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Truckload Shipment Planning and Procurement

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Abstract

This dissertation presents three issues encountered by a shipper in the context of truckload transportation. In all of the studies, we utilize optimization techniques to model and solve the problems. Each study is inspired from the real world and much of the data used in the experiments is real data or representative of real data.

The first topic is about the freight consolidation in truckload transportation. We integrate it with a purchase incentive program to increase truckload utilization and maximize profit. The second topic is about supporting decision making collaboration among departments of a manufacturer. It is a bi-objective optimization model. The third topic is about procurement in an adverse market. We study a modification of the existing procurement process to consider the market stochastic into marking decisions. In all three studies, our target is to develop effectively methodologies to seek optimal answers within a reasonable amount of time.

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Dedication

To Hazel and Yen.

Contents

Published papers

Chapter 2, published.

Nguyen, H. N., Rainwater, C. E., Mason, S. J., Pohl, E. A., 2014. Quantity discount with freight consolidation. Transportation Research Part E (66), 66–82.

Chapter 1

Introduction

In this dissertation, we study three contemporary supply chain problems that involve multiple parties. These problems focus on truckload transportation to deliver freight from shippers to customers. We apply optimization techniques to model and solve the problems. Our objective is to maximize shippers' benefits.

Our first topic is an integrated quantity discount and freight consolidation problem. The problem consists of a shipper and its customers. Customers regularly place partial truckload orders. The shipper delivers products to its customers and pays the transportation cost. The shipper looks for freight consolidation opportunities to save transportation cost. Based on the average size of the orders, the potential savings is marginal. The shipper implements a purchase incentive program which will give a discount on product price when a customer places a larger order. We formulate the problem and apply optimization techniques to find the best configurations for the incentive program.

Our second topic is a multi-objective problem where a shipper is interested in total profit and output fluctuation. The shipper utilizes a cost saving program to maximize its profit and to re-distribute its warehouse's output during a planning horizon. The control of the warehouse output represents the coordination among multiple departments within the shipper. The cost saving program enables the shipper to decide the preferable size of the deliveries. We formulate the problem as a vehicle-routing-program with quantity discount option. We develop a GA-based methodology to find the efficient frontiers.

Our third topic is a truckload procurement program with stochastic bid packages to model a shipper-unfavorable market. Shippers expand their core carrier base to include small carriers in addition to national carriers. Small carriers add supply elasticity and shipping flexibility which shippers could utilize to negotiate better rates with national carriers. We look for the optimal re-configurations to adjust the procurement process corresponding to the market conditions. Our objective is to maximize the transportation savings for shippers.

Chapter 2

Quantity Discount with Freight Consolidation

Nguyen, H. N., Rainwater, C. E., Mason, S. J., Pohl, E. A., 2014. Quantity discount with freight consolidation. Transportation Research Part E (66), 66–82.

2.1 Introduction

In this chapter, we study the integrated problem of an inventory-vehicle-routing problem and a quantity discount problem. The problem is encountered in practice by a manufacturer which wants to maximize truckload shipments and make savings by consolidating delivery orders into multi-stop truck routes and by offering discounts to increase customers' order size. The problem consists of a manufacturer and multiple customers (or a seller and multiple buyers in general).

The seller pays transportation cost on a shipment basis. The seller does not own or operate a truck fleet. It buys hauling services from common carriers. Each truckload shipment's cost is calculated by a rate per mile and the shipping distance. A rate per mile is defined by the combination of the origin and the destination of a shipment.

A half-full truckload shipment costs as much as a full truckload (FTL) shipment does because a whole truck is dedicated for the service in both cases. FTL shippers do not pay shipping cost based on their load metrics such as weight, volume, and pallet count. Table 2.1 shows the typical maximum freight weight by mode. Parcel carriers usually do not accept shipments of 100 pounds or more. Less-than-truckload (LTL) carriers generally do not accept shipments of 20,000 pounds or more. From a shipper's perspective, an LTL shipment that weighs more than 13,000 pounds usually costs less when shipped as an FTL shipment. This chapter considers full and partial truckload orders, each of which is economically shipped as an FTL shipment. This chapter does not consider LTL and parcel modes. When an order is qualified for an LTL or parcel shipment, its shipping cost is calculated based on its weight and volume. Therefore, there is likely no motivation for increasing shipment size associated with these modes. On the other hand, splitting an order into multiple LTL shipments is not a viable option. Customers strongly discourage the option because it complicates their warehouse management.

Table 2.1: Typical maximum freight weight by mode

Transportation Mode	**Maximum Weight (pounds)**
Parcel	100 - 150
LTL	13,000 - 20,000
FTL	53,000

Multiple partial truckload orders can be economically shipped in the same truck to customers in a market area. This is called a multi-stop truckload shipment. The shipping cost is calculated based on the number of delivery stops and the rate-per-mile charge. Figure 2.1 illustrates the stop-off charges of a typical multi-stop truckload shipment. In a standard stop-off charge schedule, the first delivery stop is free and each subsequent stop is charged more than its previous one. The primary reason for an increasing stop-off charge schedule is that it takes significant effort for a truck driver to arrive on time at all stops. A late arrival at a stop will jeopardize all subsequent appointments.

Figure 2.1: Example stop-off charges in a multi-stop shipment

In this problem, buyers are assumed to use a (Q, r) inventory policy. They will place replenishment orders of size Q to the seller when their inventory levels drop to r. The seller can deliver the size-Q orders by direct truckload shipments or consolidate them into multi-stop truckload shipments. Some shipments are full or almost full truckloads. Other shipments are much smaller than FTLs but cost as much as FTLs do. In order to better utilize the truck capacity, the seller offers these buyers discounts for additional order quantities beyond their original replenishment quantities Q. The cost saved is the marginal transportation cost less the discount amount.

The problem studied in this work is encountered in many industries such as building material, office products, and canned food. These types of products have low value per unit weight (e.g. pound). Transportation cost is usually a high percentage of total sales revenue. In other industries, such as toys and electronics, the products have high value. When transportation cost savings is compared to the total revenue, it is only a small percentage which makes our problem less relevant.

In order to illustrate the problem, an example is discussed below (see Figure 2.2). There is one seller and three buyers. Each buyer places a half FTL order, 25,000 pounds, every week. In scenario 1, the seller delivers the orders separately. It costs the seller six direct truckload shipments to deliver the orders to three buyers in two weeks. Each shipment is a truckload shipment. In scenario 2, the seller offers discounts to all three buyers and increases their orders to FTLs, 50,000 pounds each. The seller uses only three direct truckload shipments to supply three buyers with sufficient inventory for two weeks. It saves three direct truckload shipments while its sales revenue reduces because of the discounts it offers for the additional three half FTL order quantities. Scenario 2 presents a suboptimal solution. Scenario 3 presents the optimal solution. The seller utilizes direct and multi-stop truckload shipments to determine which discounts need to offer. Buyer 1 is offered a discount and receives its FTL replenishment stock every other week. Buyers 2 and 3 receive their half FTL orders every week by a multi-stop truckload shipment. Scenario 3 represents the balance of using discounts and multi-stop truckloads to maximize the total profit.

The quantity-discount-with-freight-consolidation (QDFC) problem can be summarized as below:

- There is one seller and multiple buyers. Buyers have constant demand and are supplied by the seller.

- Each buyer uses a (Q, r) inventory policy, in which Q must be less than a truck's capacity. Each buyer is replenished by only one truck in a time period.

- The seller can deliver more than Q to a buyer for a replenishment. The amount beyond Q will have a discount.

- There is no limit on the number of routes. Each route starts at the seller and ends at its last buyer.

- The problem's objective function is the seller's profit. The profit is the sales revenue less discounts and transportation cost.

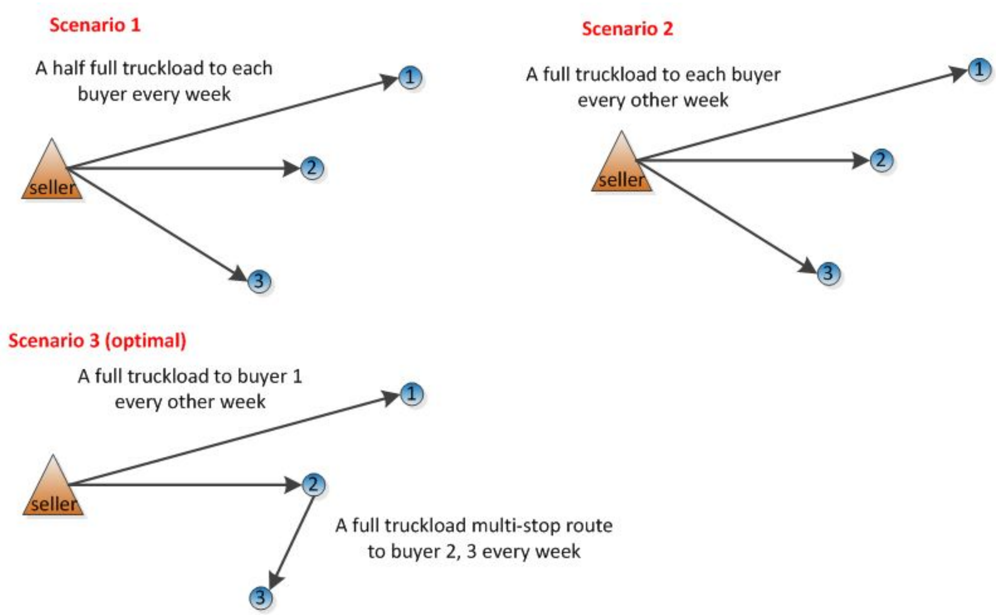

Figure 2.2: An example of a QDFC problem

The remainder of the chapter is organized as follows. Section 2 reviews the current literature. Section 3 discusses the research motivation and contribution. Section 4 introduces the model and formulations of the problem. Section 5 presents the results from experiments. Section 6 concludes the chapter.

2.2 Literature Review

The problem studied in this chapter seeks replenishment quantity and routing decisions taking into account inventory levels and locations. Therefore, this section will review vehicle-routing problems, inventory-routing problems, and quantity-discount problems.

2.2.1 Vehicle-Routing Problems

Vehicle-routing problems (VRP) have attracted a lot of research attention (see Toth and Vigo [21]). A special case of the VRP is the Open Vehicle Routing Problem (OVRP). The unique feature of an OVRP is that trucks do not return to a depot or their first pick-up locations after finishing the last delivery of a route. In other words, return legs are not included in the problem's cost function; therefore, a vehicle-routing problem can be converted into an OVRP by setting the transportation unit costs of return legs to zero. The OVRP is used when shippers do not own trucks and pay for the hauling service load by load. For example, Del Monte, a food producer, hires the truckload hauling service provided by J. B. Hunt to deliver a load from its plant in Hanford, California to a distribution center in Bentonville, Arkansas. Del Monte will pay for the truck from Hanford to Bentonville. After the load is delivered in Bentonville, Del Monte releases the truck and has no interest in returning it back to Hanford or sending it to somewhere else. That is the fundamental difference between common carriers and private carriers. Walmart, for example, owns and operates its private trucks. It must plan how to utilize its trucks and is responsible for all of its fleet's costs. The OVRP is encountered in practice very often as several manufacturers and shippers hire trucks to ship their products. Figure 2.3 illustrates the difference between VRP and OVRP.

The OVRP is an NP-hard problem. Sariklis and Powell [20] proposed a heuristic to solve a

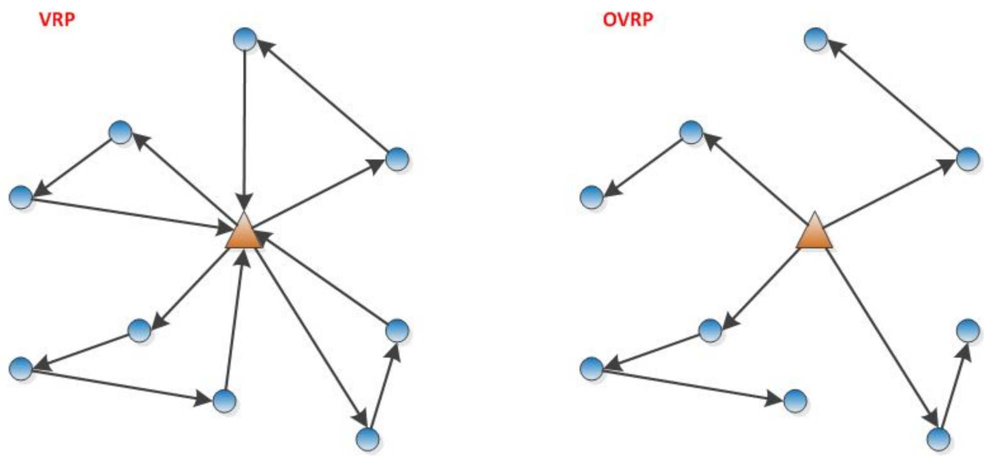

Figure 2.3: A Comparison of VRP to OVRP

generic OVRP. Their heuristic methodology utilized clustering and minimum spanning trees to generate routes. Computational results showed their proposed methodology was better than the best methodology in the literature in 8 out of 14 cases. Brandao [2] proposed a Tabu Search heuristic to solve a generic OVRP. Li et al. [12] reviewed algorithms and computational results of OVRP publications in the literature. Seven of the algorithms being reviewed were heuristics including Tabu Search and Adaptive Large Neighborhood Search. Salari et al. [19] proposed a solving methodology which randomly removed customers from a feasible solution and then used Integer Linear Programming to generate a new solution. Fung et al. [7] used a memetic heuristic to solve a variant of OVRP, called open capacitated arc routing problem.

There are few publications proposing exact algorithms to solve the OVRP. Letchford [10] proposed a branch-and-cut algorithm. The algorithm was tested with instances having at most 101 locations. We have not seen another more recent paper addressing the OVRP and using an exact algorithm.

When inventory levels at locations are taken into consideration, we have the inventory routing problem (IRP). Campbell and Clarke [4] studied an outbound transportation problem, a Vendor Managed Inventory problem. A supplier made replenishment decisions based on the inventory levels at its customers and delivered products using multi-stop routes. Oppen et al. [18] studied an inbound transportation problem in which a processing plant collected materials from its suppliers. They proposed a column generation algorithm to solve the problem and tested it on instances with up to 25 locations. In the problems studied by Gronhaug et al. [9], Mutlu and Çetinkaya [17], and Zhao et al. [23], inventory levels at both shipping and receiving locations were taken into consideration. Coelho and Laporte [6] proposed a branch-and-cut algorithm to solve many variants of the IRP. For a comprehensive review, readers should refer to Andersson et al. [1].

2.2.2 Quantity Discount Problems

Quantity discount problems are mostly seen applied in the inbound transportation or procurement context, where the decision maker is a buyer. Many problems studied in the literature are not NP-hard and were solved by analytical methodologies. A few others are NP-hard. The vendor selection problem is a prevalent topic. The common objective function includes transportation cost and procurement cost after discount. Goossens et al. [8] proposed a customized branch-and-bound technique to address the lot-sizing problem with an all-unit quantity discount schedule. Mansini and Tocchella [15] studied a vendor selection problem where they utilized piecewise quantity discount schedules. The problem was NP-hard. The study did not consider inventory holding cost. Transportation cost was truckload based. The research objectives were to test the performance of their proposed interactive rounding heuristic and the impacts of problem settings (discount structure, the number of products, and truckload capacity) on the solutions. Manerba and Mansini [13] studied the all-unit quantity discount, multi-product capacitated vendor selection problem. The authors proposed a branch-and-cut algorithm to solve the problem. The authors solved a problem instance having 100 vendors and 500 products. Manerba and Mansini [14] proposed an improved methodology which consisted of the Variable Neighborhood Decomposition Search as the master problem and the Mixed Integer Linear Programming as sub-problems. The authors showed the new methodology's performance was superior to the existing methodologies.

Discount schedules are commonly linear, all-unit, and incremental. Burke et al. [3] studied the capacitated supplier selection problem taking into account different discount schedules. Chen and Robinson [5] compared an all-unit discount schedule with an incremental one in the supplier-retailer quantity discount problem. The authors showed that supplier would benefit more from utilizing an incremental discount schedule than from an all-unit one. The authors used an analytical approach in this study.

There are also variants of the quantity discount problem considering factors such as resell options, price-sensitive demand, and supply risk. Li et al. [11] studied the all-units discount lot sizing problem with pre-defined discount structure where the buyers had the option to resell their remaining stock. Yin and Kim [22] studied container shipping services where demand was price-sensitive and the shipping slots were not storable. Meena and Sarmah [16] studied the quantity discount procurement problem taking into account the supply disruption risk.

None of the quantity discount problems currently available in the literature is applied in the outbound transportation context, where the decision maker is a seller.

2.3 Research Motivation and Contribution

The QDFC problem arose when we discovered the challenges a building material manufacturer faced when trying to fill up their trucks. One of the company's main products was fiberglass insulation, which was low value and bulky. The transportation cost was a significant portion of the total cost incurred in the process from making the product to having it delivered to a customer site. Therefore, savings from transportation cost became very attractive. In recent years, the company has spent about $\$10 - \$20$ million annually on transportation. The company observed that most customer orders were not full-truckloads and consequentially most truckload shipments were not full or near capacity. It implemented an incentive purchase program which offered a discount for an additional order amount beyond a customer's original order amount. Transportation cost savings would come from saving a delivery to a customer in the future as it ordered more in the present. After the incentive was implemented, key performance indicators showed that it actually increased the average size of truckload shipments however savings was not as expected. The company had two hypotheses for the unexpected outcome: 1) the discount was so high that it offset most of the transportation savings; 2) its customers were aware of the incentive program; therefore, they purposely ordered less than they needed in order to receive discounts.

The challenge is modelled as a QDFC problem. The problem is intended to verify the company's first hypothesis. The second hypothesis is left for future research. The problem is proven to be NP-hard in Section 2.4.2. There is not a known commercial software product to solve this problem efficiently. Logility and SAP split the problem into transportation management and demand planning packages. Their demand planning packages forecast demand for inventory management rather than manage customer demand. On the academic side, the problem is split into a quantity discount problem and an inventory-vehicle-routing problem. Each of the split problems has been studied widely. However, we have not seen the research that addresses the combined problem.

This chapter makes the following contributions:

- The chapter introduces a new variant of the inventory-vehicle-routing problem and quantity discount problem into the literature. The problem is encountered in practice and it is NP-hard. There are many directions to further develop and study the problem in the future.

- The chapter proposes the use of route elimination rules which make it possible for an available optimization solver to solve the problem in an acceptable duration. Despite its effectiveness, the rules are simple; therefore, they can be applied quickly and widely in practice. The rules are developed based on constraints in the mathematical formulation. Therefore, the computational time is improved without compromising solution quality. The route elimination rules improve the computational time by reducing the size of the route set and therefore the complexity of the constraints. Beside the route elimination rules, the chapter also proposes simple and effective tune-up techniques.

- The proposed methodology is efficient. On average, the more time it takes to solve an instance to optimality, the more savings will be returned. Even with the largest instance that was tested, the methodology quickly reached optimality when the savings was small. For example, it took 10 seconds to solve an instance to optimality and the savings was 2.2%. For another same size instance, it took $1,917$ seconds to solve and the savings was 14.1% (see Section 3.6).

- The chapter provides a means to study the interaction among discount offers, truck utilization, and total profit. The results found in the research can potentially support making operational decisions in the incentive purchase program. Total savings could be up to 18%.

2.4 Models

2.4.1 Model

The section introduces a formulation of the problem. The intent is to describe the problem mathematically.

Sets

- I : the set of buyers i and a sole seller $i = 0$.
- $I' = I \setminus \{0\}$: the set of buyers i.
- N : the number of delivery stops allowable in a route.
- R : the set of routes r.
- T : the set of time periods t.

Parameters

- C_i : the inventory capacity of buyer $i \in I'$.
- F : the maximum distance that can be travelled in a time period. The parameter is used to exclude multi-stop routes whose delivery stops cannot all be visited within a day. Both shippers and common carriers usually prefer to have this constraint in place because of the difficulties in executing multi-day multi-stop routes to customers.
- M : a very large number.
- (Q_i, r_i) : the buyer i's inventory policy. Q_i is the original replenishment order size of buyer i. Without loss of generality, it is assumed that $r_i = 0 \ \forall i \in I'$.

- c_{ij} : the transportation cost of the lane from i to j including only rate per mile cost.

- d_i : the demand of buyer i per time period.

- f_{ij} : the distance from i to j.

- g_n : the total stop-off charge for a route that has n delivery stops.

- p_i : the full price of a product unit for buyer i.

- m : the truckload capacity.

- u_i : the incremental discount for the additional replenishment amount beyond Q_i.

Variables

- $s_i^t \in \mathbb{R}^+$: the inventory level of buyer i at the end of period t.

- $q_i^{rt} \in \mathbb{R}^+$: the replenishment quantity for buyer i delivered by route r in time period t.

- β_i^t : the additional replenishment amount beyond Q_i from buyer i in time period t, i.e.

$$\beta_i^t = \begin{cases} \sum_{r\in R} q_i^{rt} - Q_i & \text{if } \sum_{r\in R} q_i^{rt} \geq Q_i \\ 0 & \text{otherwise} \end{cases}$$

- x_{ij}^{rt} defines the relationship of arc (i,j) and route r in time period t, i.e.

$$x_{ij}^{rt} = \begin{cases} 1 & \text{if route } r \text{ uses arc } (i,j) \text{ in time period } t \\ 0 & \text{otherwise} \end{cases}$$

- y_i^{rt} defines the relationship of buyer i and route r in time period t, i.e.

$$y_i^{rt} = \begin{cases} 1 & \text{if route } r \text{ visits buyer } i \text{ in time period } t \\ 0 & \text{otherwise} \end{cases}$$

- γ_i^t indicates whether buyer i places its replenishment order in time period t, i.e.

$$\gamma_i^t = \begin{cases} 1 & \text{if buyer } i \text{ places its order in time period } t \\ 0 & \text{otherwise} \end{cases}$$

- η_n^{rt} indicates whether there are n delivery stops in route r in time period t, i.e.

$$\eta_n^{rt} = \begin{cases} 1 & \text{if route } r \text{ in time period } t \text{ has } n \text{ delivery stops} \\ 0 & \text{otherwise} \end{cases}$$

The preliminary formulation (denoted as QDVRP) of the QDFC problem is as follows:

$$Maximize \sum_{i \in I'} (p_i \sum_{t \in T} (\sum_{r \in R} q_i^{rt} - u_i \beta_i^t)) - \sum_{\substack{i,j \in I: i \neq j \\ r \in R \\ t \in T}} c_{ij} x_{ij}^{rt} - \sum_{\substack{r \in R \\ t \in T \\ n \in N}} g_n \eta_n^{rt} \tag{2.1}$$

s.t.

$$s_i^0 = s_i^T = 0 \qquad \forall i \in I' \tag{2.2}$$

$$s_i^{t-1} + \sum_{r \in R} q_i^{rt} = d_i + s_i^t \qquad \forall i \in I', \forall t \in T \tag{2.3}$$

$$s_i^t \leq C_i \qquad \forall i \in I', \forall t \in T \tag{2.4}$$

$$s_i^{t-1} - d_i \leq M(1 - \gamma_i^t) \qquad \forall i \in I', \forall t \in T \tag{2.5}$$

$$Q_i\gamma_i^t \leq \sum_{r\in R} q_i^{rt} \leq M\gamma_i^t \qquad \forall i \in I', \forall t \in T \tag{2.6}$$

$$q_i^{rt} \leq m y_i^{rt} \qquad \forall i \in I', \forall r \in R, \forall t \in T \tag{2.7}$$

$$\sum_{i\in I'} q_i^{rt} \leq m y_0^{rt} \qquad \forall r \in R, \forall t \in T \tag{2.8}$$

$$\beta_i^t \geq \sum_{r\in R} q_i^{rt} - Q_i \qquad \forall i \in I', \forall t \in T \tag{2.9}$$

$$\sum_{r\in R} y_i^{rt} \leq 1 \qquad \forall i \in I', \forall t \in T \tag{2.10}$$

$$\sum_{j\in I:j\neq i} x_{ij}^{rt} = \sum_{j\in I:j\neq i} x_{ji}^{rt} = y_i^{rt} \qquad \forall i \in I', \forall r \in R, \forall t \in T \tag{2.11}$$

$$\sum_{i,j\in J:i\neq j} x_{ij}^{rt} \leq \sum_{i\in J} y_i^{rt} - y_k^{rt} \qquad \forall J \subset I', \forall k \subset J, \forall r \in R, \forall t \in T \tag{2.12}$$

$$\sum_{n\in N} \eta_n^{rt} \leq 1 \qquad \forall r \in R, \forall t \in T \tag{2.13}$$

$$\sum_{i\in I'} y_i^{rt} = \sum_{n\in N} n\eta_n^{rt} \qquad \forall r \in R, \forall t \in T \tag{2.14}$$

$$\sum_{i,j\in I':i\neq j} f_{ij} x_{ij}^{rt} \leq F \qquad \forall r \in R, \forall t \in T \tag{2.15}$$

$$s_i^t \in \mathbb{R}^+ \qquad \forall i \in I', \forall t \in T \tag{2.16}$$

$$q_i^{rt} \in \mathbb{R}^+ \qquad \forall i \in I', \forall r \in R, \forall t \in T \tag{2.17}$$

$$x_{ij}^{rt} \in \{0,1\} \qquad \forall i,j \in I, \forall r \in R, \forall t \in T \tag{2.18}$$

$$y_i^{rt} \in \{0,1\} \qquad \forall i \in I, \forall r \in R, \forall t \in T \tag{2.19}$$

$$\beta_i^t \in \mathbb{R}^+ \qquad \forall i \in I', \forall t \in T \tag{2.20}$$

$$\gamma_i^t \in \{0,1\} \qquad \forall i \in I', \forall t \in T \tag{2.21}$$

$$\eta_n^{rt} \in \{0,1\} \qquad \forall r \in R, \forall t \in T, \forall n \in N \tag{2.22}$$

The objective function (2.1) maximizes the seller's profit which is equal to the sales revenue less transportation cost. The total sales revenue is the total full price of sold products less the discount for the additional amount beyond Q_i. The transportation cost consists of rate-per-mile cost and stop-off charges. Constraint (2.2) sets the initial inventory level and the remaining inventory level at the end of the planning horizon to zero. Constraint (2.3) calculates the inventory level in an intermediate time period. This amount is equal to the inventory level in the previous time period plus total replenishment quantity less demand in the time period. Constraint (2.4) enforces the inventory capacity at buyers.

Constraint (2.5) enforces a buyer not to place an order when its inventory is enough to cover the demand in a time period. Constraint (2.6) enforces no product to be delivered to a buyer when it does not place an order. If a replenishment order is placed, it must be at least Q_i. Constraint (2.7) establishes the relationship between a route and the orders it delivers: a route will deliver some product to a buyer if it visits the buyer. Constraint (2.8) requires that total freight delivered by a route (a truck) must not exceed the truck capacity. Constraint (2.9) calculates the amount of an order beyond Q_i. Constraint (2.10) requires the replenishment order to a buyer not be split into multiple deliveries. Constraint (2.11) and constraint (2.12) are the degree of connection and sub-tour elimination of a typical vehicle routing problem. Constraint (2.13) and constraint (2.14) calculate the number of stops per route. Constraint (2.15) limits the total delivery distance so that all deliveries happen within a time period.

2.4.2 Complexity

In this section, we will prove that QDVRP is NP-hard by showing that a generic OVRP, a well-known NP-hard problem, is a special case of QDVRP. We assume:

- $T = \{0, 1\}$: The problem has only one time period. Constraint (2.2) fixes variable $s_i^t = 0 \ \forall i \in I', \forall t \in T$. Therefore, s_i^t can be replaced by constant 0. Hence, constraints (2.4) and (2.5) are always satisfied.

- $d_i^t = Q_i \quad \forall i \in I'$: Constraint (2.3) becomes:

$$\sum_{r \in R} q_i^{rt} = d_i \qquad \forall i \in I', \forall t \in T \tag{2.23}$$

Replacing $\sum_{r \in R} q_i^{rt}$ in constraint (2.6) by d_i, we have $\gamma_i^t = 1 \; \forall i \in I', \forall t \in T$. Constraint (2.6) can be removed. Replacing $\sum_{r \in R} q_i^{rt}$ by d_i in constraint (2.9), we have $\beta_i^t = 0 \; \forall i \in I', \forall t \in T$. Constraint (2.9) can be removed. The first term in the objective function is:

$$\sum_{i \in I', t \in T} (p_i(\sum_{r \in R} q_i^{rt} - u_i \beta_i^t)) = \sum_{i \in I', t \in T} p_i d_i \tag{2.24}$$

It is a constant and can be removed from the objective function.

- $g_n = 0 \quad \forall n \in N$: All stop-off charges are free. There is no need to calculate the number of stops. Variable η_n^{rt} and constraints (2.13) and (2.14) can be removed.

- $F = +\infty$: There is no limit of the delivery distance of a route. Constraint (2.15) becomes redundant and can be removed.

Since we have one time period, the time period index is dropped for simplicity. The reduced QDVRP is:

$$Minimize \sum_{\substack{i,j \in I: i \neq j \\ r \in R}} c_{ij} x_{ij}^r \tag{2.25}$$

s.t.

$$\sum_{r \in R} q_i^r = d_i \qquad \forall i \in I' \tag{2.26}$$

$$q_i^r \leq m y_i^r \qquad \forall i \in I', \forall r \in R \tag{2.27}$$

$$\sum_{i \in I'} q_i^r \leq m y_0^r \qquad \forall r \in R \tag{2.28}$$

$$\sum_{r \in R} y_i^r \leq 1 \qquad \forall i \in I' \tag{2.29}$$

$$\sum_{j \in I: j \neq i} x_{ij}^r = \sum_{j \in I: j \neq i} x_{ji}^r = y_i^r \qquad \forall i \in I', \forall r \in R \tag{2.30}$$

$$\sum_{i,j \in J: i \neq j} x_{ij}^r \leq \sum_{i \in J} y_i^r - y_k^r \qquad \forall J \subset I', \forall k \subset J, \forall r \in R \tag{2.31}$$

$$q_i^r \in \mathbb{R}^+ \qquad \forall i \in I', \forall r \in R \tag{2.32}$$

$$x_{ij}^r \in \{0, 1\} \qquad \forall i, j \in I, \forall r \in R \tag{2.33}$$

$$y_i^r \in \{0, 1\} \qquad \forall i \in I, \forall r \in R \tag{2.34}$$

The problem is a typical OVRP. According to Brandao [2], the problem is NP-hard. Hence, the QDVRP is NP-hard.

2.4.3 Route-based Model

The computational performance of the previous QDVRP formulation is not adequate. In an experiment which has eight buyers, Gurobi took ten hours to reach 3% MIP gap. ILOG CPLEX's performance was not better. We chose to continue with the exact approach. In this section, the problem is reformulated based on pre-defined routes. All possible routes are checked against the route elimination rules discussed below. Route elimination rules do not compromise the problem's objective value for computational performance. They remove the infeasible routes and feasible routes that would not be selected in an optimal solution. Table 2.2 illustrates the elimination rules in an instance of five buyers. 325 open routes could be created to visit five buyers from a seller. 315 routes are eliminated by the rules. The remaining 10 routes are used to solve the problem. The rules are:

- Rule 1: Eliminate multi-stop routes where all delivery stops cannot be finished within a day. Both shippers and carriers do not prefer routes whose delivery stops span into multiple days. It increases the chance for a truck not to arrive at a delivery stop by the appointment time. Carriers sometimes pay a penalty for missing an appointment while the shipper encounters service level reduction at the customer.

- Rule 2: Eliminate more-stop routes when the same set of buyers can be visited by multiple less-stop routes at the same or less total cost. For example, a three-stop route r_1 visits buyers i_1, i_2, and i_3 and costs \$500. A two-stop route r_2 visits buyers i_1 and i_2 and costs \$350. A single-stop route r_3 visits buyers i_3 and costs \$100. The three buyers can be visited by either one route r_1 or one route r_2 and one route r_3. The former option costs \$50 more than the latter option. Therefore, route r_1 will not be selected in the optimal solution and is eliminated.

- Rule 3: Eliminate over-capacity routes based on the minimum replenishment order quantities of buyers in the routes. Assuming that a candidate route r^0 visits buyers in $I^0 \subset I'$, we have the route's total weight:

$$m^0 = \sum_{i \in I^0} q_i^{r^0 t} \qquad \forall t \in T \tag{2.35}$$

  From the definition of β_i^t and constraint (2.10), we have:

$$q_i^{rt} = Q_i + \beta_i^t \qquad \forall i \in I' : y_i^r = 1, \forall t \in T$$

(2.36)

$$\Leftrightarrow q_i^{r^0 t} = Q_i + \beta_i^t \qquad \forall i \in I^0, \forall t \in T \tag{2.37}$$

Therefore,

$$m^0 = \sum_{i \in I^0} (Q_i + \beta_i^t) \qquad \forall t \in T \tag{2.38}$$

From constraints (2.8) and (2.20), we have:

$$m \geq m^0 \geq \sum_{i \in I^0} Q_i \qquad \forall t \in T \tag{2.39}$$

$$\Rightarrow m \geq \sum_{i \in I^0} Q_i \qquad \forall t \in T \tag{2.40}$$

A candidate route r^0 that does not satisfy inequality (2.40) will be eliminated.

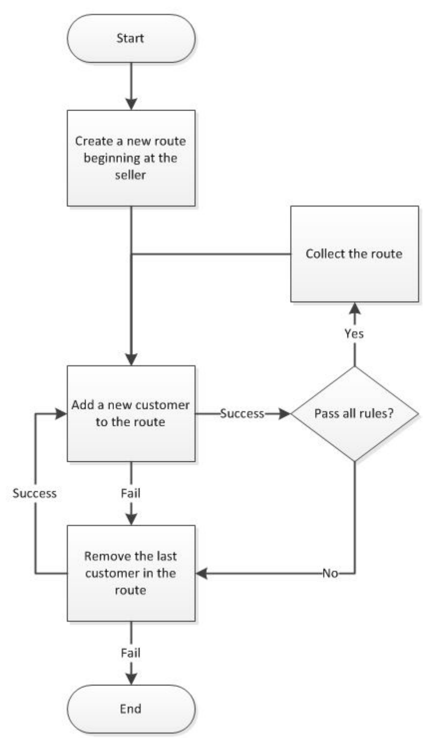

Figure 2.4: Route generation procedure

Figure 2.4 shows the flow chart of our route generation procedure. We start building a route with zero customers. Each time a new customer is added to the route, the route is checked against the elimination rules. If it passes the rules, it is collected into the list of eligible routes for optimization and the flow goes back to the step of adding one new customer. The route has more and more customers if it keeps passing the rules. If it violates a rule, the last customer is removed from the route and the process goes back to the step of adding one new customer. It fails to add a new customer when all customers have been checked. It fails to remove a customer from the route when there is no customer in the route. In order to prevent checking a combination of customers more than once, all customers are listed in an order. The next customer to be checked is behind the newly removed customer in the list. The algorithm implicitly eliminates many routes before generating them. Due to this feature, it takes about 100 seconds to generate all eligible routes for an 70-customer instance. We make the following two assumptions in the algorithm:

- If it is impossible to add one more customer to a route, it is impossible to add more than one customer to the route.
- A route that is created by adding one customer to an ineligible route is ineligible.

Besides the advantage of eliminating routes, the route-based approach also effectively considers increasing stop-off charges. Shippers are usually charged based on an increasing stop-off charge schedule: a subsequent stop is charged higher than a preceding one in a multi-stop route. The route-based formulation makes it simpler to incorporate the increasing stop-off charge schedule than a node-based formulation.

The route-based formulation has new and modified parameters and variables:

Parameters

- c^r : the transportation cost of route r including stop-off charges.

- v_i^r defines the relationship of buyer i and route r, i.e.

$$v_i^r = \begin{cases} 1 & \text{if route } r \text{ visits buyer } i \\ 0 & \text{otherwise} \end{cases}$$

Variables

- x^{rt} indicates whether route r is used in time period t, i.e.

$$x^{rt} = \begin{cases} 1 & \text{if route } r \text{ is used in time period } t \\ 0 & \text{otherwise} \end{cases}$$

The route-based formulation (QDVRP_RB) is as follows:

$$Maximize \sum_{i \in I'} p_i(\sum_{t \in T}(\sum_{r \in R} q_i^{rt} - u_i \beta_i^t)) - \sum_{\substack{r \in R \\ t \in T}} c^r x^{rt} \tag{2.41}$$

s.t.

$$s_i^0 = s_i^T = 0 \qquad \forall i \in I' \tag{2.42}$$

$$s_i^{t-1} + \sum_{r \in R} q_i^{rt} = d_i + s_i^t \qquad \forall i \in I', \forall t \in T \tag{2.43}$$

$$s_i^t \leq C_i \qquad \forall i \in I', \forall t \in T \tag{2.44}$$

$$s_i^{t-1} - d_i \leq M(1 - \gamma_i^t) \qquad \forall i \in I', \forall t \in T \tag{2.45}$$

$$Q_i \gamma_i^t \leq \sum_{r \in R} q_i^{rt} \leq M \gamma_i^t \qquad \forall i \in I', \forall t \in T \tag{2.46}$$

$$q_i^{rt} \leq m v_i^r x^{rt} \qquad \forall i \in I', \forall r \in R, \forall t \in T \tag{2.47}$$

$$\sum_{i \in I'} q_i^{rt} \leq m x^{rt} \qquad \forall r \in R, \forall t \in T \tag{2.48}$$

$$\beta_i^t \geq \sum_{r \in R} q_i^{rt} - Q_i \qquad \forall i \in I', \forall t \in T \tag{2.49}$$

$$\sum_{r \in R} v_i^r x^{rt} \leq 1 \qquad \forall i \in I', \forall t \in T \tag{2.50}$$

$$s_i^t \in \mathbb{R}^+ \qquad \forall i \in I', \forall t \in T \tag{2.51}$$

$$q_i^{rt} \in \mathbb{R}^+ \qquad \forall i \in I', \forall r \in R, \forall t \in T \tag{2.52}$$

$$x^{rt} \in \{0, 1\} \qquad \forall r \in R, \forall t \in T \tag{2.53}$$

$$\beta_i^t \in \mathbb{R}^+ \qquad \forall i \in I', \forall t \in T \tag{2.54}$$

$$\gamma_i^t \in \{0, 1\} \qquad \forall i \in I', \forall t \in T \tag{2.55}$$

Table 2.2: Route elimination rules' impact on a 5-buyer instance

	# of Routes	**Percentage of Total Routes**
Total possible routes	325	100%
Routes eliminated by Rule 1	92	28%
Routes eliminated by Rule 2	132	41%
Routes eliminated by Rule 3	91	28%
Accepted Routes	10	3%

2.4.4 Formulation Improvement

In this section, the route-based formulation will be simplified in order to further improve its computational performance.

Eliminate inventory level variables

From constraint (2.42) and constraint (2.43), we have:

$$s_i^t = \sum_{k \in T : k \leq t} \sum_{r \in R} q_i^{rk} - t d_i \tag{2.56}$$

Eliminate order period variables

Table 2.3 compares the values of γ_i^t and $\sum_{r \in R} v_i^r x^{rt}$.

Table 2.3: Eliminating order period variables

$\sum_{r \in R} v_i^r x^{rt}$	γ_i^t	**Notes**
0	0	$\surd$
0	1	infeasible
1	0	feasible but not optimal
1	1	$\surd$

- Row 1 of Table 2.3: $\sum_{r \in R} v_i^r x^{rt} = 0$ means that none of the selected routes in time period t will visit buyer i. Therefore, buyer i will not receive a replenishment order, $\gamma_i^t = 0$. This case is feasible.

- Row 2: Buyer i will receive a replenishment order in time period t, $\gamma_i^t = 1$, while none of the selected routes will visit buyer i in time period t, $\sum_{r \in R} v_i^r x^{rt} = 0$. This case is infeasible.

- Row 3: Buyer i will not receive a replenishment order in time period t, $\gamma_i^t = 0$, even though there is a selected route, for example r_1, that will visit buyer i in time period t, $\sum_{r \in R} v_i^r x^{rt} = 1$. This case is feasible but will not be in the optimal solution. Another route r_2 that visits all buyers in route r_1 but buyer i is cheaper than route r_1.

- Row 4: Buyer i will receive a replenishment order in time period t, $\gamma_i^t = 1$, when there is a selected route that will visit buyer i in time period t, $\sum_{r\in R} v_i^r x^{rt} = 1$. This case is feasible.

Therefore, we replace γ_i^t by $\sum_{r\in R} v_i^r x^{rt}$.

Eliminate big M

From constraint (2.44) and constraint (2.45), we have

$$s_i^{t-1} - d_i \leq C_i - d_i \leq M(1 - \gamma_i^t) \tag{2.57}$$

Therefore, big M in constraint (2.45) can be replaced by $C_i - d_i$.

From the right part of constraint (2.46), we have

$$\sum_{r\in R} q_i^{rt} \leq M\gamma_i^t = M \sum_{r\in R} v_i^r x^{rt} \tag{2.58}$$

Applying constraint (2.50) to the above inequality, we have:

$$\sum_{r\in R} q_i^{rt} \leq M \tag{2.59}$$

On the other side, from constraint (2.47), we have:

$$\sum_{r\in R} q_i^{rt} \leq m \sum_{r\in R} v_i^r x^{rt} \leq m \tag{2.60}$$

From both inequalities (2.59) and (2.60), we can remove the right part of constraint (2.46) from the formulation because it is not as tight as constraint (2.47).

Initial Solution

The problem is solved to optimality by Gurobi for one time period. The solution is replicated for multiple time periods and introduced into the main problem as an initial solution.

Weight Granularity

Fundamentally weight is a real number. In practice, the order of a product must be however measured by integer units such as box, case, pallet, roll, and drum. Without loss of generality, we introduce the incremental unit of weight equal to 1% of an FTL. The current formulation is an MIP because routes are selected by binary variables and orders are measured by weight. The formulation therefore becomes an IP. The introduction of weight granularity helps improve the computation but the magnitude of the weight granularity itself does not have a noticeable impact. There is not a consistent computation difference between 1% and 5% weight granularities.

Computational Performance

Each aforementioned improvement was tested with a few instances, less than ten, to evaluate its performance. Table 2.4 summarizes the results. $\surd$ is for significantly shorter run time and $\times$ otherwise. While removing γ_i^t does not improve the computational performance, the discovery of the equation helps to remove one of the big M, which improves the computational performance.

Table 2.4: Improvement evaluation

Improvement	**Evaluation**
$s_i^t = \sum_{k \in T: k \leq t} \sum_{r \in R} q_i^{rk} - t d_i$	$\times$
$\gamma_i^t = \sum_{r \in R} v_i^r x^{rt}$	$\times$
$M_{constraint\ (2.45)} = C_i - d_i$	$\surd$
Remove the right term of constraint (2.46)	$\surd$
Initial solution: optimal solution for $T = 1$	$\surd$
MIP $\longrightarrow$ IP	$\surd$

2.4.5 Final Formulation

After all of the observations and efforts to improve the problem's formulation, the final formulation is represented below and is used for experiments to evaluate its performance. The improved route-based formulation (QDVRP_IM) is as follows:

$$Maximize \sum_{i \in I'} p_i (\sum_{t \in T} (\sum_{r \in R} q_i^{rt} - u_i \beta_i^t)) - \sum_{\substack{r \in R \\ t \in T}} c^r x^{rt} \tag{2.61}$$

s.t.

$$s_i^0 = s_i^T = 0 \qquad \forall i \in I' \tag{2.62}$$

$$s_i^{t-1} + \sum_{r \in R} q_i^{rt} = d_i + s_i^t \qquad \forall i \in I', \forall t \in T \tag{2.63}$$

$$s_i^t \leq C_i \qquad \forall i \in I', \forall t \in T \tag{2.64}$$

$$s_i^{t-1} - d_i \leq (C_i - d_i)(1 - \gamma_i^t) \qquad \forall i \in I', \forall t \in T \tag{2.65}$$

$$Q_i \gamma_i^t \leq \sum_{r \in R} q_i^{rt} \qquad \forall i \in I', \forall t \in T \tag{2.66}$$

$$q_i^{rt} \leq m v_i^r x^{rt} \qquad \forall i \in I', \forall r \in R, \forall t \in T \tag{2.67}$$

$$\sum_{i \in I'} q_i^{rt} \leq mx^{rt} \qquad \forall r \in R, \forall t \in T \tag{2.68}$$

$$\beta_i^t \geq \sum_{r \in R} q_i^{rt} - Q_i \qquad \forall i \in I', \forall t \in T \tag{2.69}$$

$$\sum_{r \in R} v_i^r x^{rt} \leq 1 \qquad \forall i \in I', \forall t \in T \tag{2.70}$$

$$x^{rt} \in \{0, 1\} \qquad \forall r \in R, \forall t \in T \tag{2.71}$$

$$\gamma_i^t \in \{0, 1\} \qquad \forall i \in I', \forall t \in T \tag{2.72}$$

$$s_i^t \in \mathbb{Z}^+ \qquad \forall i \in I', \forall t \in T \tag{2.73}$$

$$q_i^{rt} \in \mathbb{Z}^+ \qquad \forall i \in I', \forall r \in R, \forall t \in T \tag{2.74}$$

$$\beta_i^t \in \mathbb{Z}^+ \qquad \forall i \in I', \forall t \in T \tag{2.75}$$

2.5 Experiments

We developed the experimental design shown in Table 4.2 to evaluate the performance of the final model of the quantity discount with freight consolidation problem. The experiments were run on a Windows server that had an Intel Xeon CPU X7350 2.98GHz and 16GB RAM. The optimization solver used was Gurobi Optimizer version 5.1.

There were three replications per each level of combination. Buyers were randomly selected from the list of customers of two suppliers in the relevant industries. The data was provided by a 3PL company. Buyers' demand and inventory capacity were generated by random discrete distributions. In total, 324 instances were solved. In each instance, the seller was in the center of the area and the buyers surrounded it. Figure 2.5 illustrates an instance that had 70 buyers in a circle with a 400-mile radius. One time period represented one day. There were 24 time periods corresponding to one month of planning in each instance. Each buyer would place its order daily to cover daily demand.

In this experiment, Gurobi was configured to use specific parameters rather than its defaults. These configurations were:

- Optimality gap: Gurobi was set to stop when reaching a 1% MIP gap.
- Root relaxation method: Gurobi was directed to use a Barrier method instead of its automatic choice. On average, runtime was improved by a factor of eight.

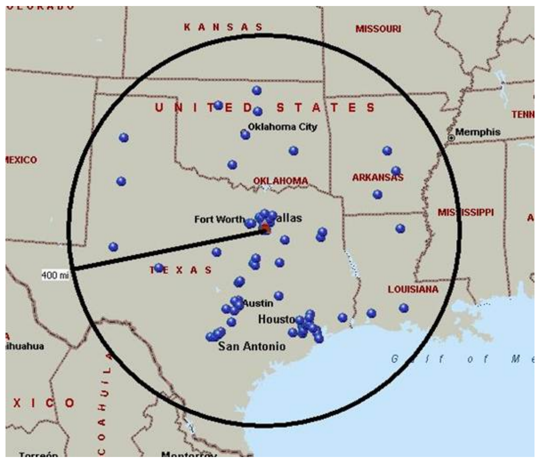

Figure 2.5: An instance used in the experiment

Table 2.5: Experimental Design

Factor	**Level Description**	**Random Distribution**	**Levels**
I'	50, 70		2
Buyers density (radius in miles)	200, 300, 400		3
d_i (% of FTL)	$\bar{d} = 40\%, 50\%, 60\%$	Discrete	3
u_i	5%, 10%, 20%		3
C_i (times of FTL)	$\bar{C} = 2, 3$	Discrete	2
Replications per level combination			3
Total number of problem instances			324

All instances were solved to optimality. The easiest instance took 2 seconds to reach optimality. The hardest instance took less than 45 minutes. Tables 2.7, 2.8, 2.9, 2.10, 2.11, and 2.12 summarizes the results. Each table represents the results of instances of the same number of buyers and buyer density factor stated in the table's caption. The first three columns of each table (average demand, discount, and inventory capacity) describe instances' characteristics. Average demand is the average demand of all buyers in an instance. It is represented as a percentage of a FTL quantity. Discount is a discount percent off the full price. Inventory capacity (of buyers) is in the unit of a FTL quantity. The next three columns (savings, discounted orders, and MS routes) are the metrics associated with the solutions. Savings is the additional profit realized by using the QDFC model compared to not having a model in place, where each order was shipped in a separate truckload shipment and there was no discount. The discounted order is the percentage of the number of discounted orders over all orders. The multi-stop routes, MS routes, column is the percentage of the number of multi-stop truckload shipments over all shipments. The last three columns (RGT, MT, TT) summarize the methodology's runtimes in second. RGT stands for Route Generation Time. MT stands for Modelling Time. TT stands for Total Time. The last row in each table shows average values.

The tables show an expected correlation between the number of discounted orders and multi-stop routes and savings. The higher the number of discounted orders and multi-stop routes are, the higher the savings. In Table 2.7, average savings is 2% and discounted orders and multi-stop routes are 20% and 18% respectively. In Table 2.9, average savings is 10% and discounted orders and multi-stop routes are 38% and 33% respectively. The increase of the number of buyers and the buyer radius increases total computation time. Average total computation time is from 49 seconds in Table 2.7 to 564 seconds in Table 2.12. There is not a significant correlation between the buyer radius and route generation time. The increase of the buyer radius increases average savings.

For each average demand in each table, the savings is higher when the discount is lower. The savings is lower when the average demand increases, which is as expected. A larger order leaves less opportunities to improve truck utilization than a smaller one does. The buyer inventory capacity did not have a significant impact on the savings or the runtime. More orders were offered a discount when the discount was low and more multi-stop truckload shipments were utilized when the discount was high. On average, savings decreased from 7% to 5% when discount increased from 5% to 20%.

The more time it took to solve an instance to optimality, the more savings was realized. The seven easiest instances each took 23 seconds to be solved. Their average savings was 1%. The top five hardest instances each took $2,873$ seconds on average to be solved and their average savings was 13%.

2.5.1 Model Comparison

This section presents the experimental results to compare the QDVRP_IM model with the QDVRP model. There were nine instances run by both models. All instances had the same length of planning horizon. Both models were limited to two hours of computation. Table 2.6 summarizes the results. Both models returned almost the same objective values for the first six instances. The QDVRP_IM model took at most one minute for an instance while the QDVRP model ran for two hours for each instance. When the instance's size increased, the QDVRP model ran out of memory (OOM) quickly.

Table 2.6: Performance comparison between QDVRP and QDVRP_IM models.

Instance	# of Buyers	QDVRP Model Objective Value	QDVRP Model Time (Sec.)	QDVRP_IM Model Objective Value	QDVRP_IM Model Time (Sec.)	Objective Value Difference
1	5	476,568	7,200	476,568	1	0.00%
2	5	487,067	7,200	487,067	2	0.00%
3	5	609,231	7,200	609,290	7	0.01%
4	10	944,750	7,200	963,624	1	1.96%
5	10	1,083,761	7,200	1,085,146	3	0.13%
6	10	1,236,077	7,200	1,242,806	52	0.54%
7	15	OOM	N/A	969,429	11	N/A
8	15	OOM	N/A	1,149,630	5	N/A
9	15	OOM	N/A	1,036,089	14	N/A

2.6 Conclusions and Future Work

The QDFC problem is an interesting problem that is encountered in practice. The problem is NP-hard and current optimization solvers are not able to solve problems of practical size and scale. The chapter proposes an effective tool to study the problem and potentially help design an incentive purchase program with realizable savings. The proposed methodology can solve large problems within an hour. It is a divide-and-conquer strategy in which the optimal solution is not compromised for computational time.

Relevant existing models in the literature can be put into two categories. In the first category are models that addressed vehicle routing problems from a shipment planning perspective (e.g, Salari et al. [19]). In the second category are models that addressed quantity discount problem from buyer's perspective (e.g., Mansini and Tocchella [15]). Models in both categories lack key attributes discussed in Section 2.2; therefore, it prevents them from being adopted to address the problem studied in this chapter.

The major contributions of the chapter to the literature therefore are:

- Introducing a new variant of the inventory-vehicle-routing problem and quantity discount problem into the literature.
- Proposing the non-compromising route elimination rules and other techniques which significantly improve the computational time of a commercial optimization solver.
- Providing an effective tool to support strategy and operational decision making.

There are plenty of directions for future research. The specific problem described in Section 2.3 has additional elements that warrant further investigation. Specifically, opportunities exist to include buyer reactions into the modelling framework. Additional work might lead to models that incorporate elements of negotiation between buyers and sellers. Such a model, to be utilized effectively, would need to arrive at solutions in a short time period.

2.7 Experiment Results

Table 2.7: 50 buyers and 200-mile radius

Average Demand	**Discount**	**Inventory Capacity**	**Savings**	**Discounted Orders**	**MS Routes**	**RGT**	**MT**	**TT**
40%	5%	2	5%	81%	5%	18	54	72
40%	5%	3	5%	76%	2%	18	73	91
40%	10%	2	4%	12%	30%	18	102	121
40%	10%	3	4%	14%	29%	19	106	125
40%	20%	2	3%	0%	36%	20	40	60
40%	20%	3	3%	0%	34%	20	67	87
50%	5%	2	2%	44%	11%	20	8	28
50%	5%	3	3%	55%	11%	20	9	29
50%	10%	2	2%	0%	29%	20	8	28
50%	10%	3	2%	0%	29%	19	9	28
50%	20%	2	2%	0%	33%	20	10	30
50%	20%	3	2%	0%	36%	20	11	31
60%	5%	2	1%	38%	3%	20	4	24
60%	5%	3	1%	41%	1%	20	4	24
60%	10%	2	0%	0%	7%	20	4	24
60%	10%	3	1%	0%	11%	20	5	25
60%	20%	2	1%	0%	9%	20	4	24
60%	20%	3	1%	0%	11%	20	6	26
			2%	**20%**	**18%**	**19**	**29**	**49**

Table 2.8: 50 buyers and 300-mile radius

Average Demand	**Discount**	**Inventory Capacity**	**Savings**	**Discounted Orders**	**MS Routes**	**RGT**	**MT**	**TT**
40%	5%	2	14%	83%	7%	20	177	197
40%	5%	3	13%	81%	6%	21	94	115
40%	10%	2	11%	21%	53%	20	199	219
40%	10%	3	12%	28%	49%	20	269	289
40%	20%	2	10%	0%	56%	20	270	289
40%	20%	3	10%	5%	58%	20	244	263
50%	5%	2	8%	62%	11%	20	30	50
50%	5%	3	8%	72%	9%	20	24	44
50%	10%	2	6%	5%	51%	20	19	39
50%	10%	3	7%	7%	49%	20	36	56
50%	20%	2	6%	1%	56%	19	25	44
50%	20%	3	6%	0%	55%	21	17	38
60%	5%	2	4%	62%	1%	20	6	26
60%	5%	3	4%	63%	2%	20	5	25
60%	10%	2	4%	36%	18%	20	8	28
60%	10%	3	3%	42%	12%	20	6	26
60%	20%	2	2%	4%	15%	20	9	29
60%	20%	3	1%	0%	9%	20	6	26
			7%	**32%**	**29%**	**20**	**80**	**100**

Table 2.9: 50 buyers and 400-mile radius

Average Demand	Discount	Inventory Capacity	Savings	Discounted Orders	MS Routes	RGT	MT	TT
40%	5%	2	17%	88%	8%	20	161	181
40%	5%	3	18%	86%	3%	20	184	204
40%	10%	2	14%	31%	60%	20	228	248
40%	10%	3	16%	35%	58%	20	320	341
40%	20%	2	13%	9%	72%	20	339	359
40%	20%	3	13%	7%	74%	20	227	247
50%	5%	2	11%	75%	9%	19	32	51
50%	5%	3	10%	71%	11%	19	24	43
50%	10%	2	10%	22%	49%	20	68	88
50%	10%	3	9%	22%	51%	20	52	72
50%	20%	2	8%	0%	64%	20	26	46
50%	20%	3	9%	6%	65%	20	39	59
60%	5%	2	6%	72%	1%	19	9	29
60%	5%	3	6%	71%	2%	20	8	28
60%	10%	2	4%	49%	10%	20	7	27
60%	10%	3	5%	37%	23%	20	10	29
60%	20%	2	2%	0%	17%	20	7	27
60%	20%	3	2%	0%	12%	20	6	26
			10%	**38%**	**33%**	**20**	**97**	**117**

Table 2.10: 70 buyers and 200-mile radius

Average Demand	Discount	Inventory Capacity	Savings	Discounted Orders	MS Routes	RGT	MT	TT
40%	5%	2	6%	78%	4%	114	328	442
40%	5%	3	6%	78%	6%	115	264	378
40%	10%	2	4%	12%	38%	114	330	443
40%	10%	3	4%	9%	37%	114	454	567
40%	20%	2	4%	0%	39%	114	529	644
40%	20%	3	4%	0%	41%	116	306	422
50%	5%	2	3%	34%	22%	114	37	150
50%	5%	3	3%	43%	16%	114	30	144
50%	10%	2	3%	0%	40%	113	36	149
50%	10%	3	3%	0%	41%	114	34	147
50%	20%	2	2%	0%	38%	113	28	142
50%	20%	3	3%	0%	39%	113	28	142
60%	5%	2	2%	38%	4%	114	11	125
60%	5%	3	2%	44%	3%	110	10	120
60%	10%	2	0%	0%	9%	101	9	110
60%	10%	3	1%	0%	12%	101	11	112
60%	20%	2	1%	0%	8%	101	9	110
60%	20%	3	1%	0%	11%	101	10	111
			3%	**19%**	**23%**	**111**	**137**	**248**

Table 2.11: 70 buyers and 300-mile radius

Average Demand	Discount	Inventory Capacity	Savings	Discounted Orders	MS Routes	RGT	MT	TT
40%	5%	2	14%	89%	2%	102	882	984
40%	5%	3	14%	90%	2%	102	759	861
40%	10%	2	10%	20%	53%	102	561	664
40%	10%	3	11%	20%	54%	102	1,361	1,464
40%	20%	2	11%	0%	58%	103	1,562	1,665
40%	20%	3	10%	0%	56%	102	1,219	1,321
50%	5%	2	8%	69%	10%	102	75	177
50%	5%	3	7%	59%	14%	102	51	154
50%	10%	2	6%	6%	46%	101	52	154
50%	10%	3	6%	5%	51%	101	59	160
50%	20%	2	6%	0%	53%	101	43	144
50%	20%	3	6%	0%	51%	101	38	139
60%	5%	2	4%	55%	3%	102	13	114
60%	5%	3	4%	53%	5%	101	10	111
60%	10%	2	3%	27%	15%	100	18	119
60%	10%	3	3%	46%	8%	101	12	114
60%	20%	2	1%	0%	8%	102	9	111
60%	20%	3	2%	3%	16%	102	15	116
			7%	**30%**	**28%**	**102**	**374**	**476**

Table 2.12: 70 buyers and 400-mile radius

Average Demand	Discount	Inventory Capacity	Savings	Discounted Orders	MS Routes	RGT	MT	TT
40%	5%	2	17%	83%	7%	102	558	660
40%	5%	3	17%	85%	8%	102	584	686
40%	10%	2	15%	22%	53%	102	1,362	1,464
40%	10%	3	15%	25%	55%	102	999	1,101
40%	20%	2	14%	8%	60%	102	2,248	2,350
40%	20%	3	15%	3%	62%	106	1,883	1,989
50%	5%	2	11%	74%	7%	107	96	203
50%	5%	3	10%	67%	14%	106	92	198
50%	10%	2	9%	17%	46%	104	90	193
50%	10%	3	9%	17%	49%	101	166	267
50%	20%	2	8%	7%	59%	101	79	180
50%	20%	3	8%	0%	53%	102	73	175
60%	5%	2	6%	63%	4%	101	15	116
60%	5%	3	6%	66%	3%	101	11	112
60%	10%	2	5%	34%	14%	101	14	115
60%	10%	3	5%	40%	14%	104	13	117
60%	20%	2	2%	10%	15%	101	14	115
60%	20%	3	2%	8%	13%	101	11	112
			10%	**35%**	**30%**	**103**	**462**	**564**

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Chapter 3

Bi-objective Freight Consolidation with Unscheduled Discount

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3.1 Introduction

The problem is a bi-objective integrated quantity-discount-and-vehicle-routing problem in a two-tier supply chain consisting of a seller and multiple buyers. The seller pays for transportation cost to deliver its product to buyers. Each buyer has a constant daily demand during the planning horizon. The seller seeks a solution that is a trade-off between maximizing its total profit and minimizing its output fluctuation. Therefore, the problem is a bi-objective optimization problem.

The seller has control of its freight's transportation. In order to maximize its total profit, the seller seeks optimal routes and consolidation to minimize the total transportation cost. It is a typical vehicle-routing problem. Transportation mode considered in the problem is truckload. A truckload shipment, whose weight ranges from 20000 to 50000 pounds, incurs a cost based on the origin and destination locations and regardless of the shipment weight. The seller wishes to consolidate freight deliveries into multi-stop shipments in order to reduce the number of shipments and therefore reduce its transportation cost. Besides a line-haul cost, a multi-stop shipment also incurs an accessorial charge, called stop-off charge. Nguyen et al. [54] discussed the details of the stop-off charge schedule.

All buyers have constant daily demand rates. They utilize a (Q, r) inventory control policy. When the inventory level reaches r, they will place a Q-sized replenishment order to the seller. Q is assumed to be the full truckload amount. Because of constant demand and (Q, r) policy, the replenishments are known in advance. Therefore, deliveries can be scheduled in advance. Demand rates encountered by buyers are different. The total freight volume that the seller has to ship out will fluctuate day by day. The seller seeks a mechanism to coordinate orders among buyers to reduce the output range. In this problem, stock-out is not allowed.

In order to change a buyer's replenishment pattern the seller will ship a freight amount that is different than the buyer's order amount. The buyer will reach the next re-order point at a different time period than it would. Those deliveries are called unscheduled. In a collaborative replenishment program, unscheduled deliveries are enabled by a discount. When the seller ships a scheduled delivery to a buyer, no discount is offered to the buyer. When the seller ships an unscheduled delivery, its profit is the sales revenue minus transportation cost and minus a discount. The additional discount cost to the seller is compensated by a better consolidation which results in higher overall profit and by a better output range which eases the seller's production and inventory planning. Besides changing a replenishment pattern, an unscheduled delivery also increases the total number of deliveries to a buyer in the planning horizon.

In the collaborative replenishment program, we assume that a seller and its buyers have established a savings sharing agreement which distributes the savings realized from the program to all parties fairly. Buyers do not have to share sensitive information such as their demand rates and inventory levels. A buyer will only receive a delivery when it places a replenishment order. In the program, a seller has the option to change the replenishment amount that it prefers to deliver. Compared to the Vendor-Managed Inventory model, a seller in the collaborative replenishment program has an indirect control of its buyers' inventory. The program does not require additional information sharing between parties. Buyers are not required to change their current operation processes. Hence, the program is easier to implement in a timely manner.

Profit or cost has been widely accepted in the research community as an objective of optimization problems. Separately, service level is also a popular objective in multi-objective optimization problems. In this problem, the seller's objectives are output range as well as profit. Firstly, there is not a proved methodology to transform output range into cost. Secondly, output range represents the coordination mechanism among multiple internal departments within the seller's organization. Resource allocation and transportation procurement are directly impacted by the output range. In a lean inventory endeavor, the output range will also impact manufacturing operations and material purchase planning. Because the scope of this problem is outbound transportation planning, taking into account the output range will assist decision makers to make collaborative decisions among departments.

In brief, the bi-objective problem of freight consolidation with unscheduled delivery is itemized as below:

- There is a seller and multiple buyers in a two-tier supply chain.
- Buyers place replenishment orders based on a (Q, r) inventory control policy. The demand rates encountered by buyers are constant. The time period and the quantity of a replenishment order is known in advance. The seller's deliveries therefore can be scheduled in advance.
- When it ships unscheduled deliveries, the seller will offer a discount. Unscheduled deliveries are used to redistribute the freight output over time periods to have a favor output range for the seller's plant.
- The seller has two objectives: maximizing the total profit and minimizing the output range.

The rest of the chapter is represented as following. Section 3.2 reviews the literature. Section 3.3 discusses the research motivation and contribution of this chapter. Section 3.4 presents the mathematical formulation of the problem. Section 3.5 describes the proposed methodology. Section 3.6 discusses the experimental results. Section 3.7 concludes the chapter.

3.2 Literature Review

We classify our problem as an integrated transportation and production planning problem. On the side of transportation planning, this problem is a vehicle-routing problem. Transportation cost is shipment-based and mile-based. Freight can be consolidated into a truck and delivered to multiple buyers in a multi-stop route trip. On the side of production planning, this problem minimizes the output range, which indicates the freight output fluctuation. The problem also includes discount cost which creates a competing relationship between total profit and output range. Discount is the enabler of the collaborative replenishment program. We will review the current literature in three categories: vehicle-routing, quantity discount, and production planning problems.

Vehicle-routing problems (VRP) are popular and have rich literature in operation research. In this section, we review articles that have multiple objectives and are related to our problem.

Tan et al. [58] proposed an evolutionary algorithm to solve a VRP with time windows. The problem minimized the total cost and the number of trucks. Burke et al. [30] utilized a memetic algorithm to solve an airline scheduling problem. The problem maximized the reliability and flexibility of an airline schedule. See Jozefowiez et al. [44] for a more detail review of multi-objective VRPs.

Quantity discount has been widely used in vendor selection problems. Ng [53] studied a multi-objective vendor selection problem whose objectives were total cost and the number of vehicles. The authors utilized linear scalarization to combine the two objectives. Demirtas and Üstün [38] considered cost and defected items as the objectives. Besides cost, Amid et al. [25] minimized rejected items and late deliveries. Wu et al. [61] considered risk factors as one of their problem's objectives. The authors proposed a fuzzy methodology to solve the problem. Mafakheri et al. [50] minimized total cost and maximized the supplier-reference function using a two-stage dynamic programming. Kamali et al. [45] proposed a meta-heuristic algorithm to minimize total supply chain cost, defect items, and late deliveries.

Production output

There have been many articles addressing output impacts in production planning problems. In a broad sense, the literature considers not only source nodes (such as plants) but also any types of nodes (such as distribution centers and cross-docks) in a network. Soltani and Sadjadi [57] proposed a meta-heuristic methodology to schedule trucks in a cross-docking systems. Zhao and Goodchild [64] studied truck arrivals in a container terminal. Bolduc et al. [29] studied an integrated production, inventory, and transportation planning problem. The problem considered warehouses in plants which created a buffer between manufacturing and outbound transportation. The problem included both common carriers and a private fleet. The authors proposed a Tabu search algorithm with penalty costs for infeasible solutions to solve the problem. Chen et al. [33] studied truck arrivals at ports. Van Belle et al. [60] studied an inbound and outbound truck assignment problem in a cross-dock terminal. Konur and Golias [47] proposed a meta-heuristic methodology to study truck arrivals at a cross-dock terminal.

In a different angle of production output, Glock and Jaber [42] studied an economic production quantity problem. The authors assumed imperfect production and defects were more likely to happen in non-optimal-sized lots.

Aggregate planning problem

We classify the aggregate planning problem in the literature into single-objective and multi-objective categories. Table 3.1 reviews the current literature of single-objective aggregate planning problems. Table 3.2 reviews the multi-objective problems.

The literature also has many review papers. Adulyasak and Jans [24] reviewed the formulations and solving methodologies of single-objective problems. Chen [35] and Mula et al. [52] addressed the modeling aspect such as objective functions, parameters, and mathematical programming models. Fahimnia et al. [39] classified the problems based on the network structures. Jones et al. [43] reviewed solving methodologies.

Table 3.1: Single-objective aggregate planning problems

Article	Objective	Methodology
Chandra and Fisher [32]	production, transportation, inventory costs	local improvement heuristic
Nishi et al. [55]	production, transportation, inventory costs	Lagrangian decomposition
Keskin and Üster [46]	location, transportation costs	Tabu search, Scatter search
Geismar et al. [41]	lead time	two-stage evolutionary
Armentano et al. [27]	production, transportation, inventory costs	Tabu search
Liberalino and Duhamel [49]	production, transportation cost	Greedy heuristic
Yan et al. [62]	production, transportation, inventory costs	analytical method
Zhang et al. [63]	profit	LINGO
Toptal et al. [59]	transportation, inventory costs	Tabu search

Table 3.2: Multi-objective aggregate planning problems

Article	Objectives	Methodology	Weighted Sum
Chen and Vairaktarakis [34]	transportation cost, service level	heuristic	No
Selim et al. [56]	profit, costs	fuzzy goal programming	No
Amorim et al. [26]	production, transportation cost, lead time	decomposition	No
Cakici et al. [31]	weighted tardiness, transportation cost	GA heuristic	No
Farahani et al. [40]	production, transportation cost, lead time	Large neighborhood search	Yes
Bilgen and Çelebi [28]	production, transportation cost, lead time	optimization and simulation	No
Leung and Chen [48]	vehicles, lead time	heuristic	Yes

3.3 Research Motivation and Contribution

Facility output is a popular research area. Dock scheduling is also a prevalent research topic in transportations. We model output as a coordinating factor between transportation and other departments in a company, each of which optimizes their own operations. These departments include raw material procurement, labor allocation, and inventory management.

All of the research publications we have reviewed considered the output as a constraint and ignored variation of the figure. We find that it is necessary not only to satisfy a facility capacity but also to maintain a stable output over time. In this research, output together with profit is an optimization objective function. Our model facilitates the decision making process by estimating the entire efficient frontier of the bi-objective problem.

This research provides a seller an effective model to identify potential buyers that would fit in its collaborative replenishment program. The seller usually implements the program in two phases. The first phase involves only the seller and its consultant party. The consultant provides expertise in modelling and change management. The model proposed by this research is used to identify potential buyers for the program. In the second phase, the seller persuades the target buyers to join its program. The seller then establishes goals and agreements of the program with the target buyers. It usually happens that some target buyers do not fully participate in the program. The opportunity pool is therefore smaller than it is in the first phase. Once the participants of the program are known, the seller preferably wants to know the revised optimal solution. However, it usually happens that they skip it to focus on implementing the program and realizing the program's benefits as soon as possible.

This research gives the following contributions to the current literature:

- We study a new variant of the bi-objective problem of vehicle routing and quantity discount in which profit and output are optimized simultaneously.
- We propose a matheuristic algorithm that effectively builds the efficient frontier of the problem.

3.4 Formulation

This section presents the mathematical formulation of the problem. We have adopted the formulation proposed by Nguyen et al. [54] and developed it to fit this research problem.

Sets

- I : the set of buyers i and a sole seller $i = 0$.
- $I' = I \setminus \{0\}$: the set of buyers i.
- R : the set of routes r.
- T : the set of time periods t.

Parameters

- C_i : the inventory capacity of buyer $i \in I'$.
- a_i : the initial inventory level of buyer i.
- c^r : the transportation cost of route r including stop-off charges.
- d_i : the demand of buyer i per time period.
- m : the truckload capacity.

- p_i : the full price of a product unit for buyer i.

- (Q_i, r_i) : buyer i's inventory policy. Without loss of generality, we assume that the reorder point $r_i = 0$ and the original order size is m $\forall i \in I'$. Q_i is the minimum replenishment amount of unscheduled deliveries.

- $$v_i^r = \begin{cases} 1 & \text{if route } r \text{ visits buyer } i \\ 0 & \text{otherwise} \end{cases}$$

- u_i^t : the all-unit discount for the unscheduled delivery to buyer i in time period t.

Variables

- $s_i^t \in \mathbb{Z}^+$: the inventory level of buyer i at the end of time period t.

- $q_i^{rt} \in \mathbb{Z}^+$: the delivery quantity for buyer i delivered by route r in time period t.

- $$x^{rt} = \begin{cases} 1 & \text{if route } r \text{ is used in time period } t \\ 0 & \text{otherwise} \end{cases}$$

- $$\gamma_i^t = \begin{cases} 1 & \text{if buyer } i \text{ receives a replenishment in time period } t \\ 0 & \text{otherwise} \end{cases}$$

- $\mu \in \mathbb{Z}^+$: the maximum output in the planning horizon.

- $\nu \in \mathbb{Z}^+$: the minimum output in the planning horizon.

$$Maximize \quad \sum_{i \in I'} p_i(\sum_{t \in T}(\sum_{r \in R} q_i^{rt}(1-u_i^t))) - \sum_{r \in R, t \in T} c^r x^{rt} \tag{3.1}$$

$$Minimize \quad (\mu - \nu) \tag{3.2}$$

s.t.

$$s_i^0 = a_i \qquad \forall i \in I' \tag{3.3}$$

$$s_i^T = 0 \qquad \forall i \in I' \tag{3.4}$$

$$s_i^{t-1} + \sum_{r \in R} q_i^{rt} = d_i + s_i^t \qquad \forall i \in I', \forall t \in T \tag{3.5}$$

$$s_i^t \leq C_i \qquad \forall i \in I', \forall t \in T \tag{3.6}$$

$$s_i^{t-1} \leq C_i(1-\gamma_i^t) \qquad \forall i \in I', \forall t \in T \tag{3.7}$$

$$Q_i \gamma_i^t \leq \sum_{r \in R} q_i^{rt} \leq m\gamma_i^t \qquad \forall i \in I', \forall t \in T \tag{3.8}$$

$$q_i^{rt} \leq m v_i^r x^{rt} \qquad \forall i \in I', \forall r \in R, \forall t \in T \tag{3.9}$$

$$\sum_{i \in I'} q_i^{rt} \leq m x^{rt} \qquad \forall r \in R, \forall t \in T \tag{3.10}$$

$$\sum_{r \in R} v_i^r x^{rt} \leq 1 \qquad \forall i \in I', \forall t \in T \tag{3.11}$$

$$\nu \leq \sum_{i \in I', r \in R} q_i^{rt} \leq \mu \qquad \forall t \in T \tag{3.12}$$

$$x^{rt} \in \{0,1\} \qquad \forall r \in R, \forall t \in T \tag{3.13}$$

$$\gamma_i^t \in \{0,1\} \qquad \forall i \in I', \forall t \in T \tag{3.14}$$

$$s_i^t \in \mathbb{Z}^+ \qquad \forall i \in I', \forall t \in T \tag{3.15}$$

$$q_i^{rt} \in \mathbb{Z}^+ \qquad \forall i \in I', \forall r \in R, \forall t \in T \tag{3.16}$$

$$\mu \in \mathbb{Z}^+ \tag{3.17}$$

$$\nu \in \mathbb{Z}^+ \tag{3.18}$$

The problem has two objective functions. Objective function (3.1) maximizes total profit which is total sales revenue less transportation cost and discount. Objective function (3.2) minimizes the output range in the planning horizon. Constraint set (3.3) and (3.4) fix the inventory level at the beginning and the end of the planning horizon. Constraint set (3.5) guarantees the delivery amount is sufficient and in time for demand. Constraint set (3.6) enforces the buyers' inventory capacities. Constraint set (3.7) enforces no delivery to a buyer when it has sufficient stock. Constraint set (3.8) enforces the size of all deliveries. They must be equal to or greater than the buyer's minimum replenishment amount and less than or equal to the truckload capacity. Constraint set (3.9) establishes the relationship between q_i^{rt} and x^{rt}. If buyer i is not on route r, q_i^{rt} must be zero. Constraint set (3.10) enforces the truckload capacity m to each route. Constraint set (3.11) guarantees no split delivery. Constraint set (3.12) calculates the maximum and the minimum outputs of the seller in the planning horizon. Constraint sets from (3.13) to (3.18) define the value domains of all variables.

3.5 Methodology

In this chapter, we propose a new variant of the matheuristic methodology, which combines exact and heuristic algorithms to solve complex problems. See Manerba and Mansini [51] for a review of matheuristic and application in supplier selection problems. The master iteration of our methodology is a Genetic Algorithm which will determine deliveries for all buyers. The route assignment subproblems which select routes to service all deliveries are solved to optimality by Gurobi optimization solver.

Figure 3.1 illustrates the design of the matheuristic. After q_i^{rt} is determined, Gurobi optimally solves the subproblems of route assignment to determine x^{rt}. At this point,

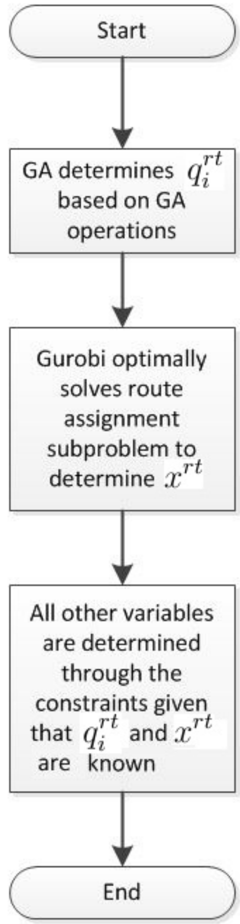

Figure 3.1: The matheuristic design

deliveries' quantity and time period are known. Each time period becomes independent from other time periods in terms of the remaining undetermined variables. The subproblems of time periods are solved independently. The mathematical formulation of the subproblem is represented below. Variable $x^r = 1$ if route r is selected. Time period index is removed for simplicity.

$$Minimize \quad \sum_{r \in R} c^r x^r \tag{3.19}$$

s.t.

$$\sum_{r \in R} v_i^r x^r = 1 \qquad \forall i \in I' \tag{3.20}$$

$$x^r \in \{0, 1\} \qquad \forall r \in R \tag{3.21}$$

The subproblem's objective function (3.19) is to minimize total route cost. Constraint set (3.20) guarantees that all buyers have to be serviced and there is no split delivery. Buyers that do not have a delivery in the time period are excluded from set I' of the subproblem. Constraint set (3.21) defines the value domain of variable x^r. The truckload capacity constraint for routes is implicitly enforced. Set R of the subproblem includes only feasible routes given known q_i^{rt}.

After q_i^{rt} and x^{rt} being determined, s_i^t are calculated based on constraint set (3.5). γ_i^t are calculated based on constraint set (3.7). ν and μ are calculated based on constraint set (3.12).

Figure 3.2 illustrates the GA design. P is the current population. N is the population size. A gene is a binary bit representing that whether there is a delivery to a buyer in a time period. A chromosome has $T \times I'$ genes. Genes of a chromosome are arranged in order of buyers and then time periods to represent a string of binary bits. A delivery quantity is implicitly determined by the distance between two consecutive deliveries to a buyer and the buyer's demand rate. p_c is the cross-over probability. We use one-point cross-over operation from two parents to reproduce two off-springs. p_m is the mutation probability. p_m represents the probability that a gene is mutated. The mutation operation will flip a binary bit between 0 and 1.

Because of cross-over and mutation operations, a new off-spring is not guaranteed with feasibility. The infeasibility cause is that two consecutive deliveries are so far from each other. That requires one delivery bigger than a full truckload amount to meet demand

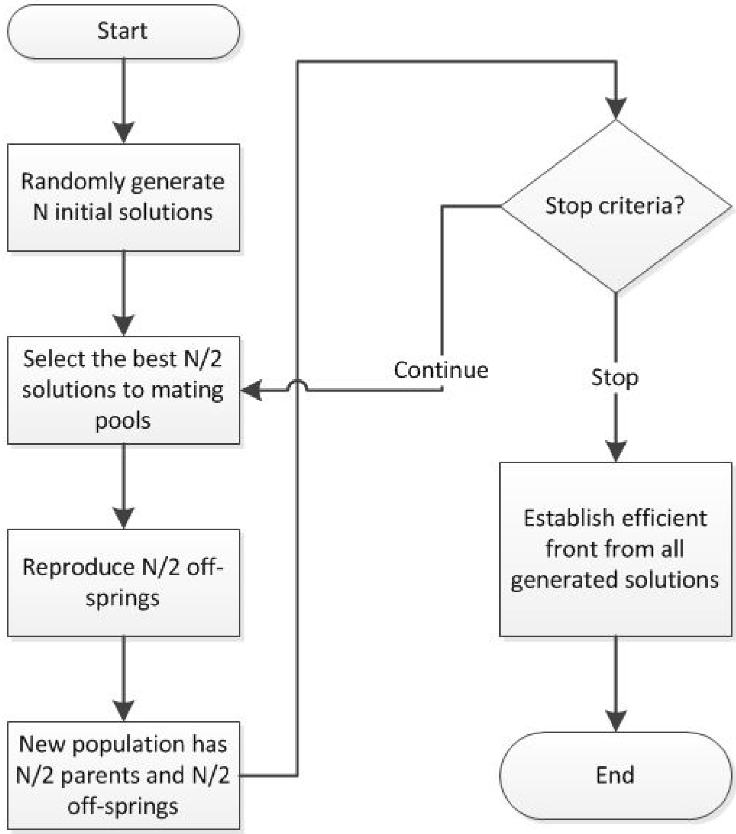

Figure 3.2: Genetic algorithm

within the time frame. We fix an infeasible off-spring by randomly adding more deliveries within the time frame.

We adopt the reproduction selection method NSGA-II proposed by Deb and Pratap [37]. In each GA iteration, we select the best $N/2$ chromosomes based on NSGA-II to create a mating pool. NSGA-II assigns two attributes to each chromosome ξ in the population: non-dominated set F_i and crowding distance $w(\xi)$.

Before discussing the details of non-dominated set F_i and crowding distance $w(\xi)$, it is necessary to denote the comparison relationship of chromosomes. We define the comparison $\prec$ such that $\xi \prec \xi'$ if and only if ξ dominates ξ'. We also have $\xi \nprec \xi'$ if and only if ξ does not dominate ξ'. We have $\xi \nsim \xi'$ if and only if $\xi \nprec \xi'$ and $\xi' \nprec \xi$.

The non-dominated set F_i where $i \in \mathbb{Z}^+$ are defined:

- $\forall \xi, \xi' \in F_i \Rightarrow \xi \nsim \xi'$
- $\forall i, j \in \mathbb{Z}^+ : i < j, \forall \xi \in F_i, \forall \xi' \in F_j \Rightarrow \xi \prec \xi'$

The crowding distance index $w(\xi)$ is calculated by Algorithm 1.

Algorithm 1 Crowding distance calculation

for each $i \in \mathbb{Z}^+$ **do**
    for each objective m **do**
        `sort` $\forall \xi \in F_i$ `in ascending order of` m`:` $\xi_0, \xi_1, ..., \xi_{max}$
        $w(\xi_0) \leftarrow +\infty$
        $w(\xi_{max}) \leftarrow +\infty$
        for k from 1 to $(max - 1)$ **do**
            $w(\xi_k) \leftarrow w(\xi_k) + \frac{\xi_{k+1}(m) - \xi_{k-1}(m)}{\xi_{max}(m) - \xi_0(m)}$
        end for
    end for
end for

We define the comparison $\prec_n$:

$$\xi \prec_n \xi' \quad \Leftrightarrow \quad \left[\begin{array}{l} \xi \in F_i, \xi' \in F_j : i < j \\ \xi, \xi' \in F_i : w(\xi) > w(\xi') \end{array} \right.$$

When $\xi \prec_n \xi'$, chromosome ξ is better than chromosome ξ'. NSGA-II selects the best $N/2$ chromosomes of the current population P into a mating pool to reproduce $N/2$ off-springs.

3.6 Experiments

In this section, we evaluate the quality of the efficient frontiers established by our proposed GA methodology. Firstly, we will assess the quality of the efficient points and the coverage of the frontiers based on the run time. Secondly, we will compare the GA's frontiers with

NISE's (Non-Inferior Set Estimation method proposed by Cohon et al. [36]) and points obtained from the exact frontier using Gurobi. Our experiments were run on a Windows server that has Intel Xeon CPU X7350 2.98GHz and 16GB RAM.

3.6.1 Problem instances

We collected the location information from a construction material manufacturer. Transportation costs are based on current market rates in the U.S. The remaining data are randomly generated as described in Table 4.2. Even though there are many other factors defining an instance such as the number of buyers and the buyer's inventory capacity, we decide to focus on the three factors mentioned in Table 4.2. It is not obvious that any of the three factors has a stronger impact to an instance's tractability than the others do. The number of the buyers obviously has the strongest impact. We believe other factors' tractability impact is much less significant than the three studied factors' are.

Table 3.3: Experimental Design

Factor	**Level Description**	**Random Distribution**	**Levels**
Buyer density (radius in miles)	200, 300		2
d_i (% of FTL)	$\bar{d} = 40\%, 50\%$	Discrete	2
u_i	5%, 20%		2
Total number of problem instances			8

Instance names are coded by a three-digit number. The first digit on the left represents the buyer density factor. Value 1 is for the low level of the factor and value 2 is for the higher level. The second and third digits respectively represent the factors of customer demand d_i and discount u_i.

3.6.2 GA's performance

We have completely solved the eight instances. Each instance is run for at most 60 hours. Figures 3.3-3.26 summarize the results. There are three charts for each instance. Chart A shows the number of additional efficient points (blue points) found every two hours. The red points represent the number of dominated solutions that are at most 0.5% worse than a blue point. In the case when the solver does not find a solution better than the first one it finds, the solution is a red point and a blue point. Chart B compares the efficient frontiers established after five-hour, ten-hour, and the maximum running time in terms of objective values. Chart C compares the efficient frontiers in terms of coverage. The linear trend line approximates the efficient frontier. For all of the eight instances, the efficient frontiers established in the first ten hours are high quality given that the 60-hour efficient frontiers are high quality.

Chart A's of instances 121, 122, 221, and 222 show that we found most of the high quality solutions, at most 0.5% worse than the efficient points, in the first two hours. Chart A's of instances 111 and 211 show that we found most of the high quality solutions in the first six hours. Chart A's of instances 112 and 212 show that it took longer, ten hours or more, to find high quality solutions. Chart B's of all instances show that there is not a significant improvement in terms of solution quality among five-hour, ten-hour, and sixty-hour run time. In all instances, the frontiers tend to be a straight line toward the higher value of output range. Chart C's of all instances show that the 60-hour frontiers have the best coverage compared to the 5-hour and the 10-hour ones.

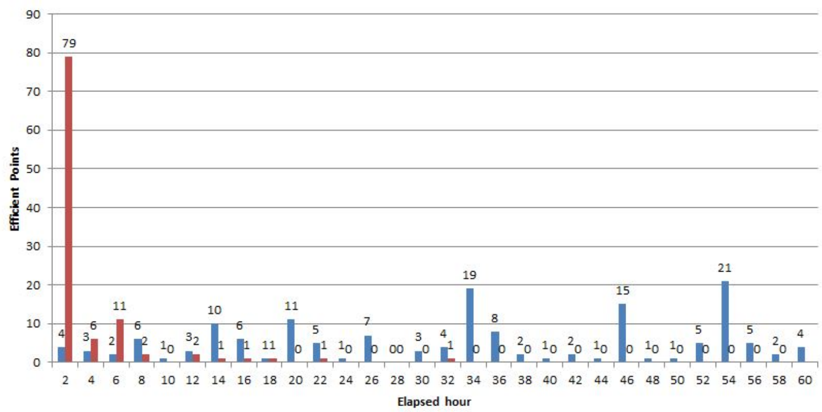

Figure 3.3: Instance 111's chart A

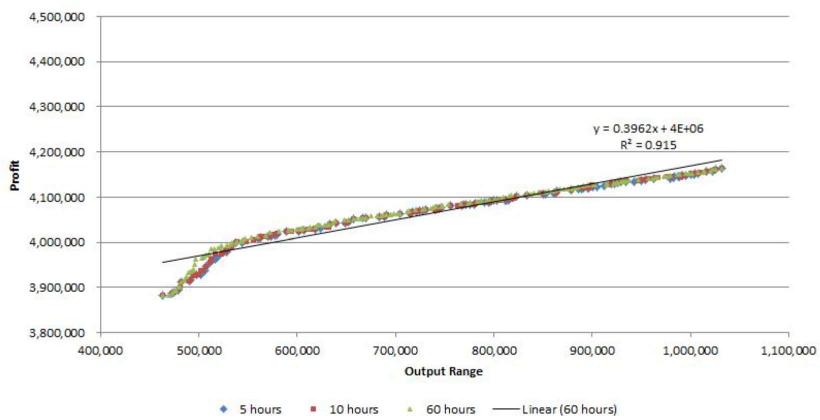

Figure 3.4: Instance 111's chart B

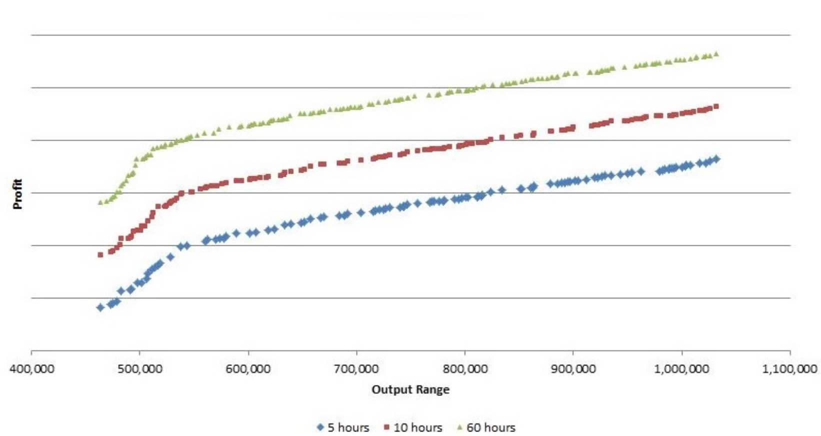

Figure 3.5: Instance 111's chart C

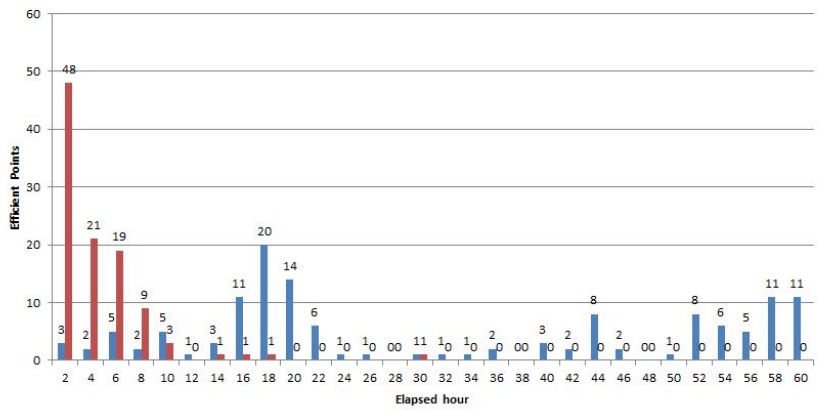

Figure 3.6: Instance 112's chart A

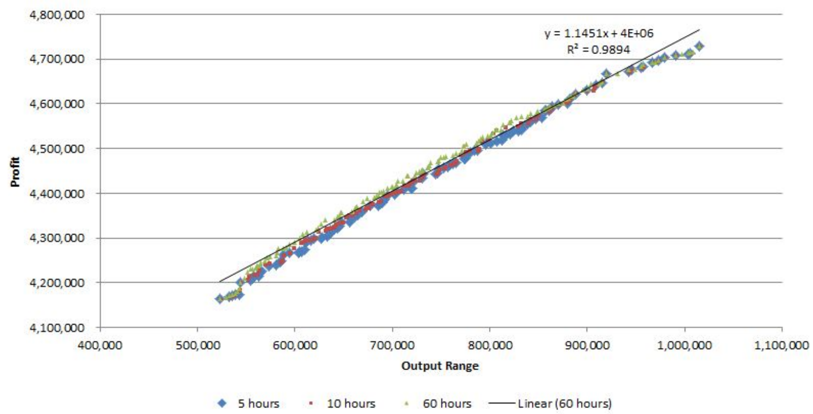

Figure 3.7: Instance 112's chart B

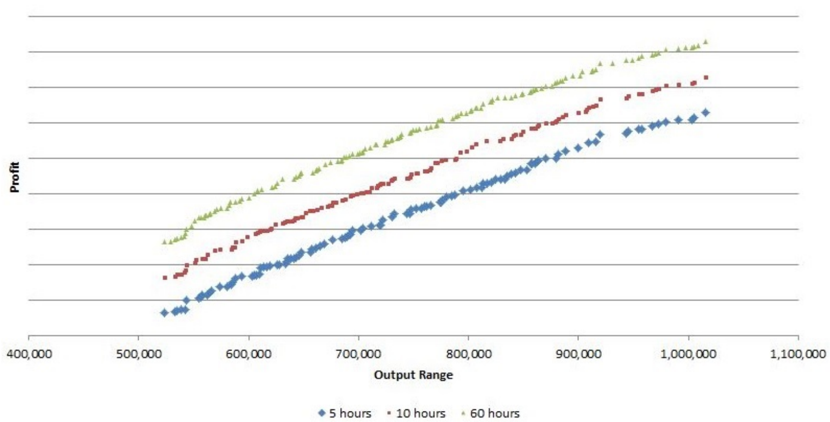

Figure 3.8: Instance 112's chart C

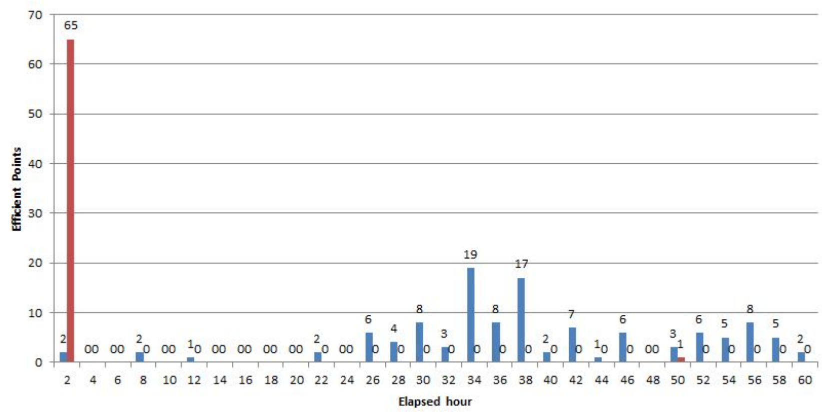

Figure 3.9: Instance 121's chart A

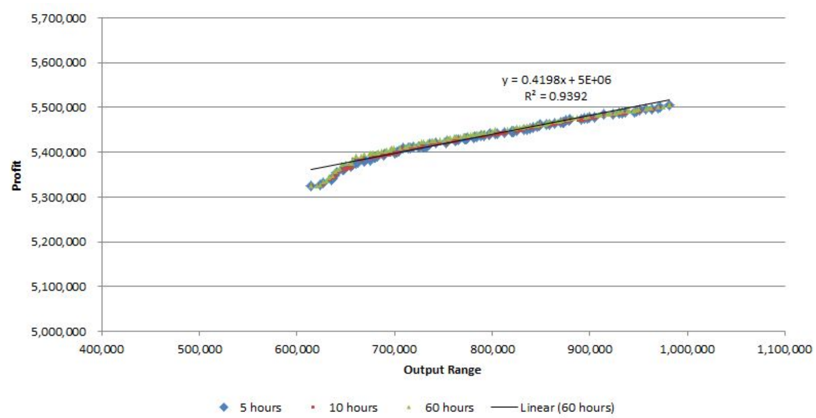

Figure 3.10: Instance 121's chart B

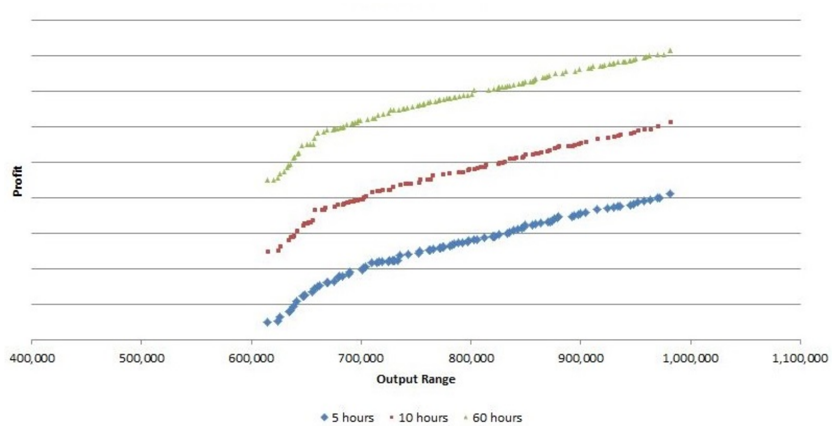

Figure 3.11: Instance 121's chart C

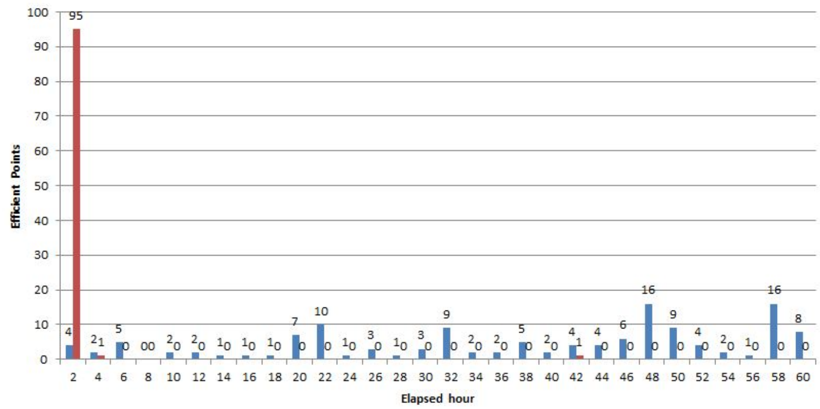

Figure 3.12: Instance 122's chart A

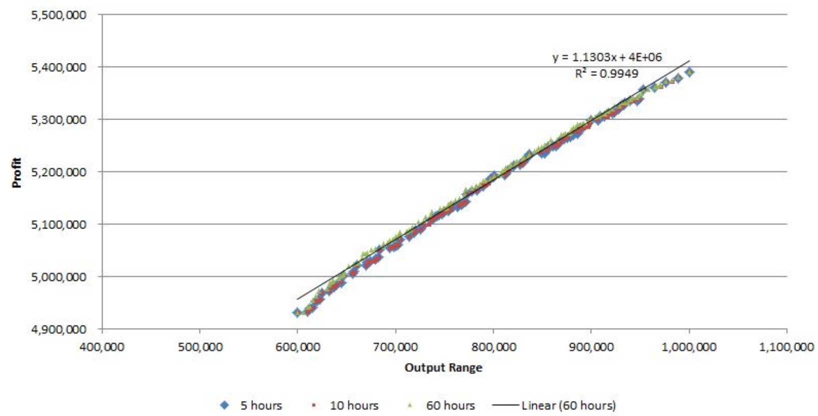

Figure 3.13: Instance 122's chart B

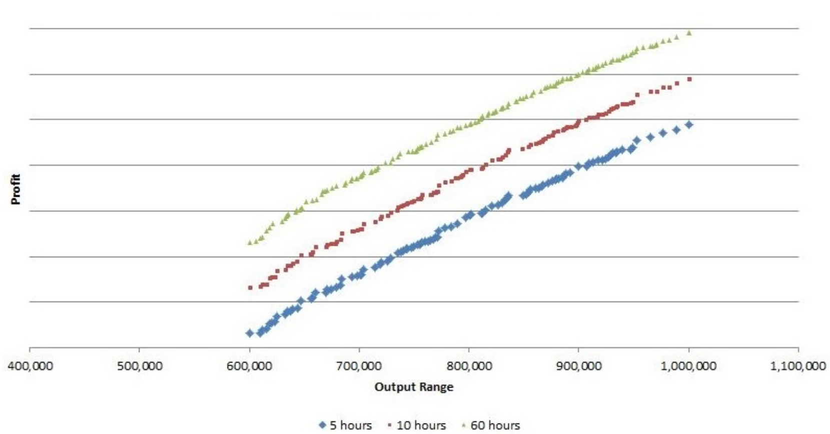

Figure 3.14: Instance 122's chart C

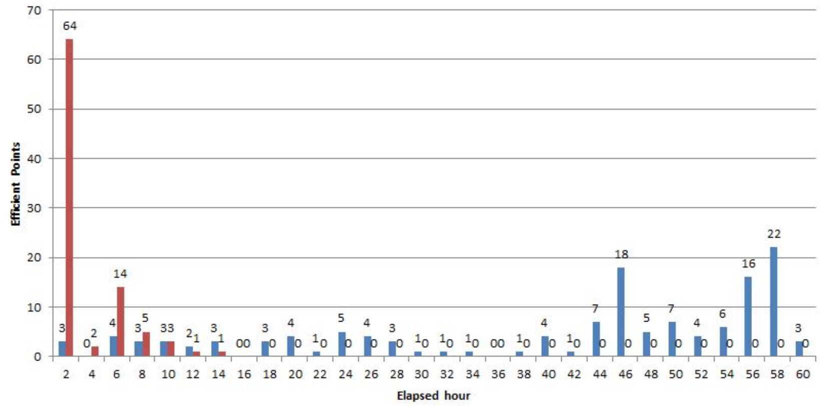

Figure 3.15: Instance 211's chart A

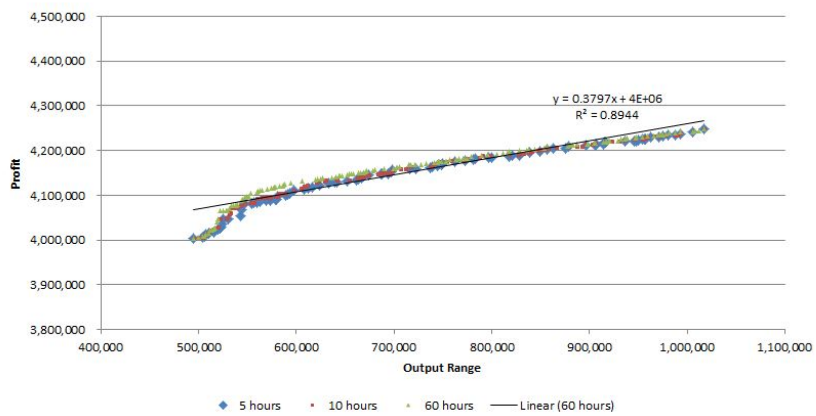

Figure 3.16: Instance 211's chart B

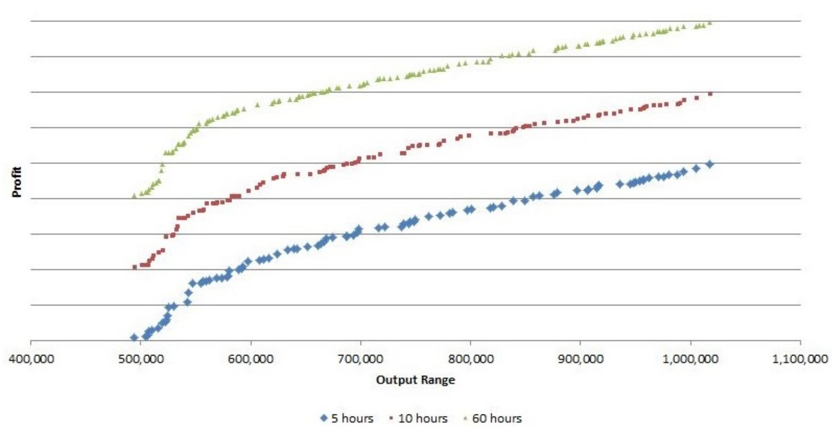

Figure 3.17: Instance 211's chart C

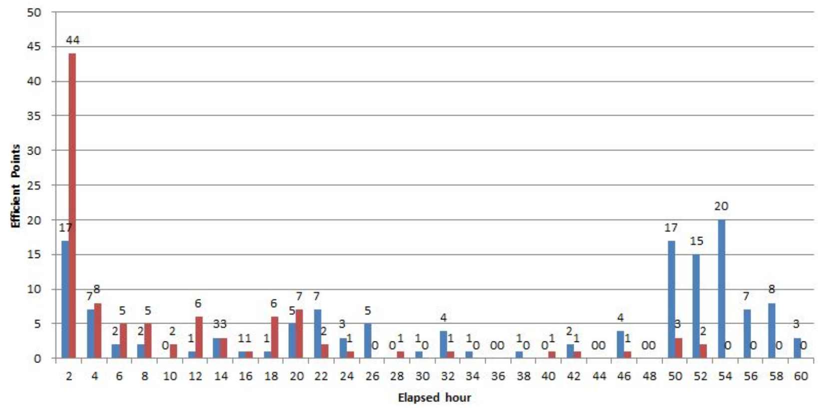

Figure 3.18: Instance 212's chart A

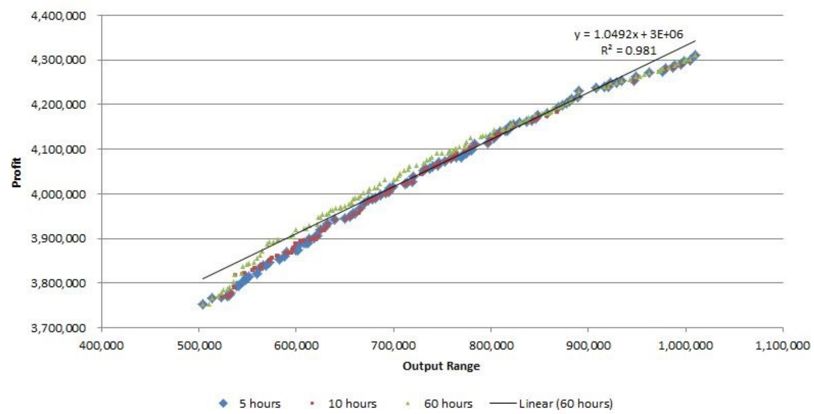

Figure 3.19: Instance 212's chart B

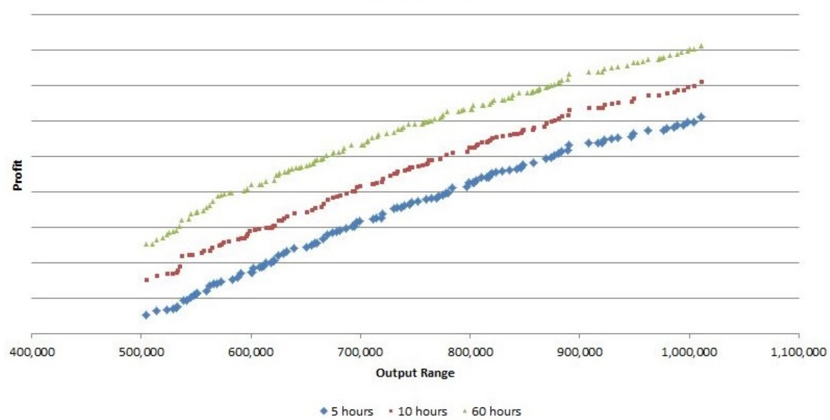

Figure 3.20: Instance 212's chart C

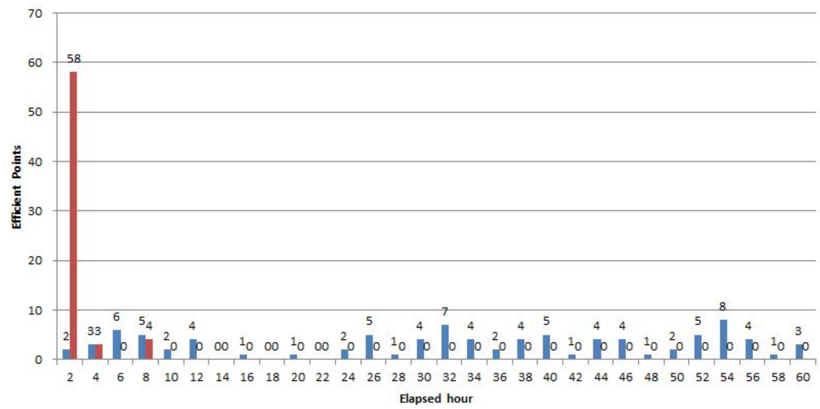

Figure 3.21: Instance 221's chart A

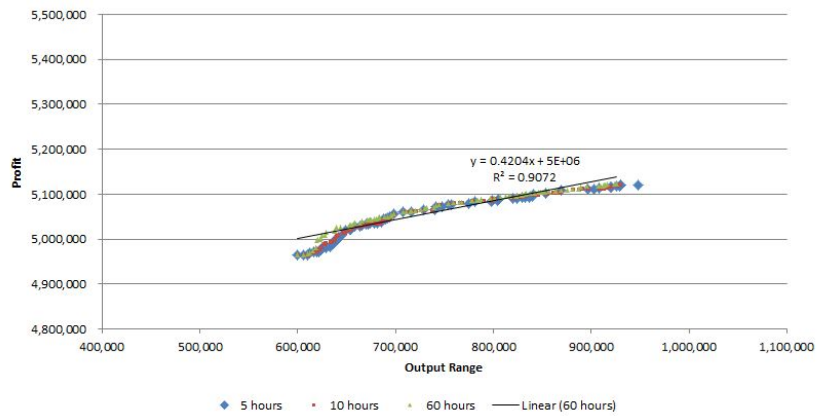

Figure 3.22: Instance 221's chart B

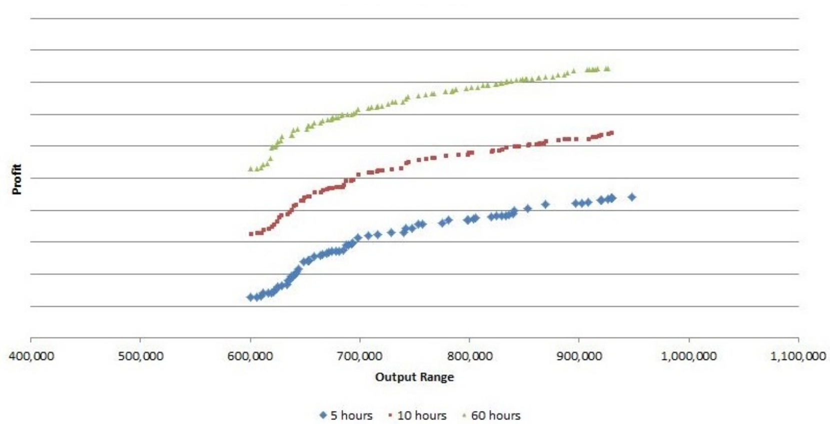

Figure 3.23: Instance 221's chart C

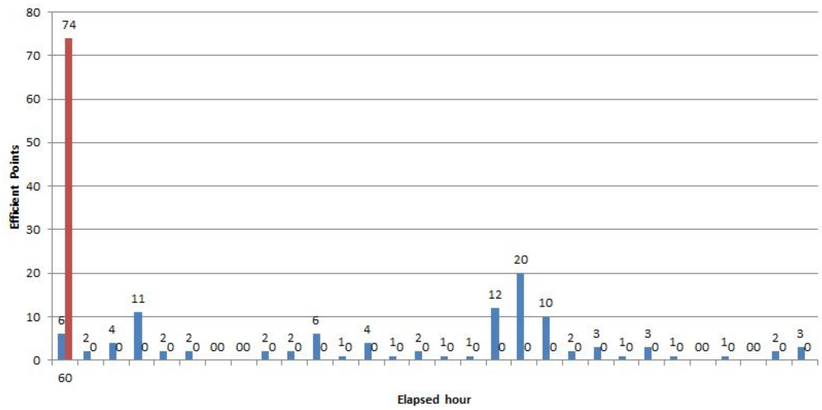

Figure 3.24: Instance 222's chart A

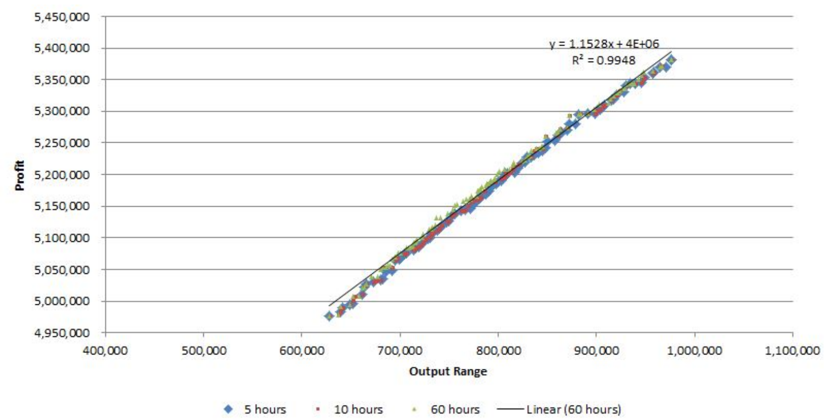

Figure 3.25: Instance 222's chart B

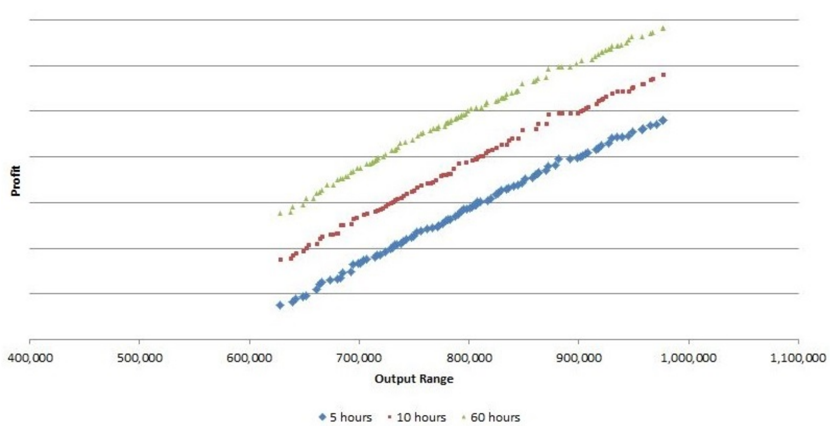

Figure 3.26: Instance 222's chart C

We use the run time it takes to find the high quality solutions, at most 0.5% worst than the best ones, to measure the hardness of the three experimental design factors. We define two indexes: the two-hour index is the percentage of the number of high quality solutions found in the first two hours compared to all of the high quality solutions. The ten-hour index is the similar percentage measured in the first ten hours. A low value index shows that there are not many high quality solutions found within the time amount. Therefore, the corresponding factor makes it harder or longer to solve the instance. Table 3.4 presents the two indexes for the eight instances. The demand of 40% makes the instances significantly harder. The combination of demand 40% and discount 20% has the second strongest impact because their two-hour indexes have the lowest values. The discount factor individually does not have a noticeable impact.

Table 3.4: Factors' intractability impacts

Instances		111	112	121	122	211	212	221	222
2-hour %		75%	46%	98%	98%	71%	44%	89%	100%
10-hour %		93%	96%	98%	99%	98%	64%	100%	100%
Buyer density	200	✓	✓	✓	✓				
	300					✓	✓	✓	✓
Demand	40%	✓	✓			✓	✓		
	50%			✓	✓			✓	✓
Discount	5%	✓		✓		✓		✓	
	20%		✓		✓		✓		✓

3.6.3 Gap comparison

Next, we compare the efficient frontiers established by our methodology to the efficient frontiers established by Gurobi Optimization Solver. In order to use Gurobi to find the efficient points, we add an equality constraint to the mathematical formulation presented in Section 3.4. The constraint enforces a predefined output range. Gurobi solves the problem to maximize the profit at each output range amount. We limit Gurobi's runs to 1% MIP gap and 12-hour solving. For each instance, Gurobi looks for 26 efficient points equally distributed in the output range established by the GA.

Because Gurobi takes a significant amount of time to establish an efficient frontier for an instance, we also exploit another option that is more practical to evaluate the quality of our efficient frontiers. We utilize the NISE methodology proposed by Cohon et al. [36] to approximate the frontiers.

We adopt the notations from Cohon et al. [36] as much as possible. $Z_1(\ )$ denotes the output range objective. $Z_2(\ )$ denotes the profit objective. ξ denotes a solution. $\Psi_{i,i+1}$ denotes the maximum possible error between two solutions ξ_1 and ξ_2. T denotes the maximum allowable error. $\bar{F}_d(\xi)$ denotes the feasible region of the problem. The NISE algorithm is described:

- *Step 1*: Obtain the two extreme points of the problem's efficient frontier, ξ_1 and ξ_2. Compute $\Psi_{1,2}$. Figure 3.27 illustrates the formula for $\Psi_{1,2}$ calculation.

$$\Psi_{1,2} = (Z_2(\xi_2) - Z_2(\xi_1)) \sin(\arctan(\frac{Z_1(\xi_2) - Z_1(\xi_1)}{Z_2(\xi_2) - Z_2(\xi_1)})) \tag{3.22}$$

- *Step 2*: Stop if $\Psi_{i,i+1} \leq T \quad \forall i = 1, 2, ..., n-1$.

- *Step 3*: Solve integer relaxation of the problem below to find ξ_{n+1}:

$$min((Z_2(\xi_1) - Z_2(\xi_2))Z_1(\xi) - (Z_1(\xi_1) - Z_1(\xi_2))Z_2(\xi)) \qquad \forall \xi \in \bar{F}_d(\xi) \tag{3.23}$$

  where $\Psi_{i,i+1} > T$.

- *Step 4*: Sort the solution set (ξ_1, ξ_2, ..., ξ_{n+1}) in the descending order of output range and profit. Go back to step 2.

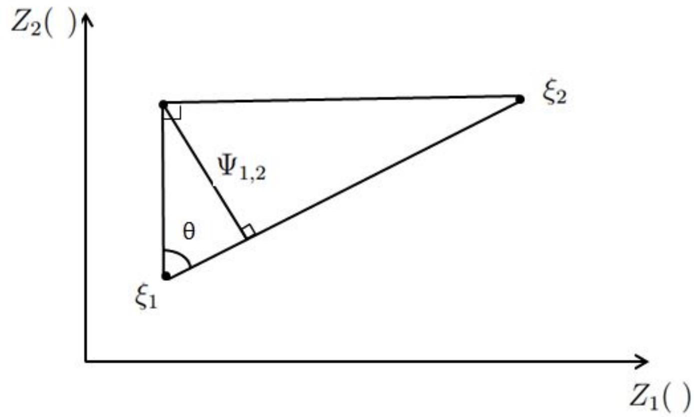

Figure 3.27: $\Psi_{i,i+1}$ calculation

According to step 2, NISE methodology will stop when all the maximum errors $\Psi_{i,i+1}$ are less than or equal to the limit T. Because all instances' objective values are about 5000000, we set $T = 5000$, nominally 1% gap. The value is about from 1.1% to 2.3% of the maximums of all $\Psi_{i,i+1}$ of all instances.

Figures 3.28-3.35 illustrates the gaps among NISE's approximated frontier, Gurobi's best bounds, Gurobi's best solutions, and GA's frontier for each instance. The gap is bigger when the output range is low and it is smaller when the output range is higher. On the high output range side, NISE's frontier stretches further than GA's frontier does.

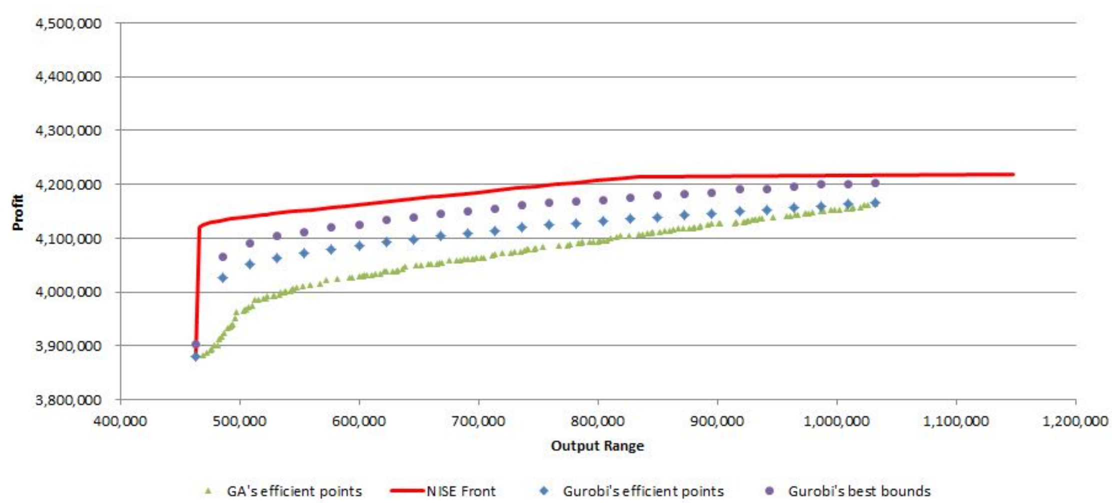

Figure 3.28: Instance 111's gap comparison

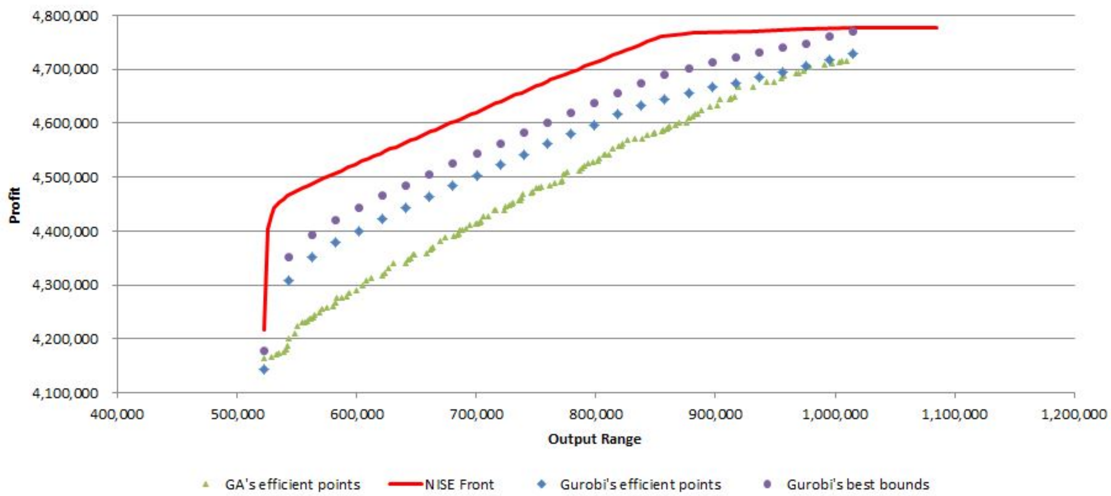

Figure 3.29: Instance 112's gap comparison

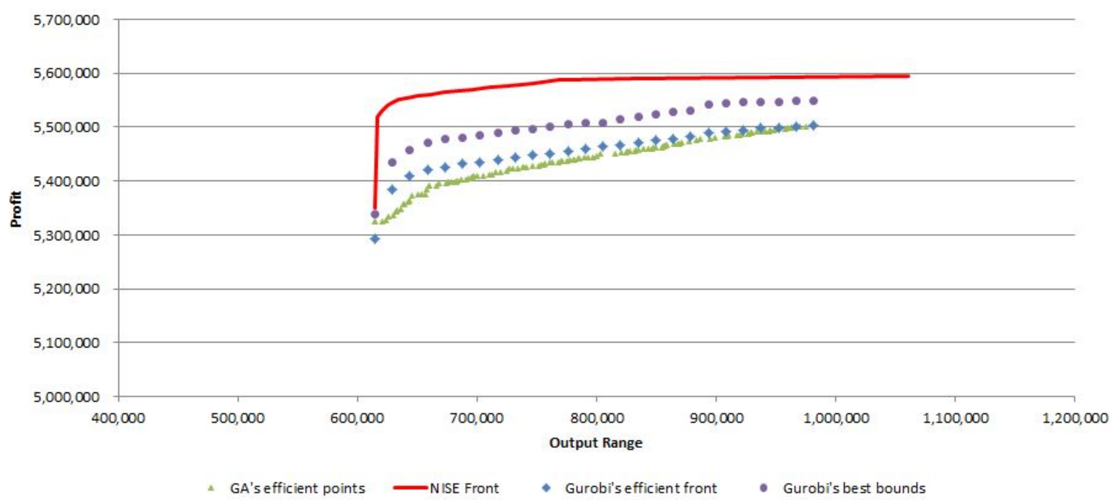

Figure 3.30: Instance 121's gap comparison

Table 3.5 summarizes the gaps of GA's frontiers compared to NISE's and Gurobi's. On average, the minimum gap is 0.6% and the maximum gap is 5.6% when compared to NISE's. The minimum gap is 0.1% and the maximum gap is 1.9% when compared to Gurobi's. Instance 211 is the hardest. Each Gurobi's efficient point of the instance took more than 12 hours to generate.

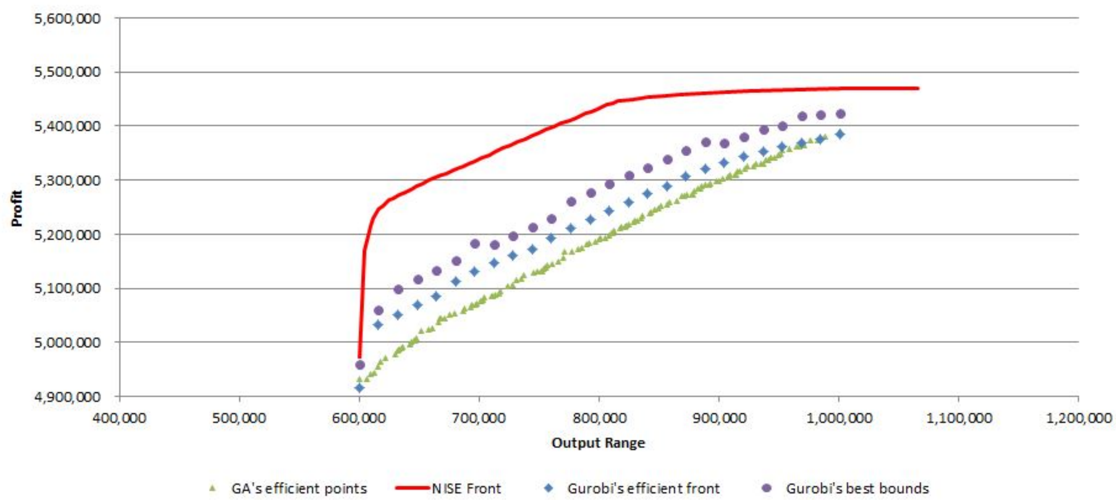

Figure 3.31: Instance 122's gap comparison

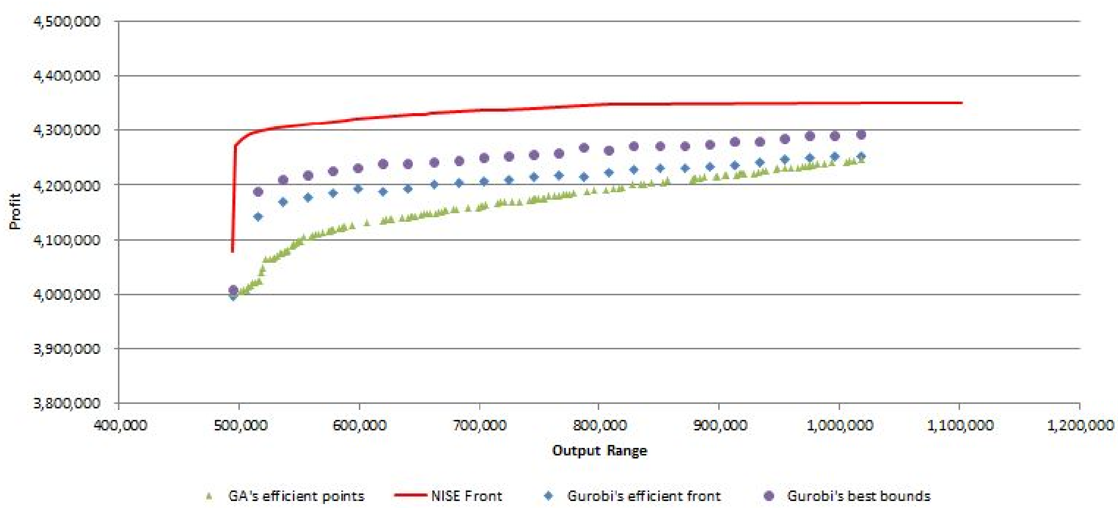

Figure 3.32: Instance 211's gap comparison

Table 3.5: GA frontier's gaps compared to NISE's and Gurobi's

Instance	Compared to NISE's		NISE's runtime	Compared to Gurobi's		Gurobi's runtime
	Min Gap	Max Gap	(minutes)	Min Gap	Max Gap	(minutes)
111	0.0%	5.8%	24	0.0%	2.3%	2,774
112	0.8%	6.1%	56	0.0%	2.5%	1,131
121	0.6%	3.3%	2	0.0%	0.9%	30
122	0.8%	5.2%	13	0.0%	1.5%	63
211	2.0%	6.5%	101	0.0%	3.9%	7,367
212	0.8%	7.4%	24	0.5%	3.3%	3,909
221	0.0%	4.6%	4	0.0%	1.4%	105
222	0.0%	5.7%	7	0.0%	1.2%	56
Average	0.6%	5.6%	29	0.1%	2.1%	1,929

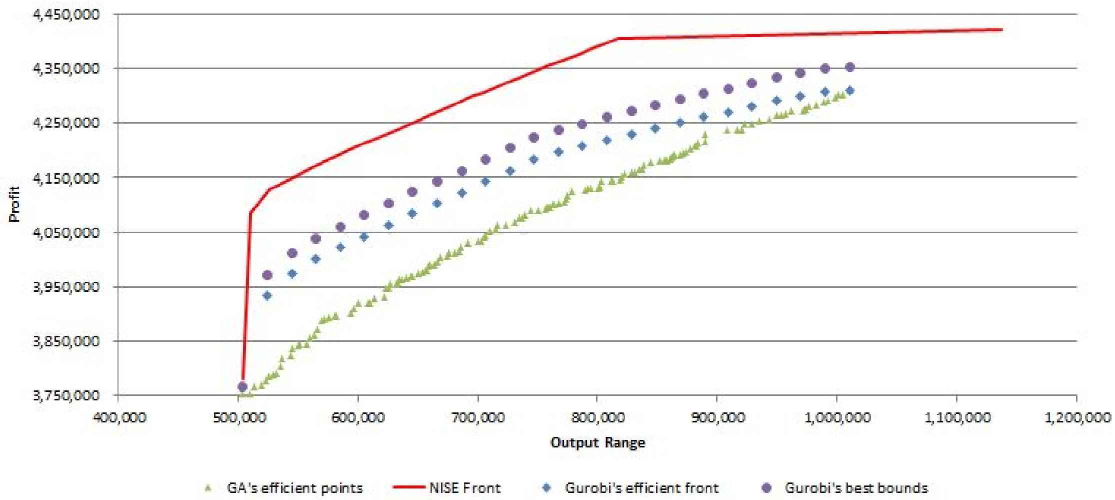

Figure 3.33: Instance 212's gap comparison

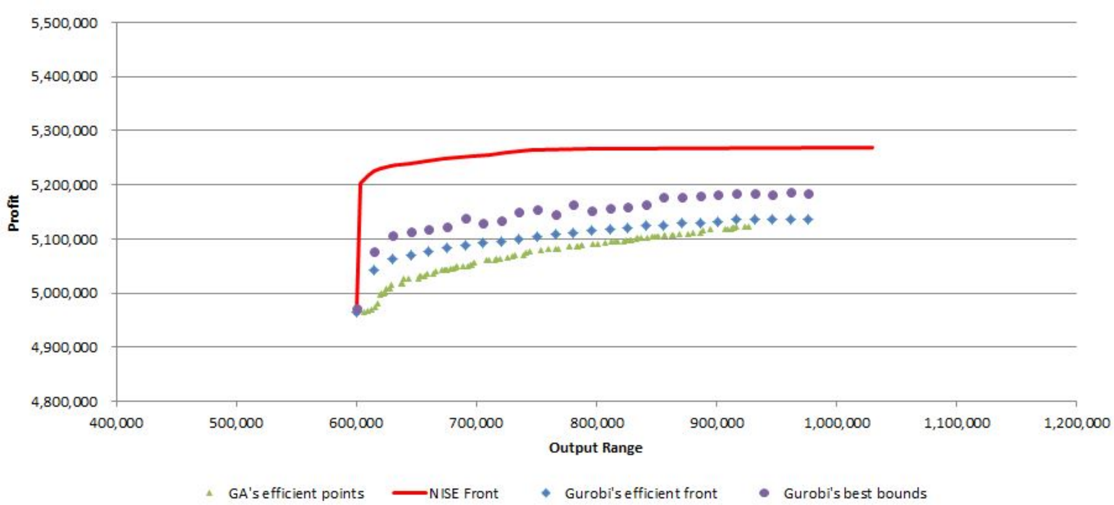

Figure 3.34: Instance 221's gap comparison

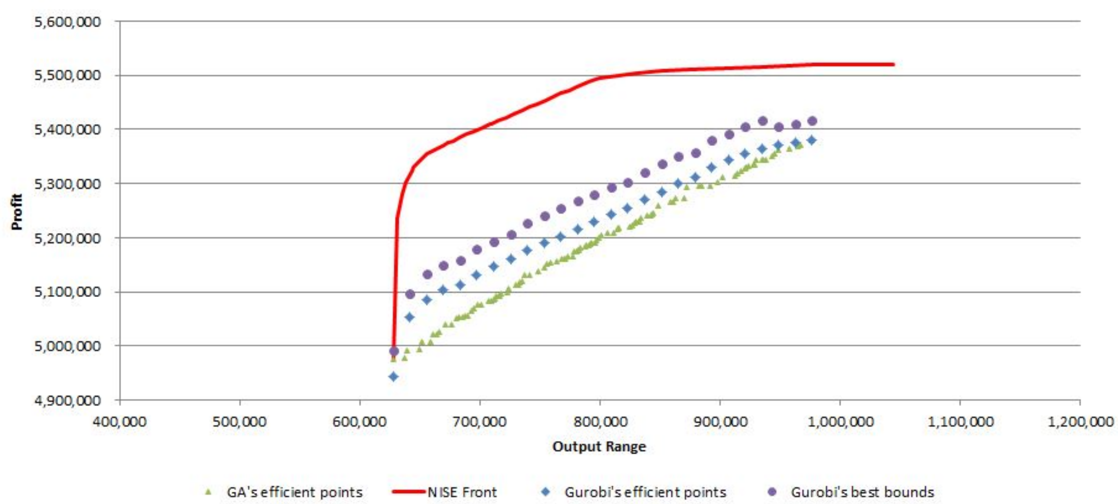

Figure 3.35: Instance 222's gap comparison

3.7 Conclusion

The bi-objective freight consolidation problem with unscheduled discount supports collaboration among multiple departments with a company to make the best decisions overall. The problem captures both transportation planning and facility output. We propose a GA-based matheuristic methodology that builds high quality frontier in terms of both coverage and optimization gap. Experiments show that the average gap between a solution and linear relaxation bound ranges from 0.6% to 5.6%. Gurobi's frontiers provide more accurate gaps, ranging from 0.1% to 1.9% on average. Our methodology takes 60 hours to produce on average 129-point frontiers which is significantly faster than Gurobi does.

We use discount to coordinate transportation with throughput and prevent stock-out at customers. The mechanism might need to be validated in reality. High service-demanding customers might not be interested in the discount program. Perhaps, we might want to introduce multiple shipment lead time levels to persuade these customers into the program.

3.8 References

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Chapter 4

Truckload Procurement in Market with Tight Capacity

Nguyen, H. N., Rainwater, C. E., Mason, S. J., Pohl, E. A.

4.1 Introduction

The U.S. economy is steadily recovering since the recent 2007-2009 recession. Recently, the American Trucking Associations (ATA) released its U.S. Freight Transportation Forecast. The forecast projected an annual growth of 3.2% through 2018 for truckload volume and 1.1% thereafter through 2024. In the time period from 2010 to 2013, the market favored shippers because trucking service demand was still low from the recession and the manufacturing volume rose from its bottom of the recession. Starting in 2014, shippers reportedly saw truckload capacity tightening. Figure 4.1 from Wolfe Research illustrates the market change from January 2014 to March 2014, SCDigest [85]. The horizontal axis of the charts categorizes the market condition based on a scale from 1 to 10. 1 is for extremely loose and 10 is for extremely tight. The vertical axis represents the percentage of surveyed shippers. The capacity condition turned substantial tighter in the first quarter of 2014. According to Tompkin's report by Ferrell [76], the capacity condition of the market will be continue in the same direction in coming years.

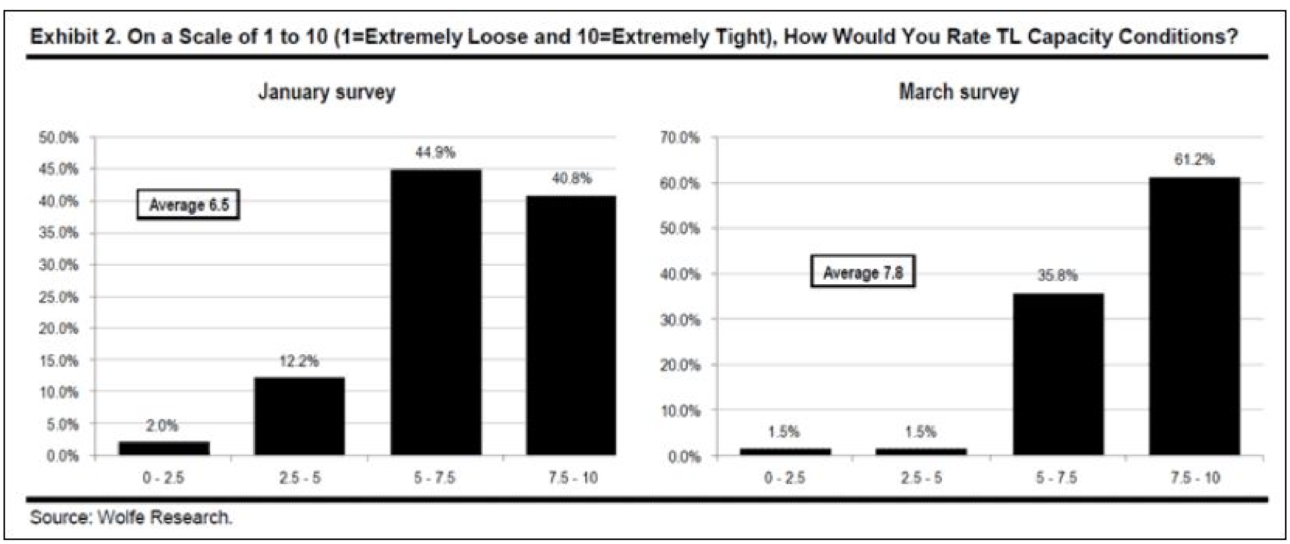


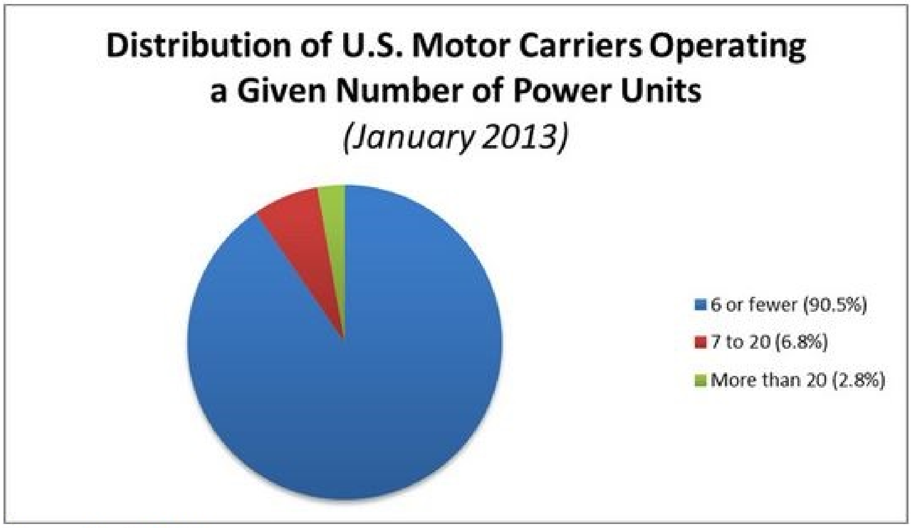
Figure 4.1: Truckload capacity conditions in Q1 2014

In the current market, carriers have more opportunities for better contracts. Those who have contractually committed rates start to reject tendered loads more frequently and look for freight in the spot market where they can easily find better deals. The situation immediately puts shipper's transportation operations under pressure. It also disrupts the current established processes of transportation management. It used to take less that a minute to have a tendered load accepted. Now shippers spend more resource to find capacity that is reasonably rated and meets the delivery window.

Encountering capacity tightening, shippers turn to alternatives such as dedicated fleet and intermodal solutions. A dedicated fleet solution guarantees capacity. Shippers are charged for the fixed amount corresponding to the size of the fleet regardless of utilization. The dedicated fleet solution is only feasible for lanes or areas with high and consistent volumes. On the other hand, an intermodal solution offers lower rates and lower fuel surcharge than over-the-road transportation. However, only lanes that are close to intermodal hubs can be converted to intermodal. Intermodal transit time is usually 50% more than truckload transit time.

Even though shippers have alternatives such as those aforementioned, they still rely on truckload transportation. Alternatives diversify shipper's transportation structure and cannot completely replace common truckload transportation. In the market with tightening capacity, shippers have to revise and redesign their procurement processes to better buy truckload services. In the past, shippers preferred to have the mix of only national carriers because of network coverage and service consistency. According to Raetz [83], only 2.8% of the truckload carriers in the U.S. are large, having 20 trucks or more, see Figure 4.2. Shippers who expand their carrier mix to include smaller carriers will encounter rate increases less substantial than those who do not. Small carriers will service market areas neglected by national carriers and leverage shipper's rate negotiation power.

Having more carriers in the mix does not only benefit shippers but also complicates the procurement process. Small carriers have coverage in selective markets. Their service and capacity are inconsistent across a shipper's freight network. They are more likely to change



Source: C. H. Robinson

Figure 4.2: U.S. Motor Carrier Profile

or withdraw their bid rates during a procurement event.

In this research, we study the impacts of small carriers in a procurement event. We model the standard procurement process that has been utilized by a 3PL. The process was designed to maximize total transportation savings for shippers through multiple negotiations with bidding carriers when the market favored shippers in the recent years. We evaluate the performance of the process in the market with capacity tightening condition. We study a modification to the existing procurement process to help shippers execute their events in an adverse market.

4.2 Literature Review

Transportation procurement auction is an active research area. There have been several papers that address the problem from multiple perspectives. In this section, we review the current literature and categorize it into three subsections: carriers' perspective, shippers', and uncertainty problems.

Caplice and Sheffi [69] described a common transportation procurement process. It consists of three phases: preparation, execution, and analysis and assignment. The first two phases do not involve any optimization. The last phase evaluates the bids submitted by carriers and makes assignment or volume awarding decisions. This phase is usually supported by an optimization tool which considers multiple business constraints and simultaneously finds the optimal assignment solution. Caplice and Sheffi [69] reviewed standard mathematical formulations for the optimization problem in the third phase. They are usually called winner determination problems (WDP). The authors described common business rules for the third phase such as favoring incumbent carrier and volume limitation. In a combinatorial auction, carriers are allowed to bundle lanes and bid. Sheffi [86] described a common combinatorial auction process. Caplice and Sheffi [70] described in more detail a transportation procurement process. The authors compared traditional to combinatorial auctions. They discussed shippers' and carrier's behaviors and objectives in the process. Abrache et al. [65] reviewed many settings of combinatorial auctions and the problem's formulations as a WDP.

Lee et al. [80] considered a carrier's entire current network and contracted volume to optimally generate bid bundles in a procurement. The problem included repositioning cost and service levels. The author adopted a vehicle routing model and column generation to solve the problem. Aral et al. [66] studied the order allocation for local and in-transit carriers in a combinatorial auction. The in-transit carriers bided for lower rates on average because of their cost advantage but their services were lower than the local ones'. Chang [71] utilized a minimum cost flow model to study the bid generation problem for a carrier. The problem considered the bid information and the carrier's current network and volume to generate a bid package that was competitive to the market. Day and Raghavan [74] proposed a matrix scheme for carriers to concisely present their bid bundles in a combinatorial auction. The matrix bids improved the tractability of the WDP.

Ergun et al. [75] studied a shipper collaboration problem. Multiple shippers combined their freight networks to offer the best bid bundles to carriers. Tours created from the combination of shippers' networks helped to reduce carriers' repositioning costs. The authors formulated the problem as a set covering formulation and proposed a greedy algorithm to solve it. Garrido [77] studied the transportation market in which significant low back-haul rates generated shipping demand. Carriers who needed to return the equipment back to their domicile offered low rates for the back-haul lanes. Collins and Quinlan [73] quantified the impacts of the bundling option in procurement auction. The authors used a statistic approach to show that bid bundling reduced bid rates and ultimately benefited shippers. Kafarski [78] studied the correlation between demand volatility and rate fluctuation. The authors proposed a demand aggregation presentation to utilize in a procurement auction which would improve the shipment volume stability. Turner et al. [88] studied the contractual agreements between a shipper and a carrier when diesel price fluctuated. The current fuel schedule widely utilized in the industry does not perfectly pass on the fuel cost from carriers to shippers. The authors proposed a lane assignment optimization model to help shippers limit their exposure to fuel price fluctuation. Xu et al. [89] studied a combinatorial auction problem in which carriers were allowed to create bundles of lanes. The auction process had multiple stages. The shippers followed a just-in-time manufacturing model. Xu et al. [89] studied a combinatorial auction problem with inventory management.

In the current literature, most of truckload procurement problems which consider uncertainty factors address the shipment volume uncertainty. Ma [81] studied truckload transportation procurement from carriers' perspective and from both carriers' and shipper's simultaneously. Their problem considered shipment volume uncertainty. The problem from the carriers' perspective was to select the best fit bidding lanes considering their existing lanes. Ma et al. [82] evaluated the solution quality when utilizing a combinatorial auction model with uncertain shipment volume. The authors proposed a two-stage stochastic integer programming model to address the problem. They showed the solutions obtained from the stochastic model were up to 7.15% better than the ones obtained from a similar deterministic model. Remli and Rekik [84] proposed a robust optimization programming to optimize a combinatorial auction problem when the shipper's volume is uncertain. The authors focused on the solution quality in the worst case scenario. The authors' methodology outperformed a static model in 65% of the experiments. Zhang et al. [90] solved a combinatorial auction problem with shipment volume uncertainty and penalty cost. Triki et al. [87] studied a load bundle generation problem for a carrier to bid on in a truckload transportation procurement auction. The problem considered the winning probability of bidding rates and generated routes for existing and potential shipments that the carrier handled.

The current literature has the great amount of studies that assumed the transportation market is at least balance between supply and demand or favorable for shippers. Well established auction processes were designed to perform the best in the market condition. As aforementioned, the market has changed since the beginning of 2014. Carriers are now able to increase rates up 5% on average compared to last year. Demand is higher than supply in some periods of the year. Auction processes need to be revised and redesigned to continue adding value to shippers' procurement. The literature review shows that our studied problem has never been addressed before. When considering uncertainty in a truckload procurement problem, most papers addressed shipment volume uncertainty. We see just one paper that considered carrier's capacity as our problem does. Chen [72] studied the bid generation problem of a carrier. The study considered the uncertainty of back-haul capacities and costs. The author modelled the problem as a two-stage stochastic integer program. We study the procurement problem from the shippers' perspective.

4.3 Procurement

An existing truckload procurement process is illustrated in Figure 4.3. Most shippers execute their truckload procurement event annually. Winning carriers, which are also referred to as awarded carriers, are awarded the truckload service contracts for one year. There are a few shippers that award two or three-year contracts. Shippers usually start the procurement event three to four months in advance of the current contracts' expiration date. Sometime they may start earlier to provide more time for gathering and processing data when there are acquisitions after the last procurement.

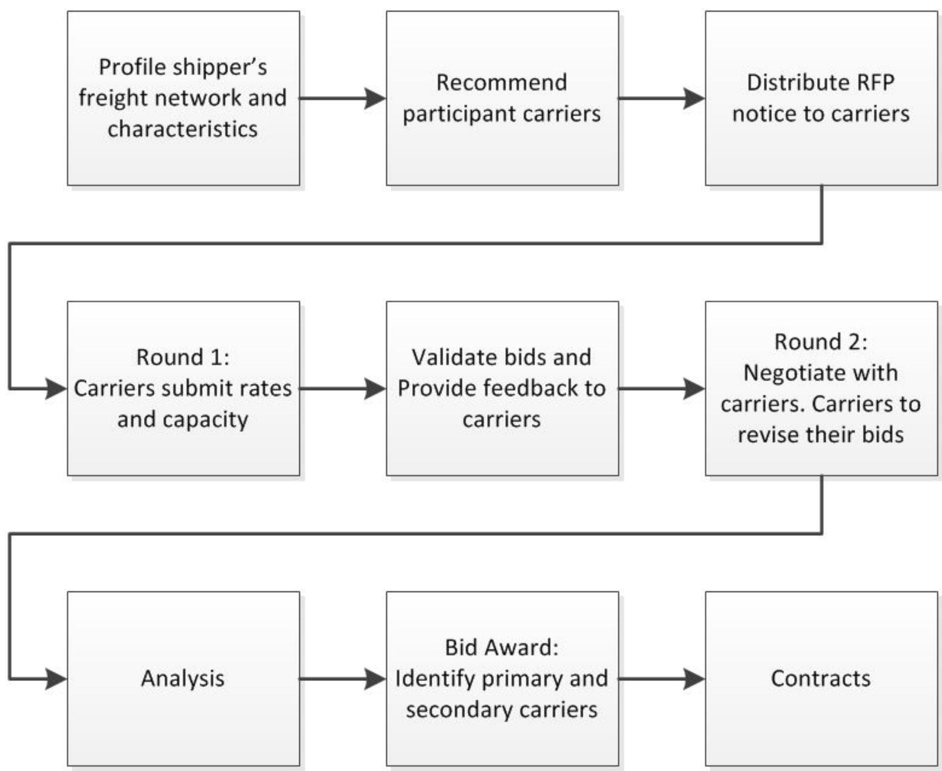


Figure 4.3: Procurement process

A procurement process begins with collecting data to describe the current status of a shipper's freight network. In this step, the shipper will review their transportation expense, equipment requirements, plants' and warehouses' profiles. They will validate their freight flow from top vendors to top customers or distributors. They will remove some lanes due to vendor switching during the last year. They will also add new lanes due to winning new customers and existing customer's network expansion. Reviewing historical data will also help the shipper to evaluate their current carriers' performance. They may want to eliminate some carriers due to their low performance and introduce new carriers with which they started to have better relationships. After the first and the second steps, the shipper will determine lanes and volume putting in the procurement event as well as the list of carriers to be invited to participate in the event. In the third step, the shipper will package all required documents into a Request For Proposal (RFP) and distribute to the recommended carriers.

Once receiving RFP package from the shipper, carriers will study the bidding network and submit their bid rates and capacity on lanes. In the first round of bidding, carriers usually have two weeks to submit their initial bids. After the time frame, the bid is closed. Carriers are not allowed to change, add, or remove their bids. The shipper begins to validate and review the initial submitted bids. They will usually highlight lanes with potential high savings. The second round of bidding starts once feedback of the initial bids are provided to carriers. The shipper starts negotiations with carriers based on potential improvement areas identified after the first round. After negotiations, most carriers may update their bids.

Valid bidding rates in the procurement event after the second round are considered carriers' final submission. The shipper begins to analyze the bidding rates and the capacity. They usually utilize an auction optimization platform to help building what-if scenarios and make decisions. In this analysis step, the shipper will incorporate their business rules and carrier relationship strategy into the analysis scenarios. They usually award most incumbent carriers to maintain long term relationship and consistent service level to major customers. They also introduce a limited number of new carriers to exploit low cost opportunity and diversify the carrier mix. In the last step of a procurement process, winning carriers are awarded the contracts which state committed rates and capacity from carriers and committed volume from the shipper.

4.4 Example

In this section, we present an example to illustrate the performance of the existing procurement process in the shipper or carrier-favoring markets. We also highlight the change to the process that we propose and address in this chapter.

There is a simple procurement event that looks for carriers to provide truckload service for a lane with 100 loads annually. Two carriers, A and B, are invited and participate in the event. A bid package consists of two parameters: the rate per load and the capacity. In the first round of bidding, carrier A's bid package is \$1300 per load and 100 loads annually. We denote the bid package as $(\$1300, 100)$. Carrier B's bid package is $(\$1050, 50)$. We study four scenarios:

Scenario 1: The procurement event does not let any carrier revise their submitted rates and capacity. After the first round's completion, the shipper analyzes the bids and selects carriers and assigns volume. Carrier A is awarded 50 loads. Carrier B is awarded 50 loads. The total transportation cost is \$117500.

Scenario 2: After the first round's completion, carriers are provided feedback on their initial bid packages. The feedback is usually about the competitiveness of their rates compared to others'. Both carriers A and B still want to win the business. Therefore, they potentially reduce their bid rates and resubmit their revised bid packages in the second round. Carrier A reduces its rate by 5%. Carrier B reduces its rate by 1%. After the second round's completion, the shipper makes its optimal assignment decision: carrier A is awarded 50 loads; carrier B is awarded 50 loads. The total transportation cost is \$113725.

Scenario 3: This scenario is similar to scenario 2. However, the market is more favorable for the carriers. Carrier A which is a national carrier with large capacity still wants to win the business. Contrastingly, carrier B has contractually committed its capacity to another shipper in the meantime. Therefore, carrier B withdraws its bid package in the second round. The shipper has to award 100 loads to carrier A. The total transportation cost is $123500.

Scenario 4: This scenario is similar to scenario 3 where carriers are allowed to revise their bid packages and the market is favorable for carriers. Knowing the market condition, the shipper proactively locks in carrier B's capacity. Because the available loads for bidding in the second round reduces from 100 to 50, carrier A reduces its bidding rate by 3% instead of 5%. The total transportation cost is $115550.

Table 4.1: Savings comparison among scenarios

Scenario	**Cost**	**Savings compared to Scenario 1**	**Description**
1	$117,500		Shipper with lowest buying power.
2	$113,725	3%	Shipper with highest buying power.
3	$123,500	-5%	Market dynamic counter-benefits negotiation.
4	$115,550	2%	Proactive strategy identifies improvement area.

Table 4.1 summarizes the differences among the four scenarios. Scenario 2 represents the past market where shippers have higher buying power than carriers. The existing procurement process was designed to best perform in scenario 2. Scenario 3 represents the situation where the market shifts to favor carriers more and shippers do not change their procurement practice. Negotiation which is utilized to compress the transportation cost further conversely causes capacity loss. Carriers may withdraw from the procurement because they have better opportunity to sell their services somewhere else. In this research, we propose a change in the existing procurement process: adding a decision step in between

the two bidding rounds to lock in capacity whose rates appear competitive already. Scenario 4 represents our proposed modified procurement process.

4.5 Models

4.5.1 Deterministic model

Sets

- L : the set of lanes l.
- T : the set of carriers t.

Parameters

- M : a large number.
- c_t^l : the price asked by carrier t on lane l.
- d^l : the number of truckloads to be hauled on lane l.
- m_t^l : the capacity of carrier t on lane l.

Variables

- y_t^l : the amount of truckloads on lane l awarded to carrier t.

The deterministic model of a procurement event is:

$$Minimize \quad \sum_{l \in L} \sum_{t \in T} c_t^l y_t^l \tag{4.1}$$

s.t.

$$\sum_{t \in T} y_t^l \geq d^l \qquad \forall l \in L \tag{4.2}$$

$$y_t^l \leq m_t^l \qquad \forall l \in L, \forall t \in T \tag{4.3}$$

$$y_t^l \in \mathbb{Z}^+ \qquad \forall l \in L, \forall t \in T \tag{4.4}$$

The problem's objective function (4.1) minimize total transportation cost. Constraint (4.2) guarantees the satisfaction of all demands. Constraint (4.3) enforces carriers' capacity.

4.5.2 Robust counterpart

In robust optimization, decisions of the procurement event are made in two stages. We redefine certain parameters and variables as below to allow us for robust analysis.

Parameters

- c_t^l : the price asked by carrier t on lane l in the second round. We have:

$$c_t^l = \bar{c}_t^l - \theta_t \hat{c}_t^l \tag{4.5}$$

  where $\theta_t \in \mathbb{R}$ and $0 \leq \theta_t \leq 1$. We assume that carrier t provides discounts consistently over lanes l. Therefore, θ_t is independent from lane l. $\bar{c}_t^l$ is the price asked by carrier t on lane l in the first round. $\hat{c}_t^l$ is the rate discount offered by carrier t after the negotiation in the second round.

- m_t^l : the capacity of carrier t on lane l in the second round. We have:

$$m_t^l = \bar{m}_t^l - \lambda_t^l \hat{m}_t^l \tag{4.6}$$

where $\bar{m}_t^l$ is the capacity of carrier t on lane l in the first round. In the second round of the procurement process, we assume there are only two possibilities. A carrier either withdraws completely from the procurement event or offers a rate discount. Therefore, λ_t^l is the same as $\lambda_t \in \{0, 1\}$ and $\hat{m}_t^l = \bar{m}_t^l$. Equation (4.6) can be written as:

$$m_t^l = \bar{m}_t^l - \lambda_t \bar{m}_t^l \tag{4.7}$$

- Γ_θ : the budget of uncertainty for cost defined by Bertsimas and Sim [68]. Decision makers use Γ_θ to describe the level of stochasticity in the market.

$$\sum_{t \in T} \theta_t \leq \Gamma_\theta \tag{4.8}$$

- Γ_λ : the budget of uncertainty for carriers' capacity.

$$\sum_{t \in T} \lambda_t \leq \Gamma_\lambda \tag{4.9}$$

Variables

- x_t^l : the amount of truckloads on lane l awarded to carrier t in the first round.
- α_t : equal to 1 if carrier t is awarded in the first round.
- y_t^l : the amount of truckloads on lane l awarded to carrier t in the second round.
- β_t : equal to 1 if carrier t is awarded in the second round.

The robust counterpart of the deterministic model is:

$$Minimize \quad \sum_{l \in L} \sum_{t \in T} \vec{c}_t^l x_t^l + opt(R(x, \Gamma_\theta, \Gamma_\lambda)) \tag{4.10}$$

s.t.

$$\sum_{t \in T} x_t^l \leq d^l \qquad \forall l \in L \tag{4.11}$$

$$x_t^l \leq \bar{m}_t^l \qquad \forall l \in L, \forall t \in T \tag{4.12}$$

$$x_t^l \in \mathbb{Z}^+ \qquad \forall l \in L, \forall t \in T \tag{4.13}$$

Objective function (4.10) minimizes the total cost which consists of the total cost in the first round and the worst of the total cost in the second round. Constraint (4.11) enforces that the awarded volume on a lane is at most equal to the demand of the lane. Demand is not required to be completely satisfied in the first round. Constraint (4.12) enforces that the awarded volume is at most equal to the capacity of the carrier on the lane.

$opt(R(x, \Gamma_\theta, \Gamma_\lambda))$ is the optimal objective value of the recourse problem (the negotiation round).

$$opt(R(x, \Gamma_\theta, \Gamma_\lambda)) = \max_{m_t^l \in \mathcal{U}(\Gamma_\lambda), c_t^l \in \mathcal{U}(\Gamma_\theta)} Q(x, \Gamma_\theta, \Gamma_\lambda) \tag{4.14}$$

where $\mathcal{U}(\Gamma_\theta)$ and $\mathcal{U}(\Gamma_\lambda)$ is the uncertainty set of c_t^l and m_t^l. $Q(x, \Gamma_\theta, \Gamma_\lambda)$ is the winner determination problem in the second round where all uncertainty factors are realized.

$Q(x, \Gamma_\theta, \Gamma_\lambda)$ is defined as below:

$$Minimize \quad \sum_{l \in L} \sum_{t \in T} c_t^l y_t^l \tag{4.15}$$

s.t.

$$\sum_{t \in T} y_t^l \geq d^l - \sum_{t \in T} x_t^l \qquad \forall l \in L \tag{4.16}$$

$$y_t^l \leq m_t^l \qquad \forall l \in L, \forall t \in T \tag{4.17}$$

$$\sum_{l \in L} y_t^l \leq M\beta_t \qquad \forall t \in T \tag{4.18}$$

$$\beta_t \leq 1 - \alpha_t \qquad \forall t \in T \tag{4.19}$$

$$y_t^l \in \mathbb{Z}^+ \qquad \forall l \in L, \forall t \in T \tag{4.20}$$

$$\beta_t \in \{0, 1\} \qquad \forall t \in T \tag{4.21}$$

Objective function (4.15) minimizes the total cost in the second round. Constraint (4.16) enforces that the awarded volume must meet the demand. Constraint (4.17) enforces that the awarded volume must not exceed the carrier's capacity. Constraints (4.18) establishes the relationship between y_t^l and β_t. If carrier t is awarded in the second round, β_t must be 1. Constraint (4.19) enforces that a carrier can only be awarded in either the first or the second round.

4.5.3 Model transformation and algorithm

We adopt the solution methodology proposed by Ben-Tal et al. [67] and Remli and Rekik [84]. Firstly, we will convert $Q(x, \Gamma_\theta, \Gamma_\lambda)$ to its dual program. Different from the recourse problem in Remli and Rekik [84], $Q(x, \Gamma_\theta, \Gamma_\lambda)$ is not a linear program because of constraints (4.20) and (4.21). Constraint (4.20) can be replaced by constraint (4.22) because y_t^l must be an integer number in an optimal solution.

$$y_t^l \geq 0 \qquad \forall l \in L, \forall t \in T \tag{4.22}$$

β_t can be eliminated by combining the two constraints (4.18) and (4.19) into a new constraint:

$$\sum_{l \in L} y_t^l \leq M(1-\alpha_t) \qquad \forall t \in T \tag{4.23}$$

Let e^l, f_t^l, and g_t be the dual variables corresponding to constraints (4.16), (4.17), and (4.23). The dual $Q^d(x, \Gamma_\theta, \Gamma_\lambda)$ is:

$$Maximize \quad \sum_{l \in L} d^l e^l - \sum_{l \in L} \sum_{t \in T} x_t^l e^l - \sum_{l \in L} \sum_{t \in T} m_t^l f_t^l - \sum_{t \in T} M(1-\alpha_t) g_t \tag{4.24}$$

s.t.

$$e^l - f_t^l - g_t \leq c_t^l \qquad \forall l \in L, \forall t \in T \tag{4.25}$$

$$e^l \geq 0 \qquad \forall l \in L \tag{4.26}$$

$$f_t^l \geq 0 \qquad \forall l \in L, \forall t \in T \tag{4.27}$$

$$g_t \geq 0 \qquad \forall t \in T \tag{4.28}$$

Based on the strong duality, we have:

$$opt(R(x,\Gamma_\theta,\Gamma_\lambda)) = \max_{m_t^l \in \mathcal{U}(\Gamma_\lambda), c_t^l \in \mathcal{U}(\Gamma_\theta)} Q^d(x,\Gamma_\theta,\Gamma_\lambda) \tag{4.29}$$

Therefore, $R(x,\Gamma_\theta,\Gamma_\lambda)$ is:

$$Maximize \quad \sum_{l\in L} d^l e^l - \sum_{l\in L}\sum_{t\in T} x_t^l e^l - \sum_{l\in L}\sum_{t\in T} \bar{m}_t^l f_t^l + \sum_{l\in L}\sum_{t\in T} \bar{m}_t^l \lambda_t f_t^l - \sum_{t\in T} M(1-\alpha_t) g_t \tag{4.30}$$

s.t.

$$e^l - f_t^l - g_t + \hat{c}_t^l \theta_t \le \bar{c}_t^l \qquad \forall l \in L, \forall t \in T \tag{4.31}$$

$$e^l \ge 0 \qquad \forall l \in L \tag{4.32}$$

$$f_t^l \ge 0 \qquad \forall l \in L, \forall t \in T \tag{4.33}$$

$$g_t \ge 0 \qquad \forall t \in T \tag{4.34}$$

$$\theta_t + \lambda_t \le 1 \qquad \forall t \in T \tag{4.35}$$

$$\sum_{t\in T} \theta_t \le \Gamma_\theta \tag{4.36}$$

$$\sum_{t\in T} \lambda_t \le \Gamma_\lambda \tag{4.37}$$

$$0 \le \theta_t \le 1 \qquad \forall t \in T \tag{4.38}$$

$$\lambda_t \in \{0,1\} \qquad \forall t \in T \tag{4.39}$$

Constraint (4.35) establishes the relationship between cost and capacity uncertainty. If a carrier withdraws from the procurement, i.e. $\lambda_t = 1$, there is no cost change, i.e. $\theta_t = 0$.

$R(x, \Gamma_\theta, \Gamma_\lambda)$ is not a linear program because of the variable multiplication in the objective function. Adopting the technique used by Remli and Rekik [84], we make a stricter assumption about the value domain of θ_t and introduce variable s_t^l to replace $\lambda_t f_t^l$.

$R(x, \Gamma_\theta, \Gamma_\lambda)$ is:

$$Maximize \quad \sum_{l \in L} d^l e^l - \sum_{l \in L} \sum_{t \in T} x_t^l e^l - \sum_{l \in L} \sum_{t \in T} \bar{m}_t^l f_t^l + \sum_{l \in L} \sum_{t \in T} \bar{m}_t^l s_t^l - \sum_{t \in T} M(1 - \alpha_t) g_t \quad (4.40)$$

s.t.

$$e^l - f_t^l - g_t + \hat{c}_t^l \theta_t \leq \bar{c}_t^l \qquad \forall l \in L, \forall t \in T \quad (4.41)$$

$$\theta_t + \lambda_t \leq 1 \qquad \forall t \in T \quad (4.42)$$

$$\sum_{t \in T} \theta_t \leq \Gamma_\theta \quad (4.43)$$

$$\sum_{t \in T} \lambda_t \leq \Gamma_\lambda \quad (4.44)$$

$$s_t^l \leq M \lambda_t \qquad \forall l \in L, \forall t \in T \quad (4.45)$$

$$s_t^l \leq f_t^l \qquad \forall l \in L, \forall t \in T \quad (4.46)$$

$$e^l \geq 0 \qquad \forall l \in L \quad (4.47)$$

$$f_t^l \geq 0 \qquad \forall l \in L, \forall t \in T \quad (4.48)$$

$$s_t^l \geq 0 \qquad \forall l \in L, \forall t \in T \quad (4.49)$$

$$g_t \geq 0 \qquad \forall t \in T \quad (4.50)$$

$$\theta_t \in \{0, 1\} \qquad \forall t \in T \quad (4.51)$$

$$\lambda_t \in \{0,1\} \qquad \forall t \in T \tag{4.52}$$

Adopting the constraint-generation algorithm proposed by Kelley [79] and Remli and Rekik [84], we will design the solving methodology for the robust problem. Let $(e^\sigma, f^\sigma, s^\sigma, g^\sigma) \in \mathcal{S}$ the extreme points of the recourse problem $R(x, \Gamma_\theta, \Gamma_\lambda)$. Our robust problem (RWDP)is:

$$Minimize \quad \sum_{l \in L} \sum_{t \in T} \bar{c}_t^l x_t^l + A \tag{4.53}$$

s.t.

$$A \geq \sum_{l \in L} d^l e^{l\sigma} - \sum_{l \in L} \sum_{t \in T} x_t^l e^{l\sigma} - \sum_{l \in L} \sum_{t \in T} \bar{m}_t^l f_t^{l\sigma} + \sum_{l \in L} \sum_{t \in T} \bar{m}_t^l s_t^{l\sigma} - \sum_{t \in T} M(1-\alpha_t) g_t^\sigma \tag{4.54}$$

$$\forall (e^\sigma, f^\sigma, s^\sigma, g^\sigma) \in \mathcal{S}$$

$$\sum_{t \in T} x_t^l \leq d^l \qquad \forall l \in L \tag{4.55}$$

$$x_t^l \leq \bar{m}_t^l \qquad \forall l \in L, \forall t \in T \tag{4.56}$$

$$\sum_{l \in L} x_t^l \leq M\alpha_t \qquad \forall t \in T \tag{4.57}$$

$$x_t^l \in \mathbb{Z}^+ \qquad \forall l \in L, \forall t \in T \tag{4.58}$$

$$\alpha_t \in \{0,1\} \qquad \forall t \in T \tag{4.59}$$

In the objective function (4.40) of $R(x, \Gamma_\theta, \Gamma_\lambda)$, the last term is a multiplication of large number M. In order to maximize the objective function, this term which is always negative must be zero. Therefore, g_t must be zero in an optimal solution. We will eliminate g_t from

$R(x, \Gamma_\theta, \Gamma_\lambda)$. Constraint (4.57) becomes redundant and therefore is removed.

The constraint-generation algorithm is:

- **Step 1:** Initialization.

  We have the point $(0, 0, 0) \in \mathcal{S}$.

  $LB = -\infty$, $UB = +\infty$.

  Denote $r = 1$ the iteration index.

  Go to step 2.

- **Step 2:** Solve the master problem RWDP^r at iteration r and update lower bound.

  Solve the master problem based on the extreme points (e^i, f^i, s^i) where $i = 0..r-1$ that have been found so far.

$$Minimize \quad \sum_{l \in L} \sum_{t \in T} \bar{c}_t^l x_t^l + A \tag{4.60}$$

s.t.

$$A \geq \sum_{l \in L} d^l e^{li} - \sum_{l \in L} \sum_{t \in T} x_t^l e^{li} - \sum_{l \in L} \sum_{t \in T} \bar{m}_t^l f_t^{li} + \sum_{l \in L} \sum_{t \in T} \bar{m}_t^l s_t^{li} \tag{4.61}$$

$$\forall i = 0..r-1$$

$$\sum_{t \in T} x_t^l \leq d^l \qquad \forall l \in L \tag{4.62}$$

$$x_t^l \leq \bar{m}_t^l \qquad \forall l \in L, \forall t \in T \tag{4.63}$$

$$x_t^l \in \mathbb{Z}^+ \qquad \forall l \in L, \forall t \in T \tag{4.64}$$

Denote x_t^{lr} the optimal solution found in this step.

Update the lower bound:

$$LB = max\{LB, \sum_{l \in L} \sum_{t \in T} \bar{c}_t^l x_t^{lr} + A\}$$

- **Step 3:** Solve the recourse problem and update upper bound.

  Solve the recourse problem $R(x, \Gamma_\theta, \Gamma_\lambda)$ based on the optimal solution x_t^{lr} found in the previous step.

  Update the lower bound:

$$UB = min\{UB, \sum_{l \in L} \sum_{t \in T} \bar{c}_t^l x_t^{lr} + A'\}$$

  where A' is the newly found objective value of the recourse problem. Unlike Remli and Rekik [84], our problem fixes a portion of the overall total cost in the *"here and now"* stage. Therefore, the upper bound is not only established by the recourse problem's objective value.

- **Step 4:** Termination criteria.

  If $LB = UB$, the optimal solution is x_t^{lr}. The algorithm is completed.

- **Step 5:** Add constraint and return to step 2.

  If $LB < UB$, add the below constraint to the RWDP^r:

$$A \geq \sum_{l \in L} d^l e^{lr} - \sum_{l \in L} \sum_{t \in T} x_t^l e^{lr} - \sum_{l \in L} \sum_{t \in T} \bar{m}_t^l f_t^{lr} + \sum_{l \in L} \sum_{t \in T} \bar{m}_t^l s_t^{lr}$$

  Increase the iteration index by 1: $r = r + 1$

  Return to step 2.

4.6 Experiments

4.6.1 Experimental Design

In a procurement for a paper packaging company that we had access to its data and results, there were $9,000$ lanes and 300 participant carriers. In the minimum cost scenario without any business constraints, the annual freight spend was approximately $200 million. This shipper is a multi-billion company. Its procurement's complexity is beyond the average of the market. In this chapter, we intend to create problem instances that represent most of the procurement events in terms of size and complexity.

Table 4.2: Experimental design

Factor	**Level Description**	**Levels**
$\|L\|$-$\|T\|$	100-40, 200-40, 200-80	3
$\frac{\Gamma_\lambda}{\Gamma_\theta+\Gamma_\lambda}$ (Withdrawal ratio)	20%, 50% , 80%	3
$\Gamma_\theta + \Gamma_\lambda$ (Stochastic level)	20%, 60%, 100%	3
Replications per level combination		5
Total number of problem instances		135

$|L|$-$|T|$ represents the size of a procurement event: the numbers of bidding lanes and participant carriers. The factor has three levels to illustrate medium size procurement events. The withdrawal ratio represents the favoring degree of the market to carriers. The higher the value is, the less buying power a shipper has in the market. The stochastic level factor represents the degree of changes happening in the negotiation round. The higher the value is, the more stochastic the market is. A high Γ_θ represents a higher likelihood that carriers will offer discount in the negotiation round. A high Γ_λ represents a higher likelihood that carriers will withdraw from the procurement event when entering the negotiation round.

Lane l's mileage is uniformly distributed within the range $[200, 3500]$ miles. The mileage band represents all regional and coast-to-coast transportation in the U.S. Carrier t is characterized by a network coverage index which ranges within $[10\%, 100\%]$. A 100% network coverage index indicates that carrier t is a national carrier who can service all lanes of the shipper. A low network coverage index indicates a regional carrier. There are a few national carriers and many regional ones in a problem instance, see Figure 4.2. Carrier's asking price $\vec{c}_t^l$ uniformly distributes within $[\$0.70, \$2.50]$ per mile and is factored by its network coverage index. This feature simulates the market in which national carriers usually ask for high price and they can provide services covering the entire country while regional carriers offer competitive prices but can provide services within a few regions. $\hat{c}_t^l$ uniformly distributes within $[5\%, 15\%]$ of $\vec{c}_t^l$. We assign a very large number to $\hat{c}_t^l$ in order to model that carrier t does not bid for lane l. Demand d^l of lane l uniformly distributes within $[12, 1325]$. The lower bound refers to one shipment per month and the upper bound refers to five shipments per day. Lanes that have less than one shipment per month usually are not included in a procurement because carriers do not prefer to commit their service for those lanes. Carrier capacity $\bar{m}_t^l$ uniformly distributes within $[30\%, 100\%]$ of the lane demand d^l. National carriers have sufficient capacity to meet the demand while regional carriers do not.

The problem can be infeasible when there is not a carrier that bids on a lane in the negotiation round. Infeasibility will make the slave problem unbounded. The algorithm will not generate any more extreme points of the slave problem and will stick at the current optimal gap. In order to deal with infeasibility, we introduce a backup carrier that bids on every lane of the procurement. Its bid rate is higher than any other carriers' bid rate on the same lane. In addition, we introduce a constraint into the master problem to prevent selecting the back up carrier in the first round. Therefore, we can always select the carrier for any lane that are not covered by any carrier in the negotiation round. In reality, when a lane is not bid on by any other carrier in the procurement, the shipper will use spot market to provide capacity for the lane. Spot market rate is usually higher than contracted rates.

4.6.2 Results

We solve all experiments in the Windows server that has Intel Xeon CPU X7350 2.98GHz and 16GB RAM. We set two stopping criteria: 1% optimal gap and 10 hours of computation.

Tables 4.3-4.5 summarize the results. The first two columns define a combination. There are five instances randomly generated based on the same combination. The savings column shows the average savings realized by the proposed procurement process compared to the existing process. The gap/time limit column indicates the number of instances reaching the optimality gap limit and the time limit. The stop gap column shows the average optimality gap. The iteration column shows the number of iterations needed to solve an instance. The column of master problem time is the average run time required to solve an instance. The column of master problem time CV measures the coefficient of variance of the master problem's run time. The last two columns are the time and coefficient of variance for the slave problem.

The greater the numbers of lanes and carriers are, the harder an instance is. The number of instances reaching the time limit increases when $|L|$-$|T|$ goes from 100-40 to 200-80. The greatest optimality gap of an instance when its run time reaches the limit is less than 5%. The number of iterations has a high correlation with the hardness of an instance. When an instance is hard and reaches the time limit before the optimality gap, there is usually fewer number of iterations.

Overall, the master problem is easy to solve. Adding constraint into the master problem after each iteration does not significantly make it harder. The slave problem is the bottleneck of the methodology. The slave problem accounts for 84% to 99% of the total run time. Its run time varies greatly by iteration. It appears that the hardness of the slave problem significantly depends on the master problem's solution in each iteration. On average, the coefficients of variance of the master problem's total run time are high because of its low magnitude. The coefficients of variance of the slave problem's total run time range from 1% to more than 100%.

The proposed procurement process shows significant savings compared to the existing process. With a given withdrawal ratio, the higher the stochastic level is, the higher the savings is. When the withdrawal ratio increases, the savings also increases. The results demonstrate that locking some capacity before the negotiation round helps to achieve greater savings during an adverse market.

Table 4.3: Results of 100-40 instance set

Withdrawal ratio	**Stochastic level**	**Savings**	**Gap/Time limit**	**Stop gap**	**Iterations**	**Master prob. time (seconds)**	**Master prob. time CV**	**Slave prob. Time (seconds)**	**Slave prob. time CV**
20%	20%	5%	5/0	0.7%	29	27	58%	149	46%
20%	60%	8%	5/0	1.0%	126	408	153%	3,802	99%
20%	100%	11%	4/1	1.1%	211	794	57%	25,269	31%
50%	20%	7%	5/0	0.9%	98	222	132%	1,340	74%
50%	60%	16%	0/5	2.8%	115	231	82%	36,051	1%
50%	100%	27%	3/2	1.6%	239	749	52%	22,363	56%
80%	20%	9%	5/0	0.8%	159	510	124%	7,556	66%
80%	60%	25%	3/2	1.7%	206	637	88%	23,000	59%
80%	100%	46%	5/0	1.0%	200	474	66%	1,578	74%

Table 4.4: Results of 200-40 instance set

Withdrawal ratio	**Stochastic level**	**Savings**	**Gap/Time limit**	**Stop gap**	**Iterations**	**Master prob. time (seconds)**	**Master prob. time CV**	**Slave prob. Time (seconds)**	**Slave prob. time CV**
20%	20%	6%	5/0	0.9%	58	154	77%	634	27%
20%	60%	9%	5/0	1.0%	201	1,635	76%	10,456	34%
20%	100%	14%	0/5	2.6%	158	770	45%	35,324	1%
50%	20%	8%	5/0	1.0%	146	924	70%	4,333	41%
50%	60%	19%	0/5	4.0%	121	580	99%	35,792	3%
50%	100%	27%	2/3	1.8%	271	1,560	68%	32,519	13%
80%	20%	10%	4/1	1.1%	238	2,086	68%	25,166	18%
80%	60%	26%	2/3	2.1%	196	710	48%	24,831	57%
80%	100%	39%	1/4	2.2%	389	4,792	76%	29,437	22%

Table 4.5: Results of 200-80 instance set

Withdrawal ratio	**Stochastic level**	**Savings**	**Gap/Time limit**	**Stop gap**	**Iterations**	**Master prob. time (seconds)**	**Master prob. time CV**	**Slave prob. Time (seconds)**	**Slave prob. time CV**
20%	20%	5%	5/0	0.9%	115	969	27%	7,576	43%
20%	60%	14%	0/5	4.1%	17	20	40%	37,078	3%
20%	100%	17%	0/5	2.2%	12	12	46%	37,446	4%
50%	20%	12%	0/5	4.3%	34	82	95%	36,707	2%
50%	60%	22%	0/5	3.5%	9	8	26%	39,985	5%
50%	100%	34%	3/2	1.6%	8	9	140%	19,897	87%
80%	20%	16%	0/5	3.1%	13	12	60%	40,983	4%
80%	60%	34%	3/2	1.9%	13	16	138%	20,988	80%
80%	100%	53%	2/3	3.1%	55	396	214%	22,553	90%

4.7 Conclusion

We study an auction problem in the market that is unfavorable for shippers. The existing procurement process does not perform well in the market condition. We propose adding a step in between the two rounds of bidding to lock in some capacity to accommodate the stochasticity of the market. We model the problem as a two-stage robust optimization. After linearizing the formulation, we adopt a constraint-generation algorithm from Remli and Rekik [84] to solve the problem to optimality. Our experiments of 135 instances demonstrate a significant savings that could be realized by using the proposed procurement process.

In this study, our contributions are:

- Introducing to the literature the original auction problem with stochastic rates and capacity to model an unfavorable market for shippers.
- Implementing the linearization and constraint-generation algorithm to solve the problem effectively.

The problem studied in this chapter is a simplified version that does not consider business constraints and strategies. To further develop this problem, we may add popular business constraints found in procurement events such as limiting carriers' revenue, favoring incumbent carriers, maintaining a strategic carrier mix, and limiting new carriers' awarded volume. These constraints may change the complexity of the problem. Linearizing the problem will be more challenging when stochastic variables are on the left hand side of the problem's constraints.

Relationship between a shipper and its carriers is not just about rate and capacity. It is a strategic relationship which requires the shipper to consider carriers' goals into its decision process. Even when a contract has been established between a carrier and a shipper, the carrier has the right to reject tendered loads if it does not fit its operations at the moment. Perhaps the development of the problem in this direction could help solving more industry problems effectively and promptly.

4.8 References

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Chapter 5

Conclusion and Future Research

5.1 Conclusion

In this research, we study the three problems that arise in the transportation industry. The research topics focus on helping shippers to better design, implement, and execute their processes to manage their truckload transportation. Nowadays, shippers usually work with third party logistics (3PL) to outsource the daily execution and to seek expert recommendations to improve their supply chain. A typical 3PL invests heavily in technology to provide state-of-art platforms to tackle all sorts of problem along a supply chain. While giving away the control of their daily operations, shippers usually retain the strategic control of their supply chain.

In the first topic, we study an integrated problem of quantity discount and vehicle routing. It covers truckload transportation and sales management. The problem seeks optimal decisions to both design a purchase incentive program and operate truckload shipments. We learnt this problem from a manufacturer who want to improve their truckload utilization while their current average shipment size is so small to ship by truckload mode but too big to ship by less-than-truckload mode. We model the problem as an Integer Program with pre-generated routes. Combinatorially, the number of all possible routes is extremely large. Therefore, we propose the set of route elimination rules which identify infeasible routes even before fully generating them. The route elimination rules as well as other improvements in the formulation have improved the tractability of the problem. We use Gurobi Solver to perform our experiments with 324 instances. All instances are solved within 45 minutes. The experiments demonstrate significant savings that could be realized by using the proposed model.

In the second topic, we expand the first topic into a bi-objective problem whose total profit and output range are simultaneously optimized. The problem incorporates facility management into transportation management. It enriches the area of the literature where output is usually modelled as a constraint and its statistic is omitted. We propose a GA-based matheuristic methodology to solve the problem. In order to assess the quality of our methodology, we implement the NISE algorithm to find the linear-relaxation fronts. We also introduce equality constraint of the output range into the formulation in order to use Gurobi to establish the 1%-optimal efficient fronts. Compared to NISE's fronts, the fronts established by our GA have the highest gap of 5.6%. Compared to Gurobi's fronts, our highest gap is 1.9%. NISE overestimates the fronts partially because of its integer relaxation. The experiments demonstrate the high performance of our proposed methodology.

In the third topic, we address the procurement problem in the contemporary market. The existing procurement process is designed to minimize transportation cost for shippers in the market where they have high buying power compared to carriers. In the current year of 2014, the transportation market has changed fundamentally to the state where carriers have higher power in selling their transportation services. We propose a change to the existing procurement process to quickly lock in capacity with competitive rates. We model the problem as a two-stage robust optimization. In order to use an exact method to solve the problem, we linearize the model and adopt a constraint-generation algorithm. Our experiments with 135 instances demonstrate significant savings that could be realized by the proposed process compared to the existing one. We limit each instance to be solved within 10 hours. The maximum optimal gap of all instances is less than 5%. The experiments also show that the master problem in the constraint-generation algorithm is very easy to solve while the slave problem can be very hard depending on the solution fed by the master problem.

5.2 Future research

There are many areas for future research based on the three topics studied in this dissertation. In the first topic, we may incorporate buyers' costs into the problem. It can be a constraint to limit the buyers' costs at certain level or guarantee no loss when taking a discount offer. In a different approach, the problem can optimize the total cost of the supply chain. We may model the fairness for each individual entity (seller or buyer) in order to avoid the optimal state where some entities in the supply chain benefit significantly while others have marginal savings or losses. We may develop the problem into a game so that each entity can try to optimize their own interest.

In the second topic where we study a bi-objective problem of truckload transportation and facility management, we may expand the problem to model the facility operation. It will become an integrated problem of vehicle routing and warehouse scheduling. The total profit will include warehouse operation costs. We still have a bi-objective optimization problem whose objectives are both profit and output range. In a different direction, we may seek optimal shipping schedules for buyers instead of dynamic replenishment. In reality, a fix and regular replenishment schedule is much preferred by buyers. A buyer will know in advance that its freight will arrive, for example, Wednesday or Thursday each week. In order to minimize the output range, the problem will find the best replenishment schedules to evenly distribute the freight with a cycle.

In the third topic where we study a procurement process in an adverse market, we may run experiments with larger instances such as 3000 to 5000 lanes and more than 200 carriers. This scale represents the middle of the large shippers in the industry. The experiments will reveal more development areas. In order to effectively solve the large instances, one of the areas that needs further research is the computation of the slave problem. The slave problem is the bottleneck of our methodology. It consumes up to 99% of the total run time.

Its computational time depends significantly on the structure of its objective function, which is defined by the master problem's solution in each iteration.