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Dark matter and weak signals of quantum spacetime

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In physically motivated models of quantum spacetime, a $\mathcal{U}(1)$ gauge theory turns into a $\mathcal{U}(\infty)$ gauge theory; hence, free classical electrodynamics is no longer free and neutral fields may have electromagnetic interactions. We discuss the last point for scalar fields, as a way to possibly describe dark matter; we have in mind the gravitational collapse of binary systems or future applications to self-gravitating Bose-Einstein condensates as possible sources of evidence of quantum gravitational phenomena. The effects considered so far, however, seem too faint to be detectable at present.

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I. INTRODUCTION

One of the main difficulties of present-day physics is the lack of observation of quantum aspects of gravity. Quantum gravity has to be searched without a guide from nature; the observed universe must be explained as carrying traces of quantum gravitational phenomena in the only "laboratory" suitable to those effects, i.e., the universe itself a few instants after the big bang.

Looking forward to see those traces in the cosmic gravitational wave background (for the analysis of quantum linearized perturbations, see the pioneering work [1] and also [2]), one can ask whether an expected consequence of quantum gravity, the quantum nature of spacetime at the Planck scale, might leave observable traces.

Indeed, quantum spacetime (QST) [3] would explain some aspects, such as the horizon problem [4], that is usually explained by inflation, without having to make that hypothesis. However, are there effects that *only* QST would explain?

Free classical electromagnetism on quantum spacetime would be no longer free: the electromagnetic field and potential F, A, would fulfill

$$\partial_\mu F^{\mu\nu} - i[A_\mu, F^{\mu\nu}] = 0,$$

where the commutator would not vanish due to the quantum nature of spacetime.

This fact was noticed [5] at the very beginning of searches on quantum spacetime. Its first consequence was also noticed: plane waves would still be solutions, but their superpositions would, in general, not be—they would lose energy in favor of mysterious massive modes (see also [6]).

A naive computation showed, by that mechanism, that a monochromatic wave train passing through a partially reflecting mirror should lose, in favor of those ghost modes, a fraction of its energy—a very small fraction, unfortunately, of the order of one part in 10^{-130} [5]. This looked too small to be worth a more accurate computation.

However, QST should reveal itself, as discussed here below, causing an electromagnetic interaction of neutral fields. This was noticed at the beginning as well, but looked to have even less promises of visible consequences (see, however, [7]).

Recent years, however, have brought increasing evidence of the role of dark matter, and the possibility of collapse of huge dark-matter binary systems; near the collapse, could those systems emit a sizable amount of electromagnetic radiation, and thus show a signature of the quantum nature of spacetime at the Planck scale?

In this paper we discuss this problem, and show that a primitive, semiclassical evaluation of that emission gives again a very small result: the fraction of the mass of such a system converted into electromagnetic radiation per unit time by the mechanism envisaged here would be less than 10^{-89} s^{-1}; this is nothing comparable to the few percents of the total mass converted into gravitational-wave radiation in the recently observed merges of binary black holes, GW150914 and GW151226, which inspire the numerical input of our calculation.

Our discussion proceeds as follows. In Sec. II, after having recalled the main terminology, notations, and results for the model of QST that we use, we discuss the action of the gauge group of QST on a neutral scalar field, and derive

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the interaction of the latter with the electromagnetic field by the covariant derivative prescription. Moreover, we show that such interaction can be described in terms of a magnetic moment associated to the scalar neutral field. Then, in Sec. III, we evaluate the electromagnetic energy emitted in a state describing the precession of a stellar member of a collapsing binary system; that energy is then computed, once the magnetic moment is evaluated, according to classical electrodynamics. We also comment on another manifestation of that magnetic moment, which at first glance potentially gives rise to more visible effects, as they would be only quadratic in the Planck length (but hard to be detected anyway, see comments below): the electromagnetic (in addition to the gravitational) deviations of charged particles by a massive stellar object of dark matter interposed between us and a distant source. In the case of the previously used data, however, we find a contribution to the angular deviation of the order of 10^{-34}.

II. THE MAGNETIC MOMENT OF A NEUTRAL SCALAR FIELD INDUCED BY QUANTUM SPACETIME

The model of QST adopted here is suggested by the principle of gravitational stability against localization of events [3,8]. This principle implies spacetime uncertainty relations

$$\Delta q_0 \cdot \sum_{j=1}^{3} \Delta q_j \gtrsim 1, \qquad \sum_{1\le j<k\le 3} \Delta q_j \Delta q_k \gtrsim 1 \tag{2.1}$$

for the coordinates q_μ of an event, which must be implemented by spacetime commutation relations

$$[q_\mu, q_\nu] = i\lambda_P^2 Q_{\mu\nu}, \tag{2.2}$$

where λ_P is the Planck length and where $Q_{\mu\nu}$ satisfies appropriate quantum conditions.

The simplest solution is given by

$$[q^\mu, Q^{\nu\lambda}] = 0, \tag{2.3}$$

$$Q_{\mu\nu} Q^{\mu\nu} = 0, \tag{2.4}$$

$$((1/2)[q_0, \ldots, q_3])^2 = I, \tag{2.5}$$

where

$$[q_0, \ldots, q_3] \equiv \det \begin{pmatrix} q_0 & \cdots & q_3 \\ \vdots & \ddots & \vdots \\ q_0 & \cdots & q_3 \end{pmatrix} \equiv \varepsilon^{\mu\nu\lambda\rho} q_\mu q_\nu q_\lambda q_\rho = -(1/2) Q_{\mu\nu} (*Q)^{\mu\nu} \tag{2.6}$$

[notice that $Q_{\mu\nu}Q^{\mu\nu}$ is a scalar and $Q_{\mu\nu}(*Q)^{\mu\nu}$ is a pseudoscalar, hence we square it in the quantum conditions].

Called for brevity the basic model of quantum spacetime, this model implements exactly the spacetime uncertainty relations and is fully Poincaré covariant.

The noncommutative C^* algebra $\mathcal{E}$ of quantum spacetime can be associated to the above relations by a procedure [5,8] that applies to more general cases.

Assuming that the q_λ, $Q_{\mu\nu}$ are self-adjoint operators and that the $Q_{\mu\nu}$ commute strongly with one another and with the q_λ, the relations above can be seen as a bundle of Lie algebra relations based on the joint spectrum of the $Q_{\mu\nu}$.

Regular representations are described by representations of the group C^* algebra of the unique simply connected Lie group associated to the corresponding Lie algebra, with the condition that I is not an independent generator but is represented by the unit operator. They obey the Weyl relations

$$e^{ih_\mu q^\mu} e^{ik_\nu q^\nu} = e^{-\frac{i}{2}h_\mu Q^{\mu\nu} k_\nu} e^{i(h+k)_\mu q^\mu}, \qquad h, k \in \mathbb{R}^4. \tag{2.7}$$

The C^* algebra of quantum spacetime is the C^* algebra of a continuous field of group C^* algebras based on the spectrum of a commutative C^* algebra.

In our case, that spectrum—the joint spectrum of the $Q_{\mu\nu}$—is the manifold Σ of the real-valued antisymmetric 2-tensors fulfilling the same relations as the $Q_{\mu\nu}$ do: a homogeneous space of the proper orthochronous Lorentz group, identified with the coset space of $SL(2, \mathbb{C})$ mod the subgroup of diagonal matrices. Each of those tensors can be taken to its rest frame, where the electric and magnetic parts $\boldsymbol{e}$, $\boldsymbol{m}$ are parallel or antiparallel unit vectors, by a boost, and go back with the inverse boost, specified by a third vector, orthogonal to those unit vectors; thus Σ can be viewed as the tangent bundle to two copies of the unit sphere in three-space—its base Σ_1.

Irreducible representations at a point of Σ_1 identify with Schrödinger's p, q in two degrees of freedom. The fibers are, therefore, the C^* algebras of the Heisenberg relations in two degrees of freedom—the algebra of all compact operators on a fixed infinite dimensional separable Hilbert space.

The continuous field can be shown to be trivial. Thus, the C^* algebra $\mathcal{E}$ of quantum spacetime is identified with the tensor product of the continuous functions vanishing at infinity on Σ and the algebra of compact operators.

The mathematical generalization of points are pure states. Optimally localized states minimize

$$\Sigma_\mu (\Delta_\omega q_\mu)^2,$$

where the minimum is 2, reached by states concentrated on Σ_1, at each point coinciding (if optimally localized at the origin) with the ground state of the harmonic oscillator. Such states are the proper quantum version of points; the classical limit of quantum spacetime is then the product of Minkowski space and Σ_1. Thus, extra dimensions, described by the doubled two-sphere, are predicted by quantum spacetime. Optimally localized states are central in the definition of the quantum Wick product, which removes the UV divergences in the Gell-Mann–Low expansion of the S matrix for polynomial interactions on QST [9].

The mentioned minimum (of the order of the squared Planck length in generic units) for the sum of the four squared uncertainties in the coordinates of an event is the first manifestation of a broader fact: the minimum Euclidean distance between two independent events in quantum spacetime is of the order of the Planck length in all reference frames. More generally, for each geometric operator, e.g., distance, area, three volume, or four volume, the sum of the squares of all spacetime components is, in each reference frame, at least of the order of the appropriate power of the Planck length [10].

These are mathematical results on the quantum geometry of quantum Minkowski space. But dynamics, already at the level of a semiclassical treatment of gravity, strongly suggests that the minimal distance between two independent events ought to have a dynamical meaning, diverging when a singularity is approached [4]. This fact allows for a possible solution of the horizon problem [4], and will play a role in our discussion in the final section.

Our first task is now to formulate and analyze gauge theories on the model of quantum spacetime just described. On ordinary classical spacetime, the gauge group of electromagnetism is the group of (regular) functions from Minkowski spacetime $\mathbb{R}^4$ to $U(1)$, which can be regarded as (a subgroup of) the group of unitaries of the algebra $C_b(\mathbb{R}^4) = M(C_0(\mathbb{R}^4))$. Going to quantum spacetime, this should be naturally replaced by $\mathcal{G} = \mathcal{U}(M(\mathcal{E}))$, the unitaries of the multipliers of the quantum spacetime algebra $\mathcal{E}$. It is therefore a rather interesting possibility that the gauge group of electromagnetism could also act nontrivially on a real scalar field $\varphi(q)$ on QST, as

$$\varphi(q) \to U\varphi(q)U^*, \qquad U \in \mathcal{G}. \tag{2.8}$$

Of course, on commutative spacetime the above action is instead trivial, because U and φ commute.

In order to find a Lagrangian invariant under the above action, we should introduce a covariant derivative D_μ, i.e., a derivation on $\mathcal{E}$ such that, under the action of $\mathcal{G}$,

$$D_\mu\varphi(q) \to UD_\mu\varphi(q)U^*.$$

This is accomplished by defining

$$D_\mu\varphi(q) \coloneqq \partial_\mu\varphi(q) - ie[A_\mu(q), \varphi(q)], \tag{2.9}$$

where e is the electron charge (see below for a discussion of this choice) which describes the coupling with the gauge field, ∂_μ is the derivation on $\mathcal{E}$ defined by

$$\partial_\mu\varphi(q) = \frac{\partial}{\partial a_\mu}\varphi(q + a\mathbb{1})|_{a=0},$$

and A_μ is the electromagnetic potential on QST, on which $\mathcal{G}$ is assumed to act as

$$A_\mu(q) \to UA_\mu(q)U^* + ie^{-1}U\partial_\mu U^*, \tag{2.10}$$

which reduces to the ordinary gauge transformation on commutative spacetime by writing $U = e^{i\Lambda}$. This also explains the choice of e in (2.9) and (2.10) as the coupling constant between the electromagnetic potential and the neutral field φ. In fact, A_μ will also interact with the electron field ψ, which transforms as $\psi(q) \to U\psi(q)$, and, therefore, the choice $D_\mu\psi(q) = \partial_\mu\psi(q) - ieA_\mu(q)\psi(q)$ for its covariant derivative gives the correct interaction. A potential problem in this respect is represented by the fact that it seems difficult to write the interaction, on quantum spacetime, of A_μ with a field of charge different from 0, $\pm e$ (like the quark fields). For a discussion in the framework of formal $*$-products and the Seiberg-Witten map, see [7,11].

The fact that (2.9) actually gives the correct definition of covariant derivative for the gauge transformation (2.8), (2.10) is easily verified: the transformed $D_\mu\varphi(q)$ reads in fact

$$\begin{aligned}
&\partial_\mu(U\varphi(q)U^*) - ie[UA_\mu(g)U^*, U\varphi(q)U^*] \\
&\quad + [U\partial_\mu U^*, U\varphi(q)U^*] \\
&= UD_\mu\varphi(q)U^* + \partial_\mu U\varphi(q)U^* + U\varphi(q)\partial_\mu U^* \\
&\quad + [U\partial_\mu U^*, U\varphi(q)U^*] \\
&= UD_\mu\varphi(q)U^*,
\end{aligned}$$

where the last equation follows from the fact that

$$[U\partial_\mu U^*, U\varphi(q)U^*] = -\partial_\mu U\varphi(q)U^* - U\varphi(q)\partial_\mu U^*,$$

which is checked using $U^*U = \mathbb{1} = UU^*$ and the fact that this identity, together with the Leibniz rule for ∂_μ, implies $U\partial_\mu U^* = -(\partial_\mu U)U^*$.

We obtain therefore the following Lagrangian covariant under gauge transformations (which therefore gives rise to an invariant action):

$$\mathcal{L} = \frac{1}{2}\eta^{\mu\nu}D_\mu\varphi(q)D_\nu\varphi(q) - \frac{1}{2}m^2\varphi(q)^2.$$

Then, expanding the covariant derivatives, interaction terms between the neutral scalar field and the electromagnetic potential are given by

$$\mathcal{L}_I = -\frac{ie}{2}\{[A_\mu(q),\varphi(q)],\partial^\mu\varphi(q)\} - \frac{e^2}{2}[A_\mu(q),\varphi(q)][A^\mu(q),\varphi(q)], \qquad (2.11)$$

with curly brackets denoting the anticommutator. We note that on classical spacetime $\mathcal{L}_I$ vanishes, as it should, since A_μ and φ commute.[1]

Therefore, we will understand $\mathcal{L}_I$ as defined through the noncommutative product in $\mathcal{E}$. This has the drawback that it will depend explicitly on the center.

If we neglect, as customary, the quadratic term in A_μ (the weak-field approximation) we obtain, for the interaction part of the action,

$$S_I = -\frac{ie}{2}\int d^4q\{[A_\mu(q),\varphi(q)],\partial^\mu\varphi(q)\} = -ie\int d^4qA_\mu(q)[\varphi(q),\partial^\mu\varphi(q)],$$

where the cyclicity of $\int d^4q$ was used. We obtain therefore the interaction of A_μ with a current $j^\mu(q) = -ie[\varphi(q), \partial^\mu\varphi(q)]$, and the classical Euler-Lagrange equations for A_μ take the form

$$\partial_\mu F^{\mu\nu} - ie[A_\mu, F^{\mu\nu}] = -j^\nu,$$

with the field strength defined by $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ie[A_\mu, A_\nu]$ and transforming as $F_{\mu\nu} \to UF_{\mu\nu}U^*$.

In order to understand the physical meaning of this interaction, we now assume $A_0(q) = 0$, $A_h(q) = \frac{1}{2}\varepsilon_{hjk} B^jq^k$, the potential corresponding to an external constant magnetic field $\boldsymbol{B}$ in the classical spacetime limit ($\lambda_P \to 0$); we again neglect in (2.11) the quadratic term in A_μ, thus obtaining

$$\mathcal{L}_I = -\frac{ie}{4}\varepsilon_{hjk}B^j\{[q^k,\varphi(q)],\partial^h\varphi(q)\} = \frac{e}{2}\varepsilon_{jkh}B^jQ^{k\mu}\{\partial^\mu\varphi(q),\partial^h\varphi(q)\}, \qquad (2.12)$$

where in the second equation we used the identity $[q^\nu, f(q)] = iQ^{\nu\mu}\partial_\mu f(q)$.

The above term then corresponds to the energy of a total magnetic moment $\boldsymbol{M}$ with components, in generic units,

$$M_j = (e/2)\lambda_P^2\int_{q^0=t} d^3q\left[\frac{1}{2}(\{\partial_l\varphi,\partial^l\varphi\}\delta_{jk} - \{\partial_j\varphi,\partial^k\varphi\})m_k - \varepsilon_{jkh}\{\partial_0\varphi,\partial^h\varphi\}e_k\right], \quad j = 1,2,3, \qquad (2.13)$$

where $e_k \coloneqq Q^{0k}$ and $m_k \coloneqq \frac{1}{2}\varepsilon_{khl}Q^{hl}$ are, respectively, the electric and magnetic components of the antisymmetric 2-tensor $Q^{\mu\nu}$. In the next section we will give some numerical estimates on the electromagnetic radiation and on the perturbations of the motion of charged particles associated to such a magnetic moment in suitable astrophysical situations.

III. SOME POTENTIALLY OBSERVABLE CONSEQUENCES

Defining, as usual, the free scalar field on QST as [3]

$$\varphi(q) = \int_{\mathbb{R}^3}\frac{d\boldsymbol{k}}{\sqrt{\omega(\boldsymbol{k})}}[a(\boldsymbol{k})\otimes e^{-ikq} + a(\boldsymbol{k})^*\otimes e^{ikq}],$$

and specializing to a point in the spectrum of the Qs where $\boldsymbol{e} = \boldsymbol{m}$, a computation yields the following expression for the total magnetic moment (2.13):

$$\boldsymbol{M}(t) = \frac{e\lambda_P^2}{2}\int_{\mathbb{R}^3}\frac{d\boldsymbol{k}}{\omega(\boldsymbol{k})} \times\{[a(-\boldsymbol{k})a(\boldsymbol{k})e^{-2i\omega(\boldsymbol{k})t} + a(\boldsymbol{k})^*a(-\boldsymbol{k})^*e^{2i\omega(\boldsymbol{k})t}] \times\cos(\omega(\boldsymbol{k})\boldsymbol{e}\cdot\boldsymbol{k})\boldsymbol{k}^2\boldsymbol{e}^\perp + 2a(\boldsymbol{k})^*a(\boldsymbol{k})[2\omega(\boldsymbol{k})\boldsymbol{k}\wedge\boldsymbol{e} + \boldsymbol{k}^2\boldsymbol{e}^\perp]\},$$

with $\boldsymbol{e}^\perp = \boldsymbol{e} - (\boldsymbol{k}\cdot\boldsymbol{e})\boldsymbol{k}/\boldsymbol{k}^2$ the component of $\boldsymbol{e}$ orthogonal to $\boldsymbol{k}$. Therefore the effective magnetic moment of a particle with sharp momentum $\boldsymbol{k}$ is given by

$$\boldsymbol{\mu}_{\boldsymbol{e},\boldsymbol{k}} = e\lambda_P^2\left[2\boldsymbol{k}\wedge\boldsymbol{e} + \frac{\boldsymbol{k}^2}{\omega(\boldsymbol{k})}\boldsymbol{e}^\perp\right]. \qquad (3.1)$$

Of course, detectable effects, if any, of the above interaction can be obtained in situations which give rise to a very large magnetic moment. To this end, it is natural to consider a compact "star" of φ particles in rapid rotation around a very massive companion, akin to a binary pulsar or black hole. Neglecting the rotation of this star around its axis, a rough estimate of the associated magnetic moment $\boldsymbol{M}_S$ can be obtained by treating such an object as composed by classical particles in uniform rotation with a given angular frequency ω, and by associating to such particles the magnetic moment obtained from (3.1).

More in detail, we choose a reference system in which the orbit lies on the (x, y) plane, and we indicate by (θ, ϕ) the spherical coordinates of $\boldsymbol{e}$ with respect to this system. Moreover, recalling that for the binary black hole giving rise to the event GW150914 the angular frequency just before the merger was $\omega \cong 471\ \mathrm{s}^{-1}$ and the radius of the

[1] Note that $\mathcal{L}_I$ would vanish also on quantum spacetime if the products appearing were interpreted as quantum Wick products [9], as $E^{(n)}(f_1(q_1)\ldots f_n(q_n))$ is independent from the ordering of the factors, and the tensor factors in $A_\mu(q)$ and $\varphi(q)$ acting on Fock space would commute again. But the quantum Wick product would violate not only Lorentz invariance, but also gauge invariance; hence, it could not be applied in the present context without first elaborating some radical modifications.

orbit was $R \cong 350$ km [12], so that the speed in natural units ($\hbar = c = 1$) was $\beta = \omega R \cong 0.6$, we may also assume that the motion of the particles is nonrelativistic and approximate $\omega(\boldsymbol{k}) \cong m$ in (3.1). We then obtain

$$\boldsymbol{M}_S(t) = e\lambda_P^2 M \left\{ 2R\omega \begin{pmatrix} \cos\omega t \cos\theta \\ \sin\omega t \cos\theta \\ -\cos(\omega t - \phi)\sin\theta \end{pmatrix} + R^2\omega^2 \left[\begin{pmatrix} \sin\theta(\cos\phi - \sin\omega t \sin(\omega t - \phi)) \\ \sin\theta(\sin\phi + \cos\omega t \sin(\omega t - \phi)) \\ \cos\theta \end{pmatrix} + \frac{1}{5}\left(\frac{r}{R}\right)^2 \begin{pmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ 2\cos\theta \end{pmatrix} \right] \right\}, \tag{3.2}$$

with M the mass of the object and r its radius. In the particular case in which $\boldsymbol{e}$ is normal to the orbital plane, $\boldsymbol{M}_S$ then precedes around it with the same angular frequency ω of the object motion.

In the general case, $\boldsymbol{M}_S(t)$ can be written as sum of a constant moment, which of course does not give rise to emission of electromagnetic radiation, and of a time-dependent moment of the form

$$\sum_i \boldsymbol{M}_i \cos(\omega_i t - \psi_i),$$

with $\omega_i = \omega, 2\omega$ and where ψ_i are suitable phases. It is then an exercise in classical electromagnetism (extending, e.g., the discussion in Sec. 9.3 of [13]) to verify that the time-averaged (classical) electromagnetic energy radiated (on classical spacetime) per unit time by this variable magnetic moment is given, in natural units, by

$$\begin{aligned} \frac{d\mathcal{E}}{dt} &= \frac{2}{9}\sum_{\omega_i=\omega_j} \omega_i^4 (\boldsymbol{M}_i \cdot \boldsymbol{M}_j)\cos(\psi_i - \psi_j) \\ &= \frac{2}{9} e^2\lambda_P^4 M^2 R^2 \omega^6 \left(1 + \sin^2\theta + \frac{1}{2}\omega^2 R^2 \sin^2\theta\right). \end{aligned} \tag{3.3}$$

Therefore, averaging over the unknown direction of $\boldsymbol{e}$, we get

$$\begin{aligned} \frac{d\mathcal{E}}{dt} &= \frac{8\pi}{27} e^2\lambda_P^4 M^2 R^2 \omega^6 (5 + \omega^2 R^2) \\ &\simeq e^2 \lambda_P^4 M^2 R^2 \omega^6 \\ &\simeq e^2 \left(\frac{\tau_P}{T}\right)^6 \left(\frac{R}{\lambda_P}\right)^2 M^2, \end{aligned} \tag{3.4}$$

where in the second equation we neglected numerical constants of order 1, and took into account that typically $\omega R \simeq 10^{-1}$ or smaller. Taking then $\mathcal{E} \simeq M \simeq Nm$, where N is of the order of the number of particles in an object of the size of the sun and density of liquid water (i.e., roughly 10^{56}), $m \simeq 1$ GeV, the rotation period $T = 10^{-2}$ s and $R = 10^3$ km (comparable to the GW150914 parameters), and recalling that the Planck time $\tau_P \simeq 10^{-44}$ s, we get that the fraction of energy radiated by the body per unit time is

$$\frac{1}{\mathcal{E}}\frac{d\mathcal{E}}{dt} \simeq 10^{-89}\ \mathrm{s}^{-1},$$

and it is therefore negligible.

To make this sizable, T should be of Planckian order, which would probably mean that our object collapsed into a black hole and no radiation is visible—and, in any case, the above Minkowskian picture would not apply.

This computation is certainly too primitive, but it suggests that the fraction of the total mass emitted as electromagnetic radiation can be expected to be negligible, and by far nothing comparable with the fraction of a few percents emitted as gravitational waves in the binary black hole collapse GW150914.

Could a more cautious approach reverse this conclusion? The question is legitimate, since a heuristic argument, whose qualitative consequences are confirmed by a more cautious analysis [4], suggests that near singularities the effective Planck length might diverge as $\lambda_P g_{00}^{-1/2}$. This might well introduce a metric-dependent factor in our formula for the electromagnetic radiation caused by the magnetic moment of neutral matter, making it considerably larger in the last instants before the collapse into a black hole; heuristically this is

$$\frac{d\mathcal{E}}{dt} = \frac{1}{g_{00}^2} e^2 \lambda_P^4 M^2 R^2 \omega^6,$$

where g_{00} is the time-time component of the background metric.

This qualitative conclusion is supported by the results in [4], which mean in particular that in a flat Friedmann-Robertson-Walker (FRW) background (which is spherically symmetric with respect to every point), with metric, in spatial spherical coordinates, $ds^2 = -dt^2 + a(t)^2[dr^2 + r^2 dS^2]$, the size of a localization region centered around an event at cosmological time t, measured by the radial coordinate r, must be at least of order $\lambda_P a(0)/a(t)$, $t = 0$ being the time of the present epoch.

The situation that we have in mind, namely, that of a neutral object rotating in the gravitational field of a collapsing one, is of course better described by a Schwarzschild metric than by a FRW one. We note that the metric of a collapsing homogenous sphere of dust is given by the Oppenheimer-Snyder solution [14,15], which is a Schwarzschild metric outside the sphere, matched with a closed FRW metric

$$ds^2 = -dt^2 + a(t)^2(d\chi^2 + \sin^2\chi d\Omega^2) \tag{3.5}$$

inside it. The scale factor $a(t)$ in the above metric can be expressed parametrically through the conformal time η,

$$a(t(\eta)) = \frac{1}{2}\sqrt{\frac{R_0^3}{2GM_0}}(1+\cos\eta), \tag{3.6}$$

$$t(\eta) = \frac{1}{2}\sqrt{\frac{R_0^3}{2GM_0}}(\eta+\sin\eta), \tag{3.7}$$

with $M_0 > 0$ the Arnowitt-Deser-Misner mass of the collapsing sphere and $R_0 \geq 2GM_0$ its initial areal radius. The conformal time at which the sphere is completely inside its Schwarzschild radius is given by $\eta_0 = \cos^{-1}(4GM_0/R_0 - 1)$.

The continuous match between the exterior Schwarzschild metric and the interior FRW one and the results of [4] recalled above seem therefore to justify the ansatz of replacing λ_P in (3.4) by $\lambda_P a(0)/a(t)$. Indeed, such an expression for the effective Planck length converges to the usual value λ_P in the limit $M_0 \to 0$ in which the FRW metric becomes Minkowski, as one can easily verify by eliminating the conformal time η from (3.6), (3.7). Then, the above formula for the radiation of a precessing neutral object would become

$$\frac{d\mathcal{E}}{dt} = e^2\left(\frac{\lambda_P a(0)}{a(t)}\right)^4 M^2R^2\omega^6. \tag{3.8}$$

This energy has to be emitted, of course, at the cost of the kinetic energy of the rotating object due to spin, precession, and orbital rotation, as well as of its potential energy, causing a faster inspiraling. For simplicity, we will consider here only the orbital kinetic term, and then

$$\frac{d}{dt}\left(\frac{1}{2}MR^2\omega^2\right) = -\frac{d\mathcal{E}}{dt}, \tag{3.9}$$

which entails that the total radiated energy can be estimated as

$$\mathcal{E} = \frac{1}{2}MR^2[\omega_0^2 - \omega(t_{\text{collapse}})^2], \tag{3.10}$$

where the integration cannot be extended beyond the hiding of our object within the event horizon of the other. Indeed, according to our formulas, the radiated power would diverge near the singularity, but it would remain trapped and would not be visible from outside.

According to (3.10), the total radiated energy can be sizable only if $\omega(t_{\text{collapse}}) \ll \omega_0$. In order to check if this is the case in typical situations, we solve (3.9), which, by inserting (3.8), becomes

$$\dot{\omega} = -e^2(\lambda_P a(0))^4 M\frac{\omega^5}{a(t)^4},$$

which can be integrated by separation of variables. To this end, we note that, by (3.7), $dt/d\eta = a$, and therefore

$$\int_{-\infty}^{t_{\text{collapse}}} a(t)^{-4}dt = \int_0^{\eta_0}\frac{d\eta}{a(t(\eta))^3} = 8\left(\frac{2GM_0}{R_0^3}\right)^{3/2}\int_0^{\eta_0}\frac{d\eta}{(1+\cos\eta)^3}.$$

Defining then, for $\eta \in [0,\pi)$,

$$F(\eta) \coloneqq \int_0^\eta \frac{dx}{(1+\cos x)^3} = \frac{\sin\eta(6\cos\eta + \cos(2\eta)+8)}{15(1+\cos\eta)^3},$$

we obtain, neglecting numerical constants of order 1,

$$\omega(t_{\text{collapse}})^2 = \left[\frac{1}{\omega_0^4} + e^2\lambda_P^4 M\left(\frac{R_0^3}{2GM_0}\right)^{1/2}F(\eta_0)\right]^{-1/2}, \tag{3.11}$$

which is smaller than ω_0^2, as it should be.

One can then observe that for $M_0 \to 0$ one has $\eta_0 = \cos^{-1}(4GM_0/R_0 - 1) \sim \pi - \sqrt{8GM_0/R_0}$, and therefore $F(\eta_0) \sim \frac{8}{5}(R_0/8GM_0)^{5/2}$, so that

$$\omega(t_{\text{collapse}})^2 \sim \frac{G^{3/2}M_0^{3/2}}{e\lambda_P^2 MR_0^2}$$

would actually be very small with respect to ω_0^2, making the total radiated energy (3.10) non-negligible. (Note that for ordinary matter the collapse would stop much before that the matter itself is hidden inside the horizon, due to the nonvanishing pressure.)

Moreover, this effect might disappear if one takes properly into account the redshift of the radiation emitted near to the horizon. This could probably be done by using the general relativistic version of the radiated power by a magnetic dipole instead of (3.3).

Conversely, for finite values of M_0, one can expand (3.11) due to the smallness of λ_P^4, and obtain for the total radiated energy

$$\mathcal{E} \simeq e^2\lambda_P^4 M^2R^2\omega_0^6\left(\frac{R_0^3}{2GM_0}\right)^{1/2}F(\eta_0).$$

Thus we see that for $2GM_0/R_0 = 1$, $F(\eta_0)$ vanishes, as it should, since the collapse takes place at the beginning. If instead $2GM_0/R_0$ is smaller than 1 but of that order, then $F(\eta_0)$ is also of the same order; e.g., if $2GM_0/R_0 = 1/2$ then $F(\eta_0) = 7/15$. Moreover, if we take as before $M_0 \simeq M \simeq 10^{56}$ GeV $= 10^{37}M_P \simeq \mathcal{E}_0$ and we recall that for $\hbar = c = 1$ we have $G = M_P^{-2}$, we deduce $R_0 = 4GM_0 \simeq 10^{37}M_P^{-1} \simeq 10^{-1}$ km, so that, assuming again $R \simeq 10^3$ km $\simeq 10^{41}M_P^{-1}$ and $T \simeq 10^{-2}$ s $\simeq 10^{42}\tau_P$, we get

$$\mathcal{E} \simeq e^2\lambda_P^4 M^2 R^2 R_0 \omega_0^6 \simeq e^2\lambda_P^4 M R^2 R_0 \omega_0^6 \mathcal{E}_0 \simeq 10^{-96}\mathcal{E}_0 \simeq 10^{-40} \text{ GeV}.$$

Note that using the same figures and multiplying the fraction of the total energy emitted as electromagnetic interaction per second, as given by the previous more brutal computation, Eq. (3.4), by the collapse time $t_{\text{collapse}} = t(\eta_0) = \frac{1}{\sqrt{2}}(\frac{\pi}{2} - 1)R_0 \simeq 10^{37}\tau_P \simeq 10^{-7}$ μs, we get an estimate of exactly the same order of magnitude.

Thus, as noticed earlier in this discussion, the fraction of the mass converted into electromagnetic radiation is negligible, unless the period T is at the Planck scale, which would probably mean that collapse took place and the emitted radiation is not visible to distant observers. As already mentioned, however, a more realistic estimate ought to treat the electromagnetic emission relativistically.

Another possibility, both more and less favorable, could be offered by a compact spinning concentrate of dark matter interposed to some distant source; spin and concentration apart, these objects exist and are revealed to us by gravitational lensing, which results from the gravitational deflection of photons that has been known experimentally for a century.

If the source emits also charged particles, say electrons, sufficiently energetic to reach us within a reasonable delay after the γ rays, their deflection ought to be modified by the magnetic field caused by the moment of our stellar object, due to quantum spacetime; this is a sort of QST–Northern Lights phenomenon. One might hope that this is a more favorable situation with respect to the one considered above because, while the energy emitted is proportional to the fourth power of the Planck length, the deviation we are mentioning now would be only quadratic in λ_P.

Nevertheless, a rough estimate of the deviation angle θ of an electron by a compact object of mass M and radius R spinning at angular velocity ω gives, using (3.2),

$$\theta \cong \frac{M_S}{m\gamma R^2} \cong \frac{e\lambda_P^2 M\omega}{m\gamma R},$$

with m the electron mass and $\gamma = (1 - v^2)^{-1/2}$. Choosing, as above, $M \simeq 10^{56}$ GeV, $R \simeq 10^{-1}$ km, and $\omega = 10^2$ s^{-1}, the deviation would be only $\theta \cong 10^{-34}$ for electrons of energy 1 TeV, which would reach us with a delay, with respect to photons, of a few hours if the source is 10^9 light years distant. The delay for protons of 10^3 TeV (still considered to be lower than the GKW limit) would be the same, but the deviation would be 10^3 times smaller. Of course, the deviation would be more important for softer electrons, which, however, would reach the Earth when nobody is there any longer.

Moreover, a less favorable aspect is that electrons are considerably influenced by the much stronger galactic magnetic field; a precise knowledge of this would be needed, together with a nearly exact location in the sky of the sources of electromagnetic radiation and of electrons, as well as a clear recognition of the coincidence of their origin.

IV. CONCLUDING REMARKS

Our discussion was based on the choice (2.9) of the covariant derivative, with e denoting the electron charge. This choice seems to be dictated by gauge invariance in a theory which includes the electromagnetic interactions of the electron, taking into account the noncommutativity of $\mathcal{G} = \mathcal{U}(M(\mathcal{E}))$, the group of unitaries in the multipliers of the algebra of quantum spacetime $\mathcal{E}$. On an $\mathcal{E}$ bimodule, only the left (respectively, right) action of U (respectively, U^*), or the trivial action, are allowed.

This poses a problem for the Standard Model (excluding quark fields). This problem has been noticed and discussed by several authors (see, e.g., [7,11]), and it deserves further discussion to see whether in our context the choice made in (2.9) is really the only choice.

According to our preceding discussion, so far there seems to be no indication of visible effects of the quantum nature of spacetime at the Planck scale, except for its role in solving the horizon problem [4] and justifying from first principles some of the assumptions made in the inflationary scenario.

The effects considered in this paper are so tiny that it would be instructive to compare them with those due to the graviton-mediated dark matter–photon interaction. Furthermore, the electromagnetic radiation emitted by a collapsing binary system due to the mechanism proposed here ought to be compared with the Hawking radiation.

The QST-induced electromagnetic interactions of dark matter might be detectable in more exotic hypothetic astrophysical objects, like self-gravitating Bose-Einstein condensates of dark matter consisting of neutral scalar particles. The stability of such objects, with a solar mass and a radius of a few dozen kilometers, has been recently investigated in both the isotropic and rotating cases; the possible formation of vortices has also been considered (cf., e.g., [16]). Smaller objects of this nature were excluded from Ref. [16] by the nonrelativistic approximation used there, but might well be relevant to manifest sizable QST-electromagnetic effects, possibly also in the form of electromagnetic vortex-vortex interactions; this might potentially change the dynamics of these hypothetical objects. These points will be dealt with in subsequent studies.

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