes the transformation process more effective and at the same time easier to implement. The proposed framework also reduces the formulation burden which has always been an obstacle to application of RO in solving operation research problems [5].

In light of the above discussion, the proposed novel transformation framework is constructed based on the RO model developed by [5]. A general two-stage stochastic recourse programming model is first developed to incorporate uncertainty in a developed formulation problem. Three different robust models are then projected using the proposed transformation framework to highlight the capability of the RO model in dealing with variability in stochastic environments. The proposed formulation can be generally used as a standard framework to transform any linear deterministic model into the stochastic robust form. The template transformation framework is then applied to a surgery planning and scheduling case of a healthcare problem in the following chapters to capture the randomness of the actual process in order to evaluate the effectiveness of the proposed framework on a realistic model and to demonstrate the applicability of the formulation. It is illustrated through the formulation that the proposed transformation framework is more practical to use than the method developed by [5]. Furthermore, the computational results confirm that the framework presented herein generates a robust allocation plan in a timely manner without requiring additional deviation variables.

2.2.1 Conventional Robust Optimization Formulation

To depict the robust optimization problem, it is assumed that $x \in R^{n1}$ is the first stage i.e. design variable vector and $y_\varepsilon \in R^{n2}$ is the second stage i.e. control variable vector. Then the basic linear programming (LP) model would be formed as follows.

$$Min\, c^T x + d^T y \qquad \forall x \in R^{n1}, y_\varepsilon \in R^{n2} \tag{2.1}$$

$$Ax = b \tag{2.2}$$

$$Bx + Cy_\varepsilon = e \tag{2.3}$$

$$x, y_\varepsilon \geq 0 \tag{2.4}$$

Equation (2.2) is the structural constraint with fixed and free of noise coefficients, whilst equation (2.3) indicates the control constraint whose coefficient is under the influence of noisy data. Equation (2.4) guarantees non-negative vector of decision variables. To define the RO formulation, a set of scenarios $\varepsilon = \{1,2,...,\Xi\}$ is introduced where under each scenario $\varepsilon \in \Xi$, the control constraint coefficients are defined as $\{d_\varepsilon, B_\varepsilon, C_\varepsilon, e_\varepsilon\}$ with predetermined probability π_ε, the occurrence probability of scenario ε, thus would be $\sum_{\varepsilon}^{\Xi} \pi_\varepsilon = 1$. In order to absorb the impact of having different values for the uncertain input data, a set of vectors containing the control variables, $\{y_1, y_2, ..., y_E\}$,is introduced. The optimal solution of the mathematical formulation (2.1) to (2.4) is considered robust when it remains ‘‘close’’ to optimality for any realization of the scenario $\varepsilon \in \Xi$, and hence termed solution robust and if it remains ‘‘almost’’ feasible for any realization of ε and thus termed model robust.

In order to measure what close-to-optimality and almost-feasibility mean in robust optimization formulation, it is required to conduct a trade-off between solution and model robustness to acquire an optimal solution that remains both feasible and optimal for all scenarios. RO overcomes the challenge of finding a solution that remains both feasible and optimal to all input scenarios, by applying concepts in multi-criteria decision making (MCDM) as follows.

$$Min\, \sigma(x, y_1, ..., y_\varepsilon) + \omega\rho(\delta_1, ..., \delta_\varepsilon) \tag{2.5}$$

$$Ax = b \tag{2.6}$$

$$B_\varepsilon x + C_\varepsilon y + \delta_\varepsilon = e_\varepsilon \qquad \forall \varepsilon \in \Xi \tag{2.7}$$

$$x, y_\varepsilon \geq 0 \qquad \forall \varepsilon \in \Xi \tag{2.8}$$

Where the set of $(\delta_1,...,\delta_E)$ contains the error vectors that measure the permitted infeasibility in the control constraints (2.7) under scenario ε. The realizations of the coefficients of the control constraints for each scenario ε comprises the set $\{d_\varepsilon, B_\varepsilon, C_\varepsilon, e_\varepsilon\}$. Furthermore, the previous objective function $\zeta = c^T x + d^T y$, becomes a random variable of value $\zeta_\varepsilon = c^T x + d_\varepsilon{}^T y_\varepsilon$ with probability π_ε. The next step would be to choose an appropriate function for $\sigma(x, y_1,...,y_E)$. In stochastic linear programming, the function that is typically used is $\sigma(.) = \sum_{\varepsilon\in\Xi} \pi_\varepsilon \zeta_\varepsilon$, which represents the mean or expected value of $\zeta_\varepsilon = c^T x + d_\varepsilon{}^T y_\varepsilon$ over all the scenarios. The second term in the objective function of the above formulations represents an infeasibility penalty function. Using the values of the realized error vectors, this function penalizes violations of the control constraints under some of the scenarios. In other words, this term would allow the model to handle scenarios in which realizations of the uncertain parameters would otherwise not be allowed for a feasible solution, although with an associated penalty for each violation of the control constraint. Hence, the first term in the objective function, (2.5), provides a measure of solution robustness, which means remaining "close" to optimal for any realization of the scenario $\varepsilon \in \Xi$, whereas the second term provides a measure of model robustness, that means remaining "almost" feasible for any realization of $\varepsilon \in \Xi$. Furthermore, the parameter ω is used as a weight to define the desired trade-off between solution and model robustness. It is obvious that assigning a value of zero to ω may result in an infeasible solution, whereas assigning a sufficiently large value to ω causes the infeasibility penalty function term to dominate the objective function, thereby resulting in a higher expected value for $\sigma(x, y_1,..., y_\varepsilon)$.

In real-world optimization problems, a high level of risk might be associated with one or more of the uncertain input parameters (e.g., availability of surgeons or arrival of emergency

cases). However, when $\sigma(.) = \sum_{\varepsilon \in \Xi} \pi_\varepsilon \zeta_\varepsilon$ is used as a cost term in the objective function of the proposed formulation (2.5-2.8) the model seeks only to minimize the expected value of the overall cost across all possible scenarios. In other words, the model does not account for the potential variability in cost (across scenarios) associated with the high risk parameter(s). Given this situation, [7] propose a mean-variance approach as one technique for mitigating the risk associated with one or more uncertain input parameters. Specifically, the revised cost function consists of the expected value of the random variable $\zeta_\varepsilon = c^T x + d_\varepsilon{}^T y_\varepsilon$ plus a constant times the variance. In other words, the cost function portion of the objective function in the proposed formulation becomes as follows.

$$\sigma(x, y_1, ..., y_\varepsilon) = \sum_{\varepsilon \in \Xi} \pi_\varepsilon \zeta_\varepsilon + \lambda \sum_{\varepsilon \in \Xi} \pi_\varepsilon \left(\zeta_\varepsilon - \sum_{\varepsilon' \in \Xi} \rho_{\varepsilon'} \zeta_{\varepsilon'} \right)^2 \qquad \forall \varepsilon \in \Xi, \forall \varepsilon' \in \Xi \qquad (2.9)$$

Clearly, as the value of λ, which is a weighting factor to trade-off between risk and expected outcome for the solution robustness, is increased, the solution becomes less sensitive to changes in the input data as defined by the scenarios. Mulvey et al. [7] point out that this inclusion of the weighted variance term enables robust optimization to account for a decision maker's preferences toward risk. Thus, robust optimization allows for a more passive management style, giving it a distinct advantage over stochastic linear programming. In other words, with variability under control, minimal adjustment to the control variables will be required when the weighted variance version of robust optimization is applied. While Equation 2.9 accounts for both expected cost and cost variability, its quadratic terms introduce the undesirable characteristic of nonlinearity into the model. To address this drawback, Yu and Li [8] propose an alternate formulation for Equation 2.9 as follows.

$$\sigma(x, y_1, ..., y_\varepsilon) = \sum_{\varepsilon \in \Xi} \pi_\varepsilon \zeta_\varepsilon + \lambda \sum_{\varepsilon \in \Xi} \pi_\varepsilon \left| \zeta_\varepsilon - \sum_{\varepsilon' \in \Xi} \pi_{\varepsilon'} \zeta_{\varepsilon'} \right| \qquad \forall \varepsilon \in \Xi, \forall \varepsilon' \in \Xi \qquad (2.10)$$

However, despite eliminating the quadratic terms of $\sum_{\varepsilon \in \Xi} \pi_\varepsilon \left(\zeta_\varepsilon - \sum_{\varepsilon' \in \Xi} \pi_{\varepsilon'} \zeta_{\varepsilon'} \right)^2$ from Equation 2.9, the formulation remains nonlinear. While a direct linearization of the absolute value term in Equation 2.10 is possible, the result is the introduction of several constraints and non-negative deviational variables into the model. They present a robust formulation of a stochastic logistics problem that reduces computational burden by adding only half of the number of variables as in the model developed by [7]. In Yu and Li [8] authors illustrate the drawbacks of the approaches taken in [7] and incorporate a novel approach to linearizing the mean absolute deviation term in the objective function. Hence, the cost term to be used in formulation 2.5 to 2.8 is transformed from a quadratic form to a much more tractable linear form. Finally, they propose an efficient methodology to minimize the objective function which is depicted in Equations 2.11 to 2.13 where θ_ε shows deviations for violations of the mean.

In objective function (2.10), $\left| \zeta_\varepsilon - \sum_{\varepsilon' \in \Xi} \pi_{\varepsilon'} \zeta_{\varepsilon'} \right|$ denotes the norm of $\left(\zeta_\varepsilon - \sum_{\varepsilon' \in \Xi} \pi_{\varepsilon'} \zeta_{\varepsilon'} \right)^2$, which can be chosen in an arbitrary way. However, its choice influences solution performance. If the norm is denoted by the variance, the quadratic terms contain numerous cross products among variables, which contribute a large computational burden. In [8] a robust model with absolute term for a logistic management Problem is proposed, and an effective method to transform the model into a linear programming model is introduced by utilizing additional deviation variables. In this study, we use the method proposed by in [8] to convert the model with the absolute term into a linear programming one.

$$Min\ Z = \sum_{\varepsilon \in \Xi} \pi_\varepsilon \zeta_\varepsilon + \lambda \sum_{\varepsilon \in \Xi} \pi_\varepsilon \left[\left(\zeta_\varepsilon - \sum_{\varepsilon' \in \Xi} \pi_{\varepsilon'} \zeta_{\varepsilon'} \right) + 2\theta_\varepsilon \right] \tag{2.11}$$

$$\zeta_\varepsilon - \sum_{\varepsilon' \in \Xi} \pi_{\varepsilon'} \zeta_{\varepsilon'} + \theta_\varepsilon \geq 0 \qquad \forall \varepsilon \in \Xi, \forall \varepsilon' \in \Xi \tag{2.12}$$

$$\theta_\varepsilon \geq 0 \qquad \forall \varepsilon \in \Xi \tag{2.13}$$

In Equation 2.11, it is notable that that if $\zeta_\varepsilon - \sum_{\varepsilon' \in \Xi} \rho_{\varepsilon'} \zeta_{\varepsilon'} \geq 0$ then finally the complete formulation of the robust objective function which includes both solution robustness and feasibility robustness is formulated as follow.

$$Min\ Z = \sum_{\varepsilon \in \Xi} \pi_\varepsilon \zeta_\varepsilon + \lambda \sum_{\varepsilon \in \Xi} \pi_\varepsilon \left[\left(\zeta_\varepsilon - \sum_{\varepsilon' \in \Xi} \pi_{\varepsilon'} \zeta_{\varepsilon'} \right) + 2\theta_\varepsilon \right] + \omega \sum_{\varepsilon \in \Xi} \pi_\varepsilon \delta_\varepsilon \tag{2.14}$$

Because of the parameter uncertainty, the model maybe infeasible for some scenarios. Therefore, δ_ε presents the infeasibility of the model under scenario set ε. In other words, δ_ε is the amount by which the control constraints is being violated under each scenario. If the model is feasible, δ_ε will be equal to 0. Otherwise; δ_ε will be assigned a positive value according to equation (2.7).

As can be seen, there is no standard framework to transform a deterministic model to the robust optimization form in the literature. This may have created a barrier on using RO as a progressive tool to tackle uncertainty in solving optimization problem in operation management content. The aim of this chapter is to provide a generalizable RO transformation formulation framework to solve operation research optimization problem. Herein, we develop a standard framework formulation to be used by decision makers as a tool to transform a deterministic

model into its robust counterpart. Such a framework can assist decision makers in solving complex optimization problem through providing an instructional guideline that makes the transformation process more effective and also easier to implement. The proposed framework also reduces the formulation burden which has always been an obstacle in application of RO in solving operation research problems

Our standard transformation technique is developed based on the RO method proposed by [8], to be employed as a general framework in order to transform linear deterministic models into their robust optimization form. To demonstrate the applicability of our proposed approach, two different set of problems, including a healthcare capacity allocation problem in an operating room department and a hybrid master surgical schedule and surgery case assignment problem is solved to provide an insight into the structural transformation framework and also the complexity of the evolved solutions in Chapter 4 and 5, respectively. To the best of our knowledge, the proposed transformation approach has not been applied to any healthcare operations problem yet. Using data of a real case study from a local hospital we demonstrate that our RO transformation framework is more efficient than the method presented in [8] as it works on a predefined framework that requires less information about the original deterministic problem while it can be solved in polynomial time.

If $\theta_\varepsilon = 0$ in the optimal solution then $Z = \sum_{\varepsilon \in \Xi} \pi_\varepsilon \zeta_\varepsilon + \lambda \sum_{\varepsilon \in \Xi} \pi_\varepsilon \left(\zeta_\varepsilon - \sum_{\varepsilon' \in \Xi} \pi_{\varepsilon'} \zeta_{\varepsilon'} \right)$. Otherwise, if $\zeta_\varepsilon - \sum_{\varepsilon' \in \Xi} \pi_{\varepsilon'} \zeta_{\varepsilon'} \leq 0$, then $\theta_\varepsilon = \sum_{\varepsilon' \in \Xi} \pi_{\varepsilon'} \zeta_{\varepsilon'} - \zeta_\varepsilon$ in the optimal solution and $Z = \sum_{\varepsilon \in \Xi} \pi_\varepsilon \zeta_\varepsilon + \lambda \sum_{\varepsilon \in \Xi} \pi_\varepsilon \left(\sum_{\varepsilon' \in \Xi} \pi_{\varepsilon'} \zeta_{\varepsilon'} - \zeta_\varepsilon \right)$. As can be seen, the solution procedure of Equations 2.11 to 2.13 is the same to Equation 2.10.

2.3 Robust Optimization vs Sensitivity Analysis and Stochastic Programming

According to Mulvey and Vanderbei [2], RO technique has several advantages over its alternatives; however, it would be quite optimistic if we do not take its deficiency into account.

Comparing sensitivity analysis (SA) with RO, it should be noted that SA, which is indeed a reactive approach to controlling uncertainty, is only influenced by the ranges of changing in input data when measuring the sensitivity of a solution and as a result cannot provide any mechanism to control the sensitivity.

Stochastic linear programming (SLP) method, on the other hand, which is a constructive approach as RO, provided the opportunity for decision makers to exploit the flexibility of resource variables. However, the SLP model optimizes only the first moment of the distribution of the objective value ζ_ε. In fact, SLP ignores higher moments of the distribution, which is quite important for asymmetric distribution and risk adverse decision makers. Moreover, taking the aim of optimizing the expected value in SLP requires management to take active role since the expected value can be remained on optimal while the large changes in ζ_ε among different scenarios might have been observed. But in RO model, both the higher moments and the variance of the distribution of ζ_ε, for instance, would be minimized while the management can take a passive style. This, in turn, requires little or no adjustment of the control variables, since the value of ζ_ε will not considerably differ among different scenarios. In this regard, RO can be viewed as an SLP, where the recourse decisions are completely restricted.

To illustrate the distinction between RO and SLP in their application domain, Mulvey and Vanderbei [2] propose an example about personnel planning problem. The authors propose that SLP design a solution for workforce that can be adjusted (by hiring or layoffs) to meet demand at lowest possible cost; however, the model was not able to maintain the employment stability. The RO model, on the other hand, is able to maintain a stable workforce to cope with

diverse demand for all scenarios. Though, the cost of the solution is higher. In fact, RO directly control variability of the solution as opposed to just optimizing its first moment.

Another main difference between RO and SLP is the handling of constraints. SLP models intend to find the design variable x so that for each scenario, a possible control variable setting y_ε satisfies the constraints. Although, for scenarios that no feasible pair of $(x, y_\varepsilon)$is possible, the SLP model is declared infeasible. RO, however, completely takes this possibility into consideration. The RO model will find a solution that violates the constraints by least amount possible through the use of error terms δ_ε and penalty function $\omega(.)$. Other advantages of RO against SLP, mentioned by [2], are stability of the respective solutions, and solutions accuracy when the number of applied scenarios is limited.

Although RO has obvious advantages over SA and SLP, it contains two main restrictions. First, RO models are parametric programs with a priori mechanism for identifying a "correct" choice for the parameter ω. This is, according to [8], a common problem in multi-criteria programming optimization. Second, the scenarios in Ξ are just one possible set of realization of data for the problem and RO does not provide a means by which the scenarios can be specified. This problem is prevalent in SLP models as well.

Even though RO model has some potential limitation, it still provides considerably improved solution framework, especially in the face of noisy data.

2.4 Robust Optimization Transformation Framework

With the aim of incorporating uncertainty into the model development process, this section describe the structure of the proposed RO transformation framework to transform the deterministic models into the robust form.

We first develop the formulation of a general two-stage stochastic recourse programming model to define uncertainty into the model and then three different robust optimization models are generated using our proposed framework.

2.4.1 Two-stage stochastic recourse programming model

A general two-stage stochastic recourse programming model can be formulated as follows:

$$Min\, c_1^T x + \sum_{\varepsilon \in \Xi} \pi_\varepsilon c_{2\varepsilon}^T y_\varepsilon \qquad \forall x \in R^{n1}, y \in R^{n2} \qquad (2.15)$$

Subject to

$$Ax = b \qquad (2.16)$$

$$B_\varepsilon x + C_\varepsilon y_\varepsilon = e_\varepsilon \qquad \forall \varepsilon \in \Xi \qquad (2.17)$$

$$x, y_\varepsilon \geq 0 \qquad \forall \varepsilon \in \Xi \qquad (2.18)$$

In the objective function (2.15), x denotes the vector of first stage (i.e. design) variables whose optimal value is determined before complete information of uncertain parameters is observed, while y_ε denotes the second stage (i.e. control) variables corresponding to realization of the scenario ε where the decisions are subject to adjustment when the realization of the stochastic parameters is observed. Therefore, the sum of the first stage costs and the second stage costs in the objective function represents the total expected costs of the stochastic recourse programing model. Under constraints, B_ε and e_ε represent random coefficient matrix and right-hand side vector, respectively. C_ε and $c_{2\varepsilon}$ denote the recourse matrix and the penalty recourse cost vector corresponding to scenario ε respectively. The equations (2.16) and (2.17) are categorized as the first stage constraints in the stochastic recourse model as the first stage constraints and the second stage constraints, respectively. The former only involves the first stage variables, while the latter

contains both first stage and second stage variables. It should be noted that in the objective function (2.15), the stochastic variable is indicated by "ε", and π_ε denotes the probability of the realization of the stochastic variables. The produced solution fails to find a trade-off between optimality and robustness of the optimum solutions.

2.4.2 Robust optimization transformation framework for two-stage stochastic recourse program

In order to incorporate the impact of having different values for the uncertain input parameters, RO model proposed by Mulvey et al. [7] is presented here to modify the objective in SP as follows.

$$Min\ c_1^T x + \sum_{\varepsilon \in \Xi} \pi_\varepsilon c_{2\varepsilon}^T y_\varepsilon + \lambda \sigma\left(x, y_1, ..., y_\varepsilon\right) + \omega \rho\left(\delta_1, ..., \delta_\varepsilon\right) \qquad \forall x \in R^{n1}, y \in R^{n2} \tag{2.19}$$

Subject to

$$Ax = b \tag{2.20}$$

$$B_\varepsilon x + C_\varepsilon y + \delta_\varepsilon = e_\varepsilon \qquad \forall \varepsilon \in \Xi \tag{2.21}$$

$$x, y \geq 0 \tag{2.22}$$

The term $\sum_{\varepsilon \in \Xi} \pi_\varepsilon c_{2\varepsilon}^T y_\varepsilon + \lambda \sigma\left(x, y_1, ..., y_\varepsilon\right)$ in the objective function denotes the solution robustness measure, where $\{y_1, y_2, ..., y_\varepsilon\}$ is introduced as a set of vectors containing the control variables and λ is intended as a measurement of the variability of the objective function. The term $\omega \rho(\delta_1, ..., \delta_\varepsilon)$ is a feasibility penalty function which is used to measure the violation of the second stage constraints where ω is a penalty weight that is used to define the desired trade-off between solution and model robustness. The term $(\delta_1, ..., \delta_\varepsilon)$ in the objective function contains the error vectors that measure the allowed infeasibility in the control constraints (2.21) under scenario ε. Using the value of the realized error vectors, the feasibility penalty function penalizes

violations of the control constraints under some of the scenarios. Equation (2.20) is the structural constraints with fixed and free of noise coefficients, whilst equation (2.21) indicates the control constraints of which the coefficients are under the influence of noisy data. The realizations of the coefficients of the control constraints for each scenario ε comprises of a set of $\{C_{2\varepsilon}, B_{\varepsilon}, C_{\varepsilon}, e_{\varepsilon}\}$. Equation (2.22) guarantees non-negative vector of the decision variables. It should be noted that the RO overcomes the challenge of finding a solution that remains both feasible and optimal to all input scenarios through a trade-off between model robustness and solution robustness by applying the concept of multi-criteria decision making procedure.

The framework proposed in this work takes into account the uncertainty of incorporating unknown parameters into the model. Here we define the term μ to measure the mean value of the objective function under uncertainty which also provides an expected second stage cost. In order to incorporate risk aversion capability into the RO model, we penalize the deviation for violation of the mean expected cost, both above and below the expected recourse cost through a deviation function represented by d^{ε}. Since variance is a systematic risk measure, penalizing the deviation from the mean provides more flexibility for decision makers regarding the degree of risk aversion that they prefer to take for a given situation. The actual deviation from the mean is penalized via the weight θ^{ε}. We employ a mean absolute deviation approach to make sure the impact of deviation from the mean is incorporated into the model (refer to Section 2.2.1).

The next terminology we use herein is represented by f^{ε}, where it denotes an equation of the control constraints. In the proposed framework, f^{ε} represent the amount by which the model allows for infeasibility to incorporate variability of the input parameters and hence maintain model robustness feature in the framework. This equation provides some flexibility for the decision makers to appropriately measure the permitted infeasibility in the control constraints. The control constraints enable the model to adjust to the impact of changes in variable input data

under different scenarios if it violates the condition of the constraints through a penalty function, γ^{ε}, in order to penalize the incurred infeasibility. The function f^{ε} penalizes violations of the control constraints under some of the scenario sets. In other words, the term f^{ε} allows the model to handle scenarios in which realizations of the uncertain parameters would otherwise not be allowed for a feasible solution, although with an associated penalty weight ω for the violation of the control constraints.

Considering the assumptions of the defined standard transformation framework, robust optimization model can be reformulated using our proposed transformation framework to measure the decision maker's risk preferences through the expected variability of the objective function and also provides a degree of trade-off between that risk and the cost of infeasibility of the second stage constraints.

2.4.3 Proposed transformation framework for the RO model with solution robustness

In realistic optimization problems, a high level of risk might be associated with one or more of the uncertain input parameters (e.g., operating rooms availability, surgery demand variability, or unpredictability of surgery duration). However, when $\sigma(.)=\sum_{\varepsilon\in\Xi}\pi_{\varepsilon}c_{2\varepsilon}^{T}y_{\varepsilon}$ is used as a second stage cost in the objective function of the robust formulation (2.19-2.22), the model only seeks to minimize the expected value of the overall cost across all possible scenarios. In other words, the potential variability in the cost functions associated with the high risk parameters across different scenarios is not taken into account. In fact, the variability that exists in the input parameters makes the transformation procedure more complicated. To provide an objective function with solution robustness that generates the optimal solutions that are less variable and are not altered substantially across different scenarios, we utilize the mean-variance approach proposed by Mulvey et al. [7] as a technique to mitigate the risk associated with the uncertain input parameters. Hence, the revised cost function consists of the expected value of the random

variable plus a constant multiplying the variance. We define ζ to represent $f(x, y)$, which is a cost function, and as a result, $\zeta_\varepsilon = f(x, y_\varepsilon)$ for scenario ε. The variable, d^ε, is defined to capture the variance of the expected cost functions of the original RO model. Therefore, the new cost function portion of the objective function that manages the risks and variability of solutions among different scenarios is formulated as follows.

$$\mu = \sum_{\varepsilon \in \Xi} \pi_\varepsilon \zeta_\varepsilon \qquad \forall \varepsilon \in \Xi \qquad (2.23)$$

$$d^\varepsilon = \zeta_\varepsilon - \mu \qquad \forall \varepsilon \in \Xi \qquad (2.24)$$

$$\sigma(x, y_1, ..., y_\varepsilon) = c^T x + \mu + \lambda \sum_{\varepsilon \in \Xi} \pi_\varepsilon \left| d^\varepsilon \right| \qquad (2.25)$$

λ is the weighting factor between zero and one that represents the trade-off between risk and expected outcome for the solution robustness. Clearly, as the value of λ is increased the solution becomes less sensitive to changes in the input data as defined by the scenarios. If a solution is resulted in too many infeasible constraints, any small change in the value of uncertain parameters can cause a huge difference in the value of the measured function. It is noted that this inclusion of the weighted variance term enables RO to account for the decision maker's preferences towards risk. Thus, proposed RO model allows for a more passive management style, giving it a distinct advantage over the stochastic recourse programming. In other words, with variability under control, minimal adjustment to the control variables will be required when the weighted variance version of RO is applied. While Equation (2.25) accounts for both expected cost and cost variability, the absolute term introduces the undesirable characteristics of nonlinearity into the model and also contributes to a large computational burden. To address this drawback, we utilize the linearization method proposed by [5] and recently demonstrated by [9]

and [10] to transform the absolute term into a linear form. In the following, θ_ε represents the deviation for violations of the mean in the robust optimization model with solution robustness.

$$Min\, Z = c^T x + \mu + \lambda \sum_{\varepsilon \in \Xi} \pi_\varepsilon \left(d^\varepsilon + 2\theta_\varepsilon \right) \tag{2.26}$$

Subject to (2.16) to (2.18)

$$d^\varepsilon + \theta_\varepsilon \geq 0 \qquad \forall \varepsilon \in \Xi \tag{2.27}$$

$$\theta_\varepsilon \geq 0 \qquad \forall \varepsilon \in \Xi \tag{2.28}$$

In the objective function (2.26), the second term μ represents the mean expected cost (i.e. second stage cost), while the third term $\lambda \sum_{\varepsilon \in \Xi} \pi_\varepsilon \left(d^\varepsilon + 2\theta_\varepsilon \right)$ defines the expected variability cost, where λ determines the severity of the variability of the objective function. Therefore, assuming no variability (i.e. $\lambda = 0$) in the objective function implies that the RO model is transformed into the two-stage stochastic recourse programming model.

In Equation (2.26), it can be seen that if $\boldsymbol{d}^\varepsilon \geq \mathbf{O}$ then $\boldsymbol{\theta}_\varepsilon = \mathbf{O}$ in the optimal solution, and thus, the objective function $Z = c^T x + \sum_{\varepsilon \in \Xi} \pi_\varepsilon \zeta_\varepsilon + \lambda \sum_{\varepsilon \in \Xi} \pi_\varepsilon d^\varepsilon$. Otherwise, if $\boldsymbol{d}^\varepsilon < \mathbf{O}$, then $\theta_\varepsilon = -d^\varepsilon$ in the optimal solution, and hence, the objective function $Z = c^T x + \sum_{\varepsilon \in \Xi} \pi_\varepsilon \zeta_\varepsilon + \lambda \sum_{\varepsilon \in \Xi} \pi_\varepsilon \left(-d^\varepsilon \right)$. As can be seen, the solution procedure of Equations 2.26 to 2.28 is the same as to Equation 2.25.

2.4.4 Proposed transformation framework for RO model with model robustness

RO can also measure the model robustness with respect to infeasibility associated with the second stage constraints. Under this situation, the violation of the second stage constrains is

allowed; however a penalty rate of ω is applied. In order to manage the infeasibility that results from the unknown input parameters under different scenarios, the variable f^{ε} is defined to capture the violation of the second stage constraints under each scenario in the proposed transformation framework. Therefore, f^{ε} represents the infeasibility of the control constraints as a result of the realization of uncertain input parameters among various scenarios. In the following, the framework for robust optimization model with model robustness is structured where the violation of the control constraints is penalized through the penalty function ω in the objective function. It should be noted that under feasibility condition the value of f^{ε} will be equal to zero, whereas, under infeasibility f^{ε} is assumed a positive value.

$$Min\, Z = c^T x + \mu + \omega \sum_{\varepsilon \in \Xi} \pi_{\varepsilon} \left| f^{\varepsilon} \right| \tag{2.29}$$

Subject to (2.16)

$$B_{\varepsilon} x + C_{\varepsilon} y_{\varepsilon} + f^{\varepsilon} = e_{\varepsilon} \qquad \forall \varepsilon \in \Xi \tag{2.30}$$

$$x, y_{\varepsilon}, \omega \geq 0 \qquad \forall \varepsilon \in \Xi \tag{2.31}$$

In the objective function (2.29), the third term $\omega \sum_{\varepsilon \in \Xi} \pi_{\varepsilon} \left| f^{\varepsilon} \right|$ represents the expected infeasibility cost, where ω is defined as a parameter to measure the penalty cost for violation of the second stage constraints. The term $\sum_{\varepsilon \in \Xi} \pi_{\varepsilon} \left| f^{\varepsilon} \right|$ measures the expected infeasibility of the control constraints, thus setting the value of $\omega = 0$ implies no penalty cost for not satisfying the second stage constraints resulting in transforming the model into the two-stage stochastic recourse programming model. Therefore, to obtain the optimal solution in the objective function, the control constraints can be violated for as much as is required. On the contrary, setting a very

large positive value for ω enforces all the second stage constraints to be satisfied due to a large penalty cost. Therefore, the RO model with a very large ω will be converted to the two-stage stochastic recourse programming model. It should be noted that assigning a sufficiently large value to ω causes the infeasibility penalty function term to dominate the objective function, thereby resulting in a higher expected value for the first stage and the second stage cost.

As noted before, we employ the linearization method developed by [5] in order to transform the absolute term in the objective function (2.29) into a linear form. A deviational variable γ^{ε} is introduced and the RO model with model robustness is reformulated as follows.

$$Min\, Z = c^T x + \mu + \omega \sum_{\varepsilon \in \Xi} \pi_{\varepsilon} \left(f^{\varepsilon} + 2\gamma^{\varepsilon} \right) \tag{2.32}$$

Subject to (2.16), (2.18), (2.30), and (2.31)

$$f^{\varepsilon} + \gamma^{\varepsilon} \geq 0 \qquad \forall \varepsilon \in \Xi \tag{2.33}$$

$$\gamma^{\varepsilon} \geq 0 \qquad \forall \varepsilon \in \Xi \tag{2.34}$$

2.4.5 Proposed transformation framework for RO model with the trade-off between solution and model robustness

The RO model can coordinate the variability and feasibility at the same time through a trade-off between solution and model robustness. The complete formulation of the robust objective function using our proposed transformation framework that includes both solution robustness and model robustness is as follows.

$$Min\, Z = c^T x + \mu + \lambda \sum_{\varepsilon \in \Xi} \pi_{\varepsilon} \left| d^{\varepsilon} \right| + \omega \sum_{\varepsilon \in \Xi} \pi_{\varepsilon} \left| f^{\varepsilon} \right| \tag{2.35}$$

In the objective function (2.35), the first term is the first stage cost, the second term is the second stage cost, the third term is the expected variability cost, and the forth term is the expected

infeasibility cost. Note that the sum of the first stage cost and the second stage cost comprises the model expected cost in stochastic recourse programming model, while the total cost consists of the sum of all the cost terms. Given the absolute term formulation to transform the objective function into the LP model, the linear RO transformation framework with the trade-off between solution and model robustness is formulated as follows.

$$Min\ Z = c^T x + \mu + \lambda \sum_{\varepsilon \in \Xi} \pi_\varepsilon \left(d^\varepsilon + 2\theta_\varepsilon \right) + \omega \sum_{\varepsilon \in \Xi} \pi_\varepsilon \left(f^\varepsilon + 2\gamma^\varepsilon \right) \tag{2.36}$$

Subject to

$$Ax = b \tag{2.37}$$

$$B_\varepsilon x + C_\varepsilon y_\varepsilon + f^\varepsilon = e_\varepsilon \qquad \forall \varepsilon \in \Xi \tag{2.38}$$

$$d^\varepsilon + \theta_\varepsilon \geq 0 \qquad \forall \varepsilon \in \Xi \tag{2.39}$$

$$f^\varepsilon + \gamma^\varepsilon \geq 0 \qquad \forall \varepsilon \in \Xi \tag{2.40}$$

$$x, y_\varepsilon, \lambda, \omega, \theta_\varepsilon, \gamma^\varepsilon \geq 0 \qquad \forall \varepsilon \in \Xi \tag{2.41}$$

To demonstrate the applicability of the proposed formulation and to provide insight into the structure of the proposed transformation framework, a surgical block allocation problem and an integrated master surgery schedule with surgical case assignment problem of a real-life healthcare delivery system is solved in chapter 4 and 5, respectively. To the best of our knowledge, this is the first time that our proposed RO approach is applied in the context of the healthcare planning and scheduling problems. Through the case study, we demonstrate that our approach outweighs the SP method. It is also shown that the proposed RO framework works more effective than the RO model presented by Yu and Li [5] due to requiring less information about the original deterministic problem while at the same time provides more flexibility for the decision makers in utilizing the formulation.

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Chapter 3

A Novel Robust Optimization Transformation Framework to Operating Room Case Mix and Capacity Planning under Uncertainty at a Publicly-Funded Hospital[1]

3.1 Introduction

Operation management at healthcare facilities is a wide area of knowledge in which conflicting objectives such as cost reduction and capacity expansion are normally against enhancing service levels and patients' satisfaction. Effective management of surgical resources, which is referred to as operating room planning (ORP), draws considerable attention from the healthcare community to reduce costs and increase revenues [1]. The ORP is a well-established literature in which different aspects of the healthcare decision makers' perspective have been studied [2]. Surgery

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capacity allocation problem is amongst the most challenging decision within the ORP that directly impacts the structural planning of patients' demand at the strategic level. In most medium to large-scale healthcare systems, surgery capacity allocation decisions are planned without considering the variability in patient arrivals. Failing to consider variable and uncertain surgery demands can rigorously degrade the hospital's performance in the real-life situation and causing extra operational costs. This study is concentrated with operating room (OR) allocation planning for multiple surgical specialties of a healthcare system on a given day, where possible mixtures of elective and emergency patients require simultaneously various surgery teams and OR blocks. Since patient arrivals are realized under uncertain circumstances, random characteristics in terms of arrival time will be observed in surgical demands of different specialties. In the OR block allocation problems, we look for the optimal combination of OR blocks from each type as well as surgical specialties that best meet unknown surgery demands. Inefficient block allocation plans could cause rescheduling of elective patients which increases patient anxiety or prolongs a patient's length of stay (LOS) before a surgery is operated. Rescheduling could also impose extra costs to the healthcare systems. Surgeries that cannot be operated as planned due to clinical resource constraints and/or the uncertain patient arrivals are either postponed to the next available time within the planning period by incurring a non-reimbursable postponement cost or become unmet and hence incur huge penalty costs. The surgery allocation is subject to the availability of alternative operating rooms in which the surgical procedures can be carried out as well as the size, fixtures, and personal requirements. The uncertainty in the decision variables of the block allocation problems is realized through the postponed demand, rescheduled surgery, and underutilized OR blocks. The objective is to minimize the initial cost of block allocation schedule and also the costs incurred by postponed/unmet surgery demands and underutilization costs of idleness of operating room hours regarding fulfillment of elective and emergency surgery demands that alter on a discrete set of scenarios. Consequently, incorporating the uncertain surgery demands into the OR block allocation problem has a significant benefit in obtaining a

robust allocation plan that minimizes operating costs and reduces service level variability as the ultimate goal of the hospital managements.

We address the allocation of ORs' capacity to surgical specialties over a finite planning horizon. We focus on two major arrival channels, namely elective and emergency patients, to allocate ORs to the specialties. While surgery operations are usually planned early in advance for the former class, treatments are carried out within an urgent situation and consequently emergency for the latter case. The allocation plan of OR block times turn out to be more complex when variety of surgical specialties and the combination of both scheduled, i.e. elective and unscheduled, i.e. emergency procedures are taken into account [3].

Surgery block allocation problems can be seen as a combination of several typical surgery planning and scheduling problems where different modeling approaches have been adapted in the literature to incorporate decision making into the modeling framework. A classical surgery planning is formulated by a deterministic mixed-integer programming (MIP) model to allocate block times of operating room capacity to various specialties' emergency and non-emergency surgery [1], [4], [5]. Blake et al. [6] proposed a cyclic timetable to control the shortage of OR capacity in a surgical unit through a MIP model. Santibanez et al. [3] provided a MIP model for the allocation of surgical blocks of elective patients under deterministic demands to determine the assignment of operating theater blocks to surgical specialties. Emergency demands are assumed to be served after the completion of elective cases through a dedicated operating room. Jebali et al. [7] developed a two-step MIP model to solve the assignment and sequencing problem of surgical operations for elective cases. Doulabi et al. [8] applied a constructive heuristic algorithm to develop an integrated operating room planning and scheduling framework to synchronize the assignment and scheduling of the surgeons. Patient arrivals were realized through a single channel of elective patients. Cardoen et al. [9] considered scheduling of prioritized patients through a multi-objective healthcare decision making process using the

branch-and-price approach. Various restricting criteria such as ORs, medical instruments, and recovering areas availability have been taken into account in their study. Fei et al. [10] and Fei et al. [11] addressed surgical case assignment problems of operating rooms using mathematical models while they proposed the use of different heuristic approaches as solution methods. An open scheduling strategy is incorporated where surgical operations are scheduled to the ORs in any available workday over the planning horizon. Despite the significant applications of mathematical models in capacity allocation problems, there would be a high risk of unsatisfied surgery demands and/or underutilized operating room hours for the hospitals when capacity decisions are being made under the deterministic environment. That arises from the inability of deterministic models to deal with variable input data. Therefore, deterministic models could result in an increased non-reimbursable costs and degraded service level due to the incurred postponed/rescheduled surgery demands.

Another stream to which the surgical block allocation problems is expanded in the literature is concentrated on the stochastic planning and scheduling models using the stochastic programming (SP) approaches [12]–[15]. A set of explicit probability distributions is constructed in this method to take the stochastic characteristics of unknown parameters into account, however, some unrealistic assumptions are assumed in the literature related to the delayed incorporation of the emergency admissions to the limited ORs in addressing emergency patients [12]; reserved resource capacities exclusively utilized by emergency surgeries [14], [16]; delayed positioning of high variance surgical operations in the schedule while patient arrivals are known [13]; assumed single class of patients in allocation of uncertain surgical block capacity [15]; and dedicated OR blocks to allocate to the surgical specialties [17] that makes the optimal solutions of most of the SP optimization problems not capable to solve the real-life healthcare issues.

Belien et al. [18] developed an integrated cyclic master surgery schedule (MSS) to model a leveled bed post-surgical schedule with the combination of MIP heuristic and metaheuristics to

control the variance of bed shortage in each day under uncertainty of both surgical durations and patient lengths of stay. However, they only focus on elective patient demands. [14] developed a SP model to study the influence of uncertain emergency surgery in surgical scheduling problem when OR capacity is shared between two competing patient classes comprised of emergency and elective patients to minimize both patient costs and overtime costs. A Monte Carlo simulation method is proposed to capture the uncertainty of emergency demand. In their work, the definition of patient costs is only limited to the surgery durations. A MIP model is developed by Min and Yih [15] that generates cyclic MSS to obtain the optimal surgery plans where both surgery durations and availability of downstream resources are uncertain parameters. A sample average approximation (SAA) algorithm is used that minimizes both patient and overtime costs, however, elective demand is the only uncertain factor considered in the problem. Denton et al. [19] proposed a two-stage stochastic MIP model for a surgery sequencing problem under uncertain surgery durations. An L-shape method and SAA algorithm are utilized to trade-off the impact of scheduling start times and waiting time of the surgical cases within a planning horizon. Erdem et al. [20] studied the impact of the uncertain emergency patient arrivals on the scheduling of elective cases to reduce the disruption costs incurred due to the adoption of elective surgeries using a genetic algorithm. The inter-arrival times are assumed to be fixed. Erdogan and Denton [21] formulated a stochastic dynamic programming model to solve an appointment scheduling problem of healthcare systems under stochastic service durations and the number of patients. Several decomposition algorithms are adapted to solve the formulated multistage stochastic linear program.

Indeed, SP is a major stream to address uncertainty associated with surgical treatment demands and surgery durations through the years. Typically, the goal in the stochastic programming approaches is to optimize the expected objective function over a range of possible scenarios for the random parameters. However, several shortcomings for such an approach exist:

(1) the method assumes that exact distributions of the uncertain data are available, however, this assumption is rarely met in practical situations of healthcare systems. (2) the behavior of the systems at some particular realization of scenarios such as the worst case scenario is ignored in SP approach. More specifically, for some scenarios unrealizable postponed demand or idleness of OR capacity might be observed by implementing the solution developed by the stochastic models, and (3) the size of the resulting optimization model extensively increases as a function of the number of scenarios which leads to the substantial computational difficulties [22]. Despite considering the stochastic characteristics of input parameters in the above research work, literature is still limited owing to the unavailability of dedicated OR blocks in the midsize healthcare systems and unrealistic assumption of devoting the OR capacity to a single class of patients. Moreover, assuming elective surgeries as the only source of demand uncertainty and surgery costs as a function of surgery duration is quite unrealistic in real healthcare systems. Furthermore, the delay in positioning of the highly variable demands can negatively impact the postponement and rescheduling of surgery cases in the hospitals. Finally, the variability of the expected cost in the objective function is not incorporated into the SP method. Robust optimization has the benefit of limiting the shortcoming of the SP method.

Robust optimization (RO) is an alternative approach to stochastic linear programming methods which applies probabilistic input data. Using a scenario-based approach in which random variables take on specified values in each scenario, the RO solution is progressively less sensitive and more flexible approach to the realization of stochastic variables. However, the application of RO in solving surgical planning and scheduling problems is very limited [23]. Denton et al. [13] presented a RO model to solve OR capacity allocation problems that minimize the maximum cost associated with an uncertain set of surgery durations. It is demonstrated that RO outperforms the SP approach in finding the optimal allocation of surgery block times. In Addis et al. [24] a surgical assignment problem of a set of elective patients to the operating room

blocks is addressed with the random surgery durations that minimize weighted costs of patients waiting time. Mannino et al. [25] solved a MSS problem that finds a robust solution approach for the allocation of surgical resources to a set of surgical groups in order to find a leveled patients queue lengths among different specialties that minimize total overtime costs. Holte and Mannino [26] a RO model is developed to address a combined surgery block allocation and a MSS problem of a real-life example from a large hospital that minimizes the patients waiting time where a single channel of elective surgery is the only source of uncertainty. Tang and Wang [27] proposed an adjustable RO model to address the surgery capacity allocation problems with demand uncertainty where the OR capacity is shared between integrated subspecialties. The uncertain emergency arrivals are allocated into the reserved OR capacities in order to minimize the revenue loss resulted from the lack of resources. Although several authors have contributed on the OR management literature using RO approach, most existing works focus on specific aspects of the OR planning and scheduling problem that consider certain constraints and or assumptions in order to reduce the complexity of the problem, such as devoting the OR capacity to a single class of patients. In Yu and Li [28] the solutions developed by the RO model are progressively less sensitive to the realization of data in a scenario sets. However, the complexity of developing the robust counterpart of an integer linear programming model is deemed a huge barrier that restricts the implementation of the RO technique in healthcare optimizations [27]. Therefore, development of a standard framework that coordinates the transformation of deterministic models into the robust optimization forms is of a great value.

In Table 3.1 we have classified the most recent contributions on OR planning and scheduling and have characterized for all these contributions the type of criteria that is taken into account, various aspect of surgery demand, as well as the modelling approach, decision types, objective functions, and solution approach in order to position our work in the growing OR

planning literature. It also provides a means to position our method in the huge state of the art of the existing models.

Table 3.1: Summary of the main literature on OR planning and scheduling

	Type of problem				Criteria					Stochastic aspects			Optimization Technique					R-l
Paper	Pln	Sch	Co	Cap	S A	Ov	Or U	Pos	Un S	El	Em	MIP	Sim	Mth	RO	Hu		
Addis et al. (2015)	√					√								√	√			
Beliën and Demeulemeester (2007)		√		√						√		√				√		
Cardoen et al. (2009)		√	√	√		√						√		√		√	√	
Cardoen et al. (2009b)		√	√	√		√						√		√		√	√	
Choi and Wilhelm (2014)	√		√	√			√		√			√		√				
Denton et al. (2007)		√	√			√	√					√	√			√	√	
Erdem et al. (2012)		√		√	√	√		√	√		√	√				√		
Fei et al. (2008)	√		√	√		√								√		√	√	
Fei et al. (2009)	√		√	√		√	√							√		√		
Jebali et al. (2006)	√	√	√	√	√	√								√				
Lamiri et al. (2008)	√		√	√		√	√				√	√	√					
Min and Yih (2010)		√	√	√		√			√			√	√			√		
Persson and Persson (2010)	√		√			√			√	√		√	√					√
Santibáñez et al. (2007)		√		√	√							√						√
Tang and Wang (2015)		√	√	√						√	√				√			√
Testi and Tànfani (2009)	√	√				√			√				√	√				√
Zhang et al. (2009)	√		√	√	√		√	√	√			√	√					√
Our research	√		√	√	√	√	√	√	√	√	√	√			√			√

Constraints typically modelled in the literature: Pln - planning; Sch - scheduling; Co - cost; Cap - capacity; S A - surgeon's availability; Ov - overtime; Or U - OR utilization; Pos - surgery postponement; Un S - unmet surgery; El - elective demand; Em - emergency demand; MIP - mixed integer programming; Sim - simulation; Mth - mathematical programming; RO - robust optimization; Hu - Heuristics; R-l - real-life case

Similar to Zhang et al. [4], Erdem et al. [20] and Tang and Wang [27] our work also focuses on allocating surgical resource capacities to the specialties in order to address the surgery needs of both elective and emergency patients. Characteristics of this problem are comprehensive to the existing methods in order to bridge the gap in various aspects of surgery demand as well as modelling and solution approaches. We develop a two-stage stochastic recourse programming

model as well as a robust optimization model that solve the stochastic surgery capacity allocation problems while considering random characteristics of patient arrivals to the hospitals. The main differences between our work and the previous works on surgery allocation with RO are as follows. (1) In addition to a single class of patient, the demand allocation plan is considered for both elective and emergency surgeries. (2) The quality of service for elective patients is introduced as a function of the surgery postponements to manage the service level in the hospitals. The random demands are modeled as scenarios with a set of discrete probability distributions along the planning horizon. The surgery operating costs are defined non-reimbursable if a proportion of surgery demand of a surgical specialty cannot be fulfilled on time. Therefore, depending on the way a surgery demand is accommodated in hospitals, the postponement or rescheduling costs is incurred. The underutilization cost, on the other hand, is incurred if the under-usage of OR capacity is realized in the hospitals. Thus, the service level is evaluated through measuring the random postponed/rescheduled demand as well as the underutilized OR capacity. Therefore, the importance of robustness is mainly recognized in terms of determining a robust surgery allocation plan by minimizing the variability in the amount of postponed/rescheduled demand sizes and the underutilized OR capacities over different scenarios for random elective and emergency surgery demands. The robustness in the developed allocation surgery plans directly reflects the risk aversion levels of the decision makers about the variability of random yields and the total costs. A real case healthcare system is used to apply the proposed RO model. The resulted combinatorial programming models are conducted on AMPL optimization software and solved by CPLEX 12 in a reasonable amount of time. The results of the two-stage stochastic programming and the robust optimization models are evaluated to provide a comparison between the variability of output measures and infeasibility of the second stage constraints. Finally, a trade-off between the variability of the performance measures and the expected total costs is performed to acquire managerial insights on the optimal allocation plans.

The main contributions of this paper can be summarized as follows. First, we propose a novel standard formulation framework to transform a deterministic capacity allocation models into the stochastic robust optimization model for the case when random variables are represented by a discrete set of independent scenarios. We improve the efficiency of the RO model by adding dominance rules to decrease the size of the problem. Our results demonstrate that the proposed RO framework significantly outperform the conventional RO model, obtaining the average optimality gap of 1.50%. We also compare our approach with a state-of-the-art stochastic recourse programming model for a novel surgery capacity allocation problem. Second, we employ RO approach as a modeling tool for a surgery block allocation problem. The robust model we develop represents considerations of the healthcare environments that have not yet been fully studied related to the uncertain parameters and the objective function. Moreover, it is shown that the problem of surgery capacity allocation is essentially a two-stage stochastic recourse problem. Consequently, it is suggested to solve the problem using the proposed framework to compare the performance of different robust models in controlling the variability and infeasibility of allocation plans with the stochastic recourse model. We also propose a trade-off function that balances the risk aversion levels of the decision makers about service level robustness and the total expected costs of the plans. Finally, we apply our framework to solve real-life instances provided by the Windsor Regional Hospital (WRH), a major hospital in Southwestern Ontario and present insight into the optimal OR capacity allocation policies in the presence of uncertain elective and emergency surgery demands.

The remainder of this chapter is structured as follows. In section 3.2, the surgery capacity allocation problem is described and its specific characteristics are introduced to build a general model aiming at minimizing the total loss resulting from the shortage of allocated resources. In section 3.3, a standard transformation framework for the RO models is proposed and the structural details of different robust models are elaborated. In section 3.4, the RO formulation for

the random surgery block allocation problems is presented using the proposed standard transformation framework. In Section 3.5, the computational results of implementing the presented RO models for a local healthcare system are provided. Moreover, a numerical analysis is conducted to evaluate the significance of the proposed robust models compared with the corresponding two-stage stochastic recourse programming model. Lastly, concluding remarks together with some outlines for future research are described in section 3.6.

3.2 Surgery capacity allocation process and specific characteristics

Surgery departments are amongst the most cost driven bottleneck in the healthcare systems while at the same time they contribute to the significant portion of the hospital total revenues [7], [13], [15], [19], [31]. The primary goal of the OR capacity allocation models is to minimize the total fixed and variable costs associated with the overall daily assignment while maintaining the required service level at the health systems. Effective OR management helps improve resource efficiency and increase the number of patients served by specialties which results in reduced postponement and patients waiting times, minimized surgery cancelations, and enhanced OR utilizations and thus the overall performance of the healthcare providers. Therefore, an efficient OR management is an inevitable key to success of hospital operations.

In most healthcare systems in North America, surgery planning and scheduling is carried out through a dedicated unit in the hospitals that builds the OR allocation templates for the operating rooms called block time schedule, where each block contains one day of staffed hours of an operating room [4]. The allocated OR capacity is then shared between various subspecialties of a surgical specialty in the hospitals. Surgeons within each specialty then determine the admission order of elective patients to their corresponding ORs. Due to the critical condition of emergency cases, emergency patients are operated as soon as their demand arises in the hospitals. Emergency cases are normally allocated to the dedicated emergency rooms, however, in case of unavailability of a dedicated OR the emergency patients are sent to the non-

emergency rooms that have already been assigned to a particular specialty which could result in some scheduled elective surgeries to be postponed to a later date or be rescheduled. Due to the resource shortages, the serviceability of the healthcare units cannot fulfil the uncertain demand arises from different surgery channels. Therefore, when the available OR capacity is saturated, surgical departments have to first postpone and then reschedule those patients that cannot be admitted into another local hospital, which leads to a revenue loss due to a non-reimbursable service cost. The goal of this model is to bridge the gap between the number of admitted and scheduled patients on each working days over the planning horizon.

In Litvak and Long [32], hospital managers normally face two different types of variations in hospitals, including natural variability which is inherent to the uncertainties associated with surgical durations, patients mix and arrivals, length of stay, and other factors, and artificial variability that originates from poor scheduling policy which leads to a longer waiting time and a lower service level. Following Tang and Wang [27], we assume that uncertain surgery demands for specialties are represented as random sets with unknown distributions for both elective and emergency surgeries that remain stationary over the planning horizon. The empirical upper bound and lower bound for the elective and emergency demand of each specialty is given based on the available historical data.

The surgery theater is assumed to comprise of multiple identical ORs that are shared between surgical specialties to perform surgeries. There exists a subset of ORs with exceptional equipment in the hospitals to cover the need of specialties that require special type of medical resources. The cost parameters related to running each subset of OR may vary from those associated with others due to the variability of the installed equipment. The available OR hours are influenced by the variability involved in the number of staffed OR and/or availability of surgeons. Hospitals are obliged to follow a governmental protocol that determines the maximum allowed waiting times for a surgery in order to be reimbursed for the operated services. The

availability of the operating rooms directly affects the number of patients receiving surgical services in the hospitals and as a result can directly impact the patient waiting times. The desired service level for the healthcare providers is also influenced by that waiting factor, and hence, is incorporated in our decision making framework. It is assumed that multiple ORs can be allocated to each specialty on a given day in order to accommodate the volatile demands and reduce the patient waiting list.

As the OR capacity allocation process is strongly characterized by unknown input parameters, we incorporate the impact of uncertain surgery demands into the model. Due to the urgent nature of the emergency cases, we assume a dedicated OR initially fulfills the emergency patient needs on each day of the planning horizon. If the emergency demand of a surgical specialty exceeds its reserved OR time, that specialty will be given a priority to be allocated to a non-emergency OR. Therefore, we consider a hybrid allocation policy in this study to accommodate the need of emergency surgeries where emergency cases are first allocated to the emergency room and then assigned to the non-emergency ORs that are shared with the elective patients. Although the real-time adjustment of the allocated operating room plans is possible, it may cause confusion among surgeons of different specialties when conflicting requirements for medical equipment or prior preparations exist. The effectiveness of any block allocation template is crucial to the robustness of the operational performance measures. Therefore, an effective robust OR capacity allocation plan is developed that efficiently allocates operating room capacities to various medical specialties where stochastic elective and emergency surgery demands are fulfilled with the aim of reducing expected loss incurred to the hospital due to the increased length of stay, rescheduled patients, and underutilized OR resources.

3.3. Proposed standard transformation framework for the robust optimization models

In recent years, the critical role of a decision maker in dealing with the real-world problems is heavily integrated with the way the marketing requirement is forecasted. The optimal

solution of most of the optimization problems is not capable to solve the real-life problems if it is obtained under the noisy information environments. Although LP has been widely applied to many optimization problems as an easy-to-implement tool, its unrealistic deterministic assumptions contradict with the real-world data, and hence, creates a huge barrier in dealing with uncertain and incomplete information of today's problems.

RO was initially developed by Mulvey et al. [33] as a proactive means of dealing with probabilistic information and in response to the limitation of LP models to absorb the effect of uncertainties in real-life optimization problems. In RO, new terminologies are defined to classify the desirable properties of the optimal solutions to the model as "solution robustness" and "model robustness" to not only reduce the variability of the objective value but to diminish the infeasibility of the formulation for any realization of the stochastic parameters. A solution to an optimal model is defined as solution robust if it remains 'close' to optimal for all input data scenarios, while the model is called robust if it remains 'almost' feasible for all data scenarios. A detailed explanations on the conceptual meanings and advantages of the robust planning is provided in Van Lendeghem and Vanmaele [34].

RO developed by Mulvey et al. [33] and Mulvey and Ruszczynski [35] is a fairly new concept that handles the trade-off associated with the expected cost and its variability in SPs. It is constructed based on the integration of goal programming formulation with the scenario-based description of problem data to take the various decision makers' risk aversions into account. RO is a proactive approach to mathematical programming that produces allocation plans which are less sensitive to the variability of unknown data. RO can enhance the agility of the healthcare systems to respond to the variability in demand when the decisions that must be made before the realization of the incomplete data is known. Therefore, RO helps to hedge the risk variability while maintaining the service level through providing a direct trade-off between the risks and the total costs in the healthcare systems. In RO approach a two-stage programming method is

employed in which the violation of control constraints is penalized in the objective function through a defined penalty function. Therefore, RO seeks to balance between solution and model robustness, and hence extends the stochastic linear programming by including higher moments of variability in the objective function and allowing for infeasibilities.

As elaborately described in section 3.2, transformation of a deterministic model into the RO form can be very complicated and at the same time lengthy process that is seen as a barrier to using RO as a progressive tool to tackle with uncertainty in solving optimization problems. The apparent lack of a transformation framework to formulate the robust model from its deterministic counterpart is seen a limitation in the operation research literature. The aim of this study is to provide a generalizable RO transformation framework for use as a tool in the context of operation research in order to generate the robust part of the deterministic models. Such a framework can assist decision makers in solving complex optimization problem through providing an instructional guideline that makes the transformation process more effective and at the same time easier to implement. The proposed framework also reduces the formulation burden which has always been an obstacle to application of RO in solving operation research problems [28].

In light of the above discussion, the proposed novel transformation framework is constructed based on the RO model developed by Mulvey and Vanderbei [36]. The proposed RO framework outperform the conventional RO model through enhanced computational efficiency (i.e. lower CPU time) as well as the utilized linearization approach (i.e. simplified model development). It also addresses the drawback of their model owing to its difficulty in obtaining information regarding numerous control variables and constraints. A general two-stage stochastic recourse programming model is first developed to incorporate demand uncertainty in the capacity allocation planning problem. Three different robust models are then projected using the proposed transformation framework to highlight the capability of the RO model in dealing with variability in stochastic environments. The proposed formulation enables adjusting the model in response to

changes in input data through incorporation of the variability of the objective function into the formulation. It can be generally used as a standard framework to transform any linear deterministic model into the stochastic robust form. The template transformation framework is then fed into a surgery capacity allocation case of a healthcare problem that captures the randomness of the actual process in order to evaluate the effectiveness of the proposed framework on a realistic model and to demonstrate the applicability of the formulation. It is illustrated through the formulation that the proposed transformation framework is more practical to use than the method developed by Mulvey and Vanderbei [36]. Furthermore, the computational results confirm that the framework presented herein generates a robust allocation plan in a timely manner without requiring addition of any additional deviation variables.

3.3.1 Two-stage stochastic recourse programming model

A general two-stage stochastic recourse programming model can be formulated as follows:

$$Min\, c_1^T x + \sum_{\varepsilon \in \Xi} \pi_\varepsilon c_{2\varepsilon}^T y_\varepsilon \qquad \forall x \in R^{n1}, y \in R^{n2} \qquad (3.1)$$

Subject to

$$Ax = b \qquad (3.2)$$

$$B_\varepsilon x + C_\varepsilon y_\varepsilon = e_\varepsilon \qquad \forall \varepsilon \in \Xi \qquad (3.3)$$

$$x, y_\varepsilon \geq 0 \qquad \forall \varepsilon \in \Xi \qquad (3.4)$$

In the objective function (3.1), x denotes the vector of first stage (i.e. design) variables whose optimal value is determined before complete information of uncertain parameters is observed, while y_ε denotes the second stage (i.e. control) variables corresponding to realization of the scenario ε where the decisions are subject to adjustment when the realization of the stochastic parameters is observed. Therefore, the sum of the first stage costs and the second stage costs in the objective function represents the total expected costs of the stochastic recourse programing model. Under constraints, B_ε and e_ε represent random coefficient matrix and right-hand side vector, respectively. C_ε and $c_{2\varepsilon}$ denote the recourse matrix and the penalty recourse cost vector

corresponding to scenario ε , respectively. The equations (3.2) and (3.3) are categorized the constraints in the stochastic recourse model as the first stage constraints and the second stage constraints, respectively, where the former only involves the first stage variables, while the latter contains both first stage and second stage variables. It should be noted that in the objective function (3.1), the stochastic entity of the stochastic variables is indicated by "ε", and π_ε denotes the probability of the realization of the stochastic variables. The optimal solution of the model (3.1) to (3.4) is feasible for all data that belong to a convex set of scenario ε . Therefore, the produced solution fails to find a trade-off between optimality and robustness of the optimum solutions.

3.3.2 Robust optimization transformation framework for two-stage stochastic recourse program

In order to absorb the impact of having different values for the uncertain input parameters, RO model proposed by [33] is presented here to modify the objective in SP as follows.

$$Min\ c_1^T x + \sum_{\varepsilon \in \Xi} \pi_\varepsilon c_{2\varepsilon}^T y_\varepsilon + \lambda\sigma\left(x, y_1, ..., y_\varepsilon\right) + \omega\rho\left(\delta_1, ..., \delta_\varepsilon\right) \qquad \forall x \in R^{n1}, y \in R^{n2} \qquad (3.5)$$

Subject to

$$Ax = b \qquad (3.6)$$

$$B_\varepsilon x + C_\varepsilon y + \delta_\varepsilon = e_\varepsilon \qquad \forall \varepsilon \in \Xi \qquad (3.7)$$

$$x, y \geq 0 \qquad (3.8)$$

The term $\sum_{\varepsilon \in \Xi} \pi_\varepsilon c_{2\varepsilon}^T y_\varepsilon + \lambda\sigma\left(x, y_1, ..., y_\varepsilon\right)$ in the objective function denotes the solution robustness measure, where $\{y_1, y_2, ..., y_\varepsilon\}$ is introduced as a set of vectors containing the control variables and λ is intended as a measurement of the variability of the objective function. The term $\omega\rho\left(\delta_1, ..., \delta_\varepsilon\right)$ is a feasibility penalty function which is used to measure the violation of the second stage constraints where ω is a penalty weight that is used to define the desired trade-off between solution and model robustness. The term $\left(\delta_1, ..., \delta_\varepsilon\right)$ in the objective function

contains the error vectors that measure the allowed infeasibility in the control constraints (3.7) under scenario ε. Using the value of the realized error vectors, the feasibility penalty function penalizes violations of the control constraints under some of the scenarios. Equation (3.6) is the structural constraints with fixed and free of noise coefficients, whilst equation (3.7) indicates the control constraints of which the coefficients are under the influence of noisy data. The realizations of the coefficients of the control constraints for each scenario ε comprises of a set of $\{c_{2\varepsilon}, B_{\varepsilon}, C_{\varepsilon}, e_{\varepsilon}\}$. Equation (3.8) guarantees non-negative vector of the decision variables. It should be noted that the RO overcomes the challenge of finding a solution that remains both feasible and optimal to all input scenarios through a trade-off between model robustness and solution robustness by applying the concept of multi-criteria decision making procedure.

The framework proposed in this work takes into account the uncertainty of incorporating unknown parameters into the model. Here we define the term μ to measure the mean value of the objective function under uncertainty which also provides an expected second stage cost. In order to incorporate risk aversion capability into the RO model, we penalize the deviation for violation of the mean expected cost, both above and below the expected recourse cost through a deviation function represented by d^{ε}. Since variance is a systematic risk measure, penalizing the deviation from the mean provides more flexibility for decision makers regarding the degree of risk aversion that they prefer to take for a given situation. The actual deviation from the mean is penalized via the weight θ^{ε}. We employ a mean absolute deviation approach to make sure the impact of deviation from the mean is incorporated into the model. The next terminology we use herein is represented by f^{ε}, where it denotes an equation of the control constraints. This equation provides some flexibility for the decision makers to appropriately measure the permitted infeasibility in the control constraints. The control constraint enables the model capable to adjust the impact of changes in variable input data under different scenarios if it violates the condition of the

constraints through a penalty function, γ^{ε}, in order to penalize the incurred infeasibility. Using the values of the realized error vectors, this function penalizes violations of the control constraints under some of the scenarios. In other words, the term f^{ε} allows the model to handle scenarios in which realizations of the uncertain parameters would otherwise not be allowed for a feasible solution, although with an associated penalty weight ω for the violation of the control constraints.

Considering the assumptions of the defined standard transformation framework, robust optimization model can be reformulated using our proposed transformation framework to measure the decision maker's risk preferences through the expected variability of the objective function and also provides a degree of trade-off between that risk and the cost of infeasibility of the second stage constraints.

3.3.2.1 Proposed transformation framework for the RO model with solution robustness

In realistic optimization problem, a high level of risk might be associated with one or more of the uncertain input parameters (e.g., operating rooms availability, surgery demand variability, or unpredictability of surgery duration). However, when $\sigma(.)=\sum_{\varepsilon\in\Xi}\pi_{\varepsilon}c_{2\varepsilon}^{T}y_{\varepsilon}$ is used as a second stage cost in the objective function of the robust formulation (3.5-3.8), the model only seeks to minimize the expected value of the overall cost across all possible scenarios. In other words, the potential variability in the cost functions associated with the high risk parameters across different scenarios is not taken into account. In fact, the variability that exists in the input parameters makes the transformation procedure more complicated. To provide an objective function with solution robustness that generates the optimal solutions that are less variable and are not altering substantially across different scenarios, we utilize the mean-variance approach proposed by [33] as a technique to mitigate the risk associated with the uncertain input parameters. Hence, the revised cost function consists of the expected value of the random variable plus a constant multiplying the variance. We define ζ to represent $f(x,y)$, which is a cost

function, and as a result, $\zeta_\varepsilon = f(x, y_\varepsilon)$ for scenario ε. The variable, d^ε, is defined to capture the variance of the expected cost functions to the original RO model. Therefore, the new cost function portion of the objective function that manages the risks and variability of solutions among different scenarios is formulated as follows.

$$\mu = \sum_{\varepsilon \in \Xi} \pi_\varepsilon \zeta_\varepsilon \qquad \forall \varepsilon \in \Xi \qquad (3.9)$$

$$d^\varepsilon = \zeta_\varepsilon - \mu \qquad \forall \varepsilon \in \Xi \qquad (3.10)$$

$$\sigma(x, y_1, ..., y_\varepsilon) = c^T x + \mu + \lambda \sum_{\varepsilon \in \Xi} \pi_\varepsilon \left| d^\varepsilon \right| \qquad (3.11)$$

Clearly, as the value of λ, which is a weighting factor to trade-off between risk and expected outcome for the solution robustness, is increased the solution becomes less sensitive to the changes in the input data as defined by the scenarios. If a solution is a high risk decision, any small change in the value of uncertain parameters can cause a huge difference in the value of the measure function. It is noted that this inclusion of the weighted variance term enables RO to account for the decision maker's preferences towards risk. Thus, proposed RO model allows for a more passive management style, giving it a distinct advantage over the stochastic recourse programming. In other words, with variability under control, minimal adjustment to the control variables will be required when the weighted variance version of RO is applied. While Equation (3.11) accounts for both expected cost and cost variability, the absolute term introduces the undesirable characteristics of nonlinearity into the model and also contributes to a large computational burden. To address this drawback, we utilize the linearization method proposed by [28] and recently demonstrated by [37] and [38] to transform the absolute term into a linear form. In the following, θ_ε represents the deviation for violations of the mean in the robust optimization model with solution robustness.

$$Min\ Z = c^T x + \mu + \lambda \sum_{\varepsilon \in \Xi} \pi_\varepsilon \left(d^\varepsilon + 2\theta_\varepsilon \right) \qquad (3.12)$$

Subject to (3.2) to (3.4)

$$d^{\varepsilon} + \theta_{\varepsilon} \geq 0 \qquad \forall \varepsilon \in \Xi \qquad (3.13)$$

$$\theta_{\varepsilon} \geq 0 \qquad \forall \varepsilon \in \Xi \qquad (3.14)$$

In the objective function (3.12), the second term μ represents the mean expected cost (i.e. second stage cost), while the third term $\lambda \sum_{\varepsilon \in \Xi} \pi_{\varepsilon} \left(d^{\varepsilon} + 2\theta_{\varepsilon} \right)$ defines the expected variability cost, where λ determines the severity of the variability of the objective function. Therefore, assuming no variability (i.e. $\lambda = 0$) in the objective function implies that the RO model is transformed into the two-stage stochastic recourse programming model.

In Equation (3.12), it is notable that that if $d^{\varepsilon} \geq 0$ then $\theta_{\varepsilon} = 0$ in the optimal solution, and thus, the objective function $Z = c^T x + \sum_{\varepsilon \in \Xi} \pi_{\varepsilon} \zeta_{\varepsilon} + \lambda \sum_{\varepsilon \in \Xi} \pi_{\varepsilon} d^{\varepsilon}$. Otherwise, if $d^{\varepsilon} < 0$, then $\theta_{\varepsilon} = -d^{\varepsilon}$ in the optimal solution, and hence, the objective function $Z = c^T x + \sum_{\varepsilon \in \Xi} \pi_{\varepsilon} \zeta_{\varepsilon} + \lambda \sum_{\varepsilon \in \Xi} \pi_{\varepsilon} \left(-d^{\varepsilon} \right)$. As can be seen, the solution procedure of Equations 3.12 to 3.14 is the same as to Equation 3.11.

3.3.2.2 Proposed transformation framework for RO model with model robustness

RO can also measure the model robustness with respect to infeasibility associated with the second stage constraints. Under this situation, the violation of the second stage constrains is allowed; however at a penalty rate of ω. In order to manage the infeasibility resulted from the unknown input parameters under different scenarios, the variable f^{ε} is defined to capture the violation of the second stage constraints under each scenario in the proposed transformation framework. Therefore, f^{ε} represents the infeasibility of the control constraints as a result of the realization of uncertain input parameters among various scenarios. In the following, the framework for robust optimization model with model robustness is structured where the violation of the control constraints is penalized through the penalty function ω in the objective function. It

should be noted that under feasibility condition the value of f^{ε} will be equal to zero, whereas, under infeasibility f^{ε} is incurred a positive value.

$$Min\ Z = c^T x + \mu + \omega \sum_{\varepsilon \in \Xi} \pi_{\varepsilon} \left| f^{\varepsilon} \right| \tag{3.15}$$

Subject to (3.2)

$$B_{\varepsilon} x + C_{\varepsilon} y_{\varepsilon} + f^{\varepsilon} = e_{\varepsilon} \qquad \forall \varepsilon \in \Xi \tag{3.16}$$

$$x, y_{\varepsilon}, \omega \geq 0 \qquad \forall \varepsilon \in \Xi \tag{3.17}$$

In the objective function (3.15), the third term $\omega \sum_{\varepsilon \in \Xi} \pi_{\varepsilon} \left| f^{\varepsilon} \right|$ represents the expected infeasibility cost, where ω is defined as a parameter to measure the penalty cost for violation of the second stage constraints. The term $\sum_{\varepsilon \in \Xi} \pi_{\varepsilon} \left| f^{\varepsilon} \right|$ measures the expected infeasibility of the control constraints, thus setting up the value of $\omega = 0$ implies no penalty cost for not satisfying the second stage constraints resulting in transforming the model into the two-stage stochastic recourse programming model. Therefore, to obtain the optimal solution in the objective function, the control constraints can be violated for as much as it requires. On the contrary, setting up a very large positive value for ω enforces all the second stage constraints to be satisfied due to a large penalty cost. Therefore, the RO model with $\omega \cong \infty$ will be converted to the two-stage stochastic recourse programming model. It should be noted that assigning a sufficiently large value to ω causes the infeasibility penalty function term to dominate the objective function, thereby resulting in a higher expected value for the first stage and the second stage cost.

As noted before, we employ the linearization method developed by [28] in order to transform the absolute term in the objective function (3.15) into a linear form. A deviational variable γ^{ε} is introduced and the RO model with model robustness is reformulated as follows.

$$Min\ Z = c^T x + \mu + \omega \sum_{\varepsilon \in \Xi} \pi_\varepsilon \left(f^\varepsilon + 2\gamma^\varepsilon \right) \tag{3.18}$$

Subject to (3.2), (3.4), (3.16), and (3.17)

$$f^\varepsilon + \gamma^\varepsilon \geq 0 \qquad \forall \varepsilon \in \Xi \tag{3.19}$$

$$\gamma^\varepsilon \geq 0 \qquad \forall \varepsilon \in \Xi \tag{3.20}$$

3.3.2.3 Proposed transformation framework for RO model with the trade-off between solution robustness and model robustness

The RO model can coordinate the variability and feasibility at the same time through a trade-off between solution and model robustness. The complete formulation of the robust objective function using our proposed transformation framework that includes both solution robustness and model robustness is as follows.

$$Min\ Z = c^T x + \mu + \lambda \sum_{\varepsilon \in \Xi} \pi_\varepsilon \left| d^\varepsilon \right| + \omega \sum_{\varepsilon \in \Xi} \pi_\varepsilon \left| f^\varepsilon \right| \tag{3.21}$$

In the objective function (3.21), the first term is the first stage cost, the second term is the second stage cost, the third term is the expected variability cost, and the forth term is the expected infeasibility cost. Note that the sum of the first stage cost and the second stage cost comprises the model expected cost in stochastic recourse programming model, while the total cost consists of the sum of all the cost terms. Given the absolute term formulation to transform the objective function into the LP model, the linear RO transformation framework with the trade-off between solution and model robustness is formulated as follows.

$$Min\ Z = c^T x + \mu + \lambda \sum_{\varepsilon \in \Xi} \pi_\varepsilon \left(d^\varepsilon + 2\theta_\varepsilon \right) + \omega \sum_{\varepsilon \in \Xi} \pi_\varepsilon \left(f^\varepsilon + 2\gamma^\varepsilon \right) \tag{3.22}$$

Subject to

$$Ax = b \tag{3.23}$$

$$B_\varepsilon x + C_\varepsilon y_\varepsilon + f^\varepsilon = e_\varepsilon \qquad \forall \varepsilon \in \Xi \tag{3.24}$$

$$d^\varepsilon + \theta_\varepsilon \geq 0 \qquad \forall \varepsilon \in \Xi \tag{3.25}$$

$$f^\varepsilon + \gamma^\varepsilon \geq 0 \qquad \forall \varepsilon \in \Xi \tag{3.26}$$

$$x, y_\varepsilon, \lambda, \omega, \theta_\varepsilon, \gamma^\varepsilon \geq 0 \qquad \forall \varepsilon \in \Xi \tag{3.27}$$

To demonstrate the applicability of the proposed formulation and to provide an insight into the structure of the proposed transformation framework, a surgical block allocation problem of a real-life healthcare delivery system is solved. To the best of our knowledge, this is the first time that our proposed RO approach is applied in the context of the healthcare capacity allocation problems. Through a case study, we demonstrate that our approach outweighs the SP method while it works as effective as the robust model presented by [28] where the predefined framework requires less information about the original deterministic problem while at the same time provides more flexibility for the decision makers in utilizing the formulation.

3.4 RO model for the surgery capacity allocation problems

To illustrate the applicability of our proposed standard transformation framework, we employ a surgery block allocation problem of a healthcare delivery system to provide valuable insights into a number of aspects of the presented framework and also the characteristics of the proposed formulation. The most important challenge of surgery capacity allocation planning is not only how to deal with the randomness of the stochastic processes, but to cope with data incompleteness and operational inefficiency of the healthcare systems as the common phenomena. Considering a surgery block allocation problem, we utilize our novel standard framework to transform a deterministic model into the RO formulation.

The total amount of available OR hours is known as a constant in this work. The goal is to provide a planning program for healthcare decision makers to develop a surgical block allocation plan for the specialties so that the total operating costs, which comprises postponement costs, rescheduling costs, and underutilization costs, are minimized. The availability of the OR blocks is under the influence of staffing availability and budget constraint. The developed decision tool can help hospital managers to allocate OR blocks to the surgical specialties in response to unknown elective and emergency surgery demands to determine the assignment of individual specialties in each OR block.

In modelling of OR allocation problems in previous work, surgery cost was defined as a function of surgery duration. This assumption, however, is quite unrealistic in OR planning of hospitals working under common wealth system as non-profit organizations. Under the publically funded system, the operating budget of a healthcare system is allocated through the governmental fund and hence public hospitals provides healthcare services (e.g. surgery operations) for their patients where the cost of surgery is not related to the length of operation, but to the duration of time between a patient is admitted to a hospital and the time when the required surgery operation is performed. This time frame plays very crucial role for the hospital managers in their attempt to control the budget through minimizing the non-reimbursable costs associated with the postponed surgery operations.

In this work, both elective and emergency surgery demands are assumed unknown parameters. The uncertainty in surgery demand stems from the uncertain reservation planning of elective surgery demand as well as incorporating the emergency cases into the model while planning for OR block allocation decisions. To capture the whole aspect of the real-world healthcare problems and to have a realistic model when OR blocks are allocated, emergency patients have to be dealt with concurrent with elective cases. Therefore, we develop an effective optimization tool through our proposed RO framework to provide flexibility for the decision makers to capture the uncertainty in their allocation plans.

In the following, we concentrate our analysis on the so called surgical capacity allocation problem. The problem comprises determining the optimum number of OR blocks X assigned to a set of surgical specialty S over a planning horizon T while the target number of surgical demand of each specialty is met. In reality hospitals deal with no-show cases as well as patients not arriving on schedule, the aim is to minimize; (1) the cost associated with the surgical postponement (i.e. waiting time) of admitted patients, (2) the penalty associated with the rescheduled surgery operations that will be met either outside of the normal staffed hours through

overtime or another hospitals, and (3) the loss incurred due to underutilization of the ORs. A block-booking scheduling approach is assumed for a multi-surgical specialty problem where each block is characterized by an OR on a day in which block type i is available. The length of available hours in each OR blocks is represented by h which explicitly represents the available capacity of OR blocks on a given day after taking the turnaround times into consideration.

To incorporate the impact of variable demand into the model, the elective and emergency surgery demand of each surgical specialty is considered as random variables where the realization of the surgical demands is determined on a discrete event scenario basis. The daily surgery demand varies between the time intervals obtained from the historical data of the hospital. The problem amounts to determining the number of OR block hours to assign to surgical specialties over a considered planning horizon aiming at maximizing the utilization of the available OR blocks while the cost of postponed demand and unmet elective surgery demand is minimized. The main trade-off in the proposed robust surgery capacity allocation model is between reducing surgery postponements and cancellations while maximizing the operating theater utilization influenced by demand variability.

A Surgery postponement cost is incurred if a scheduled surgery is postponed to a later date. Here C_{tz}^{2} represent the cost of postponement of elective surgery demand from day t to day z in the planning horizon. The cost element is a known parameter and representing the loss incurred due to an unnecessary LOS of elective demands at the hospitals. Public hospitals are not able to bill the health coverage providers for the unnecessary postponement of elective surgery demands caused by the inefficient allocation plans. Therefore, a large portion of healthcare expenditures for a hospital would not be reimbursed if unnecessary postponement happen which could eventually impact the surgery service levels. The developed cost structures presented in this work

is a general cost function comprising block allocation cost [13], [15], hospitalization costs [7], and surgeon costs [31].

Note that if a surgery is carried out on a day it is requested, no extra cost will be associated to the hospital. Hence, a patient with a longer surgery duration would not be more costly for the hospital. However, a penalty cost will be incurred on delayed surgeries to penalize the postponed surgery demand hours resulted from the limited resources. To incur the penalty cost on the postponed demand volume we propose a linear cost function that considers the penalty cost for the LOS of admitted elective patients that are waiting for their surgery in the hospital. The cost function is obtained by considering a cyclic planning horizon where the penalty cost, C_{tz}^{2}, would incur the value of $z-t$ if a surgery is postponed to a future z^{th} weekday from the same week as a day t. If a surgery is required to be postponed from a day t to the following week in either the same weekday or a day which is smaller than what it is postponed from then the penalty cost would obtain through the value of D or $D-t+z$, respectively, where D represents the total number of days in the planning horizon including the weekends. For instance, if a surgery is postponed from Wednesday (i.e., day 3) to Tuesday (i.e., day 2) from the next week in the planning horizon, the penalty cost $C_{tz}^{2}=D-t+z$ which in this case is 6 days is incurred. It should be noted that the proposed linear penalty function would consider the cost of patients stay in the hospital over the weekends, even though the surgical operations are only performed during the weekdays. Therefore, the maximum postponement days allowed in this model would be seven days, considering the cyclic capacity pattern within the weekly planning horizon.

In this work, the priority is given to the emergency patients' demand in accessing to the non-emergency ORs in the hospital when the capacity of dedicated emergency rooms is saturated. Our model is the first one addressing the surgical block allocation problems of elective and

emergency demands that considers minimizing the patients' LOS waiting for surgery as an objective under the stochastic environment when a hybrid OR allocation plan is implemented.

3.4.1 Notations

In the following, we set the notations and provide the model formulations for the surgery capacity allocation problems.

3.4.1.1 Index sets

i For operating room type $(i = 1,\ldots,O)$

s For surgical specialty $(s = 1,\ldots,S)$

t,z For planning horizon $(t = 1,\ldots,T),(z = 1,\ldots,T)$

ε For Scenarios $(\varepsilon = 1\ldots,\Xi)$

3.4.1.2 Deterministic parameters

h Planned available operating rooms hours per day for surgery operation

a_i Number of operating room of type i available in each day

k_{st} Maximum capacity (i.e. number of surgeons) available for surgical specialty s on day t

c_{is}^1 Cost associated with allocating OR block times of type i to specialty s

c_{tz}^2 Cost of postponement of elective surgery demand from day t to day z

c^3 Penalty rate associated with the unmet elective surgery demand

c^4 Cost of underutilization of operating room hours

λ Weighting penalty to measure trade-off between risk and expected outcome

ω Weighting penalty for the infeasibility of the random elective demand constraint

3.4.1.3 Stochastic parameters

e_{st}^{ε} Random parameter representing elective surgery demand for specialty s on day t under scenario ε

g_{st}^{ε} Random parameter representing emergency surgery demand for specialty s on day t under scenario ε

π_{ε} Probability of occurrence of scenario ε

3.4.1.4 First stage decision variables

X_{ist} Number of operating rooms of type i allocated to specialty s on day t

U_{st} Amount of emergency surgery demand of specialty s met in a dedicated emergency room on day t

3.4.1.5 Second stage decision variables

Y_{stz}^{ε} Amount of elective surgery demand hours of specialty s postponed from day t to day z under scenario ε

ρ_{st}^{ε} Elective surgery demand of specialty s on day t under scenario ε that is rescheduled to be met outside the normal shift operation through overtime and/or moving to another local hospital

ι_{st}^{ε} Undersupply of operating room hours allocated to specialty s on day t under scenario ε relative to its desired level

μ Expected value of the second stage cost being made after realization of the random variable is observed

d^{ε} Variability cost of deviation from the mean expected value of the objective function in each scenario ε

θ^{ε} Deviational variable for violation of the mean objective function in each scenario ε

f_{st}^{ε} Deviation variable by which the random elective demand constraints of specialty s on day t can be violated under scenario ε

$\gamma_{st}^{\varepsilon}$ Deviational variable for infeasibility of the random elective demand constraints of specialty s on day t under scenario ε

3.4.2 Formulation of the surgery block allocation problems using proposed standard transformation framework

We begin by first formulating the deterministic model for the surgery capacity allocation problem in a multi-OR context. Next, we extend the basic deterministic model to a two-stage stochastic recourse programming model. Finally, a robust optimization model is developed using our proposed standard transformation framework that incorporates surgery demand uncertainty into the model where the request for the elective and emergency surgery operations is realized over the set of random discrete scenarios.

3.4.2.1 Formulation of the deterministic surgery capacity allocation problems

Our model formulation represents issues pertain to allocation of surgical specialties to the operating rooms in a concise manner. The objective is to minimize the cost associated with the total loss of reimbursement at the hospital as well as the penalty incurred to the unmet surgical demands and the cost of idleness of OR capacity.

$$Min \sum_{i}\sum_{s}\left(c_{is}^{1} h \sum_{t} X_{ist}\right) + \sum_{t}\sum_{z}\left(c_{tz}^{2} \sum_{s} Y_{stz}\right) + c^{3}\sum_{s}\sum_{t}\rho_{st} + c^{4}\sum_{s}\sum_{t}\iota_{st} \quad (3.28)$$

Subject to

$$\sum_{s} X_{ist} = a_i \qquad \forall i,t \qquad (3.29)$$

$$\sum_{s} U_{st} \le h \qquad \forall t \qquad (3.30)$$

$$\sum_{i} X_{ist} \le k_{st} \qquad \forall s,t \qquad (3.31)$$

$$U_{st} \le g_{st} \qquad \forall s,t \qquad (3.32)$$

$$\sum_{z} Y_{stz} \le e_{st} \qquad \forall s,t \qquad (3.33)$$

$$\rho_{st} \le e_{st} \qquad \forall s,t \qquad (3.34)$$

$$h\sum_{i} X_{ist} \ge g_{st} - U_{st} + \sum_{z} Y_{szt} \qquad \forall s,t \qquad (3.35)$$

$$h\sum_{i} X_{ist} - g_{st} + U_{st} - \sum_{z} Y_{szt} + \sum_{z} Y_{stz} + \rho_{st} - \iota_{st} = e_{st} \qquad \forall s,t \qquad (3.36)$$

$$X_{ist}, Y_{stz} \text{ int} \qquad \forall i,s,t,z \qquad (3.37)$$

$$X_{ist}, Y_{stz}, U_{st}, \rho_{st}, \iota_{st} \ge 0 \qquad \forall i,s,t,z \qquad (3.38)$$

It should be noted that in the above formulation, the constraints that only involve the first stage decision variables are referred as the first stage constraints (i.e. constraints (3.29) to (3.31)). Under the first stage decision, the accurate surgery demand information of the surgical specialties is not available. Constraint (3.29) represents the operating room assignment constraints, where the number of OR blocks from each type that allocate to surgical specialties cannot exceed the total number of available OR blocks of that type. It is proven to be beneficial to the decision makers not to assign partial blocks to the surgical specialties on a given day [6]. Therefore a simple block scheduling is provided where allocated OR blocks on a given day are not allowed to be shared between surgeons from different surgical specialties.

Constraint (3.30) guarantees the availability of an emergency surgery room in a hospital to meet emergency surgery demand. It is assumed that all the emergency surgery demand has to be met on the day it arises. Therefore, accommodation of emergency patients to the non-emergency operating rooms is permitted, however, the reverse is not allowed. Constraint (3.31) states the total number of ORs assigned to a surgical specialty on a given day cannot exceed the available capacity of that specialty in the hospital. Here the capacity implies the number of

available surgeons and other staff resources that are available for each specialty on a daily basis. Constraint (3.32) ensures the allocated capacity from the dedicated emergency room to each specialty cannot exceed the realized emergency demands of that specialty.

Constraint (3.33) ensures the sum of all postponed elective surgery demand for each specialty is at most equal to the realized surgery demands on that day. Equation (3.34) limits the amount of unmet surgery demand to a desired level provided by the managements. Thus, the amount of unmet demands of a specialty that is rescheduled to be met outside of the routine shift operation cannot exceed the demand for that specialty. Constraint (3.35) is a mass balance constraint. It ensures adequate OR blocks is allocated to each specialty to meet the emergency and postponed demand in each day.

Constraint (3.36) is demand satisfaction constraint. It states that the total daily elective surgery demand of each specialty has to be met on that day, or to be postponed to a working day within the planning horizon. Here, the specialty's throughput is maximized through defining a variable surgical performance that is imposed by the target number of patient demands. A trade-off is also achieved via penalizing the unmet demand of elective patients against the cost of idle operating room hours in the healthcare systems. Constraint (3.37) and (3.38) specify the integrality of the allocation variables as well as non-negativity for all variables.

Considering the stochasticity of surgery demands, in the following the capacity allocation model is developed under the stochastic surgery demand.

3.4.2.2Formulation of the two-stage stochastic recourse programming model for uncertain capacity allocation problems

Based on the analysis in section (3.1), the following two-stage stochastic recourse programming formulation for the proposed capacity allocation problems with the uncertain elective and emergency demands is presented.

$$Min \sum_i \sum_s \left(c_{is}^1 h \sum_t X_{ist} \right) + \sum_\varepsilon \sum_t \sum_z \pi^\varepsilon \left(c_{tz}^2 \sum_s Y_{stz}^\varepsilon \right) + \sum_\varepsilon \sum_s \sum_t \pi^\varepsilon \left(c^3 \rho_{st}^\varepsilon + c^4 \iota_{st}^\varepsilon \right) \quad (3.39)$$

Subject to the first stage constraints: (3.29) _ (3.31)

$$U_{st} \le g_{st}^\varepsilon \qquad \forall s,t,\varepsilon \quad (3.40)$$

$$\sum_z Y_{stz}^\varepsilon \le e_{st}^\varepsilon \qquad \forall s,t,\varepsilon \quad (3.41)$$

$$\rho_{st}^\varepsilon \le e_{st}^\varepsilon \qquad \forall s,t,\varepsilon \quad (3.42)$$

$$h \sum_i X_{ist} \ge g_{st}^\varepsilon - U_{st} + \sum_z Y_{szt}^\varepsilon \qquad \forall s,t,\varepsilon \quad (3.43)$$

$$h \sum_i X_{ist} - g_{st}^\varepsilon + U_{st} - \sum_z Y_{szt}^\varepsilon + \sum_z Y_{stz}^\varepsilon + \rho_{st}^\varepsilon - \iota_{st}^\varepsilon = e_{st}^\varepsilon \qquad \forall s,t,\varepsilon \quad (3.44)$$

$$X_{ist}, Y_{stz}^\varepsilon \text{ int} \qquad \forall i,s,t,z,\varepsilon \quad (3.45)$$

$$X_{ist}, Y_{stz}^\varepsilon, U_{st}, \rho_{st}^\varepsilon, \iota_{st}^\varepsilon \ge 0 \qquad \forall i,s,t,z,\varepsilon \quad (3.46)$$

It should be noted that in the above formulation, the constraints that only involve the first stage decision variables are referred as the first stage constraints (i.e. constraints (3.29) to (3.31)). Under the first stage decision, the accurate surgery demand information of the surgical specialties is not available. In the objective function, the first term represents the first stage (FS) cost $FS = \sum_i \sum_s \left(c_{is}^1 h \sum_t X_{ist} \right)$ and is free of noise. The second term, however, determines how the hospital makes responses to the case where the stochasticity of the unknown parameters is realized for various scenarios, and hence is the second stage (SC) cost $\mu = \sum_\varepsilon \pi^\varepsilon \left(\sum_t \sum_z \left(c_{tz}^2 \sum_s Y_{stz}^\varepsilon \right) + \sum_s \sum_t \left(c^3 \rho_{st}^\varepsilon + c^4 \iota_{st}^\varepsilon \right) \right)$. The summation of the first stage cost and the second stage cost in (3.39) is outlined as the expected cost of the two-stage stochastic recourse programming model. The constraints that consist of both first stage variables and second stage variables are defined as the second stage constraints, i.e. constraints (3.40) to (3.44) in the two-stage stochastic recourse programming model.

3.4.2.3 The proposed formulation of the RO model with solution robustness (ROM-SR) for uncertain capacity allocation problems

We employ our novel transformation framework to develop the RO model with solution robustness for the OR blocks allocation problems of the healthcare systems under a set of surgical resource constraints. Based on the analysis in section 3.2.1, μ represents the mean objective function or the second stage cost. The deviation from the mean is captured through the term d^{ε}. Therefore, $d^{\varepsilon} = \left(\sum_{t} \sum_{z} \left(c_{tz}^{2} \sum_{s} Y_{stz}^{\varepsilon} \right) + \sum_{s} \sum_{t} \left(c^{3} \rho_{st}^{\varepsilon} + c^{4} i_{st}^{\varepsilon} \right) \right) - \mu$ represents the type of variability measure that is used in our proposed transformation framework to define the RO cost variability. In the ROM-SR model, d^{ε} it is defined as the difference between the cost of postponements, rescheduling, and underutilizations under each realization of scenario sets and the total expected cost of the two-stage stochastic recourse model. The ROM-SR model is formulated as follows.

$$Min\ FC + \mu + \lambda \sum_{\varepsilon \in \Xi} \pi_{\varepsilon} \left(d^{\varepsilon} + 2\theta^{\varepsilon} \right) \tag{3.47}$$

Subject to

The first stage constraints: (3.29) _ (3.31)
The second stage constraints: (3.40) _ (3.44)
The integrality and non-negativity constraints: (3.45) _ (3.46)

$$d^{\varepsilon} + \theta^{\varepsilon} \geq 0 \qquad \forall \varepsilon \tag{3.48}$$

$$\theta^{\varepsilon} \geq 0 \qquad \forall \varepsilon \tag{3.49}$$

The final term in the objective function (3.47) is the expected variability cost for the postponed surgery demands, rescheduled surgery demands, and the underutilized OR capacities. The term θ^{ε} represents a deviational variable to linearize the objective function and capture the negativity of the variance from the mean as elaborated in section 3.2.1.

3.4.2.4 The proposed formulation of the RO model with model robustness (ROM-MR) for uncertain capacity allocation problems

We employ our novel transformation framework to develop the RO model with model robustness for the proposed OR block allocation problems. Based on the analysis in section 3.2.2,

the infeasibility of the second stage constraints is captured through the term f_{st}^{ε}. Therefore, $f_{st}^{\varepsilon} = e_{st}^{\varepsilon} - h\sum_{i} X_{ist} + g_{st}^{\varepsilon} - U_{st} + \sum_{z} Y_{szt}^{\varepsilon} - \sum_{z} Y_{stz}^{\varepsilon} - \rho_{st}^{\varepsilon} + \iota_{st}^{\varepsilon}$ denotes the random demand constraints in equation (3.44) can be violated over some set of scenarios at the amount f_{st}^{ε}, where f_{st}^{ε} represents a deviational variable that denotes the difference between allocated OR capacities in terms of OR block times and the amount of surgery requests upon realization of uncertain surgery demands. The impact of allowing for the infeasibility of the demand constraints will be taken into account in the objective function as follows.

$$Min\, FC + \mu + \omega \sum_{\varepsilon \in \Xi} \pi_{\varepsilon} \left(\sum_{s} \sum_{t} \left(f_{st}^{\varepsilon} + 2\gamma_{st}^{\varepsilon} \right) \right) \tag{3.50}$$

Subject to

The first stage constraints: (3.29) _ (3.31)
The second stage constraints: (3.40) _ (3.44)
The integrality and non-negativity constraints: (3.45) _ (3.46)

$$f_{st}^{\varepsilon} + \gamma_{st}^{\varepsilon} \geq 0 \qquad \forall s,t,\varepsilon \tag{3.51}$$

$$\gamma_{st}^{\varepsilon} \geq 0 \qquad \forall s,t,\varepsilon \tag{3.52}$$

In the objective function (3.50), ω represents the unit penalty for the violation of the random surgery demand constraints. The term $\gamma_{st}^{\varepsilon}$ represents a deviational variable to linearize the objective function and capture the negativity of the infeasibility function as elaborated in section 3.2.2. The term $\gamma_{st}^{\varepsilon}$ also captures the amount by which the demand constraints are violated. In the objective function (3.50), when the unit weighting parameter ω increases, the penalty cost for the infeasibility of the random demand constraints also escalades. That means failure to allocate surgical demands into their required OR blocks would result in a higher cost of managing the healthcare systems through the penalty incurs as a result of growing unmet demands. Note that the number of OR allocated to surgical specialties does not contain index ε as it is scenario

independent variable, while the postponement of elective surgical demands, the quantity of rescheduled demands, and the undersupply of OR block times contain the index ε to reflect the fact that the actual value of these variables only captures after the realization of scenarios.

3.4.2.5 The proposed formulation of the RO model with the trade-off between solution robustness and model robustness (ROM-T) for uncertain capacity allocation problems

RO also provides a degree of flexibility for the decision makers by considering a trade-off between optimality and feasibility. Through this analysis, managers can explicitly realize the possible trade-off associated with the variability of different service levels and the expected cost. Therefore, the results obtained from the trade-off analysis are aligned with the level of risk that managers are willing to take. Solving for variability and the infeasibility together, the proposed RO model in this section is formulated to address the capacity allocation problems under the stochastic healthcare environment.

$$Min\ FC + \mu + \lambda \sum_{\varepsilon \in \Xi} \pi_{\varepsilon} \left(d^{\varepsilon} + 2\theta^{\varepsilon} \right) + \omega \sum_{\varepsilon \in \Xi} \pi_{\varepsilon} \left(\sum_{s} \sum_{t} \left(f_{st}^{\varepsilon} + 2\gamma_{st}^{\varepsilon} \right) \right) \tag{3.53}$$

Subject to

The first stage constraints: (3.29) _ (3.31)
The second stage constraints: (3.40) _ (3.44)
The integrality and non-negativity constraints: (3.45) _ (3.46)
The solution robustness constraints: (3.48) _ (3.49)
The model robustness constraints: (3.51) _ (3.52)

The essential parts of the objective function (3.53), the third term is incorporated to accommodate the mean-variance trade-off over scenarios, and hence the variability cost. The variability is measured in terms of fluctuations of postponed surgery demands, rescheduled hours of patient demands, and idleness of OR capacities from their total expected value. The deviation from the elective demands is expressed in the fourth term and is permitted at a penalty cost (i.e. infeasibility cost). The goal of the objective function (3.53) is to reach to a balance between solution and model robustness.

3.5 Numerical example and computational analysis

3.5.1 Case description

In order to illustrate the effectiveness of the proposed standard transformation framework for solving the uncertain OR block allocation problems with stochastic surgery demands, we employ the data obtained from WRH a local hospital sited in Windsor. WRH is a multi-faceted healthcare organization operating from two main campuses at Southwestern Ontario in Canada to provide advance care in specialized areas that include complex trauma, cardiac care, neurosurgery to mention a few supporting over 400,000 people in the community. WRH is budgeted to staff 45-bed inpatient surgical units functioning in 10 operating room theaters and two diagnostic rooms located in two different sites across the County. There are also two emergency departments to provide a range of services to meet the unscheduled and emergency healthcare needs for clients.

Based on the information from the OR surgery department, WRH provides services in specialties including General Surgery, Urology, Gynecology, Orthopedics, Ear, Nose, and Throat, Dental / Oral Maxillofacial, Plastics and Burns, Ophthalmology, Cardiovascular, and Surgical Oncology. According to its 2014-2015 annual report, WRH is one of the busiest public hospitals in the Southwestern Ontario with a record of 314,469 outpatient visits, 44,418 day surgeries, 28,898 inpatient discharges, and 128,357 emergency department visits in a year. The report shows the number of elective patients visit has increased by over 25% during the past five years while more than 65% of the admitted patients have occupied recovery beds in the hospital through the emergency department [40].

As a public healthcare provider, the hospital's budget is mostly funded by provincial programs, and hence, patients are admitted regardless of their financial status. Once a patient is discharged, the hospital is reimbursed based on a predetermined funding model that reflects the need of the patient served by the hospital. This funding model determines the amount of

compensation for the healthcare systems based on the services delivered and also the quality of services to the patient populations they serve. It is quite obvious that the hospital would not be compensated for the time that patients admitted to the hospitals and are waiting for a surgery or clinical services to be provided. So, the longer the waiting time for patients to receive services, the larger would be the profit loss for the hospital. Therefore, it is crucial from the hospital management point of view to reduce, if not completely eliminate the amount of unnecessary LOS in order to decrease costs and increase the throughputs.

At the time of this study, WRH uses 9 ORs between 8 am and 5 pm with 6 specialties and one emergency room located in its Metropolitan Campus. The six specialties are General surgery, Urology, Gynecology, Orthopedics, ENT, and Cardiovascular. We use the capacity and demand data of the 2015's fiscal year to feed the proposed standard RO framework, where both elective and emergency surgery demands are modeled as uncertain parameters to capture the stochastic nature of the healthcare environments. We have analyzed the archived data on the arrival number of elective and emergency cases to obtain the required data for input parameters. Based on the historical data in WRH, the upper bound and lower bound of surgery demand hours can be obtained. The daily emergency demand for the six specialties varies between the interval [180, 360], [90, 330], [115, 270], [45, 395], [105, 300], and [40, 390] minutes, respectively. The daily elective demand for the six specialties are within the intervals [305, 955], [280, 855], [150, 615], [210,965], [265, 1380] and [125, 1010] minutes, respectively; and the total daily OR capacity is 4800 minutes. For the sake of consistency with what is performed in practice, all times are rounded to multiples of five minutes. We assume the demand interval for each specialty is constant during a day, as the historical data is not indicated otherwise.

The hospital will look at a 1-week planning horizon, where no surgery is operated during the weekends. We assume the eight-hour operating shift, without any possible overtime. We generate random instances to evaluate the performance of the proposed RO model over various

probability of occurrences. It is assumed that uncertainty is represented by a set of possible demand situations. On the basis of historical records on surgical demands of the OR department, it can be assumed that future surgery demand scenarios will fit into one of the four possible scenarios, namely fair, good boom, and poor with associated probabilities π_{ε} of 75, 10, 7.5 and 7.5 percent, respectively. These scenarios are used to consider different range of uncertainties in the surgery demand data.

It should be noted that an in-hospital cost is incurred by measuring the delay in meeting the elective surgery demands. This cost is a function of the number of postponed surgeries by the number of days a surgery is delayed. Therefore, when a surgery demand for an elective patient arises, it can either be postponed to a working day which is no more than seven days after the demand is raised or become unmet. Emergency cases are allowed to go ahead in a non-emergency OR upon availability of resources, however, the surgery has to be completed within the normal shift operation of eight staff hours for each OR. So, no overtime work is allowed in this model. The amount of elective surgery demand that turns out to be rescheduled outside of the normal shift operations (i.e. unmet demand) will be penalized using the penalty rate c^3, which is the largest among all the penalty rates. We assume a cyclic weekly demand pattern in our model, therefore, the unmet demand for elective patients has to be met either in overtime hours or be rescheduled to another local hospital, which in either case will be penalized.

The AMPL software is used as a solution platform due to its well-known high-level modeling system for solving complex mathematical programming problems. We use CPLEX 12.6.3 with default setting to solve the proposed RO model. The problems are executed on a Pentium IV 2.66GHz CPU with 4GB RAM. The computational results of the proposed RO model are shown in the following contents. To obtain the trade-off between solution robustness and

model robustness in the RO model, the value of λ and ω have been chosen to be set to 0.1 and 75, respectively as elaborated in section 5.5.2.

3.5.2.The first stage decisions

Before realization of the accurate surgery demand for elective and emergency patients, the hospital managers have to allocate available OR blocks to the surgical specialties. The decisions that are determined before the accurate information is observed are called the first stage decisions. The result of the weekly OR capacity allocations determined by the robust mixed integer programming model is shown in Table 3.2, where block allocation plans generated by the deterministic model, two-stage stochastic recourse model, and the proposed RO transformation framework are compared. It is observed that the CPU time taken for the optimal solutions to converge is averaged around 45 seconds, 256 seconds, and 320 seconds for each model, respectively.

Table 3.2: OR capacity (hours) allocated to each specialty over the planning horizon

Surgical specialty	Deterministic model	Two-stage stochastic recourse model	Robust optimization model
General surgery	72	64	64
Urology	64	56	56
Gynecology	40	48	48
Orthopedics	56	72	64
ENT	72	80	80
Cardiovascular	56	40	48

3.5.3 The second stage decisions

The second stage decision is referred to the stage where the determination of the variables can be postponed until the realization of the stochastic parameters. At this stage, the uncertain parameters are known to the hospital managers, and hence, the decisions on the amount of postponement, rescheduled surgery demands, and idleness of OR capacity can be made which are shown in Table 3.3. The probability of occurrence for uncertain surgery demand parameters under a fair surgery demand scenario is 75%. Since the amount of surgery demand that has to be

rescheduled comprises almost 2.5% of total demand, it is very likely that the hospital manager will take the second stage decisions by accruing the overtime hours to compensate for the lost surgery demands and keep them from being rescheduled into other local hospitals if this scenario is observed.

From the results, it is obvious that all the surgery demands under the poor demand scenario will be satisfied, and hence, no unmet demand is occurred during that scenario. However, the underutilization cost for the under-usage of OR capacity is increased for all specialties under this scenario. There is no postponed demand suggested in the optimal results for the case where variability is increased. Therefore, unsatisfied demand of the cardiovascular specialty has to be realized through rescheduling in another hospitals.

Table 3.3: Postponed / unmet surgery demands and idleness of ORs capacity per week (hours)

	Scenario sets	General surgery	Urology	Gynecology	Orthopedics	ENT	Cardiovascular
Postponed demand	Fair	7	10		9	6	8
	Good	1	4				2
	Poor						
	Boom						
Unmet demand	Fair		1.7				6.2
	Good	11.2	3.8			2.6	
	Poor						
	Boom	0.5					17.7
Underutilized OR	Fair						
	Good						
	Poor	34.5	32.5	26	45.8	54.5	45.4
	Boom						

The poor demand scenario is realized with the probability of occurrence of 7.5%. In the poor scenario, all random demand constraints are satisfied. At the same time, all the elective and surgery demands are satisfied through the first stage block allocation decision in the poor scenario. Therefore, the hospital will not incur any penalty cost due to the unmet surgery

demands. However, some OR times allocated to specialties in the first stage are remained idle, as shown in table 3.3, which results in a significant underutilization cost for the hospital.

The surgery demand in boom scenario is realized with the probability of occurrence of 7.5%. If the boom scenario happens, the hospital will take the second stage postponement/unmet demand and underutilization decisions as shown in table 3.3. Since all of the OR capacities are allocated to meet the surgery demands in the first stage block allocation decision, there is no underutilization costs involved in this scenario. It should be noted that the random demand constraints in the boom scenario are satisfied with a substantial penalty costs that is incurred due to the postponed demand and/or unmet surgeries that is rescheduled outside of the normal operating shift. Obviously, there is no cost associated with idleness of OR capacity under boom scenario, as the initial allocated OR hours is not enough to meet the higher surgery demands when this scenario is observed. Therefore, it is required to add unplanned overtime or reschedule the excessive patients to other local hospitals if the trend of surgery demand tends to be realized in boom scenario.

3.5.4 Robust optimization vs. stochastic recourse programming

The performance of the proposed models in terms of quality of the solutions and CPU time required is evaluated by generating an extensive set of instances based on scenario sets encountered in the model. The properties of the objective value functions are summarized in Table 3.4, where both models are truncated after 1800 sec of computation time.

Table 3.5 provides a computational comparison in the results of the RO and the two-stage stochastic recourse programming models. Based on the analysis provided in section 3.1, the expected variability cost of the second stage decision variables is not taken into account in the two-stage stochastic recourse programing model. This is a direct result of assuming $\lambda = 0$ in equation (3.47) of the robust objective function.

Table 3.4: Model size and computation characteristics

	No. of variable			Constraint			
Model type	Binary	Integer	Linear	Equality & non equality	Nonzero	CPU time (sec)	Gap%
Two-stage stochastic	40	12	1247	601	9141	1800 s	0.23%
Robust optimization	40	12	2310	1821	10,388	1800 s	0.54%

The expected costs provided by the robust model is 0.7% more than the stochastic two-stage recourse model, however, the robust model is progressively less sensitive to the variability of the uncertain parameters as it incorporates the cost of variability into the model. The expected variability in the robust model considerably decreases by almost 79%, which resulted in generating more reliable OR block allocation plans for the hospital managers. The infeasibility cost involved in the robust model offsets the cost of not satisfying all surgery demands as it is realized over the scenario sets. When ω is set large enough, the second stage constraints in the robust model allows compensating for discrepancies in the surgery demand constraints by incurring a penalty cost per unit of infeasibility of the realized surgery demand. It is noted that increasing the penalty cost to a large number (i.e. $\omega = 100$ in the last row of table 3.5), will convert the robust model to a two-stage stochastic recourse programming model, and thus prevents the violation of the random demand constraints. Comparing this with the recourse model, the variability remains almost the same while the total cost has slightly increased by 3.8%. It means the block allocation planning proposed by the robust framework mitigates the risk of unmet surgery demands at a lower cost for the hospital when a proper value for the penalty cost is chosen. The results shown in Table 3.5 demonstrate the effectiveness and applicability of the proposed RO framework.

Table 3.5: Comparison between the results of robust model and stochastic recourse model

	Expect ed	First stage cost	Second stage	Expect ed	Expect ed	Recour se cost	Expect ed infeasi	Total cost

	recourse cost		cost (μ)	variability	infeasibility	variability	bility cost	
Stochastic recourse model	11608	7920	3688	4046	0	0	0	11608
Robust optimization model $(\lambda = 0.1, \omega = 75)$	9481	7920	1561	855	28	85	2127	11693
Robust optimization model $(\lambda = 0.1, \omega = 100)$	11651	7920	3731	4045	0	404	0	12056

3.5.5Analysis of the results

In this section, we conduct four different tests on the robust surgery capacity allocation model with arbitrarily chosen probability of occurrences for uncertain surgery demand scenarios. All other parameters are assumed constant across the four tests. The characteristic of the tests is shown in Table 3.6. Under each test condition, it is assumed that one future situation dominates the other possibilities and hence the realization of the surgery demand derives based on that scenario.

Table 3.6: Test characteristics

Test	π_1	π_2	π_3	π_4
$Test$ I	0.75	0.10	0.075	0.075
$Test$ II	0.10	0.75	0.075	0.075
$Test$ III	0.10	0.075	0.75	0.075
$Test$ IV	0.10	0.075	0.075	0.75

3.5.5.1 Analysis of the results for ROM-SR

In order to determine the sensitivity of the projected results of the robust optimization model with solution robustness to the variability of the input parameters we perform four tests in which different value is assigned to the robustness parameter, λ , to generate a range of robust optimal solutions.

It should be noted that under each test, λ reveals the hospital manager level of concern with respect to surpassing the prospective cost of postponed surgery demand, unscheduled surgery demand, and underutilized OR capacity for all scenarios. Under each test, the first row (where $\lambda = 0$) represents the results of the two-stage stochastic recourse programming model. As the value of λ increases, the recourse model transforms into the RO model in which variability is taken into account. Therefore, increasing λ reduces the variance from the mean postponed and unmet demands as well as the mean idleness of OR capacity in the hospital, although at an increased variability cost.

As shown in table 3.7, the expected recourse variability for the two-stage recourse model is always greater than that of the RO model under each test results, which directly implies the higher risk associated with the stochastic recourse model. On all the tests, the first stage cost is remained unchanged over different value of λ, with an exception of Test IV, which means the first stage decision is not affected by the decision makers' risk behavior. The second stage costs increases throughout the tests as the value of λ is growing which implies the correlation between the second stage cost and the cost of postponed demands, rescheduled surgeries, and underutilized OR blocks.

In the RO model with solution robustness, as can be observed from Table 3.7 the expected recourse variability decreases significantly as λ increases, although the mean expected cost is augmented. Compared with the recourse model, increasing the value of λ from zero to 0.9 reduces the expected variability by 2.2% in Test I, 0.65% in Test II, 15.2% in Test III, and more than 98% in Test IV. This implies the significance of the RO model in reducing variabilities as the variance of input data increases. The impact of variability becomes more severe when a sudden increase in surgery demand is observed. Test IV represents the situation where the highest surgery demand is possibly realized. Although the total cost of the robust model increased by

around 15%, the variability has greatly reduced by more than 98% which means the recourse model is way too risky to be implemented under this situation. Therefore, the obtained results demonstrate the effectiveness of the robust model in mitigating the risk arises from the variability of input parameters.

Table 3.7: Variability analysis of the objective function in ROM-SR

Test	λ	Recourse variability	First stage cost	Second stage cost (μ)	Expected variability cost	Total cost
$Test$ I	0	4046	7920	3688	0	11608
	0.1	4045	7920	3688	404	12013
	0.5	4000	7940	3673	2000	13613
	0.9	3957	7940	3561	3561	15197
$Test$ II	0	2336	7760	7465	0	15225
	0.1	2333	7760	7466	233	15459
	0.5	2322	7760	7470	1161	16391
	0.9	2321	7760	7471	2089	17320
$Test$ III	0	2710	7620	8340	0	15960
	0.1	2708	7620	8340	271	16231
	0.5	2306	7620	8362	1153	17235
	0.9	2297	7620	8469	2067	18156
$Test$ IV	0	6335	7760	18013	0	25772
	0.1	6183	7660	18115	618	26393
	0.5	5411	7420	18645	2706	28769
	0.9	103	7400	22219	93	29712

It can be concluded from the above discussion that to obtain a robust allocation plan for the OR blocks the hospital managers should select an appropriate value for λ that reflects the degree of risk aversion which is appropriate to react to the cost variability of the postponed/unmet demands and the underutilization of OR capacities.

3.5.5.2 Analysis of the results for ROM-MR

Table 3.8 shows the computational analysis of the RO model with model robustness over the predefined set of tests. We perform four tests in which the infeasibility penalty function, ω , is assigned different values to analyze the performance of the second stage demand constraints over

different penalty cost in the proposed robust model. As it can be observed from the results presented in Table 3.8, when penalty cost for the violation of the random demand constraints is ignored, the expected infeasibility is very high: 193 in Test I, 219 in Test II, 235 in Test III, and 325 in Test IV (see the first row in each test). In Test I and II, where the surgery demand has less variation, if ω gradually increases to 10, the expected infeasibility reduces significantly by 64% and 45%, while the total cost has only increased by almost 17% and 21%, respectively. Thus, a globally feasible service level can be achieved at a lower cost. When variability becomes more predominant, (i.e. Test III and Test IV), a small increase in the penalty cost to 10 cannot impact the infeasibility as such, although the total cost goes up significantly by 28% and 38%, respectively. This implies the significance of choosing the right amount for the penalty cost.

Table 3.8: Infeasibility analysis of the objective function in ROM-MR

Test	ω	Recourse infeasibility	First stage cost	Second stage cost (μ)	Expected infeasibility cost	Total cost
$Test$ I	0	193	7280	0	0	7280
	10	69	7760	92	695	8547
	75	28	7920	1561	2127	11693
	100	0	7920	3688	0	11608
$Test$ II	0	219	7280	0	0	7280
	10	121	7560	44	1210	8817
	75	38	7760	4634	2831	15279
	100	0	7760	7465	0	15225
$Test$ III	0	235	7280	0	0	7280
	10	206	7280	19	2056	9356
	75	21	7620	6729	1611	16009
	100	0	7620	8340	0	15960
$Test$ IV	0	325	7280	0	0	7280
	10	266	7360	24	2664	10049
	75	230	7660	708	17305	25792
	100	0	7760	18013	0	25772

Results in Test III represents a situation where available capacity of OR blocks outweigh the required surgery demands. Therefore the expected infeasibility implies the OR capacities that have been allocated to the specialties but remained unused (i.e. underutilized) due to the incomplete demand data. The expected infeasibility under Test IV, however, states the situation where elective surgeries have to be rescheduled to be met either through assigning the overtime hours or moving to another local hospital. Under this situation the demand for surgery is greater than the available OR blocks. The hospital managers might be interested to find a penalty cost that absorbs most of the infeasibility at a reasonable cost. The third row in each test shows a point where maximum reduction in infeasibility can be achieved. An increase to the penalty cost to 75 can reduces the infeasibility by 85%, 83%, 91%, and 29% in Test I to Test IV, respectively, which is also suggested as optimum penalty rate for this analysis.

In general, when ω increases by enough amount, both total costs and the expected infeasibility go up. One interesting result obtained from the analysis performed in section 5.5.1 and 5.5.2 is shown in the final row of each test in Table 3.8. It is noted that maximizing the penalty cost enforces all random constraints in the second stage to be satisfied resulting in the expected infeasibility to be eliminated, and hence the ROM-MR model transforms into the two-stage stochastic recourse model. Therefore, the first row of each test in Table 3.7 (i.e. $\lambda = 0$) contains the same results as the final row of each test in Table 3.8 (i.e. $\omega \approx \infty$), as both represent the case where RO model is transformed into the two-stage stochastic recourse model. Consequently, the results obtained from this analysis can provide a holistic managerial insights to the decision makers to take appropriate actions to the realization of the surgery demands towards the achievement of a higher service level while the OR utilization rate is also maximized which in turn can lead to the lower operational costs as well as controlled risk for the healthcare system.

3.5.5.3 Analysis of the results for the robust optimization model with trade-off between solution and model robustness (ROM-T)

To obtain the optimum value of the objective functions and to provide an insight into the characteristics of the output data, the optimal value of λ and ω has to be determined as a measure of trade-off between solution robustness and model robustness in the proposed RO framework. As seen before, when $\omega = 0$ the random demand constraints become infeasible in the objective function without a penalty cost, and hence the service level deteriorates due to the largest postponed and unmet surgery demands as well as the highest underutilized OR blocks incurred in the hospital. Under this situation, the resulted allocation plan is obviously not desired by the hospital managers. A very large penalty weight, on the other hand, could result in the penalty function to dominate the objective function and causes higher operational costs. Consequently, the robust model has to be solved several times each time with a different value of ω to obtain the minimum loss incur in the OR block allocation plan to find a solution that is close to an optimal solution (i.e. solution robustness) while it is almost feasible for all scenarios (i.e. model robustness). This trade-off analysis allows decision makers to acquire an optimal solution based upon an acceptable range of expected postponed/unsatisfied surgery demand, and total costs. When ω varies, the amount of infeasibility of the random demand constraints is also altered. Therefore, examining the proposed robust optimization model with various ω would provide a sense of trade-off between the risk and cost.

We analyze the proposed robust model with respect to different value of ω while λ is assumed constant to obtain the optimal block allocation plan that captures the trade-off between risk and cost. We analyze the variability, infeasibility, and the total costs trade-off in the proposed RO model over different sets of λ . Figures 3.1 to 3.3 show the computational results for Test I in terms of the total loss due to the postponement of surgery demands, the unmet demands, and the underutilized OR blocks. Figure 3.1 depicts the impact of changing ω on the expected variability

when λ is kept constant. When the lowest risk aversion policy is chosen (i.e. $\lambda = 0.1$) a small increase in ω results in a steep hike in the variability from 76 to 4045. However, the trend remains steady after ω reaches to 100. When the risk of variability becomes more costly for the decision makers (i.e. $\lambda = 0.5$ or $\lambda = 0.9$), increasing ω has a lower effect on the variability change as variability will be the factor that is limiting the rate of changes. Therefore, the variability cost absorbs the impact of penalty weight, and hence predominantly controls the objective function.

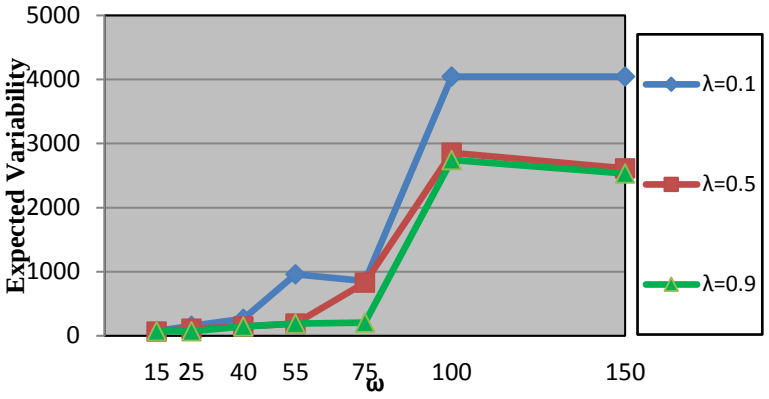

Figure 3.1: Variability analysis over constant λ

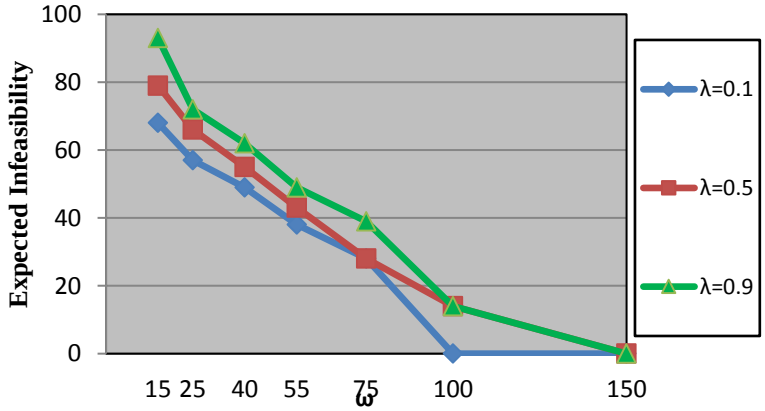

Figure 3.2: Infeasibility analysis over constant λ

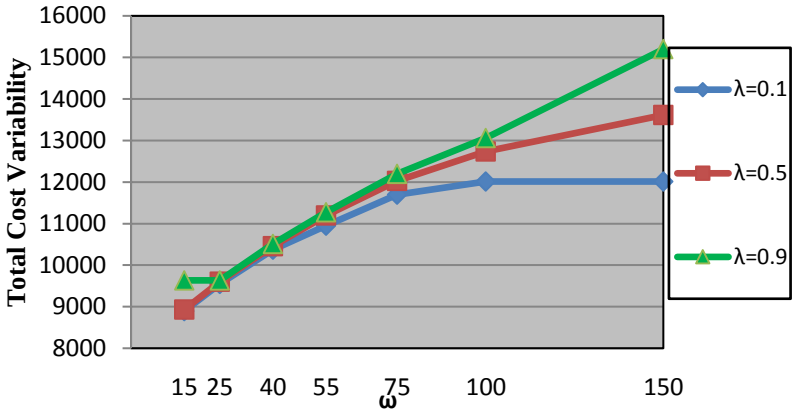

Figure 3.3: Total cost analysis over constant λ

Figure 3.2 shows the trend of infeasibility for different value of ω over $\lambda = 0.1, 0.5,$ and 0.9, respectively. It is quite obvious that as the penalty weight increases, the infeasibility decreases for all degree of risk aversion policy. It should be noted that as the variability cost increases the value of λ is likely to have a broader effect on the variability changes. Therefore, the infeasibility drops more rapidly as λ increases which illustrates the effect of risk preference level adopted by the decision makers on the total cost variability.

From the graph in Figure 3.3 it is shown that the higher risk aversion the decision makers become in the robustness of the service level as well as total costs of the allocation plans, the greater would be the operational cost of the healthcare systems. However, the speed by which the total costs increases would be the lowest when λ is set on minimum which implies the impact of variability penalty function on the total costs.

Figures 3.4 to 3.6 provide an insight into the importance of a holistic approach for the analysis of the variability, infeasibility, and the total operational cost when ω remains constant over different value of λ. As seen in Figure 4, the expected variability increases when the penalty weight changes for $\omega = 15, 25, 40, 55, 75, 100,$ and 150 while $\lambda = 0.1$. However as λ increases from 0.1 to 0.9, the variability decreases by 7% for $\omega = 15$, 55% for

$\omega = 25$, 44% for $\omega = 40$, 80% for $\omega = 55$, 75% for $\omega = 75$, 32% for $\omega = 100$, and 37% for $\omega = 150$, which illustrates the impact of risk preference level adopted by the decision makers on the total cost variability.

Figure 3.5 depicts the trend of infeasibility changes over different value of λ for $\omega = 15$,25 ,40 ,55 ,75 ,100, and 150. It is clear that the higher the value of ω is set, the lower would be the infeasibility of the control constraints. Figure 3.5 also shows the sensitivity of the expected infeasibility over different value of λ .So, when λ increases from 0.1 to 0.9 the infeasibility increases by 37% for $\omega = 15$, 26% for $\omega = 25$, 27% for $\omega = 40$, 29% for $\omega = 55$, 39% for $\omega = 75$, and remain unchanged for $\omega = 100$ and $\omega = 150$. Therefore, setting a large value for ω offset the impact of increased risk aversion policy and reduces the variability in the objective function.

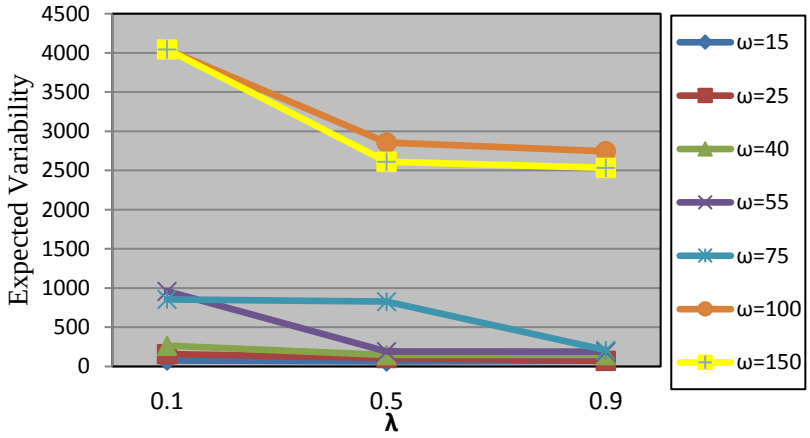

Figure 3.4: Variability analysis over constant ω

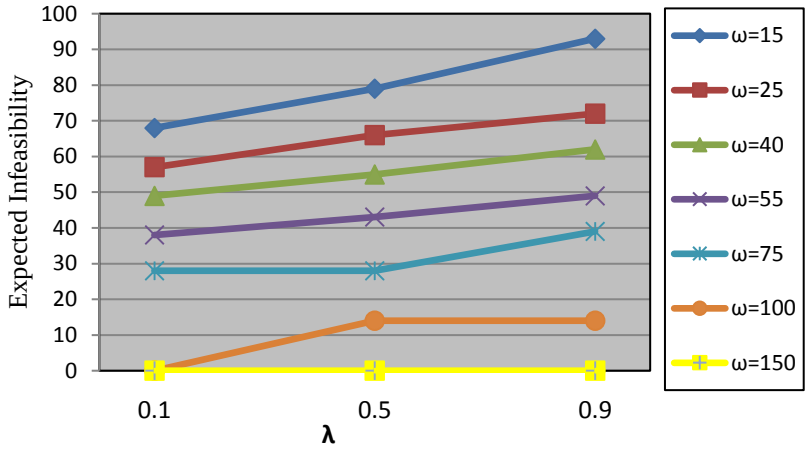

Figure 3.5: Infeasibility analysis over constant ω

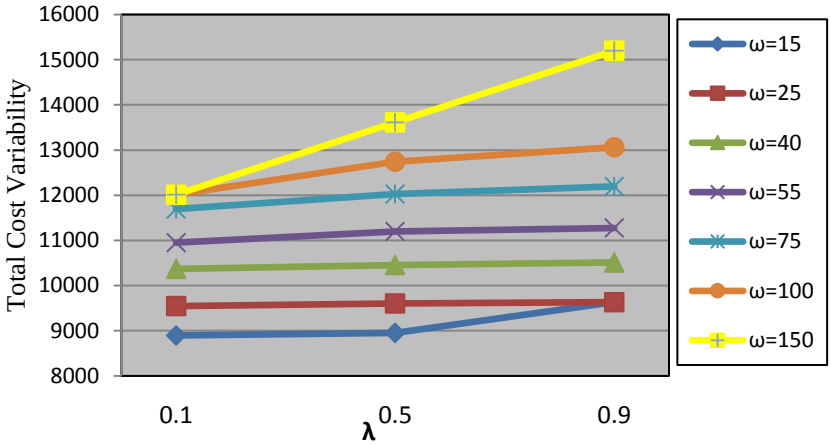

Figure 3.6: Total cost analysis over constant ω

Figure 3.6 shows the trend of total cost over different value of λ for $\omega = 15$, 25, 40, 55, 75, 100, and 150. The development in the total cost also increases by 8% when $\omega = 15$, 1% for $\omega = 25$, 1.5% for $\omega = 40$, 3% for $\omega = 55$, 4% for $\omega = 75$, 9% for $\omega = 100$, and 27% for $\omega = 150$ as the intended variability of λ increases from 0.1 to 0.9. It is noted that the rate of growth in the total cost of the objective function is significantly smaller than changes in the variability and the infeasibility depicted in figures 3.4 and 3.5. This implies the capability of the proposed robust model in providing an affordable optimal solutions with a lower risk for the capacity allocation problems of the hospital.

Figure 3.6 shows the trend of total cost over different value of λ for $\omega = 15$, 25, 40, 55, 75, 100, and 150. The development in the total cost also increases by 8% when $\omega = 15$, 1% for $\omega = 25$, 1.5% for $\omega = 40$, 3% for $\omega = 55$, 4% for $\omega = 75$, 9% for $\omega = 100$, and 27% for $\omega = 150$ as the intended variability of λ increases from 0.1 to 0.9. It is noted that the rate of growth in the total cost of the objective function is significantly smaller than changes in the variability and the infeasibility depicted in figures 3.4 and 3.5. This implies the capability of the proposed robust model in providing an affordable optimal solutions with a lower risk for the capacity allocation problems of the hospital.

Figure 3.7 gives the trade-off between the penalty weight changes and the total expected cost. The process of making the trade-off between solution robustness and model robustness is conceptually based on the RO methodology that allows for infeasibility in the second stage constraints by means of penalty as explained in section 3.2.3. When $\omega = 0$, the violation of the random demand constraints is allowed. Under this circumstance, an unrealistic allocation of OR blocks is advised in the optimal plan which results in maximum infeasibility, which indeed is not an adoptable plan [39]. In Figure 3.7, as the expected infeasibility that represents model robustness decreases, the expected total cost which represents solution robustness goes up. The infeasibility cost of the second stage constraints drops until it becomes zero as the penalty for the violation of the random demand constraints increases. However, the total costs remain steady when the penalty function reaches to a very large value. This in fact indicates the feasibility of the optimal solution for larger values of ω under any realization of the scenario data, although at the expense of a higher total costs.

It should be noted that upon reaching to the steady state situation for the infeasibility of the variable demand (i.e. $\omega \geq 75$), the impact of penalty function dominates the total objective function, and hence no significant reduction would occur in the expected infeasibility. Adopting

the best value of $\omega = 75$ in the proposed robust model, we finally obtain the optimal solution with the total annual OR operation costs of $608,000 CAD that allows for considerable cost savings for the hospital budget. Although the total cost obtained by the proposed RO model increases by 0.7% as compared with the two-stage stochastic recourse programming model, the expected variability decreases significantly by 78.8%. Therefore, it is demonstrated that RO outperforms the stochastic recourse programming on controlling the risks by generating less sensitive capacity allocation plans. As the WRH has experienced a 2.3% deficit in its 2015 annual operational budgets, the proposed robust model is of quite benefits to the hospital managers to control the budget while maintaining the service level.

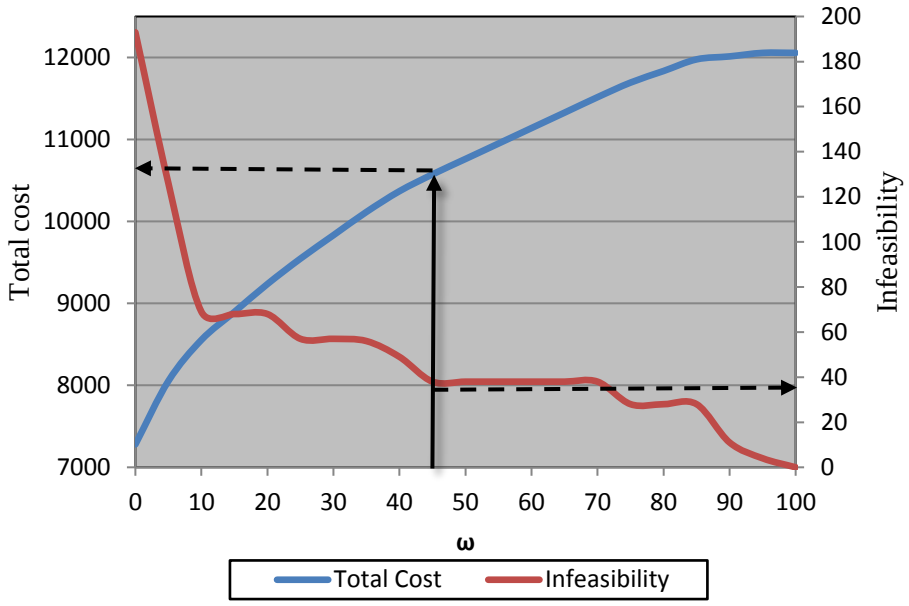

Figure 3.7: Trade-off between solution robustness and model robustness

3.6 Conclusions

The incompleteness of the elective surgery demand data and the randomness arises in the emergency surgery demand make hospital managers to look for allocation plans that fully utilize the resource capacities in order to maintain the required service level. This chapter presents an integrative solution approach to a hospital capacity allocation planning problem on the basis of

two models, including a two-stage stochastic recourse programming model and a robust optimization (RO) model that aims at advancing both resource efficiency and the health service levels. To tackle with the complexity of developing the robust counterpart of the mixed-integer linear programming models a novel transformation framework is proposed to transform a deterministic manner surgery block allocation problem into the RO form and absorb the effect of existing variability within the elective and emergency surgery demands. Three RO models with different variability measures are proposed: the RO model with solution robustness, the RO model with model robustness, and the RO model with trade-off between solution robustness and model robustness to evaluate the operational performance and to analyze the enhancement of the trade-off between efficiency and health service delivery. The computational results of addressing a capacity allocation problem of a real case situation in a Canadian hospital in which the variability of the patient arrivals is included instead of considering a known demand illustrate the advantage of the proposed RO approach over the stochastic recourse programming method in generating more robust block allocation plans and increasing the resource utilization rate while reducing the cost associated with surgery operations for the hospital. An analysis framework is also proposed to select among three RO models based on the risk aversion levels and feasibility consideration of decision makers for the robustness of postponed/unmet demand size (i.e. hospital's service level) and the increased total cost. Furthermore, the analysis of the variability and infeasibility is performed between the proposed RO model and the stochastic recourse programming model for different values of robustness term to compare the performance of those models in controlling the postponement and unmet demand size. The trade-off between the allocation plan's robustness (i.e. postponed surgery and unmet demand variability) and underutilization of OR blocks for different values of robustness is demonstrated that the proposed RO model is progressively less sensitive to the realization of the variable demand, while generating more feasible solutions as compared with the two-stage stochastic recourse programming model.

It should be noted that the approach proposed in this study can be applied on OR planning and scheduling problems in other healthcare systems where the random input parameters are deemed to be a barrier to yield the solid results. Further research will incorporate random surgeon availability as well as the integration of various decision levels to account for more realistic healthcare problems.

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Chapter 4

A Novel Robust Optimization Transformation Framework for Multi-Objective Integrated Master Surgery Schedule and Surgical Case Assignment Problems at a Publicly-Funded Hospital[1]

4.1 Introduction

Operating theatres (OTs) are among the most expensive resources in hospitals. OT management typically needs to take into account numerous factors (e.g., personnel availability, surgical instruments, intensive care units (ICU) availability and ward bed capacity, etc.) and involves the actions of different players, such as surgeons, nurses and patients. Within the operating theatres, managing surgical activities at operating rooms (ORs) can enormously impact the quality of surgery processes undergone by patients as well as the waiting time for patients.

[1] A version of this chapter will be submitted for publication. Lalmazloumian M. , Baki F. and Ahmadi M. A novel robust optimization for multi-objective integrated master surgery schedule and surgical case assignment problem at a publicly-funded hospital.

Due to having highly skilled personal, planning surgical activities also involves a noticeable operational as well as managerial costs that play a great role in the success of a healthcare system. Therefore, effective management efforts to increase performance of the OR departments are always needed.

Making plans for ORs is considered to be a very challenging task due to a number of different perspectives. The operating room department is a quite volatile environment where the uncertainty in patient arrival and surgery duration together with its interactions with other departments in the hospital makes the planning and scheduling a very complex decision [1]. The management of OR departments have been a challenging research topic that draws considerable attention over the last decades. The literature reports on exhaustive reviews [2–8] on operating room planning and scheduling problems which analyze the performance of OT and classifies the different problem versions by using multiple approaches and optimization techniques. According to [9], managing OR departments under the block booking policy can be viewed over three different phases corresponding to three decision making levels as follows:

- **Strategic level** Dividing overall operating room capacity among surgical specialties, which is known as case mix planning, with the aim of minimizing the deviation from target allocation or optimizing the benefits of the scheduled plans.
- **Tactical level** Assigning surgical disciplines to operating room sessions, referred to as the Master Surgical Schedule Problem (MSSP) over a medium-term planning horizon.
- **Operational level** Specifies the daily scheduling for each case, referred to as Elective Case Scheduling (ECS). It is divided into two steps as follows;

- **Advance scheduling:** Assigning each elective surgery to operating room sessions in the planning horizon. This is called Surgical Case Assignment Problem (SCAP).
- **Allocation scheduling:** Determining the sequence of surgeries assigned to specific time intervals for each OR block

The problem in the first stage is a strategic level problem, and it is often referred to as the case mix planning problem (CMPP) [10] with the output being the consumed resources plan for the hospital. The problem in the second stage is a tactical level problem, and it is often referred to as the master surgical schedule problem (MSSP) [5] with the output being a cyclic timetable that determines the specialty associated with each OR session. The third stage problem is an operational problem and determines the assignment of surgical cases within the OR blocks [11], and is therefore denoted as surgical case assignment problem (SCAP). Given the patients' waiting lists and various information on operating room characteristics and status, these problems aim at optimizing several performance measures, including OR utilization, throughput, surgeons' overtime, patient tardiness etc.

The efficient allocation of OR capacities to surgical specialties is a persistent problem in hospitals, especially when flat rate payments for patients based on diagnosis-related groups (DRGs) are taken into account [12]. Under the flat rate payment system, hospitals will only be reimbursed based on a pre-defined model developed by the government to establish a formal link between healthcare providers and quality. Introduction of DRG in the Canadian healthcare system forced hospitals to allocate their resources more economically. It has been emphasized in most of the previously mentioned studies that effective planning and scheduling of the OR department requires an integrated approach that concurrently considers both planning and scheduling problems of a health care system at the same time in order to acquire a combined tactical and

operational levels decision. This integration of different decision levels has been overlooked in previous literatures which we believe is one of the main problems in healthcare environment.

Integrated operating room planning and scheduling, however, has received little attention in the literature due to its intrinsic complexity. Planning and scheduling problems have been typically solved sequentially that can lead to local optimal solutions. According to a recent review made by Ferrand [2] and Van Riet and Demeulemeester [7], there are various and yet conflicting objectives involved in the OR planning and scheduling process resulted from different stakeholder criteria. The tactical and operational problems in OR planning and scheduling context has either been addressed separately in sequence [13], or the focus has been devoted to a single problem of a tactical [11,14–16] or an operational [17–20] level decision. Besides, the inherent variability in various resources impacts the trade-off between hospitals desirability to reduce cost and increase quality of health service to lower the patients waiting time. Therefore, developing an efficient model for simultaneously allocating surgical disciplines to available OR sessions and assigning surgical cases to the allocated operating room is needed.

We exploit an integrated approach to concurrently solve MSS and SCA problem to acquire optimum allocation of surgical specialty (MSS) while the assignment of patients to the OR blocks (SCA) is optimized in order to eliminate the need for changing the OR plan once developed and to obtain the optimal solution for the simultaneous planning and scheduling problem of the OR department. The main contribution of this research is to overcome the challenges rendered by inefficient plans for hospitals and unreasonable service levels for patients through a robust optimization method that address uncertainty exist in the integrated MSS and SCA problems.

4.2 Literature Review

OR planning and scheduling processes is under the influence of numerous conflicting yet interrelated factors, such as number of surgeons, availability of ORs and variety of surgery

operations, that can impact productivity and quality of service in a hospital due to the competing objectives each player will have within the system. It has received considerable attention that resulted in a wide range of research in this context due to its publicity in solving hospital management problems. Literature on both decision levels is wide and growing. Studies about the midterm tactical OR planning and control problems have thoroughly been addressed by [11,15,21–24] while the short term operational OR scheduling problems have also been well taken care of by [18,25–34] using various techniques and the objectives. Beliën and Demeulemeester [35] develop a cyclic master surgery scheduling policy using a mixed integer programming based heuristic approach with the aim of minimizing the total expected bed shortage. Although the inherent uncertainty in surgery demand and patient length of stay has been incorporated, reserved OR capacity was assumed for emergency patients. Denton et al. [53] formulate a stochastic optimization model for allocation of surgeries to OR blocks by minimizing the maximum cost associated with uncertain surgery durations. Tànfani and Testi [11] study the assignment problems of surgical wards to a given set of OR blocks on a planning horizon, taking into account both the urgency and waiting time for scheduled and not scheduled patients. A deterministic binary programming model with heuristic solution approach is developed aiming at minimizing weighted overall hospitalization costs under various resource constraints when the assignment of surgical cases depends on the corresponding expected length of stay of each patient.

There are only a few papers that focus on operational level and simultaneously solve operating room planning and scheduling problems in the literature. Agnetis et al. [51] provide an efficient decomposition approach to address MSS and SCA problem separately. Their model allocates patients to available OR blocks in combination with MSS creation taking into account surgical durations, waiting time, and priority level. Testi and Tanfani [52] develop an integrated MSS and SCA problem through a binary linear programming model where some OR are

dedicated to emergency patients with the aim of maximizing patient throughput taking into account clinical priority of patients. Agnetis et al. [9] develop an integrated integer linear programming model which concurrently solves the MSS and SCA problems to illustrate the effect of allowing flexibility in the model in order to account for the variability in elective surgical demands and priority of patients. They develop a long-term planning model that considers both the quality of solution and the organizational issues within the hospital management. Aringhieri et al. [64] study the allocation problem of OR blocks to surgical specialties together with the assignment of a subset of patients within each time block using a deterministic binary linear programming model. They consider dedicated ORs to cope with the uncertainty resulting from emergency patient arrivals.

Uncertainty is an inevitable part of operational decision in advance scheduling, and hence it impacts the effectiveness of the scheduling mechanism developed for the SCA problems [8]. Landa et al. [65] consider the allocation planning problem of elective surgery combined with sequencing of the assigned cases in each OR block when surgery durations are assumed stochastic with known distributions under a block booking system. Bruni et al. [75] a stochastic recourse programming model is developed to handle the inherent uncertainty characterized by emergency arrivals and surgery durations to adopt optimal scheduling policy of a set of elective patients with priority. A heuristic approach presents to solve the recourse problems aimed at maximizing the hospital revenues. M'Hallah and Roomi [38] develop a stochastic planning and scheduling model for elective surgeries under random surgical times with the aim of enhancing OR utilization rates. They investigate different management strategies through online rescheduling policies to optimize the flow of surgical cases using simulation models. Lamiri et al. [59] develop a stochastic programming model for advance scheduling problem where available OR capacity is shared between elective and emergency patients. They use Monte-Carlo

simulation method to absorb the effect of uncertain emergency demand in order to minimize the cost associated with scheduled elective cases.

As for the techniques, among a wide range of methodologies introduced from the domain of industrial operations research, mathematical programming models and discrete-event simulation tool are the two most commonly used techniques. While the former is utilized in allocating surgical specialties to the ORs over a planning horizon [11,15,21,22,35] as well as assignment of surgical cases to the allocated OR blocks [18,25–34], the latter is generally used to analyze the impact of changes in a resource capacity with the aim of improving existing policies [1,14,36–44]. Some approaches have been designed to treat the surgery scheduling problem as the workshop scheduling problems and hence meta-heuristics are adapted to solve the problems in the healthcare system [27,28,45–51]. With regards to the objective function, most of the studies in the literature attempt to optimize a single objective, such as maximization of operating room utilizations and minimization of the operational costs [1,13,22,24,52–57], while some other researchers consider multiple performance criteria in their study [16,26,32,47,48,54,58–60].

Some studies evaluate procedures based on the RO approach, which is a measure to incorporate variability into account [21,48,78,79]. Addis et al. [79] address a surgical case assignment problem of a set of elective cases with regard to the variability of patients' surgery durations through a cardinality constraint robust optimization approach based on Bertsimas and Sim [80]. They evaluate different level of robustness for the robust model without generating scenarios with the aim of minimizing a penalty associated with the quality of service provided to the patients. Addis et al. [78] elective patients from a set of surgery waiting list are assigned to the OR blocks under uncertain surgery durations using RO approach. The aim is to minimize the penalty associated with patients waiting time and tardiness in a planning horizon. Addis et al. [21] extend their previous work in [78,79] to the rolling horizon approach with the aim of minimizing the overall penalty rendered delay in patient service level. Hans et al. [48] address the problem of

assigning surgical case to the OR blocks to balance the trade-off between maximum OR utilizations and minimum OR overtime. Addis et al. [81] use the cardinality-constrained approach on RO method to determine the optimal assignment of surgery cases to the OR blocks that explicitly take the uncertainty of surgery durations into account where a detailed description of uncertain data is not required.

Nevertheless, studies about the integration of planning and scheduling problems to develop a hybrid tactical and operational model that address combined planning and scheduling problems of health care systems at the same time have just been emerged in the literature and are quite scarce [47,51,52,61–65]. Among all those studies, two major classes of patients that contributes to the complexity of the process are identified as elective and emergency patients. While the surgery dates for the former class is planned well in advance, those of the latter case are needed to be treated urgently, and hence unexpected demand come into play. Considering that emergency cases are among the most important sources of uncertainty that impose extensive variability to the OR department, in this study, we extend the focus to the construction of an effective combined surgery planning and scheduling policy where both elective and emergency categories are involved. The aim of this study is to develop a combined master surgical schedule with a surgical case assignment plan for a healthcare system with the intent of both minimizing patient waiting time and surgeons overtime while the operating room utilization is maximized under uncertain surgery arrivals and durations.

Despite the approach that has been undertaken in previously outlined researches where the MSSP and SCAP was divided into separate stages [13,33,58,66], we consider an integrated planning and scheduling decision where a weekly surgery planning problem is solved by concurrently assigning an OR block to each surgical discipline and the list of surgical cases associated with that specialty to be performed during each OR block in the planning horizon. Comparing with the approaches provided in the literature that simultaneously deal with more than

one planning level, our approach is different from Tànfani and Testi [11] and Agnetis et al. [51] in term of the structure of their single objective function where the focus is either on the cost of patient admission or the score of selected surgery cases in the planning horizon without considering the impact of emergency cases. Our work is also different from the one in Doulabi et al. [62], where it reflects an open scheduling policy, such that operating rooms can be shared between surgery disciplines. This study also contributes to the stochastic MSS and SCA problems in the literature in terms of modeling criteria, so despite Heydari and Soudi [63] we consider patients in the surgery waiting list that require different specialties in a non-identical operating rooms. The main limitation of the above mentioned approaches is that the inherent hierarchy between the decision levels is not considered nor any trade-off between the expected variability and feasibility of the objective function is investigated. Our aim is to extend the current literature in order to generalize existing approaches to obtain a more effective solution methodology to address joint MSS and SCA problem. The contribution of this paper is twofold. The former, more methodological, is to provide an efficient algorithmic framework to solve the joint operating room planning and advance scheduling problem. Our approach accounts for the stochastic nature of the surgical processes, such as the inherent uncertainty of surgery durations and emergency arrivals. The latter, more practical, is to provide a tool to develop robust OR schedules which consider the trade-off between reducing surgery cancellations and postponements while maximizing the operating theater utilization. The approach for generation of integrated MSS and SCA is tested with data from a local hospital in Windsor, which is one of the largest hospitals in southeast Ontario.

4.2.1 Master Surgery Schedule

Operating room planning is under the influence of numerous conflicting yet interrelated factors such as number of surgeons, availability of ORs and variety of surgeries. All those criteria can impact productivity and quality of service in a hospital due to the competing objectives each player can have within the system. While maximizing the utilization of available resources would

be desirable for the hospital managers, surgeons tend to plan surgical procedures relative to their own availability. The complication of those conflicting objectives coupled with the stochastic nature of the OR planning process that exist in surgery durations and demands, surgeon availabilities, and emergency surgical procedure results in the unproductive utilization of subsequent hospital resources and unbalanced planning for the OR department which can cause to cancelation of surgical procedures. The presence of unbalanced OR scheduling can cause fluctuations in the demand for succeeding departments such as intensive care units (ICUs). Therefore, we incorporate the limitation of ICU units in the decision making process to acquire applicable results that suit available constraints of the hospitals.

Master surgery schedule (MSS) refers to an operational planning technique assuming a block scheduling that assigns surgical procedures to ORs over the planning horizon. In other words, in MSS each specialty receives a number of OR blocks (often with a half a day or full day length) in which it schedules its surgical cases [24]. In healthcare environments, MSS generates a cyclic scheduling approach to deal with difficulties of complex planning processes of surgical procedures caused by those competing criteria that leads to a surgical timetable for the hospital that reduces demand fluctuations while increases capacity utilization rates. An MSS develop a cyclic surgical schedule of recurring surgical procedures that have to be performed in each OR in a day, which is referred to OR block in this research. An efficient MSS maximize OR utilization through providing a balanced workload for surgeons as well as the succeeding departments in the hospital such as ICU and surgical wards that results in optimized patient waiting time and throughput, surgeon overtime, as well as cancellations.

Van Oostrum et al. [22] develop a two-phase decomposition approach containing probabilistic constraints to define the mix of elective surgery procedures to be performed and schedule the surgery types under stochastic surgery durations. They employ a column generation approach as a solution method aimed at minimizing required OR capacity and leveling of hospital

bed requirements. They further investigate the effectiveness of the MSS approach in solving OR planning and scheduling problems in Van Oostrum et al. [24] and demonstrate its applicability is solving hospital problems aiming at single or multiple patient groups.

Santibáñez et al [67] discuss various trade-offs in allocation of the operating room to surgical specialties using a deterministic mixed integer programming model considering the availability of ORs and post-surgical resource constraints. They introduce planned buffers to absorb the effect of inherent uncertainty aiming at comparing different objectives including minimizing variation in OR utilization rates, maximizing throughput of patients or leveling the bed occupancy of downstream units. Fei et al. [16] use a column-generation heuristic approach to solve operating room planning problem under an open scheduling policy where surgeons are assigned to the available operating rooms based on a first-come-first-serve (FCFS) basis in order to maximize the OR efficiency. They compare multiple criteria such as overtime, cost and OR utilization with the aim of minimizing total cost of operations.

Beliën et al. [68] expand their previous study in Beliën et al. [35] and develop a decision support system for a deterministic MSS of a set of elective patients using a mixed integer programming and a simulated annealing approach. The focus of their work is devoted to developing a weighted multi-objective function in which weights can be adapted based on the importance of each criteria by the management in order to balance the bed occupancy in downstream units. Expanding previous research, Fügener et al. [15] consider an MSS problem that employs a stochastic heuristic approach that includes multiple downstream resources containing patient occupancy distributions in the surgical wards and the intensive care units (ICU) into the model with the aim of minimizing total costs.

Denton et al. [53] formulate a stochastic optimization model for allocation of surgeries to OR blocks by minimizing the maximum cost associated with uncertain surgery durations. They

compare the results of two-stage stochastic recourse programs and a robust optimization model with a heuristic model where emergency arrivals are treated through a reserved capacity upon their realization. Tànfani and Testi [11] study the assignment problems of surgical wards to a given set of OR blocks on a planning horizon, taking into account both the urgency and waiting time for scheduled and not scheduled patients. A deterministic binary programming model with heuristic solution approach is developed aiming at minimizing weighted overall hospitalization costs under various resource constraints when the assignment of surgical cases depends on the corresponding expected length of stay of each patient.

Mannino et al. [69] formulate a pattern based mixed-integer programming model to develop a robust cyclic master surgery schedule under demand uncertainty. They propose a model that focuses on minimizing patient queue lengths among surgical specialties as well as the use of overtime under a finite number of resources. Banditori et al [37] consider a mix of patients on the waiting list with the homogeneous surgery resource requirement to develop the MSS plan aimed at minimizing under and over utilization of resources. They employ a simulation optimization model to evaluate the robustness of the planed MSS against the variability of both surgery duration and patient length of stay to manage the trade-off between robustness in planning surgeries and efficiency of the actual surgery mix. Investigating the value of efficiency, balancing, and robustness by implementing alternative scheduling policies, Cappanera et al. [14] utilize a combined optimization-simulation approach through a mixed-integer linear programming and a discrete-event simulation model to address the problem of allocating surgical specialties to the OR blocks. They employ probability distribution to model the stochasticity of surgery durations and patients' length of stay.

Holte and Mannino [70] develop an adjustable robust scheduling model to handle the uncertainty of the patient demands in a cyclic surgery allocation problem. They build a robust MSS that accounts for various scarce medical resources aiming at minimizing the queue length of

patients where a column generation algorithm is utilized as a solution approach to solve the model.

There are only a few papers that focus on operational level and simultaneously solve operating room planning and scheduling problems in the literature. Agnetis et al. [51] provide an efficient decomposition approach to address MSS and SCA problem separately. Their model allocates patients to available OR blocks in combination with master surgery schedule creation taking into account surgical durations, waiting time, and priority level. They compare the solution provided for the two problems in sequence with those obtained by an exact integrated approach.

Testi and Tanfani [52] determine the allocation of OR block times to surgical specialties, and the assignment of elective patients in each block time. They develop an integrated MSS and SCA problem through a binary linear programming model where some OR are dedicated to emergency patients with the aim of maximizing patient throughput taking into account clinical priority of patients. Discuss a setting, applying the dedicated ORs to non-elective patients, Agnetis et al. [9] develop an integrated integer linear programming model which concurrently solves the MSS and SCA problems to illustrate the effect of allowing flexibility in the model in order to account for the variability in elective surgical demands and priority of patients. They develop a long-term planning model that considers both the quality of solution and the organizational issues within the hospital management. Aringhieri et al. [64] study the allocation problem of OR blocks to surgical specialties together with the assignment of a subset of patients within each time block using a deterministic binary linear programming model. Like previous studies, they consider dedicated ORs to cope with the uncertainty resulting from emergency patient arrivals and propose a metaheuristic algorithm solution approach with the aim of minimizing the costs associated with waiting time and weekend stay beds required by surgery planning.

4.2.2 Advance Scheduling

Advance scheduling is about methods to schedule patients on a surgery waiting list in advance and is referred to the decision of operating room planning problems at an operational level that consists of the assignment of a surgery date and OR block to a set of patients in a surgery waiting list to be operated over a planning horizon [65]. Since advance scheduling is decomposed into the assignment problem of surgery cases into the OR blocks, it is usually referred to as a surgical case assignment problem (SCAP). Depending on the complexity of methods developed in advance scheduling, the assignment of patients could generally be under multiple resource constraints, including available OR times, surgery teams, and equipment. Block booking, which entails reserving blocks of OR time for individual surgical specialties, has evolved in the literature as an effective mean to address SCA problems when various resource constraints are involved [5].

Fei et al. [16] formulate a deterministic column-generation-based heuristic approach to address the assignment problem of elective surgery cases in the OR blocks. The integer programming model developed under operating room and surgeon availability constraints with the aim of minimizing the costs involved in under (over) utilization of OR blocks.

Molina et al. [61] model their optimization problem as integrated advanced scheduling with the determination of the sequence of surgeries for each OR block when surgery duration is a function of the surgeon's level of experience under an open scheduling policy taking into account resource availability constraints. They propose a weighted multi-objective which maximize the number of scheduled surgical cases and their idle time. Marques et al. [71] consider a joint advance scheduling problem with a surgical case, sequencing for elective patients through an integer linear programming (ILP) model to maximize the OR utilizations. They develop a heuristic approach to improve the quality of the optimal results in order to compare them with the actual scheduling performance policies. Wang et al. [18] do a cross-comparison through

developing mixed-integer linear programing and constraint programming models to solve a surgical case scheduling problem. They take into account the applicability of each model in coping with various human and resource constraints.

Ferrand et al. [72] optimal OR blocks are allocated to elective and emergency cases through investigating different policies for dedicated and flexible resources in order to obtain a trade-off between productivity of OR resources and patient waiting times. They develop various simulation models to evaluate various resource allocation configurations that result in improved patient service level and enhanced efficiency.

In some studies, assignment of surgical cases to operating rooms present similar to the job shop makespan problems [58,73]. Jebali et al. [58] tackle both advance scheduling and sequencing of individual cases for elective patients through a hierarchical two step problem. Assuming patents in the waiting list with equal priority, they develop a mixed integer linear programing model with the aim of developing a surgical plan that maximizes the operating room utilization rates. Pham and Klinkert [73] address scheduling problem of elective cases through a job shop approach. A mixed integer linear programming model is developed to determine optimal scheduling plans a set of surgical cases that maximize resource utilization.

Molina et al. [74] multiple heuristics is presented to solve assignment problem of prioritized surgical cases on a waiting list to the OR blocks. They focus on reducing the effect of surgery postponement and to provide effective tools for the management to perform what-if analysis for determining the optimal strategy.

Uncertainty is an inevitable part of operational decision in advance scheduling, and hence it impacts the effectiveness of the scheduling mechanism developed for the SCA problems [8]. Therefore, the method that is being used to capture the existing variability in unexpected emergency arrivals and/or surgery durations is of paramount importance in the optimum results

achieved from the scheduling systems. We utilize RO approach to tackle the uncertainty at the operational levels due to its advantage in considering the higher moment of the expected optimal values when variability is involved.

Landa et al. [65] consider the allocation planning problem of elective surgery combined with sequencing of the assigned cases in each OR block when surgery durations are assumed stochastic with known distributions under a block booking system. Introducing different scenarios, they use Monte-Carlo simulation to capture the variability of the uncertain surgery duration with the aim of maximizing OR utilization rate. Bruni et al. [75] develop a stochastic recourse programming model to handle the inherent uncertainty characterized by emergency arrivals and surgery durations to adopt optimal scheduling policy of a set of elective patients with priority. A heuristic approach presents to solve the recourse problems aimed at maximizing the hospital revenues.

Min and Yih [76] address surgery scheduling problems of elective patients under surgical facility constraints and uncertain surgery demands. A stochastic dynamic programming model is developed to demonstrate the impact of patients' priority on the scheduling policy. The authors then extend their work in Min and Yih [25] to incorporate the uncertainty in surgery durations and availability of downstream resources in the model. They develop a stochastic mixed integer linear programming model where the capacity used by non-elective cases are considered a random variable to schedule multiple surgery cases with various priorities on a given planning horizon. A similar exercise for scheduling emergency cases is described by Rachuba and Werners [26]. They integrate the uncertainty of surgery durations and emergency arrivals into a scenario-based mixed-integer optimization model to develop a robust surgery scheduling with the aim of minimizing patient waiting time and the number of referrals.

M'Hallah and Roomi [38] develop a stochastic planning and scheduling model for elective surgeries under random surgical times with the aim of enhancing OR utilization rates. They investigate different management strategies through online rescheduling policies to optimize the flow of surgical cases using simulation models.

The impact of combination of uncertainty in surgery durations and patient length of stay in downstream resources on surgery scheduling is investigated by Niu et al. [34]. They propose a two stage stochastic programming model where the assignment of patients to operating room blocks is realized as first stage decision variables, while overtime and undertime of each operating room and the utilization of ICU and ward beds are the second stage decision. A sample average approximation is employed to solve the planning problem aimed at minimizing patient-related costs and expected resource utilization costs. Duma and Aringhieri [77] address an online surgery process scheduling problem where both elective and emergency patients are considered using a hybrid heuristic simulation optimization method. The main trade-off in their work in between the number of cancellations and OR utilizations while the patient quality of service is maximized.

Lamiri et al. [59] develop a stochastic programming model for advance scheduling problem where available OR capacity is shared between elective and emergency patients. They use Monte-Carlo simulation method to absorb the effect of uncertain emergency demand in order to minimize the cost associated with scheduled elective cases. Some studies evaluate procedures based on the RO approach, which is a measure to incorporate variability into account [21,48,78,79].

The rest of this paper is organized as follows. In section 3, we describe the problem of designing an integrated OR planning and scheduling under uncertainty. We propose a 0-1 programming model for the weekly operating room planning and advance scheduling problem in

section 4. After that, a two-stage stochastic programming model is then derived to perform the computational experiment. Then, a novel robust optimization framework is utilized to transform the deterministic model into the robust counterpart for the hybrid MSS and SCA problem and solve by robust optimization technique to determine the final allocation of OR block times together with the assignment of the surgical cases that have been assigned to a day in the planning horizon. Section 5 is devoted to the discussion and analysis of extensive numerical results collected from Windsor Regional Hospital, a local hospital in Windsor Ontario, to evaluate the performance of the proposed algorithm and demonstrate the capability of the utilized method. The paper is completed with some conclusions and future directions in section 6.

4.3 Problem Statement

Operating room planning problems are generally implied dealing with strategic, tactical, and operational decisions [82]. We focus on an integrated tactical and operational decision where the ORs capacity has already been fixed. The problem herein addressed is that of determining master surgery schedule (MSS) which is about the development of a cyclic timetable that determine the surgical units associated with each OR block of time (i.e. session), while addressing the surgical case assignment (SCA) problem which is described as a general assignment problem aimed at reducing costs associated with patients waiting time, surgeons over (under) time and OR capacity disruption [11].

We develop a model that simultaneously considers both midterm cyclic timetable of OR planning and short-term assignment of patient scheduling decision instead of taking them into account in successive phases. The integration of planning and scheduling levels provide some stability, in terms of repeatability of personnel schedules and predictability of bed occupancy pattern in post anesthesia care units (PACU) as well as flexibility, in terms of adaptability of weekly plans to the changing waiting lists for the decision makers. In this context, the operational decision herein addressed is about allocating a set of OR block times, available in a given

planning horizon, to a set of surgical specialties (i.e. the MSSP), together with scheduling a number of patients belonging to a waiting list to each allocated OR block time (i.e. the SCAP). We seek the trade-off between higher service capacity, which will reduce the waiting time as well as OR productivity due to under (over) utilization, and a lower capacity that result in postponement as well as ORs overtime. Our approach is demonstrated to improve patient satisfaction through reducing prioritized weighted waiting times and also improving health care efficiency by reducing overall operation costs, and hence has more societal benefits for the hospitals.

Although our proposed model considerably increases the quality of resulting plans, it will indeed be used at the expense of greater complexity in the solution methodology. However, that difficulty is reduced adopting the inherent hierarchy between the two decision levels, such that the allocation of OR block times to the specialties has direct influence on the assignment of patients to the OR sessions, but not the contrary. We also adopt the idea proposed by Tànfani and Testi [11] of utilizing societal expenses in the objective function yet stretching out with a multi-objective that incorporates both the cost associated with weighted waiting time and postponement of patients as well any deviation from surgeons' utilizations. Finding an optimal integrated MSS and SCA plan for the hospital cannot be compatible with management criteria if the uncertainty inherent in that environment is not taken into account. A 0-1 integer linear programming model is formulated to develop the stated hierarchy between decision levels. We develop a novel robust optimization (RO) framework [88] to deal with the complexity of the challenging problem and absorb the effect of inherent uncertainties. We demonstrate through extensive numerical experiment carried out on a large set of instances based on real data that the proposed RO framework can generate an effective surgical plan and analyze the impact of alternative management policies on the optimal solutions through the incorporation of risk aversion level undertaken by the management into the system. Our RO model can also be used as a tool for

analysis of different scenarios realized by the decision makers. We will show that the proposed RO approach produces near optimal solutions in a limited computer time, which is of great concerns when the problem of medium-to large OR departments is considered.

Given a set of surgical specialties, a list of patients waiting to be operated on for each specialty and a number of available OR time blocks to be assigned to each specialty, we address the problem of determining for a given planning horizon of one week: (1) the cyclic timetable that gives for each day of the planning horizon the assignment of specific OR time blocks to specialties, referred to as MSSP; together with (2) the surgery date and operating room assigned to each patient selected to be operated on, referred as SCAP. The available OR block times imply an OR that filled with complete personnel resources such as surgeon, nurses, anesthesiologists, etc., and equipped with all necessary devices. We consider two types of patients, including elective and emergency patients. While surgery time is scheduled in advance for the former group and hence can be postponed to a later period, the surgery for the latter group is realized without any plan and thus emergent. Since there exists sufficient resource in the recovery ward, the recovery rooms are not considered a bottleneck in this research work.

Two methods have been discussed in the literature as effective ways to cope with the inherent uncertainty of emergency arrivals [63]. While the first approach dedicates some ORs to the emergency cases [83], the second method offers shared ORs between elective and emergency patients to better utilize the resources [84]. Unlike the dedicated strategy, sharing OR capacity in the block booking policy might increase the flexibility for dealing with unexpected long surgery durations and emergency arrivals through adopting of the overflow principle. In this work, the given OR block times are devoted to both elective and emergency surgeries, while priority is given to the latter group due to the urgent nature of non-elective cases.

This research also considers a weighted multi-objective RO approach, which integrates optimization modules that take into account the number of scheduled surgeries, the waiting time, and tardiness of each patient associated with patient urgency factors, and weighted resource utilization rates. The multi-criteria objective function is focused on conflicting resource perspectives as well as patient perspectives at the same time. While the former is accounted for matching OR session capacity and surgical demands and hence enhance the utilization, the latter is related to having the surgery cases done within the respective due dates and thus increase the quality of service. Therefore, the objective function of the overall problem is intended to minimize the number of postponed surgeries among patients within the planning horizon to incorporate the multi-criteria nature of the advance scheduling problems into consideration. One novelty of the objective function here introduced is using a provincial guideline based on the regulated wait time target prescribed by Ministry of Health and Long-Term Care in Ontario. It has mandated the maximum length of time within which a patient should be treated in order to manage patient admission that weights the chronological waiting time with the urgency coefficient of the corresponding Urgency Related Group (URG) of each patient [89]. The wait time targets are developed with the help of clinical experts and serve as a method of accountability and provide a goal to achieve. These targets include urgency classifications and are incorporated in the regulated Wait Times Information System (WTIS). Unlike the previous researches where the number of treated patients is the main criteria, the importance of our approach is on reducing the amount of welfare loss caused by clinical deterioration or other negative consequences related to excessive waiting time within the planning horizon.

On the waiting list, patients are ranked according to their urgency coefficient factor. According to the provincial guideline, there are six different urgency group defined as L1 to L6 for which there exists a maximum time before treatment regulated by the Ontario government as 1 week, 2 weeks, 4 weeks, 6 weeks, 12 weeks, and 26 weeks, respectively. The urgency

coefficient of the patient is then defined to correspond to that factor as 26, 13, 6.5, 4.3, 2.2, and 1, respectively for each category. Some disciplines have a set of non-availability ORs. For instance, general surgery cannot be performed in OR r = 5, and orthopedic surgeries have to be performed either in room r = 3, 4, or 5. Nevertheless, these ORs are not exclusively assigned to these disciplines. There are a total number of 10 operating rooms available in the hospital under current setting. All the ORs are assumed to be identical.

Waiting time of patients on the waiting list are recorded at admission time to measure the time they spend before the required surgeries is received. The objective function in the model is derived from the performance indicator employed by the Windsor Regional Hospital (WRH) in Ontario (Canada), and it is intended to minimize a societal impact of the clinical weight related to the urgency factor of surgery operations on the hospital setting systems and reducing the deviation from optimal utilization of resources. The clinical weight depends on the linear combination of the priority of the surgery and the number of days per patient spent on the waiting list at the time.

The planning decision is subject to many resource constraints related to OR session length, available surgeons within each surgical specialty, the maximum overtime session allowed by the current collective labor agreement and hospital budget constraints, OR hours reserved for emergency cases, number of ICU beds, and available OR equipment. We assume all patients on the waiting list will be operated on the planning horizon. Therefore, we are concerned with the problem of selecting the subset of patients to be operated on each OR session such that the cost associated with patient waiting time is minimized. Moreover, we assume following restrictive assumptions in order for the solutions to be attainable and realizable in a realistic environment. In what follows, we assume that: (i) in each planning horizon there are same number of ORs and the number and length of OR sessions available for elective surgery are constant; (ii) OR sessions cannot be shared among surgical specialties; (iii) a block scheduling approach is followed; (iv)

emergency arrivals are handled along with elective cases through reserved OR sessions within the planning horizon; (v) uncertainty is considered in emergency arrivals and surgery durations within the planning horizon.

Both expected surgery durations and emergency arrivals are forecasted based on historical data and patient characteristics. A mixed-integer linear programming model is first developed where the uncertainty considerations are excluded. The deterministic model is then transformed into a two stage stochastic recourse programming model as well as a robust optimization (RO) model to incorporate the impact of uncertainty into the decision making process. A novel RO framework allows to exploit the potentialities of a linear programming model without requiring to know the probability density functions of the uncertain parameters. It requires only limited information and few general assumptions which is a realistic limitation in many real-based application [80]. We propose the details of the deterministic and robust formulation of the problem and afterwards the models are analyzed over a set of real life based instances to evaluate their behavior in terms of computational effort and solution quality. The solution quality is also reflected in the total weighted waiting time of the operated patients and the number of postponed cases. Moreover, assuming lognormal distributions for the surgery durations and a Poisson process for emergency arrivals, a set of randomly generated scenarios is used in order to compare the proposed solutions in terms of OR utilization rate and number of postponed patients. The impact of introducing overtime in the model formulation is evaluated and a sensitivity analysis on the choice of the key parameters is performed.

The contribution of this research is twofold. The former, more methodological, is to provide an efficient robust optimization (RO) framework to solve the joint MSS (i.e. planning) and SCA (i.e. scheduling) problem taking into account the inherent uncertainty of surgery durations and emergency arrivals. The latter, more practical, is to provide a tool for decision

maker to develop a robust offline OR schedules which consider the trade-off between reducing surgery cancellations and/or postponements and maximizing the operating theater utilization.

4.4 RO Model for Integrated MSS and SCA Problem

In order to develop an initial schedule that explicitly considers the preferences of different interest groups we propose a basic multi-criteria optimization model. This MC-MILP is extended to a scenario-based approach in Section 5. Basically, there are three essential types of constraints that ensure (1) capacity utilization, (2) emergency reservation and (3) feasible days for surgeries. Minimizing each goal separately obviously leads to three different schedules. We study the impact of these individual schedules in a multi-criteria context aiming at finding a balanced solution that is good with respect to every goal. The patient's waiting time decreases with a higher workload of staff which results in a higher overtime. If overtime is increased, this also leads to a lower number of deferrals. Reducing deferrals avoid scheduling patients on the last day of the planning horizon, whereas reducing waiting time implies scheduling patients as early as possible. The overall goal of our approach is to level the utilization of the different objectives at a high level and thus close to their individually optimal solutions.

4.4.1 Notations

Parameters associated with the problem size and data are firstly defined as follows.

4.4.1.1 Index sets

r Set of operating room type $(r = 1, \ldots, R)$

i Set of patient $(i = 1, \ldots, I)$

S Set of surgical specialty $(s = 1, \ldots, S)$

t Set of planning horizon $(t = 1, \ldots, T+1)$

ι Set of objective function $(\iota = 1, \ldots, \mathrm{I})$

ε Set of Scenarios $(\varepsilon = 1 \ldots, \Xi)$

4.4.1.2 Deterministic parameters

d_i Elapsed days since referral of patient i for surgery

ρ_i Urgency coefficient of patient i in days

B_s Subset of patient belong to specialty S

o_s Number of surgeon available for specialty s in the planning horizon

k_{rt} Available capacity (hours) for surgery in OR r on day t

K_{rt} Maximum available capacity (hours) for surgery in OR r on day t

$V^{\max}$ Maximum daily number of operating rooms that can reserve OR hours for emergency cases

β_t Number of available ICU beds on day t

N Subset of OR blocks not available for specialty s if $(s,r) \in N$

η_i 1 if patient i is expected to need ICU bed after operation; 0 otherwise

α_{rt} 1 if OR r is not available on day t ; 0 otherwise

w_l Weights of the objective function l given by the decision makers

$\psi^{+/-}$ Weighting factor for over (under) time of specialty

$\sigma^{+/-}$ Weighting factor for over (under) utilization of OR capacity

λ Weighting scale to measure the trade-off between risk and expected outcome

ω_1, ω_2 Weighting penalty to trade-off solution for model robustness

4.4.1.3 Stochastic parameters

p_i^{ε} Stochastic elective surgery duration of patient i under scenario ε

u_t^{ε} Stochastic emergency arrival time on day t under scenario ε

4.4.1.4 First stage decision variables

x_{irt} $\begin{cases} 1 & \text{If patient } i \text{ is assigned to OR } r \text{ on day } t \\ 0 & \text{Otherwise} \end{cases}$

y_{srt} $\begin{cases} 1 & \text{If specialty } s \text{ is assigned to OR } r \text{ on day } t \\ 0 & \text{Otherwise} \end{cases}$

v_{rt} $\begin{cases} 1 & \text{If operating room time is reserved for emergency cases in OR } r \text{ on day } t \\ 0 & \text{Otherwise} \end{cases}$

g_{rt} Operating room hours reserved for emergency surgery in OR r on day t under scenario ε

4.4.1.5 Second stage decision variables

χ_{srt}^{ε} Surgery demand of specialty s that cannot be met in OR r on day t under scenario ε

ϕ_{srt}^{ε} Undersupply of OR block times allocated to specialty s in OR r on day t under scenario ε relative to its desired level

$\zeta_{rt\varepsilon}^{+}$ Over-utilization hours of overall capacity of OR r on day t under scenario ε

$\zeta_{rt\varepsilon}^{-}$ Under-utilization hours of overall capacity of OR r on day t under scenario ε

μ Expected value of the second stage cost being made after realization of the random variable is observed

d^{ε} Variability cost of deviation from the mean expected value of the objective function in each scenario ε

θ^{ε} Deviational variable for violation of the mean objective function in each scenario ε

$f^{1}_{srt\varepsilon}$ Deviation variable by which the allocated OR block to specialty s in room r on day t can be violated under scenario ε

$f^{2}_{rt\varepsilon}$ Deviation variable by which the OR capacity utilization in room r on day t can be violated under scenario ε

$\gamma^{\varepsilon}_{srt}$ Deviational variable for infeasibility of the random allocated OR block constraint of specialty s in room r on day t under scenario ε

$\delta^{\varepsilon}_{rt}$ Deviational variable for infeasibility of the random OR capacity utilization constraint in room r on day t under scenario ε

4.5 Formulation of Integrated MSS and SCA problems using proposed standard transformation framework

4.5.1 Formulation of the deterministic Integrated MSS and SCA problems

$$Min\, Z_1 = Min \sum_i \sum_r \sum_t^{T+1} x_{irt}(t - d_i)\rho_i \tag{4.1}$$

$$Min\, Z_2 = Min \sum_i \sum_r x_{ir(T+1)} \tag{4.2}$$

$$Min\, Z_3 = Min \sum_s \sum_r \sum_{t=1}^{T+1} \left(\psi^{+}\chi_{srt} + \psi^{-}\phi_{srt}\right) + \sum_r \sum_{t=1}^{T+1} \left(\sigma^{+}\zeta^{+}_{rt} + \sigma^{-}\zeta^{-}_{rt}\right) \tag{4.3}$$

Subject to

$$\sum_r \sum_t^{T+1} x_{irt} = 1 \qquad \forall i \tag{4.4}$$

$$\sum_{i \in B_s} x_{irt} \le M y_{srt} \qquad \forall s, r, t \tag{4.5}$$

$$\sum_s y_{srt} \le 1 \qquad \forall r, t \tag{4.6}$$

$$\sum_r y_{srt} \le O_s \qquad \forall s, t \tag{4.7}$$

$$\sum_i \sum_r \eta_i \cdot x_{irt} \le \beta_t \qquad \forall t \tag{4.8}$$

$$\sum_s y_{srt} \le 1 - \alpha_{rt} \qquad \forall r, t \tag{4.9}$$

$$y_{srt} = 0 \qquad \forall (s, r) \in NS, t \tag{4.10}$$

$$g_{rt} \le M \cdot v_{rt} \qquad \forall r, t \in 1..T \tag{4.11}$$

$$v_{rt} \le \sum_s y_{srt} \qquad \forall r, t \in 1..T \tag{4.12}$$

$$\sum_{r} v_{rt} \le V^{\max} \qquad \forall t \in 1..T \qquad (4.13)$$

$$\sum_{r} g_{rt} \ge u_t \qquad \forall t \in 1..T \qquad (4.14)$$

$$\sum_{i \in B_s} p_i \cdot x_{irt} + \phi_{srt} - \chi_{srt} = k_{rt} \cdot y_{srt} \qquad \forall s, r, t \qquad (4.15)$$

$$\sum_{s} \sum_{i \in B_s} p_i \cdot x_{irt} + g_{rt} - \zeta_{rt}^{+} + \zeta_{rt}^{-} = k_{rt}(1 - \alpha_{rt}) \qquad \forall r, t \qquad (4.16)$$

$$\chi_{srt} \le y_{srt}(\mathrm{K}_{rt} - k_{rt}) \qquad \forall s, r, t \qquad (4.17)$$

$$\zeta_{rt}^{+} \le \mathrm{K}_{rt} - k_{rt} \qquad \forall r, t \qquad (4.18)$$

$$x_{irt}, y_{srt} \in \{0,1\} \qquad \forall i, r, s, t \qquad (4.19)$$

$$v_{rt} \in \{0,1\} \qquad \forall r, t \in 1..T \qquad (4.20)$$

$$\chi_{srt}, \phi_{srt} \ge 0 \qquad \forall s, r, t \qquad (4.21)$$

$$\zeta_{rt}^{+}, \zeta_{rt}^{-} \ge 0 \qquad \forall r, t \qquad (4.22)$$

$$g_{rt} \in \mathrm{N}_{\circ} \qquad \forall r, t \in 1..T \qquad (4.23)$$

The objective function in (4.1) minimizes total patients' waiting time as the sum of urgency coefficient of each patient at the time of planning multiplied by the elapsed days of that patient since its referral date. The second goal (i.e. 4.2) ensures surgical postponement is minimized to achieve a high level of OR utilization and service level. The third objective (i.e. 4.3) minimizes the loss incurred due to utilization disruptions and the amount of over (under) time for the planning horizon in order to provide a balance in surgeons' workload and to comply with their collective agreement.

Constraints (4.4) states that each patient can be admitted at most once. Constraints (4.5) ensures a patient of a surgical specialty can only be assigned to a compatible OR time block subject to allocation of adequate OR sessions to that specialty in the planning horizon. Note that M represents a large integer value that suitably defined to make the constraint non-binding whenever $y_{srt} = 1$. It can be set to the maximum number of surgeries that could be performed in the longest OR time block across all specialties and all day of the planning horizon. For example, in a context where the shortest surgery would be 30 min and the longest time block 11 hours so 660 minutes, a suitable value would be $M = 22$.

Constraints (4.6) guarantee that OR sessions are not split among surgical specialties, i.e. there are no two surgical specialties assigned to the same OR on a given day. It also prevents the OR block times to be shared among surgical specialties. Constraints (4.7) limit the number of OR sessions allocated to a specialty to the surgical teams available for that specialty per day. That typically maintain a balance between the number of parallel OR sessions a specialty can take and the number of available surgery team of that specialty.

Constraints (4.8) limit the number of assigned patients that require ICU beds to the capacity of available ICU beds on a given day. Constraints (4.9) restrict the assignment of patients to the OR blocks during the weekends. Constraints (4.10) restrict certain surgeries to be performed in a certain set of OR blocks, due to size and/or equipment constraints. This constraint is actually a symmetry-breaking constraint to speed up the computations.

Constraints (4.11) and (4.12) limit the number of OR blocks to which emergency reservations can be deducted. Since the amount of reserved emergency hours in the OR blocks is defined as an integer variable, constraints (4.13) bound the number of operating rooms that emergency reservations can be spread out per day to avoid the assignment of fraction of times to all of the active operating rooms in order to lower the undesirable overtime hours. Constraints (4.14) reserve emergency times for particular rooms to ensure emergency cases are properly being taken care of on a given day.

Constraints (4.15) calculate the deviations from the allocated OR blocks of a given surgical specialty in an aggregated level based on the time for elective patients scheduled in those blocks. It also restricts the usage of OR blocks by specialties to develop a feasible and thus executable plan. This establishes an upper bound for the duration of surgical cases that can be assigned to the same sessions to maintain a balanced workload in the health care system. The aggregated level provided through the equations ensures the block time can only be utilized by

patients belonging to a particular surgical specialty. Constraints (4.16) ensure capacity utilization of surgical OR blocks. Treatment of emergency cases is considered separately through reservation of OR block times to deal with randomly occurring emergencies and to reduce the allocated block time. A limited amount of over utilized hours is permitted to handle variation in emergency hours and surgery durations.

Constraints (4.17) limit the deviation from the surgery block allocation plan to a maximum level. The positive deviations from the daily capacity utilization, i.e. overtime, is limited to a maximum allowable amount through constraints (4.18). Constraints (4.19) and (4.20) ensure the binary property of the decision variables. Constraints (4.21) and (4.22) ensure the non-negativity of the deviations from allocated OR block times as well as the overall daily capacity of operating rooms, respectively. Finally, with constraints (4.23) the reservation time for emergency treatments is an integer.

As can be seen from these formulations, there exists a clear hierarchy between decision levels and therefore between model variables: variable y (which determine the allocation of OR block times to surgical specialties) have a strong impact on variable x (which assign individual patients to a particular OR block time) but not the reverse. Considering the stochasticity of the surgery durations and emergency arrivals, in the following an integrated MSS & SCA model is developed under both deterministic as well as the stochastic environment.

4.5.2 Formulation of the two-stage stochastic recourse programming model for Integrated MSS and SCA problems

As can be seen in the previous section, the third objective function is under the influence of uncertain variables, and hence needs to be reformulated to capture the impact of stochastic input parameters as follows.

$$Min\, Z_3 = Min \sum_{\varepsilon} \pi^{\varepsilon} \left(\sum_{s} \sum_{r} \sum_{t=1}^{T+1} \left(\psi^{+} \chi_{srt\varepsilon} + \psi^{-} \phi_{srt\varepsilon} \right) + \sum_{r} \sum_{t=1}^{T+1} \left(\sigma^{+} \zeta_{rt\varepsilon}^{+} + \sigma^{-} \zeta_{rt\varepsilon}^{-} \right) \right) \qquad (4.24)$$

Subject to

Subject to the first stage constraints: (4.4) _ (4.13)

$$\sum_{r} g_{rt} \geq u_t^{\varepsilon} \qquad \forall t \in 1..T, \varepsilon \qquad (4.25)$$

$$\sum_{i} p_i^{\varepsilon} x_{irt} \leq \mathrm{K}_{rt} \qquad \forall r, t, \varepsilon \qquad (4.26)$$

$$\sum_{i \in B_s} p_i^{\varepsilon} \cdot x_{irt} + \phi_{srt}^{\varepsilon} - \chi_{srt}^{\varepsilon} = k_{rt} \cdot y_{srt} \qquad \forall s, r, t, \varepsilon \qquad (4.27)$$

$$\sum_{s} \sum_{i \in B_s} p_i^{\varepsilon} \cdot x_{irt} + g_{rt} - \zeta_{rt\varepsilon}^{+} + \zeta_{rt\varepsilon}^{-} = k_{rt}(1-\alpha_{rt}) \qquad \forall r, t, \varepsilon \qquad (4.28)$$

$$\chi_{srt}^{\varepsilon} \leq y_{srt}(\mathrm{K}_{rt} - k_{rt}) \qquad \forall s, r, t, \varepsilon \qquad (4.29)$$

$$\zeta_{rt\varepsilon}^{+} \leq \mathrm{K}_{rt} - k_{rt} \qquad \forall r, t, \varepsilon \qquad (4.30)$$

$$x_{irt}, y_{srt} \in \{0,1\} \qquad \forall i, r, s, t \qquad (4.31)$$

$$v_{rt} \in \{0,1\} \qquad \forall r, t \in 1..T \qquad (4.32)$$

$$\chi_{srt}^{\varepsilon}, \phi_{srt}^{\varepsilon} \geq 0 \qquad \forall s, r, t, \varepsilon \qquad (4.33)$$

$$\zeta_{rt\varepsilon}^{+}, \zeta_{rt\varepsilon}^{-} \geq 0 \qquad \forall r, t, \varepsilon \qquad (4.34)$$

$$g_{rt} \in \mathrm{N}_{\circ} \qquad \forall r, t \in 1..T \qquad (4.35)$$

It should be noted that in the above formulation, the constraints that only involve the first stage decision variables are referred to as the first stage constraints (i.e. constraints (4.4) to (4.13)). Under the first stage decision, the accurate information for the surgery duration and emergency arrivals is not available. In the objective function, Z_1 and Z_2 remain unchanged as they are free of noise. In the third objective function, Z_3, the term

$Min \sum_{\varepsilon} \pi^{\varepsilon} \left(\sum_{s} \sum_{r} \sum_{t=1}^{T+1} \left(\psi^{+} \chi_{srt\varepsilon} + \psi^{-} \phi_{srt\varepsilon} \right) + \sum_{r} \sum_{t=1}^{T+1} \left(\sigma^{+} \zeta_{rt\varepsilon}^{+} + \sigma^{-} \zeta_{rt\varepsilon}^{-} \right) \right)$ represents the expected cost of the two-stage stochastic programming model and determines how well the operating rooms in a hospital are being utilized when the stochasticity of the unknown parameters is realized for various scenarios and hence is the second stage cost,

$\mu = \sum_{s} \sum_{r} \sum_{t=1}^{T+1} \sum_{\varepsilon} \pi^{\varepsilon} \left(\psi^{+} \chi_{srt\varepsilon} + \psi^{-} \phi_{srt\varepsilon} \right) + \sum_{r} \sum_{t=1}^{T+1} \sum_{\varepsilon} \pi^{\varepsilon} \left(\sigma^{+} \zeta_{rt\varepsilon}^{+} + \sigma^{-} \zeta_{rt\varepsilon}^{-} \right)$. The constraints that

consist of both first stage variables and second stage variables are defined as the second stage constraints, i.e. constraints (4.25) to (4.30) in the two-stage stochastic programming model.

4.5.3 The proposed formulation of the RO model with solution robustness (ROM-SR) for uncertain Integrated MSS and SCA problems

As elaborately discussed in [88], we employ our novel transformation framework to develop the RO model with solution robustness for the integrated MSS & SCA problems of the healthcare systems under a set of surgical resource constraints. To capture the deviation from the mean in the RO transformation framework, we define

$$d^{\varepsilon} = \left(\sum_{s} \sum_{r} \sum_{t=1}^{T+1} \left(\psi^{+} \chi_{srt\varepsilon} + \psi^{-} \phi_{srt\varepsilon} \right) + \sum_{r} \sum_{t=1}^{T+1} \left(\sigma^{+} \zeta^{+}_{rt\varepsilon} + \sigma^{-} \zeta^{-}_{rt\varepsilon} \right) \right) - \mu$$

to assess the difference between sum of utilization disruptions of each OR block and the amount of under (over) time associated with each specialty under realization of the scenario sets and their expected value in the two-stage stochastic recourse model (see [88] for further discussion about this). Therefore, the ROM-SR for the proposed integrated model is formulated as follows.

$$Min\ \mu + \lambda \sum_{\varepsilon \in \Xi} \pi_{\varepsilon} \left(d^{\varepsilon} + 2\theta^{\varepsilon} \right) \tag{4.36}$$

Subject to

The first stage constraints: (4.4) _ (4.13)

The second stage constraints: (4.25) _ (4.30)

The integrality, binary, and non-negativity constraints: (4.31) _ (4.35)

$$d^{\varepsilon} + \theta^{\varepsilon} \geq 0 \qquad \forall \varepsilon \tag{4.37}$$

$$\theta^{\varepsilon} \geq 0 \qquad \forall \varepsilon \tag{4.38}$$

The second term in the objective function (4.36) is the expected variability costs for utilization disruptions of the OR blocks and the amount of under (over) times associated with specialties. The term θ^{ε} represents a deviational variable to linearize the objective function and capture the negativity of the variance from the mean as elaborated in [88].

4.5.4 The proposed formulation of the RO model with model robustness (ROM-MR) for uncertain Integrated MSS and SCA problems

Using the standard RO framework developed in [88], $f_{srt}^{\varepsilon 1}$ is defined to capture the infeasibility of the control constraints in equation (4.27) and $f_{rt}^{\varepsilon 2}$ is defined to capture the infeasibility of the control constraints in equation (4.28). Therefore,

$f_{srt\varepsilon}^{1} = k_{rt} \cdot y_{srt} - \sum_{i \in B_s} p_i^{\varepsilon} \cdot x_{irt} + \chi_{srt}^{\varepsilon} - \phi_{srt}^{\varepsilon}$ denotes the random capacity constraints can be violated over some set of scenarios at the amount $f_{srt\varepsilon}^{1}$, where $f_{srt\varepsilon}^{1}$ represents a deviational variable that denotes the difference between allocated OR block times to a surgical specialty and its surgery demand upon realization of uncertain surgery durations. Under operational level,

$f_{rt\varepsilon}^{2} = k_{rt}(1-\alpha_{rt}) - \sum_{s} \sum_{i \in B_s} p_i^{\varepsilon} \cdot x_{irt} - g_{rt} + \zeta_{rt\varepsilon}^{+} - \zeta_{rt\varepsilon}^{-}$ denotes the random utilization constraints can be violated if the difference between surgery durations of patients associated with an operating room and the available OR hours cannot be fulfilled by under (over) utilization hours. The impact of allowing for the infeasibility of the random constraints will be taken into account in the third objective function that contains uncertain parameters as follows.

$$Min\ \mu + \omega_1 \sum_{\varepsilon \in \Xi} \pi_{\varepsilon} \left(\sum_{s} \sum_{r} \sum_{t} \left(f_{srt\varepsilon}^{1} + 2\gamma_{srt}^{\varepsilon} \right) \right) + \omega_2 \sum_{\varepsilon \in \Xi} \pi_{\varepsilon} \left(\sum_{r} \sum_{t} \left(f_{rt\varepsilon}^{2} + 2\delta_{rt}^{\varepsilon} \right) \right) \quad (4.39)$$

Subject to

The first stage constraints: (4.4) _ (4.13)

The second stage constraints: (4.25) _ (4.30)

The integrality, binary, and non-negativity constraints: (4.31) _ (4.35)

$$f_{srt\varepsilon}^{1} + \gamma_{srt}^{\varepsilon} \geq 0 \qquad \forall s, r, t, \varepsilon \quad (4.40)$$

$$f_{rt\varepsilon}^{2} + \delta_{rt}^{\varepsilon} \geq 0 \qquad \forall r, t, \varepsilon \quad (4.41)$$

$$\gamma_{srt}^{\varepsilon}, \delta_{rt}^{\varepsilon} \geq 0 \qquad \forall s, r, t, \varepsilon \quad (4.42)$$

In the objective function (4.39), ω_1 and ω_2 represent the unit penalty for the violation of the random capacity constraints and random utilization constraints, respectively. The term $\gamma_{srt}^{\varepsilon}$ and $\delta_{rt}^{\varepsilon}$ capture the amount by which the control variables are violated and that is to represent a deviational variable which linearizes the objective function and capture the negativity of the infeasibility function as elaborated in [88]. In the objective function (4.39), when the unit weighting parameters increase, the penalty cost associated with infeasibility of the second stage constraints also goes up. Therefore, any deviation from the assigned capacity to surgical specialties or the OR block time utilization rates would result in a higher societal loss for the health care system which leads to the penalty incurs as a result of growing patients waiting time as well as OR overtime. Note that the assignment of specialties and patients to the OR blocks are scenario independent variables and hence do not contain the index ε, however, the index ε is reflected in the allocation of surgical specialties to the OR blocks and utilization of on hand capacity to emphasize the fact that the actual value of these variables only captures after the realization of scenarios in the ROM-MR model.

4.5.5 The proposed formulation of the RO model with the trade-off between solution and model robustness (ROM-T) for uncertain Integrated MSS and SCA problems

RO also provides a degree of flexibility for the decision makers by considering a trade-off between optimality and feasibility. Through this analysis, managers can explicitly realize the possible trade-off associated with the variability of different service levels and the associated expected loss. Therefore, the results obtained from the trade-off analysis align with the level of risk that managers are willing to take. Solving for variability and the infeasibility together, the proposed RO model in this section is formulated to address the hybrid MSS & SCA problems under the stochastic healthcare environment.

$$Min\ \mu + \lambda \sum_{\varepsilon \in \Xi} \pi_\varepsilon \left(d^\varepsilon + 2\theta^\varepsilon \right) + \omega_1 \sum_{\varepsilon \in \Xi} \pi_\varepsilon \left(\sum_s \sum_r \sum_t \left(f^1_{srt\varepsilon} + 2\gamma^\varepsilon_{srt} \right) \right) + \omega_2 \sum_{\varepsilon \in \Xi} \pi_\varepsilon \left(\sum_r \sum_t \left(f^2_{rt\varepsilon} + 2\delta^\varepsilon_{rt} \right) \right)$$

(4.43)

Subject to

The first stage constraints: (4.4) _ (4.13)

The second stage constraints: (4.25) _ (4.30)

The integrality, binary, and non-negativity constraints: (4.31) _ (4.35)

The solution robustness constraints: (4.37) _ (4.38)

The model robustness constraints: (4.40) _ (4.42)

The essential parts of the objective function (4.43), the second term is incorporated to accommodate the mean-variance trade-off over scenarios, and hence the variability cost. The variability is measured in terms of fluctuations in utilization disruptions of the OR blocks and the amount of under (over) times associated with specialties from their total expected values. The deviation from the assignment of surgical specialties and resource utilizations is expressed in the third and fourth term, respectively, and is permitted at a penalty cost (i.e. infeasibility cost). The goal of the objective function (4.43) is to reach to a balance between solution and model robustness.

Since the first two objective functions are free of noise, they are constructed using known parameters and design variables, and hence those are excluded from the robust transformation process. To sum up, it should be noted that the final proposed robust model for the integrated MSS and SCA problems consists of Equations (4.1), (4.2), and (4.43) as the multi-objective functions and Equations (4.4)–(4.13), (4.25)–(4.35), (4.37)–(4.38) and (4.41)–(4.42) as the constraints.

4.6 Solution Procedure

Conventional deterministic optimization approaches for health care planning and scheduling problems are unable to capture the true dynamic behaviors of the real health care systems. Our novel solution approach eliminates that drawback by using a novel RO framework which provides the health care manager with a tool to handle the inherent uncertainty of the hospital environment in a more practical manner. RO is more beneficial than standard probabilistic methods which are mostly hard to implement due to the lack of historical data. Another advantage of our solution procedure is the applicability and effectiveness of the final solutions. While in deterministic approaches one optimal solution is offered for each variable, our proposed RO model generates a near optimal and yet robust plan that remains feasible over a practical range of input values at a predictable but slightly higher cost.

The solution procedure of the proposed multi-objective robust model for the integrated MSS & SCA problems is described as follows.

- In order to cope with the complexity of the conflicting objectives in our proposed robust model, the problem is separated into three individual models such that each includes a single objective function with all of the associated constraints.

- The first model aims to minimize the total loss incurred due to the patients waiting time consisting of Equation (4.1) as the objective function and Equations (4.4)–(4.13) and (4.25)–(4.35) as the constraints.

- The second model aiming at minimizing total unmet demand within the planning horizon includes Equation (4.2) as the objective function and Equations (4.4)–(4.13) and (4.25)–(4.35) as the constraints.

- The third model aims to minimize the total societal loss associated with the OR utilizations. The optimal solution of the third models is to obtain through a trade-off

analysis between expected total costs and expected utilization disruptions of the OR blocks as well as the under (over) time hours of OR associated with specialties. The robust model consists of Equation (4.43) as the objective function and Equations (4.4)–(4.13), (4.25)–(4.35), (4.37)–(4.38), and (4.41)–(4.42) as the constraints. Using the real case study data of a hospital presented in the following section, we discuss the trade-off along with the optimal solutions for the model.

- In the final step, the Lp-Metric methodology is applied. Assuming that Z_1^*, Z_2^*, and Z_3^* are the optimum solution values for the first, second, and a third model, respectively, then $Z_{Lp-Metric}$ is defined as the final integrated objective function as follows.

$$Min\, Z_{LP-Metric} = \sum_{t} w_t \frac{Z_t - Z_t^*}{Z_t^*} \tag{4.44}$$

$$\sum_{t} w_t = 1 \tag{4.45}$$

$$0 \le w_t \le 1 \qquad \forall t \tag{4.46}$$

It is worth mentioning that the main advantage of the Lp-Metric methodology is in its flexibility to investigate various weights for each objective function in order to allow the decision makers to fine tune the projected optimal solutions.

4.7 Numerical example and computational analysis

In this section the proposed algorithms for MSS and SCA problems have been tested and analyzed to obtain computational results from deterministic model, two-stage stochastic programming model, and robust optimization model. Data is provided by a local hospital and represented one year's worth of surgeries. From this data set, we are able to determine the distribution of surgery durations and the emergency arrival rates for each surgical specialty. We have carried out a series of computational experiments to evaluate the impact of the main parameters and components of the algorithms and to verify the computational consistency of the

model. Note that due to the bin packing property imposed by constraints 4.5, the problem being addressed is NP-hard [85]. However, despite the model complexity all the test problems presented in this section have been solved using CPLEX 12.6.3 with the default setting which is executed on a PC Pentium IV 2.66GHz CPU with 4GB RAM with a time limit of 600 s and average optimality gap of around 0.64%.

4.7.1 Case description

To illustrate the effectiveness of the proposed integrated MSS and SCA algorithm for solving a hybrid operating room planning and advance scheduling problem with stochastic surgery durations and emergency arrivals, we have employed data obtained from the Windsor Regional Hospital (WRH) a local hospital sited in Windsor. WRH is a multi-faceted healthcare organization operating from two main campuses in Southwestern Ontario in Canada to provide advanced care in specialized areas that include complex trauma, cardiac care, neurosurgery few to mention supporting over 400,000 people in the community. WRH is budgeted to staff 45-bed inpatient surgical units functioning in 10 operating room theaters and two diagnostic rooms located in two different sites across the County. There are also two emergency departments to provide a range of services to meet the unscheduled and emergency health care needs for clients.

Based on the information from the OR surgery department, WRH provides services in specialties including General Surgery, Urology, Gynecology, Orthopedics, Ear, Nose, and Throat, Dental / Oral Maxillofacial, Plastics and Burns, Ophthalmology, Cardiovascular, and Surgical Oncology. According to its 2014-2015 annual report, WRH is one of the busiest public hospitals in the Southwestern Ontario with the record of 314,469 outpatient visits, 44,418 day surgeries, 28,898 inpatient discharges, and 128,357 emergency department visits per year. The report shows the number of elective patients visit has increased by over 25% during the past five years while the emergency admissions through the emergency department has gone up by more than 65% in

the hospital through the emergency department (Source: http: // www.wrh.on.ca/Site_Published /wrh_internet/Document.aspx?Body.Id=65301).

As a public healthcare provider, the hospital's budget is mostly funded by the provincial programs, and hence, patients are admitted regardless of their financial status. Once a patient is discharged, the hospital is reimbursed based on a predetermined funding model that reflects the need of the patient served by the hospital. This funding model determines the amount of compensation for the healthcare systems based on the services delivered and also the quality of services to the patient populations they serve. It is quite obvious that the hospital would not be compensated for the time that patients are admitted to the hospitals, but waiting for their surgery or clinical services to be provided to them, which is normally referred to as postponement. Therefore, it is crucial from the hospital management point of view to reduce, if not completely eliminate the amount of postponing surgeries in order to decrease costs and increase the throughputs. It is also crucial from the patient point of view to consider the overall patient welfare loss caused by clinical deterioration resulted from excessive waiting. So, the longer the waiting time for patients receiving services, the larger would be the welfare loss for the hospital.

At the time of this study, WRH uses 10 ORs, which are regularly open for 7.33 hours with 7 specialties located in its Metropolitan Campus. The seven specialties are General surgery, Urology, Gynaecology, Orthopaedics, ENT, Dental (OMF), and Plastic surgery, which shares 733 hours of overall OR capacity a week with ICU availability consists of six beds. We use the capacity and demand data of the 2016's fiscal year as our input to the proposed standard RO framework, where both elective surgery durations and emergency arrivals are modeled as uncertain parameters to capture the stochastic nature of the healthcare environments. We have analyzed the archived data on both stochastic surgery durations and emergency arrivals to obtain the required data for input parameters. All input data related to patients' characteristics have been collected from the OR booking department (i.e. electronic surgery durations and emergency

arrivals) on January 2017. In particular, the characteristic of 200 patients on the waiting list is described in Table 4.1, where the first two columns give the number of patients to be operated on by each surgical specialty, while the following columns describe the distribution of patients corresponding to the regulated urgency related group (URG) and expected surgery durations (ESD) for elective cases. These characteristics are obtained from the submitted waiting list at the time of referral. The average distribution of surgery durations of the patients belonging to specialties waiting list is reported in Figure 4.1 where surgery durations are reported in hours.

Based on the historical data on the duration of more than 13,840 consecutive surgical cases performed at the WRH over the year of 2016, We have concluded that the natural logarithms of the surgery durations are normally distributed with a mean of 4.25 and standard deviation of 1.65 hours to model the surgery durations as suggested in [26,86]. That is, the surgery durations are log-normally distributed. The cases represented the elective surgeries and the average duration are taken from the empirical data to create the Log-Normal distribution models for each random variable and then regression model are used to get a standard deviation and generate scenarios from those Log-normal distributions. We also assume the emergency demand interval for each specialty is Poisson distributed between 0 and 4 hours per day [26,40,87], as the historical data is not indicated otherwise.

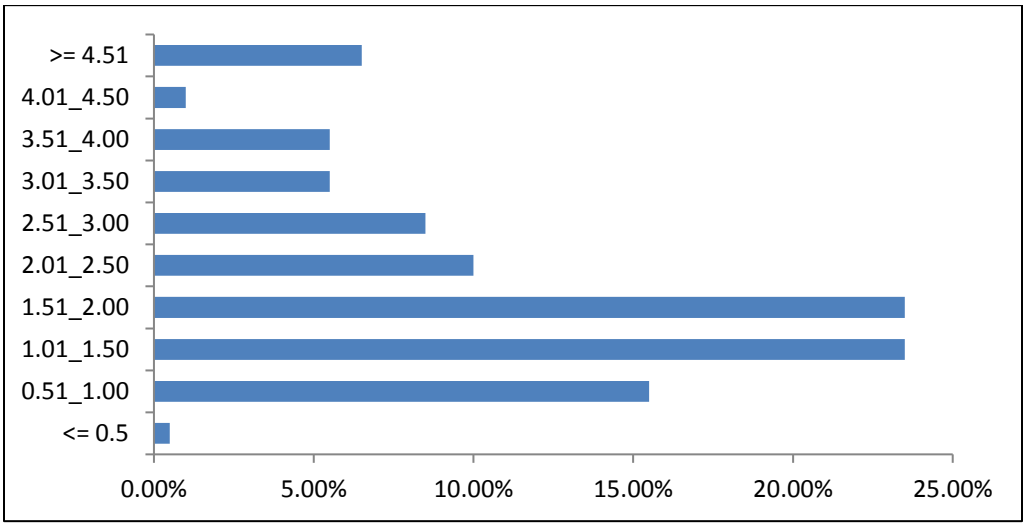

Figure 4.1: Average distribution of patient surgery durations in case study

The hospital will look at a 1-week planning horizon, where no surgery is operated during the weekends. We assume the 7.33 hour operating shift, with possible overtime of up to three hours. We generate random instances to evaluate the performance of the proposed RO model over various probabilities of occurrences. It is assumed that uncertainty is represented by a set of possible surgery duration and emergency arrival situations over Fair, High or Low scenarios. The Low scenario signifies the most optimistic future surgery durations and emergency arrivals and in contrast, the High scenario represents the extremely pessimistic case. The Fair scenario is, in fact, the most expected scenarios where surgery demand and emergency arrivals are realized as planned. According to the principles of robust optimization, the most optimistic and most pessimistic situations should be considered in addition to the fair situation in order to capture the impact of uncertainty. Following the interview with OR manager and on the basis of historical records on these stochastic parameters of the OR department, these scenarios are derived to appropriately cover different situations that happen in reality as a result of the stochastic parameters. In this study, the scenarios, indexed by $\varepsilon = 1,..,3$, include most expected (Fair), extremely pessimistic (High), and extremely optimistic (Low) with associated probability, π_{ε} of 0.6, 0.1 and 0.3, respectively, such that the sum of all four probabilities is equal to 1. Numerous meetings were held with the senior OR analysts of WRH and based on their consensual estimation and prediction of the future surgery durations and emergency arrivals outlook of the hospital, these three scenarios and their occurrence probabilities were established. These scenarios are used to consider a different range of uncertainties in the stochastic data. The characteristics of the models in terms of number of variables, the number of constraints, solution time as well as the optimality tolerance can be seen in Table 4.2.

Table 4.1: Characteristics of the 200 patients waiting for surgery in elective surgery department

Surgical specialty	# patient	URG	# patient	ESD (hours)	# patient	Patient ID	Urgency level
General surgery	76	L1	13	< 0.5	1	1-11, 13, 20	L1
Urology	35	L2	21	0.5 – 1.0	31	15-17, 21-22, 24, 26-35, 48-49, 57, 80, 87	L2
Gynecology	16	L3	62	1.0 – 1.5	47	18-19, 36-45, 12, 14, 23, 25, 47, 93, 103, 123, 134	L3
Orthopedics	22	L4	41	1.5 – 2.0	47	50-56, 58-79, 81-86, 88-91, 137-138	L3
ENT	20	L5	34	2.0 – 2.5	20	44, 94-95, 124, 125, 133, 97-102, 135-136, 139,	L4
Plastic	10	L6	29	2.5 – 3.0	17	104-112, 149, 153, 114-122, 127-131	L4
Dental	21			3.0 – 3.5	11	96, 113, 126, 132, 182,	L5
				3.5 – 4.0	11	141-148, 150-152, 155-170, 172-173	L5
				4.0 – 4.5	2	140, 154, 171	L6
				> 4.5	13	174-181, 183-200	L6

It should be noted that more scenarios would provide more comprehensive results, but given the limitations in accessing the data of the case study, three scenarios would be accurate enough for robust optimization. These scenarios are independent as each of them comprises a different set of data and they are representatives of quite different future outcomes. All the required data were obtained from historical records of the WRH.

Table 4.2: Model size and computation characteristics

Model type	Objective function	No. of variable		Constraint			CPU time (sec)	Gap%
		Binary	Integer	Linear	Equality & non equality	Nonzero		
Deterministic	Min Deter-Lp Metric	11,299	46	870	1637	48,322	600 s	0.31%
Two-stage stochastic	Min Stoch-Lp Metric	11,299	46	2610	3265	95,989	600 s	0.65%
Robust optimization	Min Robust-Lp Metric	11,299	46	5628	4835	108,609	600 s	0.94%

A prioritization algorithm based on clinical need of surgery cases is being used to calculate the welfare loss. It should be noted that welfare loss is incurred by measuring the clinical deterioration or delay in meeting the elective surgeries as they arise due to excessive waiting in a unit of prioritized wait time related loss (PWTRL). The total cost is a function of weighted patient urgency related factor, postponed surgeries and OR utilization. Therefore, patients on the waiting list of surgical specialties are assigned to the OR blocks according to their urgency score achieved relative to their priority which is obtained based on the waited time computed on referral day and the urgency coefficient. Surgery cases can either be operated within the planning horizon or be postponed to the next planning horizon. The importance of incorporating priority variable in the model functioning is not just because of its impact on measuring welfare loss that changes overtime at various speed related to urgency groups, but its influence on patient admission as a scheduling tool.

Emergency cases are allowed to go ahead in a non-emergency OR upon availability of resources, however, the surgery has to be completed within the regular shift operating hours for each OR. The amount of under (over) utilized ORs will be penalized using a penalty rate which is the largest among all the penalty rates. We assume a cyclic weekly demand pattern in our model, therefore, the unscheduled surgeries for elective patients has to be operated either in overtime hours or be rescheduled to another local hospital, which in either case will be penalized.

The AMPL software is used as a solution platform due to its well-known high-level modeling system for solving complex integer programming problems. The computational results of the integrated MSS and SCA algorithm of the proposed deterministic model, two-stage stochastic programming model, and RO model are shown in the following contents. It should be noted that throughout this study the decision variables with an optimal value of zero are not shown for the sake of making the tables clearer. To obtain the trade-off between solution

robustness and model robustness in the RO model, value of λ , $\omega 1$ and $\omega 2$ is figured to be set to 0.5, 100 and 150, respectively as elaborated in section 4.5.5.

4.7.2 Deterministic integrated MSS and SCA solution

A deterministic integer linear programming model has been used to solve the integrated MSS and SCA problem under study to generate test instances for the OR department at WRH. In particular, for each setting an optimal schedule according to every stakeholder's objective has been calculated to obtain the total welfare losses, postponed patients, and operating room capacity disruptions over the planning horizon. Then, a well-established multi-objective decision making (MODM) method (i.e. LP-Metric methodology) is applied, as elaborated in section 4.6, that aggregates multiple objective function into one dimension decision.

4.7.2.1 Deterministic results for minimizing total welfare loss

The minimum welfare loss (Z_1^*) under deterministic operative scenario is of 29,600 PWTRLs which can be seen as the price paid by the society for elective surgery in a week due to delayed treatment. Surgical activities are then planned according to the given resources to meet patients need on the waiting list as described in Table 4.3. The total cost of disruption due to under (over) time in the surgical specialties schedule and under (over) utilization of OR block capacities would be 20,888 unit.

As it was the aim of the model, the solution provides an integrated plan for the allocation of OR to surgical specialties (S) along with the number of patients from the corresponding waiting lists assigned to each OR block. For each operating room, the first row in Table 4.3 shows the surgical specialty assigned to the OR block. Within the waiting list, patients are being identified by patient ID from 1 to 200 while the urgency related characteristic corresponding to each patient is reported in Table 4.1. Note that the sequence of surgeries within each allocated block is determined by the surgeons based on the operational online planning in combination with idle time, waiting time and overtime.

Table 4.3: Integrated MSS and SCA solution for deterministic model

	Monday	Tuesday	Wednesday	Thursday	Friday	T+1
OR 1	**S3** 8, 42, 44, 45, 50, 51, 150	**S1** 25, 60, 65, 99, 100, 105	**S1** 111, 114, 156, 162	**S3** 43, 47, 148		
OR 2	**S2** 5, 6, 7, 27, 29, 30	**S2** 31, 77, 79, 135, 136, 137	**S7** 83, 87, 125	**S1** 61, 98	**S6** 117, 118, 119, 121, 122	
OR 3	**S5** 33, 35, 36, 37, 38, 39, 172	**S1** 52, 54, 64, 70, 101	**S5** 32, 40, 173	**S7** 86, 128	**S2** 144	
OR 4	**S4** 10, 12, 13, 16, 91, 93, 176	**S2** 28, 76, 80, 133, 138, 146	**S4** 92, 94, 96, 178	**S4** 14, 95, 180	**S5** 171, 174	
OR 5		**S7** 17, 18, 81, 84, 89, 126, 127	**S2** 78, 132, 134, 142, 147, 198	**S1** 108, 155, 160, 189	**S2** 141, 199, 200	
OR 6	**S1** 3, 4, 23, 53, 75, 106, 107	**S7** 82, 85, 88, 90, 131	**S1** 103, 104, 113, 153	**S2** 139, 143, 195, 196, 197	**S4** 179, 181	
OR 7	**S1** 24, 26, 55, 58, 63, 66, 69, 72, 102, 152	**S3** 9	**S7** 123, 124, 129, 130	**S4** 97, 182	**S1** 116, 157, 187, 193	
OR 8	**S1** 1, 56, 57, 59, 68, 73, 110, 112	**S5** 34, 41, 165, 169	**S4** 15		**S1** 159, 188, 190, 191	
OR 9	**S1** 2, 62, 67, 71, 74, 109	**S3** 46, 48, 49, 149, 151	**S1** 115, 158, 161, 185, 194	**S1** 154, 163, 184, 192	**S2** 140, 145	
OR 10	**S6** 19, 20, 21, 22, 120		**S5** 166, 167, 168, 170	**S1** 164, 183, 186	**S4** 11, 175, 177	

Table 4.3 also depicts the MSS for the surgery department and shows 15, 8, 4, 7, 5, 2, and 5 OR blocks allocated to surgical specialty 1, 2, 3, 4, 5, 6, and 7, respectively over the planning horizon. As compared to other specialties, surgical specialty 1 obtains more OR block than any other specialty due to its lengthy waiting list, whereas only 2 OR block times is assigned to specialty 6 over the week. However, no postponement is suggested by the model as surgery duration and emergency arrivals are assumed constant over the planning horizon. Therefore, there is no postponement reported under the column T+1.

4.7.2.2 Deterministic results for minimizing under (over) utilization of OR capacity

Next, the efficiency model for the overall utilization of OR block is solved according to the related procedure explained in Section 4.3.1 and the optimal solution of 20,888 for surgical specialties under (over) time hours as well as OR utilization's disruption is obtained. Table 4.4 shows the amount of deviation from optimal assignment of surgery operations to surgical specialties over the planning horizon in terms of under (over) time hours. Table 4.5 shows the amount of utilization disruptions for operating room over the planning horizon. As can be seen in the tables, the results are recommending no undertime for surgical specialties nor any underutilization of OR capacity is suggested by the results over the planning horizon.

Table 4.4: Deterministic under (over) time of surgery hours to specialty (hours)*

	Monday	Tuesday	Wednesday	Thursday	Friday
S 1	(14.41)	(6.73)	(2.14)	(0.74)	(0.22)
S 2	(3.56)	(6.90)	(3.58)	(3.63)	(4.07)
S 3	(2.50)	(5.80)	0.00	(2.51)	
S 4	(3.57)	0.00	(5.96)	(2.10)	(2.43)
S 5	(2.97)	(3.62)	(0.45)		(1.88)
S 6	(0.05)				(1.10)
S 7		(4.78)	(3.63)	(1.58)	

*: Numbers in bracket represent overtime

Table 4.5: Deterministic under (over) utilization of OR capacity (hours) *

	Monday	Tuesday	Wednesday	Thursday	Friday
OR 1	(3.50)	(3.30)	(0.90)	(2.51)	
OR 2	(3.56)	(3.29)	(3.36)	(0.43)	(1.10)
OR 3	(2.97)	(3.43)	(3.21)	(1.58)	(1.00)
OR 4	(3.57)	(3.61)	(3.08)	(1.47)	(1.88)
OR 5		(3.16)	(3.58)	(2.06)	(0.79)
OR 6	(3.58)	(3.62)	(0.25)	(3.63)	(2.30)
OR 7	(3.55)	(3.09)	(0.27)	(0.63)	(3.67)
OR 8	(3.61)	(3.62)	(2.88)		(0.55)
OR 9	(3.67)	(2.71)	(0.99)		(2.27)
OR 10			(0.24)		

*: Numbers in bracket represent overutilization

4.7.2.3 Deterministic results for aggregated LP metric model

This part of the analysis utilizes the optimal solution obtained from all three objective functions (see section 4.4) in order to develop one integrated objective function using the LP-Metric methodology. Therefore, we apply Z_t to represent the optimal value of each objective

function after solving the integrated model. In addition, the parameter W_t is considered to be 0.6, 0.1, and 0.3 for $t = 1,...,3$, respectively, to emphasize the importance of each objective function as per the hospital manager point of view. Using the above setting, the proposed deterministic algorithm for the integrated MSS and SCA problem is solved, resulting in the objective function of 0.8402. Since the objective function is minimized both the total welfare loss as well as utilization disruptions, a value for the Lp-Metric objective function close to zero would be more desirable

4.7.3 Stochastic integrated MSS and SCA solution

A two-stage stochastic programming model has been used to solve the integrated MSS and SCA problem under study to generate test instances for the OR department at WRH. The minimum welfare loss (Z_1^*) under stochastic operative scenario is of 30,215 PWTRLs which accounts for around 2% increase as compared with the situation where all parameters are known. As depicted in Table 4.6, the surgery operation for the total of 10 patients is postponed to the next planning horizon. Each optimal setting is evaluated individually to obtain the total welfare losses, postponed patients, and operating room capacity disruptions over the planning horizon according to table 4.6_4.8. The final optimum solution for the aggregated multiple objective is obtained through the LP-Metric methodology as we explained earlier.

Table 4.6: Integrated MSS and SCA solution for stochastic model

	Monday	Tuesday	Wednesday	Thursday	Friday	T+1
OR 1	**S7** 17, 81, 84, 88, 90, 126	**S7** 18, 82, 85, 89	**S3** 42, 44, 48	**S4** 95, 97		**S2** 141
OR 2	**S6** 20, 21, 22, 119, 122	**S5** 33, 37, 38, 39, 172	**S1** 58, 60, 71, 74, 113	**S7** 83, 127, 129	**S4** 178, 182	**S5** 174
OR 3	**S1** 3, 4, 63, 67, 68, 110	**S1** 66, 105, 107, 112, 152, 191	**S1** 59, 70, 101, 108	**S1** 61, 64, 72, 104	**S5** 168, 169, 171	**S4** 15
OR 4	**S5** 34, 35, 36, 40	**S1** 25, 62, 100, 106, 162	**S5** 32, 167, 170, 173	**S1** 116, 157, 160, 189	**S3** 46, 49, 148, 149	**S2** 140

OR 5		**S4** 11, 16, 91, 93, 180	**S2** 134, 142, 143, 147, 200	**S1** 154, 163, 185, 193	**S4** 175, 177, 179	**S4** 181
OR 6	**S2** 5, 27, 29, 77, 133, 137	**S6** 19, 117, 118, 120, 121	**S2** 80, 132, 138, 139, 197, 199	**S1** 103, 115	**S1** 183, 184, 188	**S3** 47
OR 7	**S2** 6, 7, 30, 76	**S4** 10, 12, 13	**S4** 14, 94, 176	**S4** 92, 96	**S5** 41, 165, 166	**S2** 144
OR 8	**S1** 1, 23, 24, 26, 55	**S1** 56, 69, 73, 75, 102, 109	**S7** 87, 123, 130, 131		**S2** 31, 145, 198	**S7** 128
OR 9	**S3** 8, 43, 45, 50, 51, 150, 151	**S2** 28, 78, 195	**S1** 53, 65, 111, 153	**S1** 52, 99, 156, 158, 161	**S1** 54, 155, 190, 192, 194	**S3** 9
OR 10	**S2** 79, 135, 136, 146, 196		**S7** 86, 124, 125	**S1** 98, 114, 187	**S1** 57, 159, 164, 186	**S1** 2

4.7.3.1 Stochastic results for minimizing total welfare loss

The scheduling plan for surgical activities resulted in a welfare loss of 29,775 PWTRLs which can be seen as the price paid by the society for elective surgery in a week. Surgical activities are then planned according to the given resources to meet patients need on the waiting list as described in Table 4.6. The proposed MSS for the surgery department also provides the assignment of OR blocks to the specialties as 18, 10, 5, 8, 6, 2, and 6 OR blocks allocated to surgical specialty 1, 2, 3, 4, 5, 6, and 7, respectively over the planning horizon. As compared to deterministic model, surgical specialty 1 and 2 are allocated 20 and 25 percent more OR blocks due to a higher variability of surgery durations, whereas, the allocation of OR blocks for the sixth specialty remained unchanged. The results also offer promising insights into resource optimization. Although the surgery operation of 5% of the patients on the waiting list is suggested to be postponed to the next planning horizon to incorporate the stochastic nature of surgery durations and emergency arrivals into the model, the total cost of disruption due to under (over) time in the surgical specialties schedule as well as under (over) utilization of OR block capacities dramatically decreased by almost 49% to 10,695 units.

4.7.3.2 Stochastic results for minimizing under (over) utilization of OR capacity

Table 4.7 shows the amount of deviation from optimal assignment of surgery operations to surgical specialties over the planning horizon in terms of under (over) time hours. Next, the efficiency model for the overall utilization of OR block is solved according to the related procedure explained in Section 4.3.1. Table 4.8 shows the amount of utilization disruptions for OR capacity over the planning horizon.

Table 4.7: Stochastic under (over) time of surgery hours to specialty (hours)*

	Monday	Tuesday	Wednesday	Thursday	Friday
S 1	(0.92)	(1.60)		(2.81)	(1.77)
S 2	(5.31)	(1.17)	(3.33)		(1.73)
S 3	(0.42)		(0.67)		
S 4		(0.32)	(0.24)	(0.30)	1.41
S 5	(1.57)	(0.94)	(0.37)		(1.86)
S 6	(1.18)	(0.37)			
S 7	(1.54)	(0.42)	(1.67)	(1.06)	

*: Numbers in bracket represent overtime

Table 4.8: Stochastic under (over) utilization of OR capacity (hours)*

	Monday	Tuesday	Wednesday	Thursday	Friday
OR 1	(1.54)	(0.42)	(0.67)	(1.10)	
OR 2	(1.18)	(0.94)	(2.52)	(1.06)	(1.97)
OR 3	(0.27)	(0.51)	(0.25)	(0.79)	(0.81)
OR 4	(1.57)	(0.23)	(0.37)	(0.30)	
OR 5		(0.43)	(1.66)	(0.43)	(0.62)
OR 6	(2.07)	(2.37)	(1.68)	(0.57)	(0.80)
OR 7	(1.39)		(0.24)	(1.20)	(1.05)
OR 8	(0.65)	(0.85)	(1.48)		(1.73)
OR 9	(2.42)	(1.17)	(0.27)		(0.76)
OR 10	(1.85)		(0.20)	(0.60)	(0.21)

*: Numbers in bracket represent over utilization

4.7.3.3 Stochastic results for aggregated LP metric model

The optimal solution of every stakeholder's objective is utilized in this part to develop one aggregated objective function using the LP-Metric methodology as explained in section 4.5.1.3. The parameter W_t is considered to be 0.6, 0.1, and 0.3 for $t = 1,...,3$, respectively, to emphasize the importance of each objective function as per the hospital manager point of view.

Using the above setting, the proposed stochastic algorithm for the integrated MSS and SCA problem is solved, resulting in the objective function of 0.1067. Since the objective function

is minimized both the total welfare loss as well as utilization disruptions, a value for the Lp-Metric objective function close to zero would be more desirable. Table 4.8 shows the amount of deviation from optimal assignment of surgery operations to surgical specialties over the planning horizon in terms of under (over) time hours.

4.7.4 Robust optimization integrated MSS and SCA solution

We utilize a standard robust optimization transformation framework developed by [88] to solve the integrated MSS and SCA problem under uncertainty to generate test instances for the OR department at WRH. Each optimal setting is evaluated individually to obtain the total welfare losses, postponed patients, and operating room capacity disruptions over the planning horizon.

The minimum welfare loss (Z_1^*) under robust operative scenario is about 30,058 PWTRLs which accounts for around 1.5% increase as compared with the situation where all parameters are known, which is slightly better than the result suggested by two-stage stochastic programming. The final optimum solution for the aggregated multiple objective is obtained through the LP-Metric methodology as we explained earlier. As depicted in Table 4.6, the surgery operation for the total of 9 patients is postponed to the next planning horizon which demonstrates 10% lower postponement as compared to the stochastic model. Each optimal setting is evaluated individually to obtain the total welfare losses, postponed patients, and operating room capacity disruptions over the planning horizon according to table 4.9_4.11.

Table 4.9: Integrated MSS and SCA solution for robust optimization model

	Monday	Tuesday	Wednesday	Thursday	Friday	T+1
	S2	**S2**	**S3**	**S1**		**S2**
OR 1	6, 27, 132, 136	77, 79, 80, 137	42, 49, 148, 150	65, 113, 161, 185, 189, 190		144
	S1	**S1**	**S1**	**S1**	**S5**	**S4**
OR 2	3, 4, 26, 55, 63, 67, 107	53, 59, 66, 75, 99, 110	54, 70, 102, 111, 112	108, 153, 156, 163, 184	165, 168, 171	15
	S3	**S2**	**S3**	**S1**	**S2**	
OR 3	8, 43, 44, 45, 50, 51	76, 138, 142, 143	46, 48, 149, 151	115, 154, 158, 194	139, 140, 199	

	S7	**S4**	**S6**	**S2**	**S2**	**S3**
OR 4	17, 18, 88, 90, 130	11, 93, 94, 176	117, 118, 119, 120, 122	78, 146, 147, 197	195, 196, 198	9
		S7	**S5**	**S4**	**S2**	**S7**
OR 5		86, 89, 124, 131	166, 167, 169, 170, 173	91, 96, 97	135, 145, 200	128
	S6	**S1**	**S7**	**S4**	**S4**	**S1**
OR 6	19, 20, 21, 22, 121	25, 57, 62, 64, 71, 106	87, 125, 129	95, 177, 178	179, 182	2
	S5	**S7**	**S1**	**S7**	**S1**	**S2**
OR 7	33, 35, 36, 37, 39, 172	81, 84, 85, 126, 127	103, 114, 152	82, 83, 123	104, 155, 186, 191	141
	S2	**S1**	**S2**		**S1**	**S5**
OR 8	5, 7, 28, 30	52, 56, 58, 69, 73, 109	29, 31, 133, 134		98, 157, 193	174
	S1	**S5**	**S1**	**S1**	**S1**	**S4**
OR 9	1, 23, 24, 68, 72, 74	32, 34, 38	60, 100, 101, 105	61, 116, 162	160, 188, 192	181
	S4		**S4**	**S5**	**S1**	**S3**
OR 10	10, 12, 13, 16, 180		14, 92, 175	40, 41	159, 164, 183, 187	47

4.7.4.1 Robust optimization results for minimizing total welfare loss

The scheduling plan for surgical activities resulted in a welfare loss of 30,058 PWTRLs which can be seen as the price paid by the society for elective surgery in a week. Surgical activities are then planned according to the given resources to meet patients need on waiting lists as described in Table 4.9. The proposed MSS for the surgery department also provides the assignment of OR blocks to the specialties as 17, 11, 5, 8, 6, 2, and 6 OR blocks allocated to surgical specialty 1, 2, 3, 4, 5, 6, and 7, respectively over the planning horizon. As compared to the stochastic model, the welfare loss generated by the RO model is slightly lower. Surgical specialty 1 is allocated over 5% less OR block than stochastic model due to incorporation of variability in the robust model, whereas, the allocation of OR blocks to the second specialty has increased by 10% which demonstrates the advantage of the RO model to cope with the volatile surgery durations. The results also offer promising insights into resource utilization as only 4% of the surgery operations are postponed to the next planning horizon to incorporate the stochastic nature of surgery durations and emergency arrivals into the model.

4.7.4.2 Robust optimization results for minimizing under (over) utilization

Next, the efficiency model for the overall utilization of OR block is solved according to the related procedure explained in Section 4.3.1 and the optimal solution of 3410 for operating room utilization disruption is obtained. Although the loss incurred due to under (over) time in the surgical specialties schedule as well as under (over) utilization of OR block capacities has increased in robust model by almost 15% (i.e. 12280) as compared with the two-stage stochastic recourse model, the mean objective function variability in the former model is significantly lower than the former model which justify the increased amount of loss. Table 4.11 shows the amount of deviation from optimal assignment of surgery operations to surgical specialties over the planning horizon in terms of under (over) time hours. Table 4.10 shows the amount of utilization disruptions for operating room over the planning horizon.

Table 4.10: Robust optimization under (over) time of surgery hours to specialty (hours)*

	Monday	Tuesday	Wednesday	Thursday	Friday
S 1	(1.46)	(1.39)	(0.77)	(1.53)	(0.28)
S 2			0.23	0.37	0.53
S 3					
S 4	(0.85)	(0.70)	(0.24)	(0.73)	(0.47)
S 5	(0.76)	(0.70)		(0.32)	(0.20)
S 6	(0.20)		(0.30)		
S 7	(0.50)	(0.55)	(0.40)	(0.37)	

*: Numbers in bracket represent overtime

Table 4.11: Robust optimization under (over) utilization of OR capacity (hours)*

	Monday	Tuesday	Wednesday	Thursday	Friday
OR 1	(0.20)	(0.29)	(0.32)	(2.30)	
OR 2	(1.56)	(2.26)	(1.32)	(1.26)	(0.70)
OR 3	(2.43)	(0.34)	(3.20)	(1.14)	
OR 4	(1.13)	(1.41)	(0.84)		(0.31)
OR 5		(1.24)	(0.27)	(1.58)	(0.20)
OR 6	(0.71)	(1.39)	(0.99)	(0.29)	(1.09)
OR 7	(1.50)	(0.39)	(0.68)	(0.95)	(0.78)
OR 8	(0.73)	(1.59)			(0.39)
OR 9	(1.35)	(1.41)	(0.35)	(1.17)	(3.56)
OR 10	(1.63)		(0.77)	(0.88)	(0.72)

*: Numbers in bracket represent over utilization

4.7.4.3 Robust optimization results for aggregated LP metric model

Similar to section 4.5.2.3, we utilize Lp-Metric methodology to develop one aggregated objective function in order to find the optimal solution of every stakeholder's objective. The parameter W_t is considered to be 0.6, 0.1, and 0.3 for $t = 1,...,3$, respectively, to emphasize the importance of each objective function as per the hospital manager point of view.

Using the above setting, the proposed robust optimization algorithm for the integrated MSS and SCA problem is solved resulting in the objective function of 0.0923. Since the objective function is minimized both the total welfare loss as well as utilization disruptions, a value for the Lp-Metric objective function close to zero would be more desirable.

4.8 Discussion of the results and evaluations

In this section, we provide insights into the results of deterministic, two-stage stochastic, and robust optimization model. The effectiveness of the proposed robust model is demonstrated through evaluation of different performance measures as compared with other developed models. The output solution obtained from different results can be used to assist hospital decision makers in evaluating the performance of the OR department as well as analyzing alternative situations based on the degree of decision makers' risk behavior.

First, we analyze the schedules of different models in order to calculate the resulting overtime/idle time of each specialty over the entire planning horizon. In particular, for each setting an optimal schedule according to the Lp-Metric objective has been calculated to identify the disruption of surgical team schedule from their allocated plan. Showing the amount of overtime in positive and undertime in negative values, Figure 4.2 points out that the deterministic model generates the highest planning disruption for surgical specialties followed by two-stage stochastic programming model. It is initially observed from the deterministic results that Specialties that generally have long surgeries (such as General surgery and Urology) incur more overtime, because they lack short surgeries to better utilize the available OR hours. The proposed

robust model, however, could fairly handle the over (under) time, which causes lower patients' related cost and most feasible plan for the system through allowing the hospital to stay within the current collective labor agreement and legislations. Compared with the deterministic model, the amount of overtime suggested for Dental surgery has dramatically reduced by around 82% in robust model while no overtime hours are reported for Orthopaedics. The results also shows 61 and 67% reduction in overtime hours of the robust model as compared with the stochastic programming model for ENT and Plastic surgery, respectively, which leads to reduced cost associated with overtime staffing and the tardiness of patients.

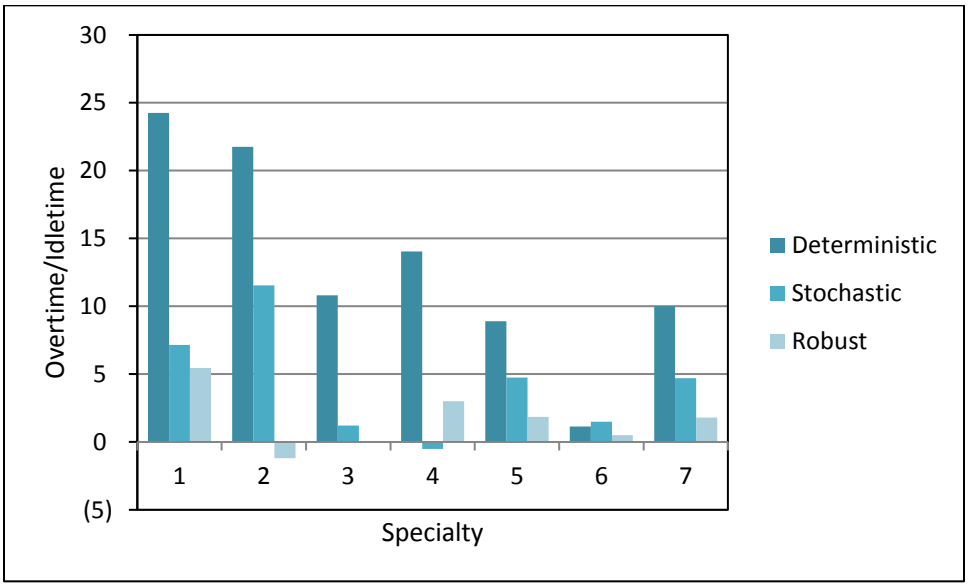

Figure 4.2: Comparison Over (under) time hours of surgical specialty in different models

As previously discussed, operating room utilization is seen as one of the main factors that contributes to poor scheduling of patients. In fact, suboptimal utilization of operating theatre time is one of the most common causes of surgery postponement or cancellation. Therefore, in this section we focus on analyzing the results based on the assessment of utilization of operating room on different models. As the third objective is aimed at minimizing the disruption from under (over) utilization of OR blocks, we analyze the schedules of different models in order to calculate

the resulting suboptimal utilization rates of each OR block that leads to postponement of surgical procedures as well as cancellations over the entire planning horizon. In particular, for each setting an optimal schedule according to the Lp-Metric objective has been calculated to identify the disruption of OR capacity from their utilized plan. Showing the amount of over utilization (as none of the models suggest any under-utilizations), Figure 4.3 points out that the deterministic model generates the highest planning disruption for OR utilizations. Due to the impact of variability within the uncertain input parameter, the proposed robust model recommended lower disruption in OR utilization in a robust optimization model which typically resulted from a degree of risk aversion policy associated with cost variability of OR overutilization in the robust model. As compared with deterministic model, the proposed robust model improves the utilization rate for OR blocks by over 54%, however robust model suggest 10% lower OR utilization rate compared with two-stage stochastic programing model. This is the direct result of variability that has not been incorporated into the stochastic programming model which results in more over utilization of OR blocks for the robust model.

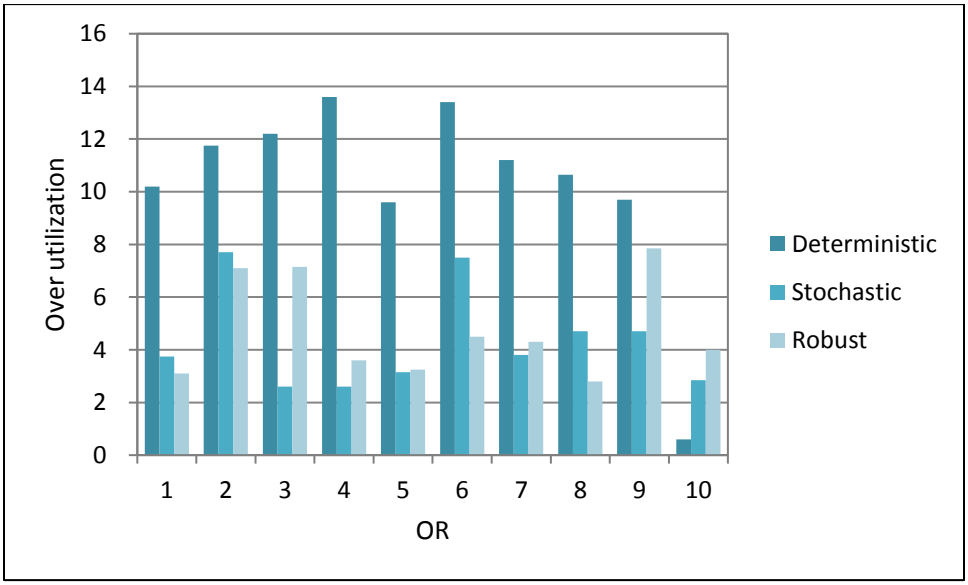

Figure 4.3: Comparison of overutilization of OR capacity in different models (hours)

As the aim of the developed models is to minimize the welfare loss due to postponed/cancelled patients while the disruption in surgical specialty hours and OR capacity utilization is minimized, it is demonstrated through the proposed robust model that this can be achieved at the cost of overutilization of OR blocks on the one hand, but less overtime for the surgery teams on the other hand. Therefore, from the above discussion, it is concluded that while the deterministic model generates schedule for surgical cases that go past the desired end time while at the same time the OR utilization rate during those hours is less than desired, the proposed robust optimization model better handle the suboptimal surgical specialties' hours at an affordable and yet optimal OR capacity utilization rate for the planning horizon.

To obtain the optimum value of the objective functions and to provide an insight into the characteristics of the output data, we conduct a trade-off between solution robustness and model robustness in the proposed robust optimization model. As seen in section 4.5.5, if $\omega_1 = 0$ and $\omega_2 = 0$ the second stage constraints in the robust formulation become infeasible in the objective function without a penalty cost, and hence the welfare loss increases due to the largest postponed and unscheduled patients as well as the highest disruption in surgical hour and OR utilization in the hospital. Under this circumstance, the resulted OR allocation plan for specialties and patient scheduling is not desired by the hospital managers. Therefore, the optimal value of λ , ω_1 and ω_2 has to be determined as a measure of trade-off between solution robustness and model robustness in the proposed RO framework. A very large penalty weight, on the other hand, could result in the penalty function to dominate the objective function and causes higher welfare loss due to an increased variability in the objective function. Consequently, the robust model has to be solved several times, each time with a different value of ω_1 and ω_2 to obtain the minimum loss incur in the integrated MSS and SCA problem in order to find a solution that is close to an optimal solution (i.e. solution robustness) while it is almost feasible for all scenarios (i.e. model

robustness). This trade-off analysis allows decision makers to acquire an optimal solution based upon an acceptable range of expected postponed patients from waiting lists at a minimized but affordable welfare loss. When ω_1 and ω_2 varies, the amount of infeasibility of the random constraints is also altered. Therefore, examining the proposed robust optimization model with various penalty costs would provide a sense of trade-off between the risk and welfare loss.

Figure 4.4 gives the trade-off between the penalty weight changes and the total expected welfare loss. The process of making the trade-off between solution robustness and model robustness is conceptually based on the RO methodology that allows for infeasibility in the second stage constraints by means of penalty as explained in section 4.5.5. When ω_1 and ω_2 are zero, the violation of the random constraints for disruption of both surgical specialty throughputs and OR capacity utilizations is allowed. Under this circumstance, an unrealistic allocation of OR blocks as well as an infeasible assignment of patients is advised in the optimal plan which results in maximum infeasibility, which indeed is not an adaptable plan. In Figure 4.4, as the expected infeasibility that represents model robustness declines, the expected total welfare loss which represents solution robustness goes up. The infeasibility cost of the second stage constraints drops until it becomes zero as the penalty for the violation of the second stage constraints is maximized. However, the total welfare loss remains steady when the penalty function reaches to a very large value. This in fact indicates the feasibility of the optimal solution for larger values of ω_1 and ω_2 under any realization of the scenario data, although at the expense of a higher welfare loss.

It should be noted that upon reaching the steady state situation for the infeasibility of the control constraints (i.e. $\omega_1 \geq 100$ and $\omega_2 \geq 150$), the impact of penalty function dominates the total objective functions, and hence no significant reduction would occur in the expected infeasibility. Adopting the best value of penalty costs in the proposed robust model, we finally obtain the optimal solution to the total welfare loss of 30,058 PWTRLs per week that allows for

considerable cost savings for the hospital budget. Although the total loss obtained by the proposed RO model increases by almost 1.0% as compared with the two-stage stochastic recourse programming model, the expected variability decreases significantly by more than 83%. Therefore, it is demonstrated that RO outperforms the stochastic recourse programming on controlling the risks by generating less sensitive allocation plan for specialties and more feasible assignment plan for patients. As the WRH has experienced a 2.3% deficit in its 2015 annual operating budgets, the proposed robust model is of quite benefits to the hospital managers to control the budget while maintaining the service level.

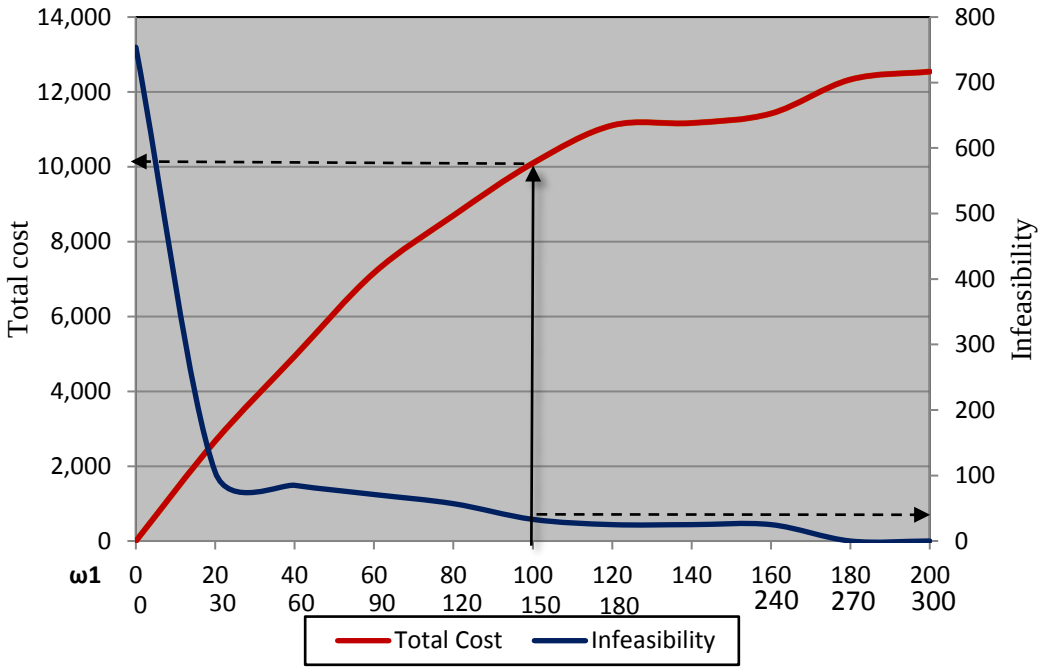

Figure 4.4: Trade-off between solution robustness and model robustness

Finally, the impact of selecting different values for weighting factor (w_t) by decision makers in Lp-Metric methodology is investigated. Here, a trade-off is actually conducted between different objective functions, including expected prioritized waiting time loss, expected postponement, and expected utilization disruptions by considering various amount of w_t to depict a spectrum of different possible future situations for the decision makers. In this regard, the

proposed robust model is solved several times when the value of w_t is changed between zero and one. Table 4.12 illustrate the range of objective functions over different values of w_t. For instance, when $w_1 = 1$, $w_2 = 0$, and $w_3 = 0$, the cost associated with the utilization disruptions would be at the highest level ($165,290) which represents the worst situation. However, the welfare loss would be on its minimum value of (28618 PWTRL) while the postponement rate is set at zero due to a null weighting factor assumed by the decision makers. On the contrary, when $w_1 = 0$, $w_2 = 0.40$, and $w_3 = 0.60$, the worst situation happens at welfare loss function and it increases by almost 9%.

Table 4.12: Trade-off between Lp-Metric objectives of postponement, utilization, and waiting time

w_1	1	0.90	0.80	0.70	0.60	0.50	0.40	0.30	0.20	0.10	0
w_2	0	0	0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40
w_3	0	0.10	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60
Welfare loss (PWTRL)	28618	29983	30133	30049	30114	30024	29905	29871	29721	29689	31184
Postponement (# of patient)	0	20	23	10	10	7	5	4	1	0	0
Utilization disruption ($)	165290	9870	9778	11546	11266	14427	16686	18203	22676	24240	24234

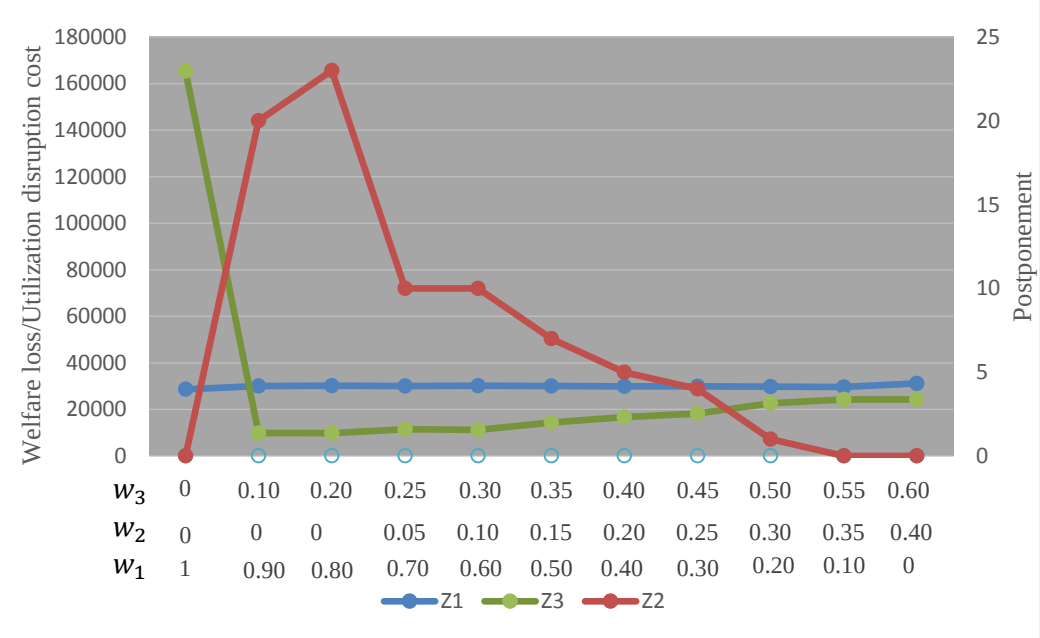

Figure 4.5: Trade-off between Lp-Metric objectives (welfare loss, postponement, utilization disruptions)

The postponement rate becomes zero as its weight has been maximized. The number postponed patients first increased to 23 and then it substantially decreased as the weighting factor grows. While the cost associated with utilization disruptions significantly drops by 94% as the weighting factor slightly increases to 0.1, the welfare loss related to the waiting time would reach to a maximum of 31184 PWTRL as its weighting factor becomes zero. In general, the amount of waiting time loss of the proposed robust model would gradually increase as the value of w_1 decreases, whereas the number of postponement and the cost of utilization disruption would increases.

As depicted in Figure 4.5, it would not be beneficial to the decision makers to lower the waiting time loss by far beyond $w_1 = 0.60$ due to its negative impact on resource utilization rate. Moreover, the cost of utilization disruptions cannot justify the loss incurred as a result of postponement beyond the $w_3 = 0.3$. Therefore, the main focus of the decision maker should be devoted to minimization of welfare loss resulted from waiting time than the cost of resource utilization. Our analysis suggest that with a controlled waiting time loss of (30114 PWTRL), a reasonable level of patients postponement of 5% can be achieved at a weekly cost of $11266.

4.9 Conclusions

In this thesis, we address the operating room (OR) planning problem at an integrated tactical and operational planning level. The novelty herein puts forward is twofold: first, we model the problem of OR planning and advance scheduling in order to support integrated master surgery scheduling (MSS) and surgical case assignment (SCA) problem in the presence of multiple objectives and stochastic surgery durations and emergency arrivals. Second, we utilized a novel transformation framework in order to transform a deterministic hybrid MSS and SCA that explicitly model the conflicting goals of patients' service level through clinical prioritization weighting factor and hospital management in terms of surgical throughputs and operating room utilizations. The incompleteness of the random surgery durations and the randomness arises in the

emergency arrivals are considered using a discrete set of scenarios. We present an extensive solution approach to integrated hospital MSS and SCA problem on the basis of two models, including a two-stage stochastic recourse programming model and a robust optimization (RO) model that aims at advancing both OR utilization and the health service levels. To tackle with the complexity of developing the robust counterpart of the mixed-integer linear programming models a novel transformation framework is proposed to transform a deterministic manner integrated MSS and SCA problem into the RO form and absorb the effect of existing variability within the stochastic parameters. The compromised allocation of OR blocks as well as the assignment of patients obtained from the RO framework was capable to handle the variability within the uncertain parameters through generating optimal scenario-dependent solutions. Three RO models with different variability measures are proposed: the RO model with solution robustness, the RO model with model robustness, and the RO model with trade-offs between solution robustness and model robustness to evaluate the operational performance and to analyze the enhancement of the trade-off between efficiency and health service delivery. The computational results of addressing integrated MSS and SCA problem of a real case situation illustrate the advantage of the proposed RO approach over the stochastic recourse programming model. The proposed model creates a robust integrated scheduling plan for specialties and assignment plan for surgical cases while reducing the loss associated with prioritized waiting time for the hospital.

An analysis of the results is performed to demonstrate the benefit of RO model in increasing the OR utilization level and throughput of the system. The trade-off between the allocation plan's robustness (i.e. postponed/cancelled surgery) and underutilization of OR blocks for different values of robustness is demonstrated that the proposed RO model is progressively less sensitive to the realization of the variable input parameters, while generating more feasible solutions as compared with the two-stage stochastic recourse programming model.

Future research on integrated MSS and SCA problem can go in two directions. First, the proposed RO framework presented in this study can be applied on OR planning and scheduling problems in other healthcare systems where the random input parameters are deemed to be a barrier to yield the solid results, such as random surgeon availability and downstream resource availability. Second, an extension of the model in which detailed surgeons' timetables and bed occupancy is considered on an open-scheduling strategy to develop a mechanism that meet the overall objective to reduce surgical waiting times.

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Chapter 5

Conclusion and Future Work

5.1 Concluding remarks

In this dissertation, we have performed a comprehensive study on operating room planning and scheduling problem [1–3]. In Chapter 2, we have developed a novel robust optimization (RO) transformation framework based on the RO model developed by [4] that outperforms the conventional RO approach by enhancing the computational efficiency through a linearization approach adopted from [5]. To the best of our knowledge this is the first standard transformation algorithm for the robust optimization method. The developed RO transformation framework addresses the drawback of the Mulvey's model owing to its difficulty in obtaining information associated with the numerous control variables and constraints by incorporating the expected variability and infeasibility functions into the conventional model. Three different robust models are then projected using the proposed transformation framework to highlight the capability of the developed model in dealing with variability of the stochastic parameters as well as the infeasibility of the control constraints in an uncertain environment. Both variability and infeasibility functions are penalized through the weighting penalty factors to trade-off between risk and expected outcome for the solution robustness and model robustness. We also

demonstrate that increasing the penalty factors would result in the solution to become less sensitive to the changes in the input data as defined by a set of different scenarios which in turn enables the proposed RO framework to account for the decision maker's preferences towards risk. Therefore, the proposed RO framework allows for a more passive management style, giving it a distinct advantage over the stochastic recourse programming. In other words, with variability under control, minimal adjustment to the control variables will be required when the weighted variance version of RO is applied. The proposed formulation also enables adjusting of the model in response to changes in input data through incorporation of the variability of the objective function into the formulation. It can be generally used as a standard framework to transform any linear deterministic model into the stochastic robust form. After developing our RO transformation framework and outlining that we can transform deterministic models into the robust form through our framework to find an optimal solution for the stochastic problems in polynomial time, we have focused on practical implementation issues in Chapter 3. We have developed three models to solve surgery capacity allocation problem of a healthcare system using mathematical programming, two-stage stochastic programming, and robust optimization model [6–8]. The standard transformation framework is then fed into a surgery capacity allocation problem of a healthcare delivery system to capture the randomness of the actual process in order to evaluate the effectiveness of the proposed framework on a realistic model and to demonstrate the applicability of the formulations. To the best of our knowledge, this is the first time that our proposed RO approach is applied in the context of the healthcare capacity allocation problems.

We have provided valuable insights into many aspects of the presented framework and also the characteristics of the proposed formulation. We have focused on data incompleteness as well as operational inefficiency of the healthcare systems as the most important challenges of the healthcare planning process to provide a decision making tool for the hospital managers to allocate OR block times to the surgical specialties in response to unknown elective and

emergency surgery demands with the aim of minimizing the cost associated with postponement and or rescheduling of patients as well as underutilization of OR blocks. We have considered a publically funded hospital where the surgery cost are determined based on the duration of time between patients are admitted to a hospital and the time when their required surgery operation is performed and not as a function of surgery duration. We define a set of discrete random scenarios to represent uncertain elective and emergency surgery demands. This incorporation has important practical implications, as the true demand distributions are often not known and only their past realizations or some samples are available. It is illustrated through the formulation that the proposed transformation framework is more practical to use than the method developed by [4]. Furthermore, the computational results confirm that the framework presented herein generates a robust allocation plan in a timely manner without requiring addition of any additional deviation variables.

Again motivated by operating room planning and scheduling, in Chapter 4 we have developed an integrated operating room planning and advance scheduling in a surgery theater comprising several specialties that share a fixed number of operating rooms and post-surgery beds [9–11]. We have jointly considered the allocation of surgical specialties to OR blocks together with the assignment of the subsets of patients from each specialty's waiting list to the OR blocks over a one-week planning horizon. We have also incorporated multiple criteria while evaluating the performance of the hybrid planning and scheduling, including OR utilization, surgeons' overtime, and patient prioritized waiting time. We have extended the stability of the scheduling process resulted from tackling simultaneously both the master surgery schedule (MSS) problems with the surgical case assignment (SCA) problems by considering both uncertain surgery duration and emergency arrivals to increases the chance of successful implementation. We have utilized the novel RO transformation framework, presented in Chapter 2, in order to transform a deterministic hybrid MSS and SCA model to a robust form that explicitly captures

the conflicting goals of patients' service level through clinical prioritization weighting factor and hospital management in terms of surgical throughputs and operating room utilizations. We considers a weighted multi-objective RO approach that focuses conflicting resource perspective as well as patient perspective at the same time, which take the number of scheduled surgeries, waiting time and tardiness of each patient associated with patient urgency factors, and weighted resource utilization rates into account. The integration of planning and scheduling levels provide some stability, in terms of repeatability of personnel schedules and predictability of bed occupancy pattern in post anesthesia care units (PACU) as well as flexibility, in terms of adaptability of weekly plans to the changing waiting lists for the decision makers. Our RO model seeks for the trade-off between higher capacity, which will reduce the waiting time as well as OR productivity due to under (over) utilization, and a lower capacity that result in postponement as well as ORs overtime. The compromised allocation of OR blocks as well as the assignment of patients obtained from the RO framework was capable to handle the variability within the uncertain parameters through generating optimal scenario-dependent solutions. Three RO models with different variability measures are proposed: the RO model with solution robustness, the RO model with model robustness, and the RO model with trade-offs between solution robustness and model robustness to evaluate the operational performance and to analyze the enhancement of the trade-off between efficiency and health service delivery.

The computational results of addressing integrated MSS and SCA problem of a real case from Windsor Regional Hospital demonstrate to improve patient satisfaction through reducing prioritized weighted waiting times and also improving health care efficiency by reducing overall operation costs, and hence has more societal benefits for the hospitals. The resulting plans provide a decision tool for the OR managers to exploit a trade-off between risk aversion level associated with the robustness of patient service level and the expected cost of surgery deferments. The key managerial insights that the proposed models provide to the planner is the

ease of dealing with uncertain data which creates an integrated MSS and SCA plan for different surgical specialties that leads to a lower surgery postponement, a higher resource utilization, and a levelled workload for surgeons. The proposed robust models could successfully absorb the variability exists in surgery durations and emergency arrivals and enables management to allocate OR blocks more accurately while limiting the negative impact of surgery overtime. It also has to be noted that the time the decision makers are required to allocate operating room capacities is reduced by the use of this method. Therefore, managerial attention can be paid to implementation of the proposed robust optimization framework to reduce the operational burden of surgery departments.

5.2 Future directions

There are exciting future directions and improvement possibilities for this research. First, the proposed RO framework presented in this study can be applied on OR planning and scheduling problems in other healthcare systems where the random input parameters are deemed to be a barrier to yield the solid results, such as random surgeon availability and downstream resource availability. Second, a non-parametric sample average approximation (SAA) approach can be used to form an empirical distribution of uncertain parameters obtained from random samples. Further research will also incorporate using simulation models for replicating and predicting surgery durations as well as using fast heuristic approaches in order to reduce the solving time and/or improve the feasibility of solutions.

One other research avenue that we consider is an extension of the model in which detailed surgeons' timetables and bed occupancy is considered on an open-scheduling strategy to develop a mechanism that meet the overall objective to reduce surgical waiting times.

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VITA AUCTORIS

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