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# A case study of effective practice in mathematics teaching and learning informed by Valsiner's zone theory

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**Abstract** The characteristics that typify an effective teacher of mathematics and the environments that support effective teaching practices have been a long-term focus of educational research. In this article we report on an aspect of a larger study that investigated ‘best practice’ in mathematics teaching and learning across all Australian states and territories. A case study from one Australian state was developed from data collected via classroom observations and semi-structured interviews with school leaders and teachers and analysed using Valsiner's zone theory. A finding of the study is that ‘successful’ practice is strongly tied to school context and the cultural practices that have been developed by school leaders and teachers to optimise student learning opportunities. We illustrate such an alignment of school culture and practice through a vignette based on a case of one ‘successful’ school.

**Keywords** Mathematics teaching · Mathematics instruction · Mathematics pedagogy · Mathematics achievement · School environment · Valsiner

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## Introduction

How schools can best support student engagement and achievement in mathematics has been a long-term focus within educational research. While aspects of teaching practice, such as pedagogical practices, mathematical knowledge and beliefs about teaching and learning, have often been central to studies related to student engagement and achievement (e.g. Ball et al. 2008; Charalambous et al. 2012; Cobb and Jackson 2011), other less direct factors, such as school environment and broader educational policy settings, have also been identified as influential (e.g. Millett and Bibby 2004).

Within the Australian context, the focus on supporting students' mathematics engagement and achievement has been sharpened by the release of the latest results from the Trends in International Mathematics and Science Study (TIMSS) (e.g. Thomson et al. 2016a, b) and the Programme for International Student Assessment (PISA) (e.g. Thomson et al. 2016a, b). These assessments indicate that performance by Australian students is declining both in absolute terms and by comparison to students in other countries. Thus, the standard achieved in mathematics by capable Australian students is a problem of national significance.

In response to concern about Australia's diminishing international performance in mathematics and decreasing enrolments in science, technology, engineering and mathematics (STEM) subjects at both secondary and tertiary level (Australian Academy of Sciences 2016; Office of the Chief Scientist 2012), the project *Building an evidence base for national best practice in mathematics education* (NBPME) was commissioned by the Office of the Chief Scientist. A major target of this project was one of the five key areas identified by the Chief Scientist as requiring attention (Office of the Chief Scientist 2012)—*Inspired Teaching*:

Inspired teaching is undoubtedly the key to the quality of our system, and to raising student interest to more acceptable levels. It is the most common thread running through the responses in every country where the issue has been assessed in any detail.

(p. 12).

In this article, we report on aspects of the NBPME project within which the educational practices of schools showing 'superior gain' (consistent with the terms of reference of the project) in the mathematical component of the annual National Assessment Program Literacy and Numeracy (NAPLAN), for school years 3, 5, 7 and 9, were documented. The specific goal of the project was to build an evidence base of national 'best' practice in mathematics education as they occurred within authentic school settings. The project is significant as it is the first attempt to make use of NAPLAN data to identify those teaching practices, at a national level, that are effective in promoting student learning outcomes. Thus, in seeking to identify the practices that characterise *inspired teaching*, we documented the attributes of teaching and the environmental circumstances that supported or hindered quality teaching practices within schools whose students exhibited *superior gain* in NAPLAN numeracy assessments. In beginning to address this aim, we sought to answer the following research question:

What are the personal and contextual influences that shape teaching practices in one school whose students have exhibited high gains in NAPLAN achievement?

This question was attended to by the following: (1) presenting a synthesis of previous research into the factors that promote or inhibit effective teaching practice, (2) outlining and describing Valsiner's zone theory as the theoretical framework used to analyse data sourced from a participating school and to guide conclusions drawn from the study, and (3) discussing the alignment of teaching practices and cultural factors within the school that optimise students' engagement and achievement opportunities.

## Background

There is now strong evidence that teachers have a substantial impact on the academic success of their students (e.g. Charalambous et al. 2012; Chetty et al. 2014). How such success is realised has generated international research interest into effective teaching practice. To date, research has tended to focus on the characteristics or attributes of effective teachers of mathematics including pedagogical content knowledge (e.g. Shulman 1986); subject matter knowledge (e.g. Ball et al. 2008; Cobb and Jackson 2011); teacher efficacy (e.g. Young-Loveridge and Mills 2009; Zambo and Zambo 2008); and teacher confidence, attitudes and beliefs (e.g. Swars et al. 2007). While this research has identified important and necessary elements of instruction, none are sufficient in themselves to explain the nature of quality teaching practice.

Other researchers have scrutinised the 'work' of teachers in order to document those practices that shape student success in mathematics classrooms (e.g. Anthony and Walshaw 2009). This research has led to the identification of a wide range of influences on student success (e.g. Danielson 2013; Hattie 2015; Schoenfeld 2014). For example, through the project Measures of Effective Teaching (MET), which involved analysis of more than 23,000 video recorded lessons, Danielson (2013) developed a framework that identified and described four domains of practice that led to successful teaching: planning and preparation, classroom environment, instruction and professional responsibilities. These domains were further divided into 22 components; for example, the domain of planning and preparation consists of the following: demonstrating knowledge of content and pedagogy, demonstrating knowledge of students, setting instructional outcomes, demonstrating knowledge of resources, designing coherent instruction and designing student assessments. Alternatively, Hattie (2015), on the basis of a meta-analysis of approximately 1200 studies, concluded that there were 196 influences on student achievement; however, '...nearly all of them have a positive impact on student learning. That is, almost everything works...claims can be made for almost every method...however...some interventions have dramatically higher impact than others' (p. 81).

One attribute of effective teachers that has been identified through multiple studies is the ability to engage students with learning (Lawson and Lawson 2013; National Council of Teachers of Mathematics [NCTM] 2014). While definitions of engagement vary (Appleton et al. 2008), most coalesce around the notions of cognitive investment, active participation and emotional commitment to specific learning tasks (e.g. Chapman 2003). Academic engagement is vital to student success as it drives learning and also predicts school success (Reschly and Christenson 2012). Factors that increase engagement at school level include voluntary choice, clear and consistent goals, student participation in school policy and management, opportunities for staff and students to

be involved in cooperative endeavours and academic work that allows for the development of product (Fredricks et al. 2004). Fredricks et al. (2004) have also identified influences that mediate engagement at the classroom level—teacher support, peers, classroom structure, autonomy support and task characteristics. The identification of so many factors related to student engagement highlights the multifaceted nature of effective teaching practice.

Research, such as that of Danielson (2013), Hattie (2015) and others (e.g. Clarke and Pittaway 2014) documents the complexity of teaching as a practice. The challenge of understanding the interplay between different aspects of teaching, for example, orchestrating instructional activity, can create a desire for reductionism in describing the role of teachers. Further, as Elmore (2006) suggested, there is often a failure to appreciate institutional realities and complexity of accountability which systems, sectors, parental bodies, parents and the general community demand. In order to deal with this complexity, some have argued (e.g. Goos 2013; Lerman 2013) that learning needs to be viewed more holistically through situated or sociocultural perspectives.

How learning is afforded or constrained has been a primary focus of sociocultural theory (e.g. Gibson 1966; Valsiner 1997; Vygotsky 1978). The idea of ‘affordances’ arose from Gibson’s work in the 1950s and 1960s as a way to understand how learning takes place through a person’s perception of, and interaction with their context. Gibson (1966) asserted that affordances emerge through the relationship between observer, object and environment, and refers to action possibilities, that is, what can be done by the observer with the object. Within these systems are effects and relations which limit the wider possibilities of the system—constraints. In considering the practice of teaching more broadly, Simon (2009) argued that affordances can be used by researchers as both a lens and a tool to gain insights into the interactions between agents with other agents, and agents with systems. These interactions can be viewed through the many activities in which teachers engage; as Timperley (2008) commented:

Teaching is a complex activity. Teachers’ moment-by-moment decisions about lesson content and process are shaped by multiple factors, not just the agendas of those looking for changes in practice. Such factors include teachers’ knowledge and their beliefs about what is important to teach, how students learn, and how to manage student behaviour and meet external demands (p. 6).

Simply knowing about successful approaches to teaching and learning does not necessarily mean that teachers can readily adopt suggested changes to their current practice. Typically, schools and education systems attempt to bring about change in teachers’ instructional practice through professional learning programs. Research on what constitutes effective professional learning is well established (Borko et al. 2009; Clarke 2007). Findings of such research include the important role of school leaders, in collaboration with teachers, in identifying core learning goals and making sure these are targeted and mutually understood (Jansen et al. 2009).

Other research (e.g. Cobb and Jackson 2011) has indicated school leaders can be instrumental in helping to facilitate a detailed vision of high-quality instruction, as part of a professional learning program, which specifies practices that contribute towards the attainment of the learning goals. Mathematics curricula can vary significantly in terms of the goals for students’ learning, the supporting tasks and the proposed

pedagogy (Stein and Kim 2009; Stein et al. 2007); thus, direction is needed if the instructional practices are to be coherent and have fidelity across a school. For instance, direction may be necessary where a school requires teachers to work from a prescribed text that may be instructionally ambiguous.

Findings from research into the effectiveness of such programs in promoting student achievement, however, are mixed. For example, Yoon et al. (2007) argued, on the basis of a meta-analysis of research into teacher professional learning, that the effectiveness of programs can be considered to have ‘substantively important’ (p. 13) effect in promoting student achievement. By contrast, other researchers (e.g. Hiebert and Grouws 2007; Watson et al. 2012) claimed that there is relatively little evidence to support such claims.

An alternative to formalised teacher professional development programs is the facilitation of school-based professional learning communities (PLC). PLCs usually require scheduled time during the school week for teachers to meet and address issues such as the integration of new ideas and tools into current practice (Cobb et al. 2009; Horn and Little 2010). In the case of mathematics instruction, a productive PLC might include such activities as sharing challenging mathematics problems and comparing solution strategies, analysing student work and rehearsing well-considered instructional practices (Borko et al. 2009; Kazemi and Hubbard 2008). Successful PLCs require direction that shapes interactions within the group(s)—typically provided by school leaders at whole school, department or year level (Cobb and Jackson 2011).

Teaching practice is also subject to extra-school influences such as policies emanating from school systems. For example, in a study aimed at empirically grounding a theory for improving the quality of teaching at a scale beyond that of the individual teacher, Cobb and Jackson (2011) proposed that there are five components of action for what they refer to as ‘district-level’ instructional improvement. That is, instructional improvement scaled beyond an individual teacher or school. These are a coherent instructional system, teacher networks, coaching, school instructional leadership and district instructional leadership. Cobb and Jackson (2011) argued that if improvements are to be made to the quality of teaching on a wide scale, then attention needs to be paid to elements which range from dealing with the individual teacher, through supporting and leadership in schools and to broader systemic support and policy leadership.

Helping teachers to adopt assessment practices that can inform ongoing improvement of instructional practices is another area in which schools and systems must provide support (Cobb and Jackson 2011). Support may take the form of system and sector-wide involvement that promotes teachers’ use of diagnostic assessments that in turn informs the development of quality of classroom instruction (e.g. Confrey 2011; Confrey et al. 2009). Tellingly, Confrey (2011) observed that the use of diagnostic assessments requires a significant reorganisation of practice for most teachers and would therefore require considerable professional learning opportunities.

The research presented above indicates the high level of complexity associated with the practice of teaching. Within this complexity lie affordances and constraints, shaped by culture, the environment and people in it, that corral adopted approaches to teaching and learning. The extent to which the unique affordances and constraints associated with a particular school shape or are shaped by a school community is likely to determine how well the complexity of teaching and learning is managed.

## Theoretical framework

This article is concerned with teachers' instructional practices and how these are shaped by the cultural contexts of their schools, the people they work with and broader sociopolitical educational environments. Consistent with this focus, a sociocultural perspective was adopted as a theoretical frame to analyse case studies of participant schools as, from within this stance, the "role of culture, motives, values, and social and discursive practices are central, not secondary" (Lerman 1996, p.4). The origin of sociocultural theory is generally attributed to Vygotsky's (1978) work on child intellectual development in the early twentieth century; however, this work has been extended over time to incorporate the role of cultural practices, institutional contexts, personal histories, beliefs and values in attempting to understand and describe interactions central to teaching and learning (Goos 2014).

One such extension of sociocultural theory is the contribution of Valsiner (1997) who reconceptualised Vygotsky's zone of proximal development (ZPD), a theoretical space that represents a learner's intellectual readiness to develop, to include two additional 'zones' that address the roles of social settings and the goals and actions of individuals. In doing so, Valsiner redefined the ZPD, narrowing it to accommodate the zones of free movement (ZFM) and promoted action (ZPA). Valsiner saw the ZPD as a space for an individual's potential development defined by their knowledge and beliefs and shaped by past interactions with others and their environment. The ZFM was conceived as the ways in which an individual is permitted or enabled to act within their learning environment and the ZPA as the influences within an environment that promote action. While initially conceived as a theory of child development, Valsiner's zone theory has also been used to describe teacher's professional learning as Goos (2013) explained:

Like Valsiner, I take "development" to mean more than the formation of higher mental functions in children; instead, it refers to the emergence of new domains of action and thinking and new cultural frameworks that organise a person's social and psychological functioning.

(p. 523).

This perspective on Valsiner's zone theory has yielded different research foci within mathematics education such as students as learners (e.g. Blanton et al. 2005), teachers as learners (e.g. Goos 2007) and teachers' numeracy identities (e.g. Bennison 2015). In this article, we seek to explore issues related to teachers' identity and agency that influence or determine choices they make in relation to the instructional approaches they adopt in their mathematics classrooms. Thus, we focus on teachers' actions in relation to the promotion of their students' learning and engagement rather than their own personal learning about teaching. In this construction, the ZPD is influenced by knowledge, beliefs, attitudes and values teachers bring to their schools/classrooms that shape potential for action. Accordingly, the ZFM is related to constraints, both environmental and personal, that limit teacher's actions, thinking and learning. According to Goos (2005), constraints can be constructed as personal ZFMs, rather than being externally imposed, as a result of an individual's own interpretation of the constraints or affordances available within or imposed by their external environments. Aspects of a

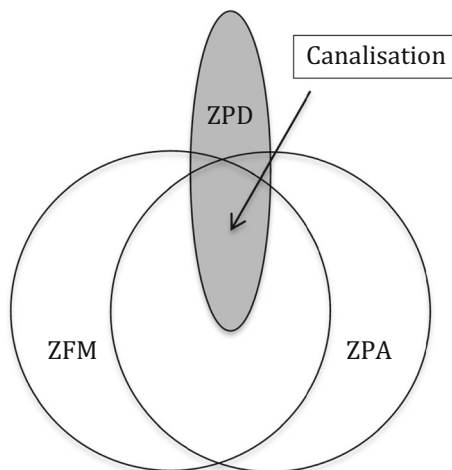


teacher's ZFM could include the following: indirect effects of policies adopted by an educational system, a school's interpretation of educational system policies, curriculum requirements, beliefs about how to engage students in learning and the resources available to support teaching and learning in a school. By contrast, the ZPA relates to how an individual's actions, thinking and learning can be advanced or promoted through aspects of their environment or the actions of others, such as their teacher peers or school leaders. A teacher's ZPA might include professional learning opportunities or policies enacted within a school with the intent of managing teaching approaches in a particular direction.

The ZPD, ZFM and ZPA form a complex (an interconnected and interactive combination of these zones) that represents the dynamic interaction between possibilities and limitations. Teachers' resulting actions are the outcome of their own potential to develop, interpretation of what they see as non-negotiable requirements and other opportunities to progress activities or agendas within the realms of what is believed possible. Thus, teachers' actions are determined by their potential to adopt different approaches to instruction, what is permitted within the environmental constraints and cultural expectations of schools, and the particular teaching practices that are supported and/or promoted. The effect of the ZFM/ZPA complex, those things that are promoted within what is permitted, on an individual's actions is referred to as the canalisation of development within the ZPD (Blanton et al. 2005; Oerter 1992). Canalisation is often represented in diagrammatic form as in Figure 1.

## Research design and methodology

This article reports on an aspect of a national cross-sectional study that aimed to document instructional practices employed by schools that had demonstrated 'success' in NAPLAN numeracy outcomes. While the larger study employed both surveys and case studies as the means of data collection, this article drew on case study data alone as these provided insight into the relationship between teachers' practices and school-



**Fig. 1** Canalisation of the ZPD



based influences that potentially led to successful student outcomes within sampled schools.

Within the project specifications, successful schools were defined as those which showed superior gain (more than one standard deviation above the mean) on overall numeracy results in both the time periods 2011–2013 and 2012–2014 when compared with like schools and schools with similar starting scores. Initial scrutiny of the data, however, revealed very few schools met this condition; thus, preliminary selection was based on superior gain when comparison was made with like schools *or* schools with similar starting scores or both conditions. Additionally, relatively few schools had made superior gain during both time periods of the targeted time frame. As a result, case studies were conducted in schools that had made superior gain during at least one of the defined time periods (i.e. 2011–2013 or 2012–2014, or 2011–2013 and 2012–2014). Final selection also took into account the need to balance location, sector, level of schooling (primary/secondary) and Index of Community Socio-educational Advantage (ICSEA) scores (Australian Curriculum, Assessment and Reporting Authority, ACARA 2013). It is important to note that because the primary criteria for school inclusion was gain, many of the schools included were not high-performing schools as it is possible to make significant gain from a low base. This means that some schools included in the sample were performing below the national average. These schools are ‘closing the gap’ and hence are of interest in terms of national policy settings and initiatives.

A total of 55 case studies were conducted in 52 different schools (some schools having been selected for both secondary and primary levels). The sites of case studies were recruited via invitation. Where invitations were not accepted, like school, replacements were chosen from a reserve list. Case studies were conducted by experienced mathematics education researchers in the latter half of 2015. Data collection instruments included classroom observations documented via field notes, semi-structured interviews with school leaders, teachers and focus groups of students after working on a selected problem task. In some schools, a focus group of parents was also available for interview.

To ensure consistency across such a large number of case studies, a detailed protocol was developed including interview questions and lesson observation templates. The interview protocol was informed by Bronfenbrenner’s (1989) Ecological Systems Theory with questions structured around the levels of context and background, policy, school, classroom and student. Some questions were asked of all participants, for example, ‘What makes a good mathematics lesson?’ while other questions were aligned with the role of respondents. Examples of the focus of questions posed to different types of participants appear in Table 1. Since NAPLAN testing is conducted in years 3, 5, 7 and 9, teachers and students of these year levels were interviewed whenever possible—as was permitted by prior school commitments and timetabling constraints. Ideally, observations were of typical mathematics lessons taught by the interviewed teachers.

Interviews were audio recorded and transcribed as soon as possible after a school visit. Case studies were developed via a synthesis of researchers’ lesson observation field notes, semi-structured interviews and student responses to the problem task. After a school visit (usually one full day in a school), each case study researcher analysed data using a descriptive coding approach (Miles et al. 2014). Descriptive codes allowed researchers to seek patterns in the data allowing for interpreting and theorising about

**Table 1** Role and interview question foci

Role	Exemplar question foci
Principal	<ul style="list-style-type: none"> <li>• School mission statement as this relates to mathematics/numeracy</li> <li>• Management of NAPLAN assessment processes within the school</li> <li>• Views about the best identifiers of student achievement in mathematics/numeracy</li> </ul>
Head teacher of mathematics/ curriculum coordinator	<ul style="list-style-type: none"> <li>• Characteristics of effective mathematics teaching,</li> <li>• Organisation and management of mathematics within the school,</li> <li>• Professional learning opportunities available for teachers of mathematics</li> </ul>
Classroom teachers	<ul style="list-style-type: none"> <li>• Strategies for accommodating individual difference</li> <li>• Assessment practices</li> </ul>
Students	<ul style="list-style-type: none"> <li>• A typical mathematics lesson</li> <li>• A mathematics lesson they had enjoyed</li> <li>• If they considered themselves ‘good’ at mathematics</li> </ul>

the data (Richards 2015). This approach led to the identification of key themes that emerged within the levels of context and background, policy, school, classroom and student. Emergent themes included teaching and learning approaches, collegiality and mentoring, professional learning, the student experience and approaches to mathematics assessment.

In the next section, we present an analysis of a case study using Valsiner’s zone theory as an illustration of how the relationship between affordances and constraints emanating from school culture and organisational structure result in the alignment and focus of teaching practice within one school. The case study has been selected because this alignment appears to have been an essential element in the long-term success of the school in terms of student mathematics achievement. Data collection for this particular study consisted of lesson observation field notes from two separate classes and semi-structured interviews with the school principal, the head of mathematics and an additional mathematics teacher.

## Case study—Eucalyptus High School

### The school teaching and learning context

Eucalyptus High School (EHS) was selected for this case study as one of the few schools that met the condition of superior gain in both the time periods 2011–2013 and 2012–2014 when compared with like schools and schools with similar starting scores. Further, this school was also one of only eight schools from an invited 19 to agree to participate in the case study phase of the project within their educational jurisdiction.

EHS is an all-boys, academically selective public school (enrolment only available to high-performing students) situated in a New South Wales metropolitan area. There is strong competition for places at the school as many students in the catchment area nominate EHS as a first priority when taking the selective schools examination in year 6. The Index of Community Socio-Educational Advantage (ICSEA) for the school was greater than 1150 (MySchool Website) in 2015—well above the standardised mean.

In 2014, there were approximately 1000 students enrolled in the school, with more than 90% from diverse cultural backgrounds—Chinese being the largest group. About 75% of students resided out of the school's immediate location with some students travelling up to 2 hr to attend classes. The school has very high retention rates from year 10 (15–16 years of age) to year 12 (17–18 years of age), typically >90%, a rate superior to the state average of around 60%. All year 12 students in 2014 were successful in gaining an offer to an Australian University with more than 60% going to the University of New South Wales and more than 20% to the University of Sydney—both highly prestigious institutions. While the chosen degree programs for this cohort of students were diverse, the most popular were business, medical-related programs, science, IT and environmental and applied science. Thus, the school caters for students with high-academic aspirations with a future focus on university education. At the same time, the school aims to provide a holistic education for the boys with a diverse curriculum and an extensive co-curricular program.

The school's NAPLAN results revealed all students score in the top band for numeracy (MySchool website)—leading to a school view that NAPLAN numeracy tests are trivial and thus practice or preparation for these tests each year are unnecessary. There is a focus on literacy across the school because of the high non-english speaking background (NESB) student population. None-the-less, the mathematics head teacher believes they could do more to support the lowest performing students as well as the highest performing students in mathematics—even though it is a selective high school.

The school is attempting to diversify students' educational experiences through activities in which they are challenged to apply their learning in unfamiliar contexts. For example, a 2-week block of problem-based learning in year 7 has been recently trialled. During this activity, students worked in teams to develop a TED-talk style activity in which they presented approaches to redesigning a local park while taking into account the environmental and social impacts on the local community. A positive evaluation of the learning and engagement outcomes stemming from the activity has resulted in the extension of the initiative to include year 7 and year 8 students working on different projects involving developing a built environment for boys' education and an ethical investigation to include business and government requirements.

In order to encourage teachers to consider new pedagogies and develop a common language as they observe each other and provide feedback, a whole school professional learning approach has been adopted based on the NSW Quality Teaching Framework (NSW Department of Education and Training 2003). This initiative began with a small group of teachers in 2014 but was expanded in 2015 to a cross-faculty initiative. As the initiative was seen as successful, whole school implementation was being considered for 2016—consistent with the trend within the school for professional learning to have a whole school focus rather than be based around faculty/department structures. As noted by the mathematics head teacher, three of his staff recently attended the annual conference for the Mathematical Association of New South Wales but such activity is 'not encouraged'.

### Teaching and learning mathematics

The mathematics head teacher reported that teachers in his department are collegial and supportive of their students. Teachers displayed a personal engagement with mathematics as they shared resources and frequently *did* mathematics together, that is, they

talked about mathematics in their staff room as well as the students they are concerned about. According to the principal, the mathematics teachers were passionate, have deep discipline knowledge, cater for gifted and talented students and are able to differentiate the curriculum.

All students choose to study mathematics for the higher school certificate, which is not a school requirement. Additionally, almost all students are enrolled in the most mathematically demanding courses available within the NSW curriculum (*Extension 2* and/or *Extension 1 Mathematics*). The non-calculus inclusive course General Mathematics is not offered at the school. The school imposes a limit of three *Extension 2* classes in year 12 (72 students), even though more may nominate to enrol. This is despite a demand for more classes based on student requests. Some students were very disappointed in this decision—one of the interviewed mathematics teachers indicated those who miss out are ‘devastated’—an indication of students’ emotional commitment to the study of mathematics.

Mathematics classes are taught in heterogeneous groupings in years 7 and 8 but ‘streamed’ thereafter—in years 9 and 10, there are two classes for students displaying the highest achievement from year 8 and the remaining three classes are organised heterogeneously. As such school structures reflect an emphasis of high achievement, the head of mathematics reminds teachers in his department that they should not assume all students are able to keep up with the very high expectations set by the school—they still have a range of students and some do struggle in this environment. To support students who struggle with mathematics, the head of mathematics has also established fortnightly before school classes in year 8.

The mathematics program is designed for teachers to make explicit connections to topics students will encounter in later years, that is, where the mathematics will lead to next. Teachers believe most year 7 and 8 students arrive at the school with very good understanding of the K-6 curriculum and frequently with good understanding of the years 7 and 8 curricula and so instruction is fast paced in order to maintain students’ engagement and learning trajectories. The scope and sequence of the program reflects the main topics of the NSW syllabus for years 7 and 8 with links from each topic to other areas in the curriculum and with ideas for enrichment and challenge. Mathematics is allocated seven 52 min lessons per fortnight in year 7, increasing to eight lessons in year 8, nine lessons in year 9 and eight lessons in year 10—an indication of the importance with which mathematics is viewed within this school.

According to both the principal and the head of mathematics, lessons were typically ‘traditional’—teacher directed with an emphasis on teacher exposition followed by student practice of skills. The head of mathematics described this approach as ‘the Eucalyptus High School way’. At the same time, lessons always included an element of challenge with the intent of pushing students forward to reach their potential. While the approach to teaching was typically traditional in an organisational sense, teachers did not rely on one textbook as the basis of instruction but drew on a large set of resources to find questions teachers judged as both appropriate and challenging for students. In an attempt to broaden this approach, the head of mathematics has encouraged teachers to include more open-ended questions, mathematical investigations and to make greater use of digital technologies (e.g. Geogebra).

*We don't do enough of unfamiliar problem solving. We need to do more to broaden that experience. We do the AMT [Australian Mathematics Trust]*

*enrichment and I want to push that further because it encourages more than one solution.*

To encourage teachers to adopt this practice, he has attempted to set an example by including at least one ‘non-traditional’ lesson with each of his own classes per fortnight.

I try to do one lesson per topic which is different and a bit open ended – that’s my personal goal. I’m building them up over time and when I came here it took me a while to convince the kids these were worth doing.

He indicated some teachers are more willing than others to embrace such approaches.

Typically, formative assessment takes the form of regular topic tests administered in mathematics lessons or weekly assignments with teacher feedback given as quickly as possible in either case. Most assessment involves traditional pen and paper tests, and there is one common test per term—half-yearly examinations in term 2 and the yearly examinations in term 4. When assessments are designed, teachers discuss the range of possible solutions to problems and debate how they will allocate marks—they recognise the importance of valuing alternate solutions. The head of mathematics is attempting to broaden the current approach to assessment by introducing an alternative item (contributing 10% to an overall grade) in the form of engagement in activities such as competitions or reading mathematics books.

Despite attempts by the head of mathematics to encourage changes in practice, there was still a strong emphasis on developing competence with mathematical techniques, with performance on assessment in mind. He indicated students appeared satisfied with this approach.

Most classrooms would be teacher shows something, the discussion with students, and then worked examples and practice. The students are very happy with that approach. Most respond well to that.

This approach was evident in the practice of one of the mathematics teachers, Kevin. Before beginning his teaching career, Kevin had completed a degree as a petrochemical engineer but did not enjoy the work as much as he had tutoring students while he was at university. As a consequence, he decided to change career and completed a graduate teaching qualification and eventually found a position at EHS.

When planning for lessons, Kevin makes use of the faculty program, a range of resource materials including textbooks and enrichment ideas (e.g. NRich website), as well as digital technologies such as Geogebra and spreadsheets. In one observed lesson, Kevin encouraged students with expressions such as ‘practice makes perfect’ when they were experiencing difficulties with work related to the use of the quadratic formula and worked through the questions on the whiteboard highlighting common errors. He emphasised the importance of accuracy and careful presentation of solutions, and indicated where marks may be deducted in an examination. When interviewed, Kevin indicated he believed learning involves being shown the method and procedures, practising, applying and finally looking at derivations. Thus, he left the derivation of formulae until after the students had become competent with their use and application.

Kevin had adopted this approach on the advice of a very experienced mathematics teacher in the department—a mentoring practice consistent with that identified by the head of mathematics:

Our teachers are experts because there is a lot of knowledge in the Faculty and they help the new ones. They have knowledge of all the HSC questions for years and they share that knowledge.

Kevin believed that this type of support, combined with his high level of personal mathematical knowledge, was all that was required to develop as a mathematics teacher:

I am confident with my maths ability and I don't feel the need to do professional development for maths. I do ask my colleagues for assistance so it might be one-on-one in the Faculty. We do share resources and materials which is how it should be.

He differentiates the program for different learners by assisting individuals if they are having difficulty and providing challenge for those who are able to complete the work quickly. Every Friday, he gives each of his classes a short pen-and-paper test (15–20 min) which they correct in class—he does not record any marks but believes this approach provides constructive and formative feedback—he also noted the boys are very competitive and enjoy this approach.

### Teacher focus and development

Kevin's teaching practice was consistent with the approach described by the head of mathematics as the *Eucalyptus High School* way, that is, a focus on mathematical facts, skill development and fluency. The potential for development of his teaching and learning practice (ZPD) was influenced by factors related to the school environment, its culture, the influence of teaching colleagues as well as his personal mathematical knowledge, teaching capabilities and beliefs about teaching and learning practice. Kevin's training as a petrochemical engineer provided a solid background for the mathematical knowledge required for teaching at a school with high expectations of students' mathematical performance. His colleagues were supportive in terms of his development as a teacher of mathematics. As a relatively novice teacher, he was developing beliefs about effective approaches to teaching and learning and his practice. The ZFM/ZPA complex offered by the school, however, appeared to strongly canalise Kevin's development in a direction tied to the schools' history and culture.

Kevin's ZFM was characterised by highly motivated students with a history of achievement in mathematics, as this was a selective public school classified as high ICSEA. Students were focused on their studies and well behaved. Quality resources were freely available including both written teaching materials and digital technologies. He was working within a mathematics department team that was supportive of his development as a teacher of mathematics, readily providing advice upon request. The head of mathematics provided encouragement for variations from the "traditional" approaches to teaching practice and assessment adopted across the Mathematics Department—thus there was some licence to trial new ideas and approaches to teaching

and learning. However, what was actually *permitted* (ZFM) was largely shaped by what was *promoted* (ZPD) (see Table 2).

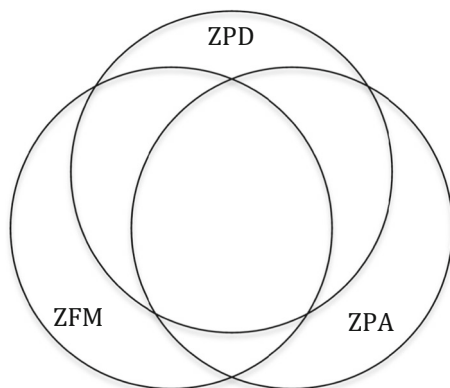
Kevin's perception of what approaches to instruction engaged students and enhanced their performance in mathematics in addition to his beliefs about the expectations of parents, teachers within the mathematics department and the school leadership *promoted* (ZPA) a learning culture that was robustly focused upon assessment. Kevin's declared approach to teaching—showing the method, practising, applying and finally looking at derivations after students had become competent with more procedural aspects of mathematics—had been adopted on the advice of a more experienced teacher. Such an approach, aligned with a practice of discussing where marks would be allocated in a test context, appears to be focused on developing students' capabilities to perform mathematical procedures effectively with speed and efficiency—a capability generally accepted to be advantageous in timed test situations. Additional attention to providing 'challenging' problems also provided experience for the type of questions on assessments that were designed to discriminate in terms of mathematical performance. The focus on performance was also evident in organisational and cultural practices such as the academic streaming of students into a predetermined number of 'top' classes. These practices were consistent with a belief that the employed approach was needed for the engagement of highly motivated and ambitious students.

Mathematics staff members were clearly hard working and enthusiastic about their subject, evident by unstructured discussions around mathematics in the staffroom. The focus, however, seemed to be on students' understanding of mathematics as a self-referenced system rather than its role in the workplace, and personal and civic life. Thus, what was promoted (ZPA) shaped what was *effectively permitted* (ZFM) indicating a large overlap between the ZFM and the ZPA (see Fig. 1). This in turn appears to have shaped Kevin's practice to that consistent with the established culture of teaching within the school. That is, the ZFM/ZPD complex as a manifestation of the school's approach to mathematics teaching canalised Kevin's ZPD into teaching and

**Table 2** Influences on Kevin's practice in EHS (adapted from Goos 2014)

Valsiner's zones	Exemplars of aspects of 'zones'
Zone of proximal development (possibilities for developing new teacher knowledge, beliefs, goals and practices)	<ul style="list-style-type: none"> <li>● Mathematical knowledge</li> <li>● Pedagogical content knowledge</li> <li>● Skill/experience as mathematics teachers</li> <li>● Beliefs about teaching and learning</li> </ul>
Zone of free movement (structures teachers' access to different areas of the environment, availability of different objects within an accessible area and ways the teacher is permitted or enabled to act with accessible objects in accessible areas)	<ul style="list-style-type: none"> <li>● Perceptions of students (engagement, motivation, behaviour, social economic status, potential for achievement)</li> <li>● Access to resources</li> <li>● Curriculum and assessment requirements</li> <li>● Organisational structures and cultures</li> </ul>
Zone of promoted action (activities, objects or areas in the environment in respect of which the teacher's actions are promoted)	<ul style="list-style-type: none"> <li>● Perceptions of student and parent expectations</li> <li>● Professional development</li> <li>● Head of mathematics initiatives</li> <li>● Peer interactions with teaching colleagues</li> <li>● Cultural structures and expectations—school</li> <li>● Cultural structures and expectations—mathematics department</li> </ul>





**Fig. 2** Kevin's ZPD/ZFM/ZPA complex

learning practices that had proved successful in promoting student achievement in state-wide assessment regimes (Fig. 2).

## Discussion and conclusion

The practices that best support student engagement with and achievement in mathematics have been a long-term research focus within mathematics education. This research has identified factors related to teaching practices that appear to impact on student engagement and achievement (e.g. Danielson 2013; Hattie 2015) within a specific school context. The complex nature of teaching, however, means that no single approach is likely to be successful across widely variable school contexts (Elmore 2002). The important role of context, including existing schools' culture and established teaching practices, may explain the limited success of many professional development programs (e.g. Watson et al. 2012).

Each unique school context generates its own affordances and constraints that promote or limit aspects of teaching practice. How such affordances and constraints combine or interact can corral approaches to teaching and learning to those acceptable within specific school settings. The social cultural practices that lead to a dominant approach to teaching and learning within a school noted for high student achievement have been the focus of this article.

Teachers at EHS appeared to employ what might be considered a 'traditional' approach to instruction. This approach is embedded in the culture of the school and as the new teachers enter the community, it is appropriated from existing staff—as was the case when Kevin took the advice of a more experienced staff member in adopting a pedagogy typified by the steps—showing the method, practising, applying and finally looking at derivations after students had become competent with more procedural aspects of mathematics. Given that this approach to instruction appeared to be typical, it is reasonable to surmise that all teachers joining the school were subject to the same process of enculturation—resulting in a highly focused, consistent and cohesive approach to the teaching of mathematics across the school in a manner that supported students in meeting their potential through a combination of challenge and support.

This approach was also consistent with a belief of what was needed to engage highly motivated students. The alignment of teachers' ZPDs with the ZFM/ZPA complex shaped by the school appears to permit a focus that simplifies some aspects of the complexity of teaching and learning. This simplification and resulting focus appears to lead to successful outcomes for students in terms of mathematics achievement—a significant priority for the school community.

The way Kevin's ZPD is canalised is a result of a ZFM/ZPA complex that is a product of the expectations of parents, school leaders and teachers made manifest by an explicit focus on performance on both school and state-wide assessment. Within the mathematics department, teachers frequently talk about mathematics and how they can promote students' performance on assessments. Interestingly, externally conducted professional development in mathematics teaching is not encouraged by the school leadership—resulting in a nearly 'gated' community of educators, further promoting their concentration on and refinement of existing methods that remain successful against criteria accepted as valid within the school. This approach is consistent with the function of a professional learning community in that activities that address mathematics problems, compare solution strategies, analyse student work and rehearse well-considered instructional practices are shared (Ball et al. 2008; Borko et al. 2009; Kazemi and Hubbard 2008; Sherin and Han 2004). This community is different, though, from typical PLCs in that it appears to be self-sustaining and functions without specific times set aside by school leaders for such activity (Cobb et al. 2009; Horn and Little 2010). There is a danger, however, that shunning external input may lead to a level of insulation that limits the potential for further enhancement of practice through the appropriation and enculturation of new ideas. Further, because teaching practice is subject to extra-school influences such as regional or nationwide changes in policy (Cobb and Jackson 2011), insular approaches may leave schools at a disadvantage when attempting to adapt to change. These issues appeared to have been recognised by the head of mathematics through his attempts to introduce small incremental changes—such as broadening the range of assessments used in the early year levels of the school. With this initiative comes the risk that he will not meet students' expectations of what it takes to be successful in mathematics and so threaten the high level of engagement evident in classrooms.

There are a number of reasons why schools may make gains, some of which are contextual rather than related to best practice. For example, schools that have a large intake of students who come from backgrounds where English is not the first language or are new to Australia may make large gains because the students become more proficient in English. Similarly, high schools that have a very diverse intake may have somewhat depressed scores in year 7, because the students are still settling in to a new learning context, but show good gains by year 9 because the students are now settled into the high school situation. Nevertheless, these schools have something to offer a study of this nature because they are able to mitigate the students' initial disadvantage. In the same way, we make no claim that the case study presented in this article describes an approach that should be adopted by other schools; rather, it provides insight into how students' engagement and achievement in mathematics might be best fostered by pedagogical approaches that align with the specific goals of school communities. This alignment resulted in a consistent and cohesive approach that included high expectations of performance of both teachers and students in which all

are challenged and supported. Further to this, we believe our use of Valsiner's zone theory is a contribution to new knowledge as a way of providing insight into the alignment between teaching practice and the goals of school communities. Our approach points the way to further research, firstly to the validation of our use of Valsiner's zone theory and secondly into how the identification of alignments or misalignments with the ZPD, ZFM and ZPA within a learning context can lead to improved approaches to teaching and learning which result in improved learning outcomes for students.

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