Beginner mathematics teachers learning to teach and assess advanced problem solving

Other

BEGINNER MATHEMATICS TEACHERS LEARNING TO TEACH AND ASSESS ADVANCED PROBLEM SOLVING

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SUPPORTING ADVANCED MATHEMATICS PROJECT

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FINAL REPORT, NOVEMBER 2016

CONTENTS

SUMMARY

In line with new GCSE and A Level mathematics specifications, secondary mathematics teachers in England are expected to develop teaching and assessment of problem solving in ways which have not been common in English schools or initial teacher education in recent years.

This study asked what beginner teachers bring to this process, including knowledge, skills and beliefs, and what should and can be further developed, in the specific context of problem solving for advanced school mathematics. Beginner teachers were asked to reflect on their preparedness for assessing A Level (and AS) Mathematics. This informed the design of a short intervention workshop designed to increase their awareness of the issues implicit in assessing and teaching for problem solving in mathematics. Key parts of the workshop were repeated with three groups of experienced teachers, and data was collected from written responses, transcribed audio recordings of workshop discussions and follow-up interviews with four beginner teachers.

The study suggests that the intentions of the change are well-aligned with the professional beliefs of many teachers, both beginners and experienced. It shows that carefully structured workshop problem solving can enable beginner teachers to acquire specific assessment-related skills, to replay their classroom experiences when considering new questions and rehearse how they might improve their practice. They were able to appreciate tensions between reliability and validity in assessment, and articulate principles for resolving those tensions, drawing mainly on their own experiences of learning. They started to consider formative classroom strategies for supporting exploration, persistence and written reasoning in order to gain marks. Experienced teachers showed a wider command of such strategies, although they recognised that many of their skills relied on features of the old assessment.

Acquiring deep expertise in this area is demanding, and perceived to be daunting by both beginner and experienced teachers. Many experienced teachers found it almost impossible to re-envision their current practice so as to successfully prepare their students for problem solving.

An initial report of the study for the academic community was made in Golding and Smith (2016).

RECOMMENDATIONS

1. *For providers of ITE, FMSP and Awarding Bodies:* There is a need for professional development that supports the goals of the new A-level, goes into detail at the level of student responses in the classroom, and exposes the judgements that need to be made when teaching and assessing for problem solving. For beginner teachers, such development should be provided in a sustained and coherent way across the different sites of their initial teacher education and into the early years of their career. The need is acute for teachers in their early career, but experienced teachers also claimed that their established departmental ways of developing professionally were insufficient.

2. *For providers of ITE and others:* Beginner teachers should have opportunities to observe or support teaching in classes being prepared for high stakes examinations, and to develop reflection on the issues involved. Professional development offered to beginner teachers should include training in the use of formal mark schemes. However, deeper reflection and implications for school practice are facilitated by marking and comparing live and authentic student solutions. These activities are valuable for all teachers.

3. *For FMSP and other providers of professional development:* The FMSP and other providers can draw on considerable support amongst teachers for the values implicit in the new assessments. However teachers were unsure how to resolve tensions between these values and others they held around student inclusion, well-being and how they measured their own professional status. FMSP and other professional development should invite teachers to articulate reasons behind their judgements, invoking their knowledge of disciplinary (mathematical) values as well as their knowledge of students.

4. *The FMSP and/or Awarding Bodies* should provide and publicise a bank of short, semi-structured exemplification questions that allow multiple solution routes, along with sample student solutions and suggested mark schemes. There is a dearth of available material that exemplifies what is valuable in such problems, how they could be scored and how judgements of solutions can be made. We note the historic documents, Pupils' work assessed: Key Stage 3 Mathematics (1993) have suitable detail, although based on levelling coursework rather than examination questions.

5. *Teachers and teacher educators* should consider carefully at what stage, and with what purpose, young people can best benefit from seeing formal mark schemes when attempting problem solving in preparation for external assessment.

1. BACKGROUND

We report on a small scale exploratory study of beginner teachers reflecting on their preparedness for (formatively and summatively) assessing A Level and AS Mathematics, and then experiencing a short intervention workshop designed to increase their awareness of the issues implicit in assessing and teaching for problem solving in mathematics. The A-Level mathematics curricula in England, in common with pre-16 curricula in England and elsewhere, are being re-developed with a renewed emphasis on problem solving and reasoning. These new curricula were scheduled for introduction between September 2015 and 2017, immediately after these beginner teachers gained their qualified teacher status.

The scale of participation and success in A Level Mathematics is thought to be of national importance (Noyes, Wake, & Drake, 2011). For students it is a high stakes year 12/13 qualification, often a gatekeeper to university entrance, and its summative assessments embody (de facto) rules for the discipline. Teachers of A Level (and AS) Mathematics therefore need to be able to teach effectively for problem solving, including being able to enact formative assessment in problem solving situations. As part of this practice, they need to be expert in understanding and using summative assessment materials and their associated mark schemes. Summative assessment is significant for teaching in England as it is common practice in classrooms and textbooks (Ofsted, 2012) for examination type questions to act as a proxy for formative methods in teaching and learning: they are used to communicate curricular goals, to motivate learning and to demonstrate achievement to teachers and students. Examination questions in themselves therefore form pervasive tools for A Level teaching. Our emphasis on assessment reflects this integrated practice, reasoning that teachers should ideally be familiar with assessment materials, be able to constructively critique them, and be confident they represent opportunities to develop valued mathematical practices and knowledge.

Teachers represent and embody the discipline for their students (Fordham, 2012), in terms of its values and its rules, yet the renewed emphasis on problem solving is an aspect of mathematics that beginner teachers may not have experienced as school students because the curriculum in England for over a decade has been characterised as "concentrated on the acquisition of disparate skills that enabled pupils to pass tests and examinations but did not equip them for the next stage of education, work and life" (Ofsted, 2012, p9). Although there was significant debate in the 1980s and 1990s about teaching and assessing problem solving (eg Mason, Burton and Stacey, 1982; Niss, 1992; Noss, Goldstein & Hoyles, 1989; Brown, 1989), we know little about what our current beginner teachers know, value or believe in this area.

We use 'beginner teachers' here to refer to secondary teachers at the end of one-year postgraduate preparation courses for mathematics teaching, either school-led or Higher Education-led, about to qualify as teachers and with substantial (at least 28 weeks') classroom experience to draw on. Serendipitously, we also had opportunity to work with three groups of experienced teachers using similar materials, and this enabled a comparison across the two types of sample. The study is not, though, claimed to be representative: all participants opted into it and so were presumably interested in, and/or concerned about, the issues around teaching and assessing for problem solving at this level.

In relation to both formative and summative assessment, then, we asked

- *What knowledge, values and beliefs inform beginner teachers' approaches to the teaching and assessment of problem solving in advanced mathematics?*
- *What are their needs in relation to developing assessment of problem solving as part of their preparation for teaching (and how does that compare with the needs of more experienced teachers adapting to this change)?*
- *How could initial teacher education and early professional development be strengthened to include education for better assessment of problem solving?*

1.1 PROBLEM SOLVING

Mathematics 'problem solving' is a contested term in the literature (Schoenfeld, 2007). Clearly, what is a problem to one student might well be anything but to another: for example, summing an arithmetic series might comprise a problem to a 14 year old who, two years later, has learnt a standard algorithm that requires little thinking. We take 'problems' to be those tasks to which there is no approach or algorithm that is both familiar to the student and (s)he knows is almost certain to result in a closed solution. Instead, the task might contain unfamiliar, unstructured or complex aspects to which the student might bring one or more approaches, but without high initial confidence of success. This is an interpretation which appears to be consistent with the early specimen materials produced by A Level examination bodies in response to the English policy change (see, for example, Fig 1).

Figure 1: A historical question and a specimen question on the topic of summing sequences.

Historical question	Specimen question
A sequence of numbers $a_1, a_2, a_3, \ldots$ is defined by $a_1 = 3$ $a_{n+1} = 2a_n - c \quad (n \geq 1)$ where c is a constant. (a) Write down an expression, in terms of c, for a_2 (b) Show that $a_3 = 12 - 3c$ Given that $\sum_{i=1}^{4} a_i \geq 23$ (c) find the range of values of c.	A sequence $u_1, u_2, u_3, \ldots$ is defined by $u_1 = -2$ $u_{n+1} = \frac{2}{2 - u_n} \quad n \geq 1, n \in N$ Find the value of $\sum_{i=1}^{100} u_n$ *Source: Edexcel*

These questions require problem solving behaviours such as: familiarising oneself with notations and definitions by specialising, interpreting constraints, recognising analogous reasoning and assumptions, and being prepared to persist and to adopt a different approach if the first proves unfruitful. We sought existing evidence of the differential demands of teaching mathematics, including problem solving, at this advanced school level, but found little; Smith and Golding (2015) begins to address this issue.

1.2 ASSESSING PROBLEM SOLVING

There are two strands within the literature on assessing problem solving: a concern with assessing mathematical understanding *through* problem solving, and a concern to assess problem solving as a skill in its own right. The first concern has been formulated as designing 'authentic' assessments that reflect the nature of mathematics (Niss, 1992) and incorporate the full range of mathematical practices intended by the curriculum (Swan and Burkhardt, 2012). The second concern was addressed in England from 1988-2007 by assessing the skills of 'using and applying mathematics' separately within GCSE investigatory or practical coursework, and in less high-stakes projects such as Graded Assessment in Mathematics[1] Bowland Mathematics[2]. It is also informs the influential PISA tests that assign mathematics questions to three problem solving subscales (formulating situations mathematically, employing mathematical skills, interpreting mathematical outcomes) and report separately on performance in problem solving. The proposed changes in the A-level specification seem to us to fall into the first strand - assessing mathematical understanding through problem solving - since problem solving is specified within the assessment objectives but not reported as an outcome. Although these policy proposals have been widely publicised, there has been little change to summative assessment practices in England for over a decade, and we wondered whether beginner teachers would be aware of the detail of these (historic) debates. The intervention workshop was designed as a research-informed introduction to the new curriculum that would allow some of these issues to be raised.

It is generally accepted in the literature that it is more difficult to assess understanding through problem solving than assess recall of facts and procedures. Much of this difficulty arises from the requirement that summative assessment be both valid and reliable. Problems, by their nature, often admit multiple solution routes and uncertainty and this has several implications for assessment. It means that assessing problem solving may take longer for students and markers and not cover the whole of the target content (Swan and Burkhardt, 2012). It reduces the possibility for structuring questions into subtasks, a strategy that examination designers have used to increase scorer consistency and hence reliability (Jones, Swan & Pollitt, 2014). The levels of uncertainty within complex problem solving processes mean that students are prone to make errors and to launch into provisional calculations without explanation, making it difficult for markers to recognise their strategy and to decide its worth (Szetala, 1992). In addition, performance may be heavily dependent on the exact phrasing or context in which the problem is posed (Noss, Goldstein & Hoyles, 1989). This prevents valid comparisons about performance across tasks and so affects the coherence of the construct being reported on. It also implies that detailed mark schemes need to be written for each task, helping markers to recognise the range of student responses and decide their value in that context.

As this brief summary suggests, much of this literature exposes a tension between the goal of authentic assessment and the (mis)perception that tests can/should provide a perfectly accurate, stable measure of mathematical attainment (Black et al, 2015). A different approach has been suggested by Jones, Swan and Pollitt (2014) who show that a group of mathematics education experts can compare a class set of problem solving scripts holistically, in reasonable time, yielding a rank ordering that is as reliable as a standard scoring mark scheme. Drawing on this research, one aim of our workshop was to provide beginner

[1] www.stem.org.uk/elibrary/collection/2780
[2] www.bowlandmaths.org.uk

teachers with experiences of attempting, marking and comparing solutions to questions from traditional and specimen examinations, to allow discussion of any issues they perceived and to start framing these in terms of research constructs where appropriate. As indeed happened, we expected these issues to arise unprompted from teachers' concern with fairness.

The complementary argument in the literature is that assessing problem solving has an important formative role in providing detailed, timely information about students' strengths and areas for development during substantive mathematical work. Swan and Burkhardt (e.g. 2012) are the pre-eminent assessment designers who make this argument, drawing together evidence from the UK and the US context in their Mathematics Assessment Project (http://map.mathshell.org/). The length of a problem solving task here becomes an advantage as it allows the teacher to assess and give feedback while the enquiry is still live. The potential for multiple approaches naturally offers opportunities for knowledgeable teachers to ask questions that encourage students to reflect and communicate their reasoning. The range of responses allows teachers to appreciate the diversity that exists in student thinking and to recognise signs of common misconceptions. Teachers can plan how to respond to ongoing student assessment by differentiating the structure of tasks, providing and removing structure so as to scaffold student thinking. Thus there is a huge potential within problem solving tasks for teachers to improve their assessment, and hence students' learning, but also a substantial demand on teacher subject-specific professional competencies.

The second aim of our workshop, therefore, was to prompt beginner teachers to think about such implications for their teaching. After discussing student solutions, we asked questions such as "If your student had written this in test, what do you think (s)he understands? What would you do with him/her in class?" We did not expect that teachers would simply acquire these context-dependent skills within our workshop, but hoped that we could observe whether they were aware, from their school placements, that experienced teachers did so.

1.3 TEACHER LEARNING

We adopt Livingston and Borko's (1990) conceptualisation of mathematics teaching as a complex cognitive skill which includes improvisational performance and builds on prior experiences. Cochran-Smith and Lytle (1999) review teacher education and show that relationships between 'knowing more' about teaching and 'teaching better' are anything but straightforward. They distinguish between knowledge for/in/of practice, claiming that effective teachers need all of these. In our context, knowledge *for* practice can be seen as learning about the new assessments and the technical features of mark schemes (e.g. their codes for accuracy and method marks, and their conventions of 'follow-through'). This is the type of knowledge that can be seeded in an HE-course before teaching, but since these teachers had nearly completed their training, we hoped to achieve more. Knowledge *in* practice is embedded, maybe tacitly, in the actions and reflection of teaching. Cochran-Smith and Lytle suggest that beginner teachers need opportunities to collaborate with experienced teachers and "enhance, make explicit and articulate" such knowledge. Here, the only collaboration possible was with differently-experienced peers and with us, but we were interested in whether our beginner teachers would recall and share relevant mentoring conversations from school practice. This might then allow the third type of knowledge, knowledge *of* practice, which is generated by deliberate inquiry in, and reflection on, everyday classroom practice.

Horn (2005) has suggested that teachers develop in a social and situated way through professional interactions that focus on the classroom but take place alongside it. Informal professional conversations are key resources for teacher learning when they focus on pedagogical classifications, classroom experiences and curriculum resources that foreground the goals of a desired reform. These social learning interactions are re-presented in Horn (2010, 243) as 'replays, rehearsals, and re-envisioning of practice'. In our workshops we expected that teachers might: replay experiences of teaching to interpret students' thinking behind authentic written solutions, replay experiences of assessment to identify how the new materials were different, rehearse how they would teach differently given these changes, including how they would use examination questions in class and, perhaps, re-envision their teaching. This is perhaps over-ambitious since Berliner (2004) argues the teaching expertise is both domain-specific and a long-term goal, taking 5-7+ years to develop, if it ever does. However it provided a frame for understanding the relationship between workshop discussion and classroom knowledge.

1.4 LEARNING ABOUT TEACHING AND ASSESSING PROBLEM SOLVING

Classroom studies of early career teachers' beliefs about teaching for problem solving are sparse. Cooney's (1985) study of a newly qualified teacher suggests that strong initial beliefs in the centrality of problem solving to mathematics can be crowded out during teaching by contradictory needs and contextual norms, leaving the teacher articulating same beliefs but with few developed links with classroom practice. Little appears to be known, either, about how beginner teachers learn to engage with authentic classroom assessment. Grainger and Adie (2014) asked how key assessment areas develop when beginner teachers engage in criterion-referenced assessment of peers' coursework, and highlighted the profound influence on beliefs and understandings of teachers' background experiences. They noted that group moderation discussions resulted in preservice teachers becoming aware of a wider range of features that could be noticed and valued. They also noted the tendency of preservice teachers to want to rank scripts - moving towards norm referencing - and to ask for ever greater detail in the criterion descriptions. In our research, by focusing on examination questions and mark-schemes that are intended to be tightly specified, we aimed to study a 'critical' case where the ambiguity in the interpretation is already much reduced, leaving teachers free to consider the match between their values and the mark-scheme.

Lavi and Shriki (2014) identify the emotional impact of feedback when beginner teachers engage in peer assessment of proof, devising their own mark schemes and reports. Their beginner teachers initially had difficulty in identifying the values that might underpin criteria for constructing mark schemes, and their mathematics knowledge for teaching increased after reflection on the values of the discipline and on the worth of alternative solutions. Although the evidence on learning to assess is scarce, their work suggests that beginning teachers will need to work through feelings and beliefs related to their own experiences of assessment, as well as – and possibly before - turning their attention to drawing on their experiences of teaching.

Webb (2009) reports on middle grade *experienced* teachers engaging in written assessment design, highlighting the interpretive skills they use, especially regarding student strategies and evidence of students' understanding. He notes the extensive demands that designing assessments made on these teachers' mathematical knowledge for teaching, and how they supported their discussions with a range of informal mathematical representations and

strategies drawn from classroom experience. Such expertise is clearly demanding for all – but particularly for beginner teachers who have not yet built up knowledge of student thinking. In trialling our workshop with experienced teachers, we were looking for such differences in what they brought to the discussion, what concerns they raised and what changes they envisaged. Livingston and Borko (1999) suggest that the core differences between expert and novice teacher functioning are characterised by experts' link-making and strategic thinking, and we expected that experienced teachers would raise more connections with existing practice and envisage more developed plans for change.

1.5 TEACHER VALUES AND BELIEFS IN TIMES OF CURRICULUM CHANGE

Llinares and Kraner (2006) review current thinking on conditions for teacher change, and identify research in three areas: the social dimensions of change; the organisational contextual resources; individual teacher skills including particularly professional reflection. Our study was constructed to entail considerable reflection in the recognisable social context of a professional development workshop. Knowledge was shared amongst peers from different schools and training routes in an atmosphere where it was possible to claim authority and also to have doubts. We aimed to provide some appropriate resources and tasks to support and provoke discussion, as well as to probe what additional resources and support beginner teachers identified as necessary to continue and embed their learning in this area.

Within individual teacher characteristics, our study examined teacher beliefs and values, explicitly asking teachers how the assessment changes corresponded with what they valued in A Level mathematics. Teachers are known to hold a variety of beliefs, in a hierarchy and not necessarily consistent (Leatham, 2006). Informed professional discussion can expose those in ways which enable teachers to re-order that hierarchy (Baker and Johnson, 1998). There is considerable debate about exactly how teacher beliefs interact with practice, but it is thought that they interact in cyclical ways, often mediated by student outcomes, and there is good evidence that supportive beliefs are necessary for the embedding of teacher change (Guskey 2002, Clarke and Hollingsworth 2002). The study was therefore constructed to expose and challenge the beliefs and values beginner (and, for comparison, experienced) teachers bring to the focus curriculum development.

2. THE STUDY

In this section we outline the conduct of the study and the data collection, comprising a web questionnaire for beginner teachers, two workshops for beginner teachers, three similar workshops with experienced teachers and individual follow-up interviews. The group and individual written tasks used in workshops included responses to the examination questions in Figure 1 and, for beginner teachers, the workbooks in Appendices 2 and 3.

Our planned development provision was informed by an online questionnaire (Appendix 1) about assessment experiences sent to 190 secondary mathematics trainees on a variety of ITE routes at our own institution, which is well-respected among Education departments in England with all recent Ofsted inspections of these routes rating them 'outstanding'. Unfortunately, there were only 55 responses, about half from those for whom mathematics had formed a substantial part of their degree, and the rest from those for whom their degree had minor, or no, mathematical component. We note that similar response rates have been identified for the recent national NQT surveys (18-24%), suggesting this is not an efficient way to collect representative data from beginner teachers (NCTL, 2016).

We asked about teacher education route, teachers' own A level and degree background, confidence for teaching year 12 or year 13 A Level mathematics, experience of formal mark schemes, response to student solutions that differed from the mark scheme, and familiarity with the recently-developed papers for GCSE Mathematics, the usual foundation for A Level Mathematics.

This same group of 190 beginner teachers were invited to sign up, during their final professional day, for an opt-in session that focused on the changes in the nature of A-level examination questions and issues of teaching for these questions. The session was open to all, and participants were sent a question as preparatory work. They were requested to spend twenty minutes attempting the question and then email us an anonymised electronic version of their solutions: we underlined the learning benefit for us all of them not being able to master questions in that time, but encouraged participants to pursue question afterwards if they wanted to. Participation was on a first-come, first-served basis, but in the event we ran two parallel sessions, each two hours in length and each with about 25 participants, with some having completed the questionnaire, some worked the questions thoroughly and others not at all. The workshops were followed by four individual interviews with a purposive sample of participants, chosen amongst volunteers to represent a range of teacher preparation routes and mathematical backgrounds.

The workshops focussed on the policy-driven shift from fairly structured to semi-structured questions on AS mathematics content. As discussed above, the literature on teacher learning, assessment and problem solving informed our approach. We used past questions and new sample assessment materials to demonstrate intended changes, educate participants in the use of standard mark schemes, and probe their response to changes, both in principle as representatives of the discipline and for their implications for teaching and learning.

Beginner teachers were initially introduced to the use of standard mark schemes for fairly closed AS-level questions, since many were not familiar with these. They then used selected AS-level specimen questions to develop their understanding of teaching for such problems.

They evaluated and assessed authentic solutions (obtained from their peers and, by us, from school students) using informal ranking methods and more formal mark schemes, and were prompted to consider formative feedback. Participants attempted some questions under time pressure, peer-marked resulting solutions and reflected on both the technical challenges of answering and marking questions, and their emotional responses to less structure and to formal assessment of their work. During the workshops they had access to individual resource booklets, and they recorded responses to questions, attempts at marking etc. in either individual work booklets or, for group tasks, group work booklets. We audio recorded workshops by table, and also recorded and transcribed individual interviews. We were able to deliver, and record, broadly similar sessions (without initial introduction re use of traditional mark schemes) to a total of nearly seventy experienced A level teachers over three sessions at two teacher conferences. In each case the teachers opted into the conference and the session, so again, the sample is not representative. However, this experience enabled us to make some comparisons between the responses of beginner teachers and those of more, and sometimes highly, experienced A-Level teachers, seeking to understand commonalities and differences between groups.

Data comprised web questionnaire responses (50), written group responses (10) and individual responses (53) from workshop activities (Appendices 2 and 3, complemented by resource booklets), transcribed audio recordings of workshop activity for both beginner and experienced teachers, and transcriptions of individual post-workshop interviews (4). Responses of participants in workshops and in follow-up interviews were open- and then axially- and selectively-coded (Charmaz, 2006), resulting in analysis under the three headings knowledge and experience, concerns, values brought by teachers to the study We consider each of those in turn, and then identify the implications for teachers in response to the intended changes, and for their learning.

3. FINDINGS AND DISCUSSION

3.1 KNOWLEDGE AND EXPERIENCE

The web questionnaire and workshops showed these beginner teachers, as one might expect, brought very limited experience of A Level teaching to the study – and most had also had very limited, or no, exposure to A Level students or classrooms.

Of the 50 who responded to the web survey, about two-thirds felt confident to teach at least one year of A Level Mathematics (though in workshops we saw some very superficial understandings of content). Over half, all of them on school-led routes, had not at that stage (May/June 2015) seen a specimen assessment for either the GCSE or A-level examination, despite the fact that they were immersed in schools and would be teaching towards the new GCSE assessment from September 2015.

School experience with assessment of GCSE or A Level questions was largely confined to informal assessment of their 'correctness', without reference to formal mark schemes. Only ten of fifty questionnaire respondents had used such schemes, with eight of these marking a class set of papers: the other forty had only informal experience with existing GCSE questions at a time when all might expect to be teaching for these high-stakes examinations immediately after qualifying. Many had little experience, either, of year 11 classes, with schools largely timetabling their beginner teachers to years without external assessments. In a high-stakes assessment culture this is not surprising, but it means participants embarking on a career as qualified teachers have little knowledge of examination classes and students with a teacher lens. Additionally, in the survey and the workshops, few claimed any mentoring conversations about teaching for high-stakes examination courses.

Beginner teachers did, though, draw extensively on their own experiences at school and university when talking about learning for A Level mathematics and preparation for examinations. This has the advantage of accessing emotions and experiences with a student lens, but also the disadvantage of focusing on particular contextualised and personal experiences rather than accessing the range of student thinking ideas or an evidence base. Many remembered their two years in the sixth form as dominated by preparation for examinations. A quarter mentioned that university or career mathematics required greater attention to reasoning and problem solving.

Our beginner teacher participants all claimed an aspiration to teach at least AS Level Mathematics in the near future – hardly surprising in this self-selecting sample. However, when faced with either traditional or new-style questions on basic AS or GCSE content, many evidenced highly procedural ways of thinking about questions, apparently lacking deep conceptual understanding, or indeed core fluency. For example, one question applied knowledge about the relationship between gradients of perpendicular lines, a topic included in the GCSE specification. When asked follow-up questions about teaching (*'How do you know that? How would you convince students that's so?'*), none of the five teachers on that table could begin to attempt an explanation, despite three of them having gained first degrees in mathematics. More prosaically, and even more worryingly, some teachers converted fractions to decimal approximations in order to carry out basic calculations. These same teachers claimed themselves 'confident to teach most A Level mathematics'.

Beginner teachers quickly understood and were able to apply standard mark schemes to authentic solutions to traditional AS questions. The notion of reliability was implicit in their concern to achieve the knowledge *for* assessment that would establish them as accomplished teachers able to agree exactly on each mark allocations. They were largely able to resolve differences among themselves, deferring to the knowledge-in-practice of more experienced colleagues. They were surprised at some of the written expressions and mistakes made by students, and sometimes surprised also that inaccuracies in notation and missing steps of working were condoned. At this stage, however, the discussion stayed with knowledge-in-practice rather than considering the validity of the assessment. For example, one beginner teacher raised a concern about consistency in her school:

> *'There have been a large number of arguments […] about questions in awarding full marks for an answer. There seems to be very little understanding within the teachers at my school about when you are allowed to give just an answer all the marks[…] It's a 3 or 4 mark question. And it's a question where they would have to have some kind of working out. To get the right answer by chance is pretty much impossible. And they have only given an answer. And a teacher has got really annoyed and gone 'I don't want to give them full marks. And a teacher's gone 'Well, you have to give them full marks. They have got the answer right.' […] And I have absolutely no idea how to tell who's right.'*

There is clear alarm here, from a beginner teacher who is seeking an authoritative voice. It's also noticeable that she subjugates teachers' classroom judgements to the authority of the summative mark scheme, which influences what they are "allowed" to do in informal assessments. There is no mention here of calling on a disciplinary rationale for awarding marks to method, for example that chains of reasoning are valued in mathematics. She does however, refer to the emotions of the 'annoyed' teacher, perhaps implying transgression of a belief about the nature of mathematics. Her reported experience is some distance from the articulations of thinking called for by Cochran-Smith and Lytle to develop knowledge in practice.

The language used in the workshop discussions showed the beginning teachers' approach to be comparing a solution step-by-step with the model solution in the mark scheme, checking off individual marks and seeing which ones were lost. Perhaps because they were successful mathematicians themselves, they considered student work as a variation from the model solution. Some were able to identify errors in algebraic manipulation and make links with errors they'd seen with younger students, and when prompted were able to articulate lessons for their teaching.

More experienced teachers in the study were largely familiar with mark schemes for traditional A Level papers, and with a range of typical student errors on material in historic specifications. Many were familiar with proposed changes at a high level, but few showed evidence of having engaged with the implications in much depth. We presented them with a range of five less structured AS 'specimen' problems, including that in Figure 1. Teachers attempted those 'live' under time pressure, and then worked with the associated mark schemes. We asked them what response 'their' year 12 students would make to such questions, and whether teachers considered the mark schemes valid.

Instead of focussing on how to award marks, these teachers talked about a range of approaches to tackling errors, making links with classroom experience. As with the beginner

teachers, however, many of their approaches were essentially procedural. One teacher said *'I major on teaching them how to do it – I don't worry about why, I just teach them how to answer the questions. My students don't do why'*, and several others agreed that they did not encourage students to question the deeper foundations of the material studied. Most appeared to be very familiar with common questions in recent papers, and a number made comments about *'not taking much time over that – it doesn't come up very often'.* Experienced A Level teachers were able to quickly identify errors in attempted solutions to historic questions, and accepted inaccuracies in notation without comment, except for *'that's common though isn't it'*: in general they were well-versed in ways to optimise the marks gained by students in these high-stakes assessments.

In contrast to the beginner teachers, experienced teachers' talk showed them drawing almost exclusively on their teaching experience rather than personal experiences at school or university. Their knowledge of the assessment, and of students in relation to the assessment, appeared fluent. For some, though by no means all, this was in contrast to their exposed conceptual knowledge of the related mathematics and its pedagogy. The first reactions of many were about teaching to get the marks, not teaching the underlying mathematical ideas.

3.2 CONCERNS

The workshops started by outlining intended curriculum and assessment changes, many of which appeared unfamiliar to beginner teachers. These expressed written and spoken concerns primarily from the student perspective, and appeared to value assessments that allow students to feel secure in demonstrating what they can do. None of the beginner participants educated in the English system would have experienced terminal assessment, nor extended A Level papers, though both workshops included teachers who had such experiences in the International Baccalaureate. Most expressed concerns that terminal examinations were too 'high stakes', with little room for error, and 2- or 3-hour papers would be too demanding in terms of stamina. Similarly, they expressed unease that students would have to learn both Mechanics and Statistics, arguing that they might not need both, or that they might not be interested in both, despite evidence (Matthews and Pepper, 2007) that any choice is often made by the centre, sometimes because of teacher preferences. It is clearly challenging for these beginner teachers to move from their own experience to imagine a different system, despite the range of experiences of peers present.

In both the beginner teacher workshops, the initial concerns were about application of mark schemes, especially when these were applied to less structured problems in the new specification. These concerns were framed around consensus, and hence reliability. Teachers recognised that they needed to understand students' solution attempts in order to interpret mark schemes uniformly, but they did not initially consider the implications for teaching. Teachers were also concerned about the reliability and validity of timed examinations in showing students' ability to answer semi-structured questions. They started to invoke their understanding of mathematical values during these discussions, such as when one argued against the changes: *'It's fair to break the problem down given the time pressures of an exam. Getting a proof quickly is not the best way to find a mathematician.'* A detailed discussion of validity only developed when they were asked to compare several student solutions and notice the marks given. There were notable differences of opinion in the values that beginning teachers gave to good communication of reasoning and slips in notation, but also

to repeated calculations, using a standard but inappropriate strategy, and persistence in making sense of a problem.

Experienced teachers also voiced concerns about returning to terminal assessment, some of the most experienced referring to a *'different cohort of students from those we used to have'*. Like beginner teachers, concerns were often couched in terms of the fairness of terminal assessments for high stakes purposes. Some highly experienced, and largely apparently more confident, teachers stated that they valued the necessarily synoptic nature of terminal examinations – but were still nervous about whether students would be able to demonstrate their mathematical capabilities reliably given just one chance – and especially if questions were less predictable and less structured.

Experienced teachers were less inclined to critique the validity of mark schemes. Instead they expressed considerable concerns about their ability to teach for problem solving and reasoning at this level – and particularly in such a way as to be sure students could perform under examination conditions. Beginner teachers might not be as aware of the challenges in teaching for the new specifications, and/or they might have less to lose: some experienced teachers were overt in their expressions of feeling threatened by the changes. One experienced Head of Department said as she worked:

> *'In principle I'm in favour – part of me knows these are the sorts of things I think youngsters should be experiencing in maths, but another part knows I'm successful and efficient at getting students reasonable grades in the current A Level – and ours aren't the strongest students. I'm actually not at all confident I can do the same with this – they're going to have to understand more, and that might just make it too hard for maths to be a sensible choice for them. I'm simply not sure I can deliver on it, which is hard to face at my stage of career.'*
>
> *'I'm now panicking: how can I ever get my students to do this?'* and
>
> *'I've just gone down a blind alley on this question: I think it's a lovely question to be using in class to develop deeper understanding, but having to answer it in a limited time when it's a high-stakes assessment, is another matter and I'm not sure I can look students, or their parents, or SLT, in the eye and say yes, I think we should be encouraging them to do this.'*

Many claimed they would have to make significant changes to their teaching: *'This isn't about learning a new bit of maths, it's about a fundamental change to how I've come to teach'*, though one teacher said *'This validates the way I want to teach: I'm under pressure to produce results by whatever means, but this necessitates genuinely good teaching.'* Some readily drew implications for their teaching of younger students: 'this isn't something you can teach in two years, it's got to be developed over a student's lifetime'. These teachers, already immersed and skilled in the high-stakes nature of teaching for A Level, appeared more aware of the challenge of devising valid and reliable mark schemes for more genuine problem solving. They were willing to leave such conversations to examiners and expressed some scepticism that the intended changes would materialise.

The pedagogical implications drawn by experienced teachers largely focused on specifics of how to build up students' problem solving experiences, and where/how to find appropriate support and resources for that. These responses identify sizeable challenges for even experienced teachers, and with final specifications and assessment materials to be accredited

less than a year before first teaching, it is hard to see how many will be well prepared. However, strikingly, almost all participant experienced teachers, despite their misgivings about enactment, claimed alignment of the intended changes with their deeply-held mathematics education values: 'this is really what maths is about, isn't it – yes, you need the techniques and the knowledge, but unless you can use them they're sterile.' Cooney (1985) and Golding (2016) show such beliefs do not necessarily translate into practice if they are avoidable.

Participants, whatever their experience, were almost unanimous in saying that they would need a range of resources to support them in teaching for the new specifications: none voiced confidence they could either access or adapt sufficient existing materials to support the range of experience needed by students. Horn (2005) points to the central role of reform-driven curriculum resources in the development and embedding of teacher change, and it would appear that these teachers are not confident to attempt the change without such support. Many felt they would be unable without external input to acquire the enhanced subject knowledge needed, let alone the perhaps different pedagogical knowledge needed to engage with unfamiliar areas of the curriculum, in which they included problem solving and reasoning for all students, as well as several areas of applications.

3.3 VALUES AND TENSIONS

Beginner teachers were much less likely than their more experienced colleagues to refer without prompt to their core mathematical values in relation to the intended changes: at this stage in their career, and with the limited school experience they bring, they were more transactional in their talk, with concerns focused on the fairness and reliability of assessments, and the predictability of outcomes, rather than the potential of the new assessments to drive valued changes in classroom priorities for the learning of mathematics. Although some (e.g. Torrance, 2011) question the potential for assessments to actually drive classroom change, there is clear evidence (Golding 2014) that the absence of valid assessments undermines the intentions of highly-valued change even in apparently well-positioned contexts.

However when pressed, beginner teachers, in common with their experienced teachers, were unanimous in stating their in-principle support for the intended changes in emphasis, with most, after being prompted about the implications for learning, articulating the value of the synthesis of mathematical ideas necessary to succeed in terminal assessments. Nevertheless, for all participants but to varying degrees, there remained a tension between recognition that their core mathematical values were aligned with the intended changes, and a lack of confidence that they could reliably deliver successful teaching for that. One beginner teacher said *'I'm confident I could teach successfully for the existing A Level, but not for this – I don't know enough'* and that led to discussion of what it is that the new A Level demands differentially, at least in terms of assessment.

It is interesting that an experienced participant reflected that in principle, he already tried to teach in a way consistent with the new curriculum – but could cut corners where necessary because the assessments allowed him to do so. In his rehearsal of the new curriculum, this area of professional flexibility was denied to him, raising anxiety. Teachers live with a variety of different imperatives where their core subject values might well be in tension with professional values driving them to support students in optimising their A Level grades.

Deeper analysis of mark schemes, and scaffolded reflection on their validity, exposed areas where there was a range of values espoused. For some teachers, clarity of thought and evidence of that in clarity of communication, were valued, together with a valuing of extended and/or sophisticated reasoning over repetition: for example, there was much heated debate about a mark scheme which awarded 4 of 7 marks for routine repeated calculation. Some – and usually the experienced teachers - argued for the needs of less mathematically sophisticated students to gain reasonable marks, while others argued that this would result in comparable total marks being obtained through an unacceptable range of routes, of possibly different mathematical value. . Other differences related to beliefs, and especially the degree to which workshop participants espoused mathematical values such as the development of student deep conceptual understanding, genuine problem solving, struggle and rigorous communication, and were willing to sustain these against the very real professional pressures of school accountability and time.

When discussing or reflecting on teaching, there was some difference between beginner teachers whose first degree was mathematically intense and those for whom mathematics was a relatively small part (though in the questionnaire sample this accounted for only 11 of the 50 respondents). Most participants said they would assess student alternative solutions in the classroom in terms of their mathematical reasoning and techniques, with just 6 of 50 saying 'As long as they both get the correct answer, it doesn't matter'. Participants varied in their claimed confidence to teach the new AS or A Level, and though in general those with more intense mathematical backgrounds were more confident, there were instances of teachers with no mathematics beyond A Level themselves claiming the strongest degree of confidence (*'Although I welcome support, I would be confident to prepare myself to teach this area, including semi-structured or more probing problems'*). In some cases, the evidence from workshops is that there would be a lot of preparation needed!

As discussed above, teacher beliefs are not necessarily consistent, and they are held in a hierarchy. We know that beliefs in relation to problem solving can be fragile, and these beginner teachers might well not enact their espoused beliefs. Leatham (2006) suggests such apparent contradictions should be understood as 'sensible systems' under which teachers have not necessarily changed their beliefs, but those are trumped by other, possibly emergent, beliefs. Here, for example, those might be about an overriding imperative to optimise outcome grades, together with a belief that in a limited time that can best be achieved by focusing on procedure. This latter approach appeared consistent with the talk in experienced teacher workshops. We did not have access to their mathematical backgrounds, but many appeared to be quite fluent with the mathematical content, and to support the aspirations of the new curriculum, which include a conceptual fluency necessary for genuine reasoning and problem solving. For many, though, there were clear concerns in the hierarchy of their beliefs, and tensions with their beliefs around what is achievable in the time and with the students they have.

In beginner teacher interviews, where we were able to probe some of these ideas more deeply (Appendix 4), those from a more mathematical background perceived tensions as existing between their own discipline-steeped decision making and the shared pedagogical craft practices they realised were valuable. They gave examples such as *'how much work should be marked and in what way?', 'how much mathematical working is acceptable?', 'at what stage does a teacher 'cut out' from aiming at understanding and resort to 'teaching how to do it'?'* and *'how does a teacher respond to students asking 'is this on the syllabus?'?'*. As

beginner teachers these participants were not free to make all such decisions themselves, and talked about having to abide by department and school practices. There is some evidence that such tensions can remain an issue for teachers through the early years of their career (Smith and Golding, 2015) and that the nature of the department as a professional community can frame the relationship between teachers' core subject professional values and offer resistance to pressures to conform to short-term, easily-measurable but not always deeply-valued goals (Golding and Smith, 2015).

3.4 TEACHERS' RESPONSE TO INTENDED CHANGE

Participants, whether beginner or experienced teachers, gave unanimous support to the principles behind the intended changes, though they feel it will be demanding to teach and to learn. Many teachers, both beginner and experienced, have considerable doubts about their own preparedness or even capability to teach it effectively, and for experienced teachers this doubt manifested itself widely as 'I'm not at all sure I can teach it and get the same results as I'm getting now': they perceive a considerable element of threat to their professional success and standing as previously-'successful' teachers of A Level. For such teachers, emotional as well as practical support is therefore important. Threats to a 'successful' persona are less of an issue for beginner teachers of course, but evidence from beginner teacher workshops suggests they do need considerable support with both the content and the pedagogy involved. Both beginner and experienced teachers widely expressed the view that they are in need of large banks of suitable problems and other resources to draw on, as they begin to enact this new curriculum.

The range of teachers problematized the associated single, timed assessment window and long examinations while simultaneously recognising the inappropriateness of short examinations for genuine problem solving. They also expressed significant scepticism about students being able to achieve the intended outcomes – and also about Awarding Bodies' commitment to following through the intentions in a principled manner given that market share is a big issue for Awarding Organisations in this large entry subject. Teachers, particularly the more experienced, justified their scepticism by referring to their perceptions of the introduction of the new GCSE, where they feel that market forces are already compromising curriculum intentions.

4. LEARNING AND STRUCTURES FOR LEARNING

We report here on what beginner teachers learnt from the two-hour workshop, drawing on claims made in their spoken and written reflections and our analysis of these and subsequent interviews. It is worth noting that two of the four beginner teachers interviewed made further claims to learning deriving direct from those interviews, consistent with Baker and Johnson (1998), who conceptualise some interviews as focused 'professional talk' with the potential to support comparatively deep professional learning.

As intended, beginner teachers claimed they had gained knowledge of, and skills in, the use of formal mark schemes, as well as knowledge of a number of related pedagogical strategies that would support them in making formative use of mark schemes, including peer marking. They claimed improved knowledge of the new curriculum and its implications for their teaching, as well as techniques for teaching mathematical communication (as in Webb, 2009).

It was possible in interviews to probe in more depth individual understandings of student solutions, their rankings of different aspects of those, and the state of their related formative assessment knowledge and skills. Two identified, in passing, that collaborative approaches had enable them to understand aspects of the solutions which at this stage would not otherwise have occurred to them; three of the four identified aspects of the given mark scheme with which they disagreed because it was not consistent with their own values. They were able to talk about how, even with this awareness, their necessary engagement with assessment might lead to distortion of teaching so that it became inconsistent with their core subject- and professional values. More generally, two felt that the workshops highlighted for them the tensions experienced in school between choices *'as a mathematician'* and *'for exam results',* and the challenges in following through one's values on a daily basis. They talked about *'you really have to believe it a lot'* and the difficulty in standing out against a department, especially given their perception of overcrowded curricula.

The choice of tasks in the development of mathematics knowledge for teaching is known to be critical (Suzuka et al, 2009). Many of the curriculum-related benefits appeared to be related to the use of 'live' solutions – where participants attempted problems under time pressure then assessed with a mark scheme. Teachers talked about internalising the changes, and they were able to develop reflections on their own experience into appreciating some of the different demands on students:

> *'It strikes me as much different as a typical sequence of questions. I think we talked about it the other day as well. You look at a sequence and think, OK; what kind of sequence is this? What formula do you need to use? […] If I didn't see what was happening; maybe I would have put them as a sequence and see if there was some sort of pattern I could find. But luckily I found the recursion. So I think there you have to identify that that's important. Once you see that again... Maybe a student, maybe someone would have to have the next one to see the numbers to start repeating.'*

They also valued the use of authentic student solutions, provided by us and showing a range of errors and approaches. Teachers commonly talked about having improved their skills in interpreting and 'unpicking' student strategies, including diagnosing errors, and enhanced their understanding of effective ways to give written and oral feedback, particularly in response to semi-structured problems (as in Grainger and Adie, 2014). During the interviews we noted that beginner teachers continued to find difficulties in following student work. They

were unused to lapses or incongruous notation, such as subscripts replaced by superscripts, decimals being substituted for fractions or the grid method for multiplication being used at A-level, and they expressed a strong belief in coherent communication of reasoning. Many however treated this as a lapse in effort:

> *'So...if I was marking the question in class and I had an A-level class, I'd probably give them 5 out of 7 because I'd say, well you didn't show your workings so I'm not giving you full marks. And you know, just say to them again, I don't care if ...you know, you had enough time.' (Interview 3)*

One teacher was able in interview to consider some of the advantages for problem solving of '*just writ[ing] down what we can work out from the problem'* but also to replay episodes of teaching weaker students, and reflect that recording work linearly helps some students to self-check (*'they can at least refer back and say, well I did this here because... ')* whereas others can be frozen ('*I've got another kid who was very, very structured, but after he's written a line he's got no idea what to write next').* (Interview 4)

In summary, we see these two tasks as effective in moving beginner teachers from their initial concerns with gaining knowledge *for* teaching and for achieving reliability towards appreciating the opportunities and challenges of teaching to a more valid, but also more open-ended, test. The conflicts and the new thinking they provoked were instrumental in teachers replaying their own assessment experiences and conversations in school in order to further a shared professional understanding, while at the same time appreciating that judgements were more complex than they had expected (or hoped). In all four follow-up interviews, teachers expressed appreciation of the opportunity to talk about values and tensions in specific teaching situations, and two identified the importance to them of input that supported and challenged the discussion so that teachers' learning was deep and sustainable. One had experienced such discussions in a reflective and knowledgeable school department and felt confident that they would have opportunities for developing such knowledge of practice. The other three reported that in-school conversations for them had rarely reached beyond the superficial, partly because of perceived time- and syllabus-coverage pressures. This meant that they particularly valued the in-depth nature of the workshops experienced in the study, and the input from Higher Education.

Our study beginner teachers, as with Lavi and Shriki's (2014) participants, were outspoken about the emotional impact of attempting 'live' unstructured questions under time pressure, and of often failing to complete those, as well as the considerable vulnerability they felt when their work was marked and commented on by peers. They were able to make links to the implications for their own classrooms – throughout the secondary school, not just post-16 - though the extent to which that will be followed through in practice is unknown.

Participants also claimed a renewed, or sometimes new, appreciation of the benefits of group approaches to problem solving, of the need to build up student resilience, of the value of teacher modelling – and of the importance of teachers working A Level questions themselves, including in relation to the mark scheme. Some articulated the considerable value to them of teacher peer discussion of the issues involved in developing teaching and assessing for this re-developed new A Level curriculum.

As above, this learning might well be transient: ideally these ideas would be sustained over time, interspersed by school-based experience of their enactment since the literature

suggests this is the best way to embed teacher development (Joubert and Sutherland, 2008). There are some reasons to doubt whether this would be prioritised in the induction of these beginner teachers. No interviewees had had in-school discussion about teaching (or assessing) for problem solving at any level - yet in three months all would be teaching 14-16 year olds for a new curriculum rich in problem solving. All made repeated reference to their own mathematical background, and this appeared to frame their current approach, as in Grainger and Adie (2014). Those with a more mathematical background were able to reason quite deeply about the challenges involved in teaching for problem solving, including a need for developing robust fluency and a long term approach. After the workshops, all centralised challenges of formative assessment for problem solving in their talk, but for some, mathematical awareness of related issues such as communication and rigorous reasoning still seemed to us to be comparatively low.

5. CONCLUSIONS AND RECOMMENDATIONS

Beginner teachers in our study often appeared ill-equipped to engage with key aspects of teaching for problem solving, or with formal summative assessment.

Although we do not claim the responses are representative of all entrants into English secondary mathematics teaching we would highlight some aspects as being of concern. First, as we discuss below, workshops showed many of these beginner teachers appear to have inflated ideas of their own preparedness for teaching mathematics at a higher level. Secondly, many respondents had at that stage – about six weeks before they qualified as teachers - experienced little education in the use of formal mark schemes or of new content or assessments for courses they were about to teach. We would argue that in the context of English education, these are important facets of a teacher's work if beginner teachers are to be well-prepared to enter teaching and their students are to have confidence in teachers

A relatively short, structured workshop session enabled many to enact and critique mark schemes, and to engage quite deeply with a variety of related pedagogical issues, including those of student affect. Teachers used professional interactions with us and with their peers to 'replay' their previous or observed practice and compare and contrast that with possible enactments of the new intentions; they 'rehearsed' specific aspects of pedagogy associated with that, including both formative and summative assessment and articulated a tentative 're-envisioning of practice', in Horn's (2010) terms.

Although wide and significant claims for beginner teacher learning were made, it would be unrealistic to think that such learning was embedded and it should be further and overtly developed, including in school. Even those with apparently secure and deep subject knowledge identified as problematic the range of professional skills needed to enact envisaged changes effectively. Many experienced A Level teachers also did so, especially if they perceived themselves to have 'weaker' A Level students: expertise is in part very context-specific, although Berliner (2004) cites evidence that expert teachers adapt more quickly and effectively to new expectations. The envisaged problem-solving focus appears to be well-aligned with the mathematical values of many teachers, both beginner and experienced, but teacher preparation for such changes appears minimal, and for some experienced teachers the changes can appear a real threat to their effectiveness.

In relation to the intended policy changes, beginner teachers on reflection perceived them as exciting, promoting aspects of mathematical learning they value deeply, and that they could enjoy teaching. They will also be challenging for both teachers and students. If such ambitious change is to be realised across A Level mathematics classrooms, it would appear that substantial targeted teacher support and development is needed for both beginner and experienced teachers, including emotional support. Face to face sessions are one way of achieving this; parts of the same workshop have now been incorporated into the AMTEC course, one version of which is delivered live online. We are shortly to compare the perceived effectiveness of different modes of delivery. Ideally, such exposure would be only the beginning of a process of inter-professional work on the new curriculum and its assessment. There is a clearly articulated need for exemplification materials from the Awarding Bodies, as well as other resources to support the teaching and learning of approaches to less structured problems. It would appear, too, that the use of tasks where there are multiple reasonable approaches is particularly helpful, as are authentic student scripts which are unclear in their

strategy: teaching for the new A Level would indeed appear to be (Livingstone and Borko 1990) a complex cognitive skill requiring informed improvisational performance and robust affect.

It would seem those beginner teachers who had experience of examination classes and their assessment had acquired valuable experiences. This study suggests all beginner teachers could very productively be inducted into examination classes at appropriate levels, as observers or support if not teachers. Further, it suggests that those supporting/educating mathematics teachers in early career should also give them (preferably extended) opportunities to have challenging professional conversations focused on teaching and assessing, formatively and summatively, for high stakes qualifications – at GCSE if not at A Level. The study suggests that the harnessing of authentic student solutions and of appropriate, moderately mathematically-challenging 'live' participant attempts, are both useful tools with which to develop related subject and subject pedagogical knowledge in this context.

Our study leads to the following recommendations:

1. *For providers of ITE, FMSP and Awarding Bodies:* There is a need for professional development that supports the goals of the new A-level, goes into detail at the level of student responses in the classroom, and exposes the judgements that need to be made when teaching and assessing for problem solving. For beginner teachers, such development should be provided in a sustained and coherent way across the different sites of their initial teacher education and into the early years of their career. The need is acute for teachers in their early career, but experienced teachers also claimed that their established departmental ways of developing professionally were insufficient.

The evidence contributing to this recommendation is:

- The beginner teachers sampled, all of them about to qualify as teachers and having completed substantial school placements, lacked significant experience in the use of formal mark schemes.
- Most professed themselves reasonably confident to teach AS and/or A Level mathematics, yet workshops showed that their deep conceptual understanding of the mathematics involved was often poor, and participants frequently claimed they themselves had experienced highly procedural teaching at A Level.
- However, all teachers sampled perceive the intended changes to be demanding for both teachers and students. Experienced teachers not uncommonly expressed themselves to feel threatened professionally by the scale of the perceived challenges involved.

2. *For providers of ITE and others:* Beginner teachers should have opportunities to observe or support teaching in classes being prepared for high stakes examinations, and to develop reflection on the issues involved. Professional development offered to beginner teachers should include training in the use of formal mark schemes. However, deeper reflection and implications or school practice are facilitated by marking and comparing live and authentic student solutions. These activities are valuable for all teachers.

The evidence contributing to this recommendation is:

- Beginner teachers sampled claimed to have had few meaningful professional discussions about preparation of students for high stakes examinations, and most had little experience of examination-year classes at either GCSE or A Level.
- Short (2-hour) beginner teacher workshops, carefully structured, appeared to offer potential for considerable learning around the understanding and use of formal mark schemes, and the implications of less structured questions for teaching, learning and assessment. Teachers developed awareness of their own professional values, the role of affect in the classroom, and ways to support students in building up problem solving skills, deepening conceptual understanding, and developing communication.
- The use of authentic student solutions proved constructive in developing teachers' interpretation of student work and formative responses. Similarly, the role of 'live' teacher solutions to sample AS problems appeared to support teachers' development of a student lens on summative assessment. Both these tools were fruitful in the exposure and development of teachers' own subject and subject pedagogical knowledge.

3. *For FMSP and other providers of professional development:* The FMSP and other providers can draw on considerable support amongst teachers for the values implicit in the new assessments. However teachers were unsure how to resolve tensions between these values and others they held around student inclusion, well-being and how they measured their own professional status. FMSP training should invite teachers to articulate reasons behind their judgements, invoking their knowledge of disciplinary (mathematical) values as well as their knowledge of students. It should consider at what stage students can benefit from seeing formal mark schemes when attempting problem solving.

The evidence contributing to this recommendation is:

- The range of beginner and experienced teachers sampled for this study are supportive of the intentions inherent in the new specifications at A Level, with many branding them 'exciting'.
- Interviews and workshops showed many beginner teachers experience tensions between their core discipline-based beliefs and the everyday, performance-driven priorities evident in their schools.
- The range of teachers lack confidence in the appropriateness of timed written terminal papers for the valid and reliable assessment of genuine problem solving and reasoning.

4. *The FMSP and/or Awarding Bodies*: The FMSP should perhaps work with Awarding Organisations provide and publicise a bank of short, semi-structured questions that allow multiple solution routes, along with sample student solutions and suggested mark schemes. There is a dearth of available material that exemplifies what is valuable in such problems, how they could be scored and how judgements can be made. We note the historic documents, Pupils' work assessed: Key Stage 3 Mathematics (1993) have suitable detail, although based on levelling coursework rather than examination questions.

The evidence contributing to this recommendation is:

- Both beginner and experienced teachers feel strongly they need access to more problem solving materials from which to draw; experienced teachers widely claim

they need significant subject knowledge development, as well as subject pedagogical knowledge development, in order to be able to teach the new A Levels effectively.

5. *Teachers and teacher educators* should consider carefully at what stage, and with what purpose, young people can best benefit from seeing formal mark schemes when attempting problem solving in preparation for external assessment.

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APPENDIX 1: ITE QUESTIONNAIRE

Current A Level Mathematics papers have fairly closed questions, so students can learn a limited range of techniques and get good marks through that; the new Mathematics A Level for first teaching in September 2017 is intended to incorporate more genuine problem solving questions, with less structure and often a variety of suitable approaches. This questionnaire is designed to find out how familiar you are with current proposals for changes in Mathematics curricula, and how confident you would feel about teaching and assessing those at AS/A Level.

1. *Your ITE route:* traditional HE-led PGCE/School Direct fee paying/School Direct Salaried/Teach First/any other route
2. *Your mathematical background:* AL Mathematics grade? FM AS grade? FM A Level grade? First Degree? From which university? Anything else?

3. Preparedness for teaching A2 Pure Mathematics

- Which of these statements best indicates how confident you feel to teach the topics in A2-level pure mathematics - currently C3 and C4 (assuming you had normal preparation time and colleagues' advice)?
- I do not feel confident to teach in this area, even with support
- I feel confident to teach this area at present, with support, but not if the questions become much more demanding
- I feel confident to teach this area even if questions become significantly less structured or more probing, although I would need to seek some input from more experienced colleagues or from training
- Although I welcome support, I would be confident to prepare myself to teach this area, including semi-structured or more probing questions

5. Preparedness for teaching AS Pure Mathematics

- Which of these statements best indicates how confident you feel to teach the topics in AS-level pure mathematics - currently C1 and C2 (assuming you had normal preparation time and colleagues' advice)?
- I do not feel confident to teach in this area, even with support
- I feel confident to teach this area at present, with support, but not if the questions become much more demanding
- I feel confident to teach this area even if questions become significantly less structured or more probing, although I would need to seek some input from more experienced colleagues or from training
- Although I welcome support, I would be confident to prepare myself to teach this area, including semi-structured or more probing questions

6. . Experience of external summative assessment

What experience do you now have of GCSE, AS or A Level Mathematics summative assessment? (tick all that apply)

- None
- In-class experience of informally assessing GCSE questions
- I have used exam board mark schemes to allocate marks to GCSE questions
- I have marked a class set of GCSE past papers using the given mark scheme
- I have assessed AS or AL questions informally
- I have used exam board mark schemes to allocate marks to AS or AL questions
- I have marked a class set of AS or AL past papers using the given mark scheme

6. Response to student solutions
   Which of these best describes your response when two students approach the same examination question in different ways? (tick one) :
   - It is best if students follow the approach I have modelled
   - I check the mark scheme for the main approved method and advise them to follow that
   - As long as they get the right answer, it doesn't matter
   - I assess myself whether it is an alternative method that examiners should give marks to
   - I assess myself whether it is an alternative method that demonstrates mathematical reasoning and techniques

7. New style of GCSE questions
   New GCSE papers, for first examination in June 2017, incorporate more problem solving questions than has been the case in the last few years, for both foundation and higher tiers. Which best describes your response to that? (tick one)
   - I haven't seen papers for the new GCSE so have not formed an opinion
   - I haven't seen papers for the new GCSE but I am confident in my skills of teaching students to tackle problem solving questions
   - I have seen papers for the new GCSE and I am fairly confident I could teach most students to tackle the problem solving questions on them
   - I have seen papers for the new GCSE and I am confident I could teach almost all students to tackle the problem solving questions on them at an appropriate level

8. Some of the semi-structured problems being developed on C1/C2 material are quite interesting, and raise issues about how best to teach and (formatively and summatively) assess them. If you would like to see a selection of such questions, please put your name and preferred email address here and we will send them to you. Please read the email we send carefully!

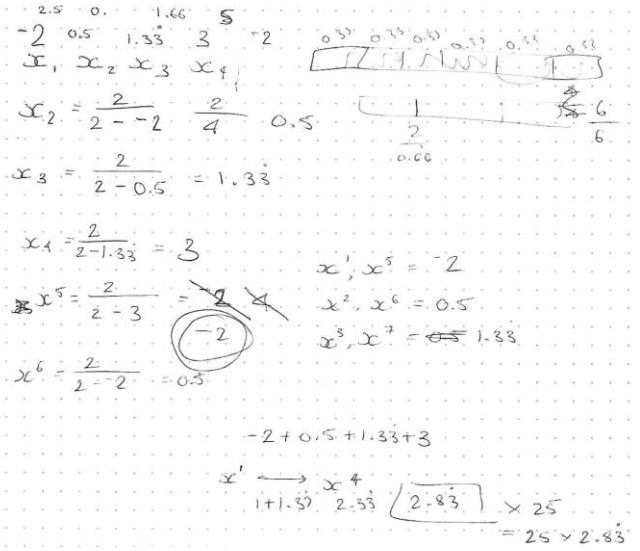
Name:

Preferred email address:

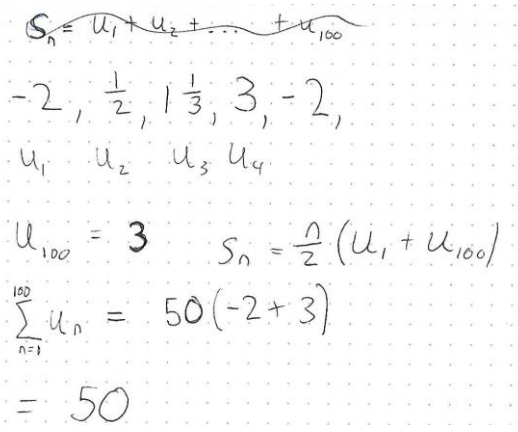
Qu1: Are the described curriculum changes consistent with what you value in A Level mathematics? Why (not)?

Qu2: Use the mark scheme in your resources booklet to try to mark your allocated solution. Note any issues.

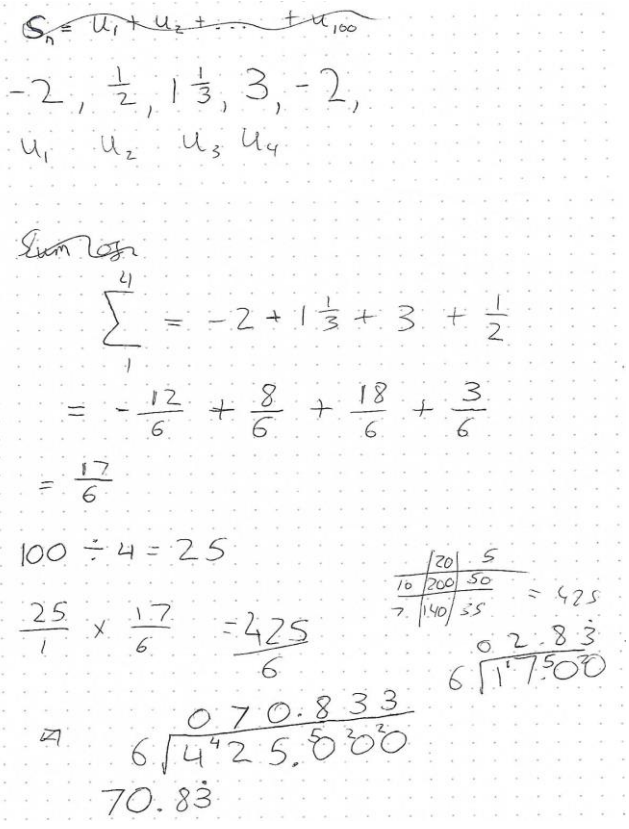
Question A solution 1:



Question A solution 2:



Question A solution 3:



Qu3: You have 2 minutes to write down a start to answering the rectangle question, below.

- Rotate solutions on your table and consult the mark scheme.
- Does the mark scheme reflect what are important aspects of the problem solving process? Were there any challenges in using it?

Qu4: What have you learned for your future teaching, from trying these questions and using the mark schemes?

APPENDIX 2: GROUP WORK BOOKLET FOR ITE WORKSHOP

Group sheet 1: Did any of you complete the problem set prior to the session? If so, do refer to those as well as to the materials here today.

What is your response to these changes in curriculum and assessment? Do you foresee any difficulties or challenges with teaching, learning and assessment of them?

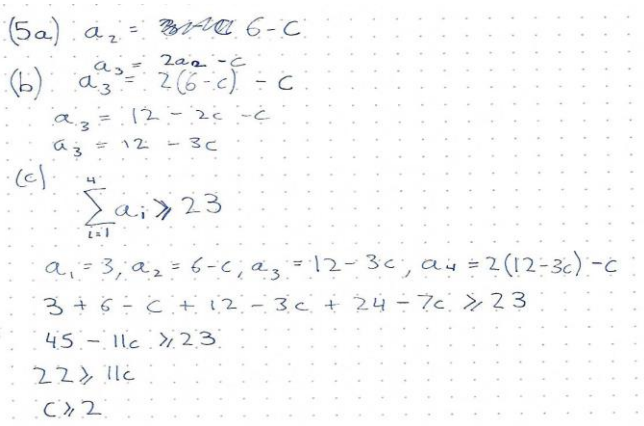
Group sheet 2: How does question B compare with question A? What is the same and what is different?

- On your table, compare and rank qualitatively the 2 given solutions to question B. What are you valuing?
- Now give solution 1 to half the table and solution 2 to the other half. Use the MS to agree a mark.

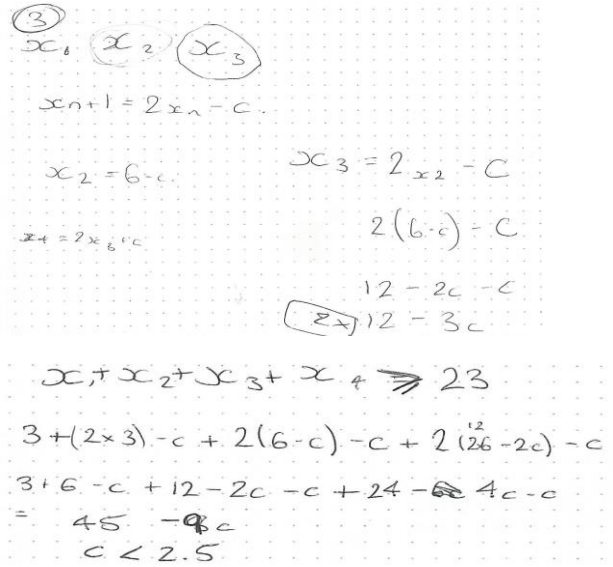
Solution 1: ***Solution 2:***

- Are there any issues?
- Do the outcomes reflect what you valued?

Question B Solution 1



Question B Solution 2



Group Sheet 3: What does your table think A Level teaching is about? What is it you value about A Level Mathematics? Record the range of responses.

Group Sheet 4: *Discuss and record:*

- How it feels to be attempting, and be marked on, these questions (refer to those we've worked today and also, if relevant, the set circulated previously)
- What demands on students are made by less structured questions?
- What implications for teaching have you identified so that students can attempt less structured questions?

Group Sheet 5: *Discuss and record* your responses to today's session as a learning experience

- Should such sessions be incorporated into ITE courses? If so, how would they need to be changed to be most effective?

- Look at the three solutions to Q3.
    - What are the strengths and weaknesses of each? How would you rank them? Why?
    - How do they compare with what you would like to see in your students' a) working strategies and b) written answers?
    - Now talk me through allocating marks according to the markscheme: are there any questions? Do the marks reflect your judgements?
    - For these three students, what do you think they need to work on next? What feedback would you give them? How would you adapt your teaching of this class if they were A-level or extension year 11?
    - How would your feedback or response be different for a GCSE class?
- Thinking about the workshop more generally: were there any surprises for you in the process of agreeing marks with other teachers?

  Prompt: Are there aspects you only noticed when others pointed them out?
- What do you feel should be the relationship between teachers and examiners? Do you see yourself as taking a future interest in discussions about what should be assessed in summative examinations?
- What kind of discussions have you had with teachers in school about problems that require students to apply their knowledge flexibly? *Have you met tasks with multiple solutions in schools? Have you developed any ideas about encouraging students to compare possible strategies, or to judge their efficiency? About how to assess formatively?*
- Who do you think assessment is for? *(prompt: formative/summative)*
- Finally, In the light of changes in the curriculum and related assessment to include more problem solving, what are your thoughts about how you need to develop your practice over the next year or more?
- *Thank you.*