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Seasonality in Irish Economic Statistics

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In analysing short-term economic movements the presence of seasonality in economic indicators is a serious obstacle as it may tend to exaggerate or conceal true changes in economic conditions. It is, therefore, of some importance, firstly, to ascertain which of the main series are affected by seasonal factors and which are not, and secondly, to investigate whether seasonality can be reliably measured and allowed for by appropriate corrections. It is with these two problems that the present study is concerned.

One limitation may be stated at the outset The quarter is held to be a sufficiently short period to study short-term economic movements, and no attempt is made to analyse month-to-month fluctuations. In as far as the data are published in the form of monthly series, they have been transformed into quarterly data by simple totalling or averaging as the case may be.

The series which have been investigated for possible seasonality are as follows:

Volume of production index (transportable goods)
Total number of new houses built (state-aided schemes)
Electricity output
Sales of insurance stamps
Persons engaged in transportable goods industries
Total live register
Percentage of insured persons on live register
Wholesale price index
Consumer price index
Agricultural price index
Import price (unit value) index
Domestic export price (unit value) index

Price index of ordinary stocks and shares

Index of weekly earnings of industrial workers (transportable goods)

Index of retail sales

New private motor cars registered

Total revenue receipts

Total exchequer expenditure

Value of imports

Value of total exports

Money supply

Bank debits (ordinary accounts)

Commercial banking assets: bills, loans and advances

Commercial banking assets: investments

External assets of banking system and departmental funds

Most of the data are published in the economic series or elsewhere in the Irish Statistical Bulletin, or in the Quarterly Bulletin of the Central Bank of Ireland; the money supply is obtained as sum of note and coin circulation and current accounts, following Oslizlok (1962-63).

In addition to the 25 series listed above, the analysis also throws light upon derived series which are obtained as ratios (or differences) between two of the original series, like import and export volume, or import excess.

For 23 out of the 25 series for which the data were available, 20 observations representing quarterly figures for the years 1958-1962 were utilised; the period 1961-1963 was used for retail sales and 1960-1963 for external assets. The observations were formally subjected to an analysis of variance between years and between quarters, thus testing the null hypothesis of no trend and no seasonal variation. Except for the two series based on shorter periods, the sum of squares between years thus had 4 degrees, that between quarters 3 degrees, and the residual sum of squares 12 degrees of freedom. The F - ratios were calculated in the usual way; in addition, the residual coefficient of variation, i.e. the square root of the residual mean square expressed as percentage of the grand mean, was computed to indicate the magnitude of the irregular movements.

At the 5% significance level, the critical F - ratios are 3.26 for the years and 3.49 for the quarters; at the 1% level, the corresponding ratios are 5.41 and 5.95 respectively. The variation between years thus appears to be significant for all series except new houses built, exchequer expenditure and possibly the agricultural and foreign trade price indices. At first sight, the variation between quarters also appears significant for all but 6 of the series analysed, but in interpreting this result, some care must be taken as some reflection will show.

Assume that the time series follows a linear trend. If the trend is rising, this fact will also be reflected in rising quarterly totals from the first to the last quarter. If the series is exactly linear, with no seasonal or irregular component, then it is easy to prove that with m years or 4 m observations, the ratio of mean squares between quarters and between years will be equal to $5/16(m + 1)$. With $m = 5$, this ratio amounts to .0521.

It follows that in cases where the F - ratio between quarters is about 5% of F between years in magnitude, a trend effect upon variation between quarters may be suspected and verified by inspection of the annual and quarterly totals. This effect appears to explain the value of F between quarters entirely in the case of the share price index and to a substantial part in a few other cases.

Table 1. Results of Analysis of Variance

Series	F - ratio		Residual coefficient of variation (%)
	Between years	Between quarters	
Volume of production	139.4	28.5	2.06
New houses built	1.3	.8	13.48
Electricity output	95.4	228.0	2.71
Sales of insurance stamps	7.6	220.9	1.58
Employment (transportable goods industries)	769.3	68.8	.41
Live register	37.4	92.8	4.95
% insured on live register	56.3	55.5	5.11
Wholesale price index	34.9	1.9	.65
Consumer price index	52.0	1.9	.89
Agricultural price index	4.2	6.1	1.59
Import price index	4.1	.9	1.00
Export price index	3.8	1.1	1.82
Price index of stocks and shares	497.7	19.7	2.75
Index of weekly earnings	303.5	31.9	1.18
Index of retail sales	126.0	138.4	1.17
New cars registered	29.9	34.1	.22
Revenue receipts	15.7	133.1	.14
Exchequer expenditure	2.5	7.0	.48
Value of imports	69.4	10.3	3.36
Value of exports	27.6	7.8	5.82
Money supply	118.2	25.0	1.60
Bank debits	65.1	8.6	3.60
Banks' bills, loans and advances	156.6	10.5	2.02
Banks' investments	80.6	2.0	1.63
External assets	65.2	7.6	1.39

With the help of Table 1 and these considerations, it is possible to classify the series into three groups. The series in the first group show no marked seasonal variation, and seasonal correction is not required. For those in the second category, the seasonal variation is very marked and clearly overshadows any irregular movements. There is a third category of series which appear to be influenced by seasonal factors, but these are relatively moderate and combined with fairly large irregular fluctuations. Clearly the series in the third category are those which may present the greatest difficulties. The classification is as follows:

1) No marked seasonal variation:

- New houses built
- Wholesale price index
- Consumer price index
- Import price index
- Export price index
- Price index of stocks and shares
- Banks' bills, loans and advances
- Banks' investments

2) Clearly marked seasonal variation:

- Volume of production
- Electricity output
- Sales of insurance stamps
- Employment
- Live register
- % insured on live register
- Index of weekly earnings
- Index of retail sales
- New cars registered
- Revenue receipts
- Money supply

3) Seasonal variation and irregular movement:

- Agricultural price index
- Exchequer expenditure
- Value of imports
- Value of exports
- Bank debits
- External assets

A mere comparison of quarterly means would, of course, not yield a satisfactory measure of seasonal variation. Various methods of measuring seasonality are available. It is to be expected that with the series in the second group, the choice of method does not substantially influence the result; larger differences may be found for the series in the third group.

From either group two series were selected for further analysis. Electricity output and sales of insurance stamps are examples of series with a large seasonal and a fairly small irregular component. Import value and export value, on the other hand, are series with only moderate seasonality and a substantial irregular element.

It will be assumed that the seasonal variation can be best represented by an index, the simple mean of the four quarters being 100; this seems more appropriate than a constant positive or negative amount for each quarter. Furthermore it is assumed that the seasonal indices may slowly change over time, and that no more than the five previous calendar years should be utilised to obtain seasonal indices which are applicable to the current period. In the present analysis, the observations over the period 1958-1962 were utilised.

There is, of course, a wide variety of procedures which may be followed to obtain seasonal indices, and no exhaustive study is made here. Three basic methods derive from the alternative assumptions that the trend can be represented as a centred unweighted four-quarter moving average, a polynomial, or a quasi-linear trend. The first two of these are standard statistical methods; the third was introduced by Leser (1963) and will be briefly described here in due course.

The moving average may be computed either for the original data or for their logarithms. In the former case, the seasonal index for a quarter is obtained as the unweighted mean of four ratios of observation to moving average multiplied by 100; in the latter case, the logarithm of the seasonal index divided by 100 is the mean of four differences between the logarithms of observation and trend. In either case, an additive adjustment is made to ensure that the indices add up to 400, or their logarithms to 0 respectively.

Further refinements may be introduced into the moving average procedure. Geary (1964 b) advocates an adjustment to give a truer representation of the trend at turning points, which may be applied to all moving averages obtained except the first and last one. Effectively, it means that a weighted moving average is now used as trend.

The construction of polynomial trends with seasonality in the data, by means of orthogonal polynomials has been described by Geary (1964 a). To obtain seasonal indices, the method should be applied to the logarithms of the data. The logarithms of the seasonal indices (divided by 100) are then obtained as means of five differences and add up to 0.

The quasi-linear trend method is designed to yield a trend and seasonal variation which minimise the sum of squares of changes in trend direction plus the sum of squares of the residuals. The computational procedure developed yields the seasonal variation estimates in the first place; the trend values may be derived afterwards if desired. To obtain seasonal indices, logarithms should be used. With 20 observations, indicating the logarithms of the data by Y_1, Y_2, ..., Y_{20} and the seasonal indices by S_1', S_2', S_3', S_4', the formulae for S_1' and S_2' are as follows:

$$\begin{aligned}\log(S_1'/100) = {} & .0607Y_1 - .0664Y_2 - .0438Y_3 - .0525Y_4 \\ & + .1780Y_5 - .0459Y_6 - .0592Y_7 - .0679Y_8 \\ & + .1730Y_9 - .0459Y_{10} - .0592Y_{11} - .0679Y_{12} \\ & + .1730Y_{13} - .0459Y_{14} - .0592Y_{15} - .0729Y_{16} \\ & + .1653Y_{17} - .0459Y_{18} - .0286Y_{19} + .0112Y_{20}\end{aligned}$$

$$\begin{aligned}\log(S_2'/100) = {} & -.0602Y_1 + .1418Y_2 - .0546Y_3 - .0577Y_4 \\ & - .0449Y_5 + .1546Y_6 - .0495Y_7 - .0602Y_8 \\ & - .0449Y_9 + .1546Y_{10} - .0495Y_{11} - .0602Y_{12} \\ & - .0449Y_{13} + .1546Y_{14} - .0495Y_{15} - .0602Y_{16} \\ & - .0551Y_{17} + .1444Y_{18} - .0469Y_{19} - .0117Y_{20}\end{aligned}$$

By writing the coefficients in the second expression in reverse order, the formula for S_3' is obtained; similarly the formula for S_4' is derived from the first expression. The formulae for the trend can be found in the paper by Leser (1963).

One further point may be mentioned. All methods using logarithms, and particularly the last-named method, yield seasonal indices S_1', S_2', S_3', and S_4' such that their geometric mean is 100. It is preferable to have seasonal indices S_1, S_2, S_3 and S_4 such that the arithmetic mean is 100. This is best achieved if the indices are divided by their sum and multiplied by 400, or

$$S_i = 400\, S_i' / \sum_{j=1}^{4} S_j' \qquad (i = 1,2,3,4)$$

If the trend values are required, they should then be inflated in the same proportion as the seasonal indices are deflated. It does not matter then for the computation of the trend and seasonal variation combined whether the original or the adjusted figures for trend and seasonal variation are used.

For the chosen series, alternative estimates of S_1, S_2, S_3 and S_4 have been derived with the help of seven methods or variants of methods; these are:

1 a) Ratio to moving average
1 b) do. with Geary's correction
1 c) Moving average of logarithms
1 d) do. with Geary's correction
2 a) Linear trend
2 b) Quadratic trend
3) Quasi-linear trend

The results are given in Table 2, together with an indicator R'^2 of goodness of fit. R'^2 is in none of the cases an ordinary coefficient of determination. It is based on the central 16 observations, since the moving average method does not yield trend values for the first 2 and last 2 observations unless special devices are introduced. Furthermore, it represents that portion of the sum of squares for the percentage deviations of the 16 observations from their mean which is explained by trend and seasonal variation.

As expected, the various estimates are fairly close to each other in the cases of electricity output and sales of insurance stamps; for value of imports and exports, wider variations are observed. In no instance are there any substantial differences between the results of methods 1 a - 1 d and between the results of 2 a and 2 b. That is to say, if a moving average method is adopted, it does not matter here much whether original data or logarithms are used, nor whether Geary's correction is made or not; if a regression method is chosen, the introduction of a quadratic term has no appreciable effect in any of the series analysed.

Table 2. Estimates for seasonal indices (1958-62) and goodness of fit

Series and method	S_1	S_2	S_3	S_4	R'^2
Electricity output:					
1a	118.3	86.8	81.4	113.5	.9962
1b	118.5	86.9	81.3	113.3	.9948
1c	118.1	86.6	81.5	113.8	.9963
1d	118.1	86.6	81.5	113.8	.9955
2a	118.2	87.0	81.4	113.4	.9943
2b	118.2	87.0	81.4	113.4	.9947
3	118.2	86.9	81.5	113.4	.9974
Sale of insurance stamps:					
1a	114.2	90.1	99.7	96.0	.9903
1b	114.2	90.1	99.7	96.0	.9914
1c	114.3	90.0	99.7	96.0	.9903
1d	114.2	90.1	99.7	96.0	.9913
2a	114.9	89.7	99.7	95.7	.9810
2b	114.9	89.7	99.7	95.7	.9838
3	114.7	89.6	99.7	96.0	.9923
Value of imports:					
1a	104.5	101.8	92.3	101.4	.9666
1b	104.6	101.6	92.4	101.4	.9691
1c	104.5	101.7	92.3	101.5	.9664
1d	104.7	101.4	92.3	101.6	.9693
2a	103.9	102.2	92.2	101.7	.9144
2b	103.8	102.2	92.3	101.7	.9136
3	104.3	102.3	92.2	101.2	.9771
Value of exports:					
1a	99.1	91.5	105.4	104.0	.9682
1b	99.6	91.3	104.9	104.2	.9660
1c	99.1	91.4	105.3	104.2	.9684
1d	99.5	91.4	105.0	104.1	.9675
2a	98.9	93.6	104.6	102.9	.8016
2b	98.9	93.6	104.6	102.9	.8029
3	97.7	93.2	105.1	104.0	.9789

Some guidance as to choice of method is given by the goodness of fit indicator. The linear or quadratic trend methods do not give a good fit in the case of imports and a poor fit in the case of exports; the long-term movements appear to be too complex to be adequately described by a simple quadratic expression. A third-degree polynomial might improve the fit, but the method would become fairly cumbersome. On these grounds, polynomial trends are apparently ruled out for general use in connection with Irish economic time series.

The biggest differences between results obtained by the moving average methods on the one hand and the quasi-linear trend method on the other are found in respect of S_1 and S_2 for the export series. The explanation for the discrepancy lies in a low import value for the first quarter and a high value for the second quarter of 1958, combined with the fact that these quarters, being at the beginning of the observation period, carry a very low weight in the estimation of seasonal variation by a moving average method. If the last two quarters of 1957 and the first two quarters of 1963 were added, method 1a would yield the following estimates:

$$S_1 = 98.1, \; S_2 = 93.4, \; S_3 = 104.9, \; S_4 = 103.6$$

which are close to the results of method 3. In such circumstances, great time stability cannot be expected from the seasonal indices estimated by any method.

Both the moving average and the quasi-linear trend are sufficiently flexible to give, together with the seasonal indices, a good fit for any of the series. The fit yielded by the quasi-linear trend method is the better one; this appears to be a general feature of the method, which is achieved at the expense of a less smooth trend.

For some purposes, the relatively large changes in direction for the quasi-linear trend might be a drawback. There is no theoretical difficulty in modifying the method so as to obtain a smoother trend at the expense of larger residuals, though no explicit formula has been worked out yet.

The main advantage of the quasi-linear trend method over the moving average is that it gives a basis for forecasting, however inadequate such a forecast may be in the absence of other information. Once the data for the last quarter of a calendar year have become available, the method yields trend values for the last two quarters from which the trend may be projected into the next year by linear extrapolation, and if multiplied by the appropriate seasonal indices, forecast values for each quarter are obtained. This has been done here for the selected series in respect of 1963. Table 3 shows the forecast, together with the actual values.

Table 3. Forecast and actual figures 1963.

Series	Quarter			
	I	II	III	IV
Electricity output (mill kwh)				
Forecast	800.6	597.1	568.8	802.4
Actual	852.4	617.1	578.2	813.0
Sales of insurance stamps (No)				
Forecast	7,124	5,514	6,078	5,791
Actual	7,815	5,774	6,285	6,290
Value of imports (£ mill)				
Forecast	79.91	82.28	77.73	89.44
Actual	69.31	79.87	69.95	87.17
Value of exports (£ mill)				
Forecast	43.85	42.62	49.06	49.51
Actual	43.09	47.67	52.47	51.98

Regarding electricity output, the forecast could not foresee the abnormally cold winter and high demand for the first quarter 1963; otherwise it is reasonably satisfactory. The trend in sales of insurance stamps showed a downturn in the second half of 1962, hence the low forecast which was belied by an upturn. Imports showed a steep rise in 1962, and the trend somewhat flattened out at the beginning of 1963 which accounts for the discrepancy. The export forecast is remarkably good for the first quarter 1963; after that, a new upswing took place and the rise was somewhat greater than predicted by the formula.

Once information and data for the new year come in, it is of course possible to modify the forecasts by introducing changes in trend direction while maintaining the values of the seasonal indices.

A task which is easier than forecasting is the seasonal correction of quarterly data as they become available, deflating by the latest estimate of the appropriate seasonal index. This permits comparison with the previous quarter and without explicitly having to work out a trend, a reasonable assessment of the most recent movements in important economic indicators may be made. An extensive application of seasonal corrections, based on the quasi-linear trend method, is intended for the Irish scene in the near future.

References

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