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Multi-Start Methods for the Capacitated Clustering Problem

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Abstract

In this work, we investigate the adaptation of the Greedy Randomized Adaptive Search Procedure (GRASP) and Iterated Greedy methodologies to the Capacitated Clustering Problem (CCP). In particular, we focus on the effect of the balance between randomization and greediness on the performance of these multi-start heuristic search methods when solving this NP-hard problem. The former is a memory-less approach that constructs independent solutions, while the latter is a memory-based method that constructs linked solutions, obtained by partially rebuilding previous ones. Both are based on the combination of greediness and randomization in the constructive process, and coupled with a subsequent local search phase.

1 Introduction

Multi-start heuristic procedures were originally conceived as a way to exploit local or neighborhood search, by simply apply the search multiple times starting from different random initial solutions. Modern multi-start heuristic methods for combinatorial optimization problems usually incorporate a powerful form of diversification in the generation of solutions to help overcome local optimality. Without this diversification, such methods can become confined to a small region of the solution space, making it difficult, if not impossible, to find a global optimum. Most of such methods perform these steps iteratively: apply a randomized constructive method followed by a local search procedure. In these methods, diversification comes from the iterative randomized construction of solutions.

Multi-start heuristic methods for combinatorial optimization can be classified as suggested by [5] in memory-based and memory-less procedures. GRASP (Greedy Randomized Adaptive Search procedure) is probably the best well-known memory-less multi-start heuristic method [8], while Tabu Search [3] is nowadays a reference for memory based approaches. In this paper, we focus on both memory-based and memory-less multi-start heuristic methods, and investigate the effect of randomization in these designs. We use the Capacitated Clustering Problem (CCP), an NP-hard combinatorial optimization problem, as a test case for our proposals and findings. Another memory-based multi-start method is the Iterated Greedy (IG). This method generates a sequence of solutions by iterating over a greedy constructive heuristic using two main phases: destruction and construction. IG is easy to implement that has exhibited state-of-the-art performance in some settings [9].

In this work, we investigate these two successful methodologies in multi-start methods: GRASP and Iterated Greedy, and their hybridization. The former constructs *independent* solutions, while the latter can be viewed as a constructive method of *linked* solutions. These are two very different approaches to construct a solution. Both methods combine greediness and randomization in different ways. The aim of this investigation is to identify ways to exploit better greediness and randomization. For our experiments, we consider the CCP, which is a difficult optimization problem. However, our objective is to identify effective strategies and patterns that could succeed in other settings. Hence, the intended contribution of this paper is to exploit greediness and randomization within the context of multi-start heuristic search effectively. In a broader sense, we can say that we are comparing memory-less and memory-based designs within constructive methods.

2 Capacitated Clustering Problem

The aim of the Capacitated Clustering Problem (CCP) is to obtain a partition of the set of points into different groups in order to optimize some weighted measure of distance among the points in the same group. The most recent applications of this problem are in the context of facility planners at mail processing and distribution centers within the US Postal Service. In particular, the design of the zones to help rationalize the bulk movement of mail, see [1]. Furthermore, Morán-Mirabal et al. in [7] applied an equivalent in the context of mobility networks.

Given a graph $G = (V, E)$ where V is a set of n nodes and E is a set of edges, let $w_i \geq 0$ be the weight of node $i \in V$ and let c_{ij} be the benefit of edge $(i, j) \in E$. The Capacitated Clustering Problem (CCP) consists of partition V into p clusters in such a way that the sum of the weights of the elements in each cluster is within some integer capacity limits, L and U, and the sum of the benefits between the pairs of elements in the same cluster is maximized. The CCP can be formulated as a quadratic integer program with binary variables x_{ik} that take the value of 1 if element i is in cluster k and 0 otherwise. The objective function adds the total benefit of all pairs of elements that belong to the same cluster.

$$
\begin{aligned}
(CCP)\ Maximize \quad & \sum_{k=1}^{p} \sum_{i=1}^{n-1} \sum_{j>i}^{n} c_{ij} x_{ik} x_{jk} & \\
subject\ to \quad & \sum_{k=1}^{p} x_{ik} = 1 & i = 1,2, \dots, n \\
& L \leq \sum_{i=1}^{n} w_i x_{ik} \leq U & k = 1,2, \dots, p \\
& x_{ik} \in \{0,1\} & i = 1, \dots, n \ \ k = 1, \dots, p
\end{aligned}
$$

3 Iterated Greedy and GRASP

The GRASP that we consider in this work, called GRASP2-1, has the constructive method in [6] but the improvement method performs 2-1 exchanges (IM2-1). This neighborhood explores the exchange of two nodes, say i and j, in the same cluster k with a node l in another cluster s. This move [6] can be simply called a 2-1 exchange, and it makes possible to swap nodes that individually are not allowed for reasons of capacity.

The Iterated Greedy method (IG) alternates between destructive and constructive phases. During the destructive phase, some elements are removed from the solution. Next, it applies a greedy constructive method to reconstruct the partial solution and obtain a new solution. Then, an acceptance criterion is applied to decide whether the new solution replaces the current solution or not. The method iterates following this pattern until a stopping criterion is met. We investigate two different IG algorithms, IG and a hybridization between IG and GRASP called IG-GRASP. Our first implementation of the Iterated Greedy methodology, called simply IG, starts from an initial solution x, built with the CM algorithm and improved with IM2-1. Then, IG iteratively alternates between destructive and constructive phases.

In the destructive phase, a percentage of the nodes are removed randomly from each cluster. Then, the constructive phase applies the greedy heuristic CM to reconstruct the solution. Additionally, the local search phase IM2-1 is applied to improve the new solution. Our hybridization initially, as in IG, it builds a complete solution with CM and then improves it with IM2-1. Then, the algorithm iteratively applies a destructive algorithm based on a greedy, then the constructive method CM, and finally the improvement procedure IM2-1. However, after a number of pre-established iterations applying these three methods consecutively with no improvement, instead of ending the procedure (as it is the case of IG), the hybrid algorithm resorts to GRASP2-1 to generate a new solution (built from scratch) to start again.

An interesting distinction between different IG methods is in the acceptance criterion to select the solution for applying the destructive method. As described in [4], in the 'Replace if better' acceptance criterion, the new solution is accepted only if it provides a better objective function value. In other words, the IG iterates over the best solution found. However, this can lead to stagnation situations of the search due to insufficient diversification. On the other hand, the "Always replace" acceptance criterion applies the destruction phase to the most recently visited solution, independently to its objective function value. This criterion clearly favors diversification over intensification, because it promotes a stochastic search in the space of local optima. We applied the latter one to our IG variants.

We compare our three new methods: GRASP2-1, IG and IG-GRASP on a previously reported benchmark (CCPLIB) with 60 instances, available at http://www.optsicom.es/ccp. The results of our experimentation provide an important lesson about the way in which greediness and randomization are combined in the different methods. It turns out that the IG approach seems more effective than the GRASP methodology to solve the CCP instances. In particular, IG is able to match 38% of best-known solutions with a percent deviation from the best value of 0.54% while GRASP2-1 only matches 2% of them with a 2.39% of deviation. As expected, improved outcomes are obtained when these methodologies are hybridized in IG-GRASP, which obtains a 70% of the best-known solutions, and exhibits a percent deviation of 0.24%.

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