



University of Dundee

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Liang, T.; Knappett, J. A.

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Corresponding Author: Mr. Jonathan Knappett, MA MEng PhD

Corresponding Author's Institution: University of Dundee

First Author: Teng Liang, PhD

Order of Authors: Teng Liang, PhD; Jonathan Knappett, MA MEng PhD

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*Highlights (for review)

Newmark sliding-block procedure for slopes containing vegetation.

Use of mobilised friction angle compatible with the strength of the input motion.

Simulation of non-associative behaviour via an equivalent associative friction angle.

Failure mechanism and yield acceleration from Discontinuity Layout Optimisation.

Potential benefits of roots in reducing slip varies with the height of the slope.

1 Newmark sliding block model for predicting the seismic performance of vegetated slopes

2 T. Liang & J. A. Knappett¹

3 **Abstract:**

4 This paper presents a simplified procedure for predicting the seismic slip of a vegetated slope. This is important
5 for more precise estimation of the hazard associated with seismic landslip of naturally vegetated slopes, and also
6 as a design tool for determining performance improvement when planting is to be used as a protective measure.
7 The analysis procedure consists of two main components. Firstly, Discontinuity Layout Optimisation (DLO)
8 analysis is used to determine the critical seismic slope failure mechanism and estimate the corresponding yield
9 acceleration of a given slope. In DLO analysis, a modified rigid perfectly plastic (Mohr–Coulomb) model is
10 employed to approximate small permanent deformations which may accrue in non-associative materials when
11 subjected to ground motions with relatively low peak ground acceleration. The contribution of the vegetation to
12 enhancing the yield acceleration is obtained via subtraction of the fallow slope yield acceleration. The second
13 stage of the analysis incorporates the vegetation contribution to the slope's yield acceleration from DLO into
14 modified limit equilibrium equations to further account for the geometric hardening of the slope under
15 increasing soil movement. Thereby, the method can predict the permanent settlement at the crest of the slope via
16 a slip-dependent Newmark sliding block approach. This procedure is validated against a series of centrifuge
17 tests to be highly effective for both fallow and vegetated slopes and is subsequently used to provide further
18 insights into the stabilising mechanisms controlling the seismic behaviour of vegetated slopes.

19 **Key words:** Analytical modelling; Centrifuge modelling; Dynamics; Earthquakes; Sand; Slopes; Vegetation;
20 **Ecological Engineering**

21

22 **1. Introduction**

23 The use of vegetation to reinforce soil on landslip-prone slopes is an ecologically and economically beneficial
24 sustainable alternative to traditional civil engineering reinforcement techniques [1-3]. The mechanical benefit of
25 roots on slope stability has been commonly accepted. Many analytical models have been developed, based on
26 small site in-situ investigation and laboratory tests, to quantify this benefit and predict its impacts on global
27 slope behaviour [4-6]. However, to the best of authors' knowledge, all of these analytical models have been
28 developed for static/monotonic use. The impacts of vegetation on seismic performance of slopes subjected to
29 earthquake ground motions are generally overlooked in preliminary design. As observed by recent physical
30 modelling studies [7-9], vegetation could highly improve the seismic performance of slopes (in terms of crest
31 settlement) especially for the case of slopes of modest height (e.g. small embankments). As a result, ignoring the
32 benefit of vegetation may lead to a conservative result and the use of more extensive remedial methods (e.g.

¹ Corresponding author: j.a.knappett@dundee.ac.uk; +44-1382-384345

33 piling, soil nailing) which may not be necessary. Analytical models which incorporate vegetation are therefore
34 required for use in seismic analysis and design [10].

35 Eurocode 8 [11], which guides the design and construction of buildings and civil engineering works in
36 seismic regions within Europe, recommends the use of established methods of dynamic analysis, such as Finite
37 Elements (FE) or rigid block models or by simplified pseudo-static methods to determine the response of slopes
38 to a design earthquake. Given the computational expense of the FE method, a complimentary simplified
39 procedure would be highly useful in preliminary design, particularly for identifying key cases for further
40 detailed study via FE. While compared with pseudo-static methods, Newmark sliding block models [12], which
41 as displacement-based methods, are aligned with modern trends in performance-based design and assessment,
42 potentially offer a useful basis for such a method, especially given their popularity. Recently, such methods have
43 been developed to incorporate the large displacement effects of continued sliding in hardening the slope
44 response [13], and also to incorporate the stabilising effects of a row of discretely spaced piles [14].

45 In this paper, an improved sliding-block procedure is developed to predict the seismic performance of
46 vegetated slopes. The procedure consists of two components. Firstly, an analysis using Discontinuity Layout
47 Optimisation (DLO [15]) is used to detect the critical seismic failure mechanism for slopes incorporating zones
48 of enhanced strength where the roots are present (i.e. the lowest upper-bound mechanism using a virtual work
49 approach and optimisation routine) and predict the contribution to the yield acceleration of a given slope
50 configuration provided by the roots. This derived yield acceleration information is then incorporated into a
51 modified limit equilibrium formulation for a sliding block to further account for the geometric hardening of the
52 slope as it flattens with slip, allowing the permanent settlement at the crest of the slope to be estimated. The
53 procedure is then validated against a database of centrifuge test results reported in [8], and subsequently used to
54 reveal further insights into the seismic behaviour of vegetated slopes.

55

56 **2. Discontinuity Layout Optimisation**

57 *2.1 Fundamental theory*

58 Discontinuity Layout Optimisation [15] is a recently developed numerical limit analysis procedure which can be
59 applied to a wide range of geotechnical stability problems involving cohesive and/or frictional soils. Compared
60 with the more traditional Finite Element Limit Analysis (FELA) technique which requires discretising the
61 problem into solid (finite) elements, DLO employs rigorous mathematical optimisation techniques to identify a
62 critical layout of lines of discontinuity which form a kinematically-admissible collapse mechanism. These lines
63 of discontinuity are typically ‘slip-lines’ in planar geotechnical stability problems and define the boundaries
64 between moving rigid blocks of material which form the mechanism of collapse. Associated with this
65 mechanism is a collapse load factor, determined via the principle of virtual work, which is an upper bound on
66 the ‘exact’ load factor according to formal plasticity theory. The core matrix formulation for seismic problems is
67 given in Appendix A, repeated from [16] for completeness.

68 2.2 Constitutive modelling of soil

69 DLO calculations were carried out using the software LimitState:GEO, v.2.0, which involves an adaptive
70 solution procedure described by Gilbert & Tyas (2003) [17] to significantly reduce memory requirements and
71 the time (of the order of a few minutes) to reach an optimised solution. The geometry of a vegetated slope
72 problem is shown schematically in Fig.1. The root-soil matrix is modelled using smeared zones with additional
73 representative shear strength (here incorporated into the soil behaviour as additional cohesion) reflecting the
74 contribution of the roots, which can vary with depth. The maximum rooting depth is denoted as h_r and the lateral
75 spread of the roots by the Critical Rooting Zone (CRZ), essentially a diameter which defines the zone of
76 dominant structural roots which have been found to provide more than 80% of the total root mass. The two-
77 dimensional (2D) plane strain model assumes that the input additional representative shear strength from the
78 roots can be modelled as an equivalent amount per metre length of the slope, accounting for the plant spacing in
79 the out-of-plane direction.

80 The current implementation of DLO uses a rigid-plastic material model based on the Mohr–Coulomb
81 model with an associative flow rule for frictional materials, and this was used in the modelling presented herein.
82 Four soil input parameters were required, namely: unit weights under saturated and dry condition and two
83 measurable effective stress strength parameters, ϕ' and c' . Although associative flow is implicitly assumed in
84 this model, such an assumption will overestimate the yield acceleration compared to the true non-associative
85 behaviour in the soil due to an overestimation of the amount of dilation, and therefore potentially overestimate
86 the yield acceleration resulting in an under-prediction of seismic slip. Hence non-associative flow should be
87 considered pre-input [18]. As the soil model is rigid-plastic, if the strength is defined by the peak friction angle
88 it will imply that slip will not occur until peak strength is exceeded, even though the soil may be substantially
89 into its non-linear elasto-plastic deformation range below this level, and therefore able to accrue small
90 permanent displacements with repetitive cyclic loading. To overcome these limitations an approximate
91 procedure is proposed below (and validated against centrifuge data later on) to account for non-associativity and
92 pre-peak accumulation of (small) deformations via an equivalent associative analysis with a mobilised friction
93 angle (ϕ'_{mob}) [19] and corresponding mobilised yield acceleration for cases where the induced seismic shear
94 stress is less than the peak shear strength of the soil to allow improved predictions of small amounts of
95 permanent displacement in smaller earthquakes.

96 2.3 Influence of non-associativity

97 Here, non-associative flow was modelled by adjusting the value of $\phi' = \phi'_{mob}$ used in the analyses from the
98 actual value for the true non-associative behaviour to an equivalent associative value ϕ^* as suggested in [20] and
99 previously used for other seismic limit analysis problems (e.g.[21],[22]), given by:

100
$$\phi^* = \tan^{-1} \left(\frac{\cos \psi'_{mob} \sin \phi'_{mob}}{1 - \sin \psi'_{mob} \sin \phi'_{mob}} \right) \quad (1)$$

101 where ϕ'_{mob} is a mobilised friction angle which takes a value between a lower bound of ϕ'_{cs} at critical state and
 102 an upper bound of ϕ'_{pk} if the seismically induced shear stresses would be sufficient to exceed the peak soil
 103 strength. Considering the limiting case of $\phi'_{mob} = \phi'_{pk}$, ϕ'_{pk} can be written in terms of dilation angle ψ' as:

104
$$\phi'_{pk} = \phi'_{cs} + 0.8\psi' \quad (2)$$

105 for plane strain after [23]; ϕ'_{pk} can also be given as a function of the relative dilation index I_R :

106
$$\phi'_{pk} - \phi'_{cs} = AI_R \quad (3)$$

107 where A is a dimensionless factor to account for strain type ($A = 5$ for plane strain) and I_R is given by:

108
$$I_R = I_D(Q - \ln p') - R \quad (4)$$

109 where I_D is the relative density of the soil, Q , R are fitting parameters that depend on the intrinsic sand
 110 characteristics and p' is the mean confining stress, which can be expressed in terms of the vertical and horizontal
 111 effective stresses using:

112
$$p' = \frac{1}{3}(\sigma'_v + \sigma'_h) = \frac{1}{3}(\sigma'_v + 2K_0\sigma'_v) \quad (5)$$

113 where σ'_v is the vertical effective stress, σ'_h is the horizontal effective stress, and K_0 is the earth pressure
 114 coefficient at rest, which for normally consolidated soils may be estimated using:

115
$$K_0 = 1 - \sin \phi'_{cs} \quad (6)$$

116 Q and R can be simplified to 10 and 1 when $0 < I_R < 4$, while at very low confining stress level ($I_R > 4$), Q and R
 117 can be calculated as $7.1 + 0.75 \ln p'$ (for plane strain) and 1, respectively [24].

118 **2.4 Mobilised friction angle accounting for pre-peak deformations**

119 The dilation angle utilised by this approach and expressed via Eq. (2) is the maximum dilation angle,
 120 corresponding to a capping yield surface. The state of soil is very strongly dependent on its stress history
 121 [25],[26] and the shape of the yield surface is determined by the maximum stress the soil has ever experienced.
 122 For smaller earthquake motions, the magnitude of the induced shear stresses may not be sufficient to push the
 123 effective stress path within the soil to the capping yield surface, though there may be accumulation of small
 124 plastic strains due to inelastic stress-strain response of the soil pre-failure. This is here represented by an
 125 expanding yield surface (described by ϕ'_{mob}) for the non-associative soil, which the induced shear stresses from

126 the combined effects of the ground slope and the earthquake will just reach. It is then assumed that Eq. (2) is
 127 also valid below peak strength, i.e.:

$$128 \quad \phi'_{mob} = \phi'_{cs} + 0.8\psi'_{mob} \quad (7)$$

129 Eq. (1) is then used to approximate the non-associative values of ϕ'_{mob} and ψ'_{mob} as an equivalent associative
 130 value. When a ground motion is large enough to push the mobilised yield surface to the capping yield surface
 131 the soil will dilate to the maximum (capping) condition and any further increase in ground acceleration and
 132 seismically induced shear stress will not further change the shape of yield surface. Compared to recent previous
 133 sliding block models ([13],[22]) which considered strong ground motions with peak accelerations large enough
 134 to easily exceed the peak strength, the use of ϕ'_{mob} here extends the range of applicability to smaller ground
 135 motions, a feature which will be useful in the later validation against centrifuge data.

136 To incorporate the model of soil behaviour described above into a slope stability problem, it is necessary to
 137 estimate the peak induced cyclic shear stresses in the ground such that the mobilised friction angle ϕ'_{mob} can be
 138 estimated. For a slip plane at depth z beneath the slope surface (fallow soil) and parallel to it (i.e. infinite slope
 139 failure), under uniaxial horizontal shaking (i.e. plane strain – see Fig.2), the applied down slope shear stress
 140 $\tau_{applied}$ is:

$$141 \quad \tau_{applied} = \gamma z \sin \beta \cos \beta + k_h \gamma z \cos^2 \beta \quad (8)$$

142 where the first term relates to the static shear stress due to the ground slope, and the second term relates to the
 143 additional peak dynamic shear stress induced by the earthquake, (here, γ is the soil unit weight, β is the slope
 144 angle and k_h is the horizontal seismic acceleration coefficient). The effective normal stress σ' on the same slip
 145 plane is:

$$146 \quad \sigma' = \gamma z \cos^2 \beta - k_h \gamma z \sin \beta \cos \beta - u \quad (9)$$

147 where u is pore water pressure. In dry cohesionless soil, as modelled in the centrifuge testing described later, $u =$
 148 0. Then the mobilised friction angle (for a cohesionless soil) may be estimated as:

$$149 \quad \tan \phi'_{mob} = \frac{\tau_{applied}}{\sigma'_n} = \frac{\gamma z \sin \beta \cos \beta + k_h \gamma z \cos^2 \beta}{\gamma z \cos^2 \beta - k_h \gamma z \sin \beta \cos \beta} = \frac{\tan \beta + k_h}{1 - k_h \tan \beta} = \tan[\beta + \tan^{-1}(k_h)] \quad (10)$$

150 or alternatively,

$$151 \quad \phi'_{mob} = \beta + \tan^{-1}(k_h) \quad (11)$$

152 Eq. (11) is applicable while $\phi'_{cs} < \phi'_{mob} < \phi'_{pk}$.

153 The model described in this section is shown schematically for the simplified case of $c' = 0$, $u = 0$ in Fig.3,
 154 with some indicative cyclic loading shown in the positive quadrant of a shear stress-strain plot in Fig.3 (b)-(d).
 155 The model essentially assumes that for the purposes of predicting plastic slip, a soil with a dilative peak strength

156 can be idealised as being elastic, accruing no plastic strain while $\phi'_{mob} < \phi'_{cs}$ (Fig.3(b)). This captures the slope
 157 being initially stable under static conditions and demonstrates that the slope can sustain a small ground motion
 158 (low k_h) without inducing slip. Once $\phi'_{mob} > \phi'_{cs}$ the model assumes that the soil will be well into its non-linear
 159 elasto-plastic range, even though $\phi'_{mob} < \phi'_{pk}$, with Eq. (11) describing the value of ϕ'_{mob} as a function of the
 160 initial slope angle (β) and the size of the earthquake shaking (k_h) (Fig.3(c)). The form of Eq. (11) implies that
 161 stronger earthquakes will induce greater slip for a given slope angle. Once $\phi'_{mob} = \phi'_{pk}$, the model reduces to a
 162 conventional slip model based on the soil initially having its peak strength. Therefore the key new feature of this
 163 model is that a low-to-moderate strength earthquake can now potentially induce some slip within a sliding block
 164 model. Previous models, even those with sophisticated strain-softening behaviour (e.g.[13][27]) required the
 165 peak strength to be exceeded before any amount of slip could take place, even if it subsequently softened rapidly
 166 to the critical state condition, and therefore could potentially predict zero slip in cases where the earthquake is
 167 moderately strong and inducing a highly non-linear elasto-plastic response within the soil. The new model
 168 therefore potentially makes sliding block analysis applicable to a wider range of earthquake motions.

169 2.5 Geometric-hardening and vegetation

170 In the forgoing section, it has been proposed that through the use of smeared zones with additional shear
 171 strength from the roots (Fig.1), and through careful selection of mobilised friction angles, DLO could be used to
 172 determine the critical failure mechanism and corresponding yield acceleration in vegetated slopes over a wide
 173 range of input motion strengths. However, one drawback of the DLO procedure (and indeed all limit analysis-
 174 type procedures for seismic problems) is that it does not immediately provide a direct measure of slope
 175 performance (e.g. seismically-induced slip) and only provides a measure of the instantaneous yield acceleration
 176 for the initial pre-earthquake slope conditions and therefore cannot account for an increase of yield acceleration
 177 due to geometric hardening of the slope with slip (defined as the benefit from the slope flattening) [13] without
 178 performing many repeat analyses on cases with reduced slope angles. In this section the sliding block method
 179 introduced by [12] and modified by [13] to allow for geometric hardening in fallow slopes is further developed
 180 to estimate the permanent deformation response of vegetated slopes, utilising only initial yield accelerations
 181 derived from DLO.

182 The mechanism of earthquake induced slope displacement is the sliding of an essentially rigid block (for
 183 shallow translational slips such as shown in Fig.2) along a well-defined slip surface. From Newmark's original
 184 method, sliding occurs when the shaking induced acceleration $a(t)$ exceeds the yield acceleration, k_{hy} :

$$185 \quad a_{slip} = a(t) - k_{hy} \quad (12)$$

186 where a_{slip} is the acceleration of the sliding mass. Those portions of the recorded acceleration that exceed the
 187 yield acceleration are integrated to obtain the cumulative displacement history of the block, $s(t)$, using the
 188 following equations:

189

$$v_i = \begin{cases} \int_{t_i}^{t_{i+1}} (a(t) - k_{hy}) dt, t \in [t_i, t_{i+1}] \\ 0, otherwise \end{cases} \quad (13)$$

190

$$d_i = \int_{t_i}^{t_{i+1}} v(t) dt, t \in [t_i, t_{i+1}] \quad (14)$$

191

$$s = \sum d_i \quad (15)$$

192

193

where v_i is the slip velocity in the time step t_i , d_i is an increment of slip in this time step and s is the cumulative soil slip.

194

195

196

197

198

199

Aside from the DLO approach mentioned earlier, the horizontal yield acceleration of a shallow translational slip in fallow soil may be estimated using standard limit equilibrium techniques, incorporating pseudo –static acceleration components due to the seismic ground motion, as shown previously in Fig.2 [28][29][30]. Here, the shear strength of the soil along the slip plane τ_{ult} in the fallow case, assuming that the soil failure can be described by the Mohr-Coulomb failure criterion using an equivalent associative strength parameter ϕ^* , is given by:

200

$$\tau_{ult} = c' + (\gamma z \cos^2 \beta - k_h \gamma z \sin \beta \cos \beta - u) \tan \phi^* \quad (16)$$

201

202

where c' is the soil cohesion (due to cementation or structure effects). The soil yields when $\tau_{applied} = \tau_{ult}$, where $\tau_{applied}$ was previously defined in Eq. (8), resulting in:

203

$$k_{hy(fallow)} = \frac{c' + (\gamma z \cos^2 \beta - u) \tan \phi^* - \gamma z \sin \beta \cos \beta}{\gamma z \cos^2 \beta + \gamma z \sin \beta \cos \beta \tan \phi^*} \quad (17)$$

204

205

206

207

In a vegetated slope, the mechanism is potentially more complicated given that the profile is now non-homogenous (having rooted zones and non-rooted zones of defined geometry, as shown in Fig.1). An initial assumption may be to average out the effect of the roots across the whole slope face and include it along with any true soil cohesion in the c' term in Eq. (17), that is

208

$$c' = c'_{soil} + \Delta \tau \quad (18)$$

209

210

211

212

213

214

where c'_{soil} is the apparent cohesion of the soil itself and $\Delta \tau$ is the additional shear strength provided by the roots. However, such an approach was used in FE simulations presented in [7] and it was found that it highly over-predicted the reinforcing effect of roots on slope performance and highly under-predicted seismically induced slip when compared to centrifuge test data. However an alternative way of expressing Eq. (17) with Eq.(18), is by dividing it into two parts, one attributed to the fallow slope and the other attributed to the mobilisation of root resistance. This gives:

215
$$k_{hy(\text{rooted})} = \frac{c'_{soil} + (\gamma z \cos^2 \beta - u) \tan \phi^* - \gamma z \sin \beta \cos \beta}{\gamma z \cos^2 \beta + \gamma z \sin \beta \cos \beta \tan \phi^*} + \frac{\Delta \tau}{\gamma z \cos^2 \beta + \gamma z \sin \beta \cos \beta \tan \phi^*} \quad (19)$$

216 or, alternatively:

217
$$k_{hy(\text{rooted})} = k_{hy(\text{fallow})} + \Delta k_{hy} \quad (20)$$

218 where Δk_{hy} is the increase of yield acceleration due to the presence of the roots. As both rooted and fallow yield
219 accelerations can be determined using DLO, the root contribution can be estimated from:

220
$$\Delta k_{hy} = k_{hy(\text{rooted})}^{DLO} - k_{hy(\text{fallow})}^{DLO} \quad (21)$$

221 Eq. (20) and Eq. (21) may show small differences in the values of k_{hy} between the limit equilibrium (Eq. (20))
222 and DLO-derived (Eq. 21) versions depending on the appropriateness of the infinite slope limit equilibrium
223 model for a particular slope geometry, with the DLO value more appropriately capturing the true geometry of
224 the failure mechanism.

225 The slope angle will decrease with slip as crest settlements make the slope shallower (re-grading, RG). A
226 simplified model for re-grading is shown schematically in Fig.4 after [13]. The instantaneous slope angle β_{i+1}
227 can be estimated by the following equation,

228
$$\beta_{i+1} = \tan^{-1} \left(\frac{H_i - d_i \sin \beta_i}{H_i \cot \beta_i + d_i \cos \beta_i} \right) \quad (22)$$

229 where H_i is the height of the slope at the previous time step of the Newmark analysis. For the initial time step, d_0
230 = 0, $H_i = H$ and $\beta_i = \beta_0$ (initial slope angle). It is assumed here that once the slope has deformed to a new, smaller
231 value of β , the failure mechanism will continue to be of the translational type, with a new slip surface parallel to
232 the new slope surface. Then the slope angle can be re-calculated at each time step to account for the regrading of
233 the slope based on the increment of slip occurring in the previous time step using Eq. (22). For the case of a
234 vegetated slope, as the rooted zones are near-surface it is here assumed that they will move with the surrounding
235 soil and that $k_{hy(\text{rooted})}$ is affected by re-grading in the same way as $k_{hy(\text{fallow})}$ (i.e. that the effect is related purely to
236 the external geometry of the slope), such that Δk_{hy} in Eq. (21) will remain constant throughout the analysis. It is
237 therefore proposed that Eq. (20) can be modified to incorporate re-grading by multiplying the $k_{hy(\text{fallow})}$ value
238 from DLO by a 're-grading reduction factor' determined from the limit equilibrium method without recourse to
239 further DLO, recalculated in each time step as the slope flattens out, according to:

240
$$k_{hy(\text{rooted},i)} = k_{hy(\text{fallow})}^{DLO} \left[\frac{k_{hy(\text{fallow})}(\beta_i, \phi^*, c'_{soil}, \gamma, u_i)}{k_{hy(\text{fallow})}(\beta_{i-1}, \phi^*, c'_{soil}, \gamma, u_{i-1})} \right] + (k_{hy(\text{rooted})}^{DLO} - k_{hy(\text{fallow})}^{DLO}) \quad (23)$$

241 with $k_{hy(\text{fallow})}$ from Eq. (17). Eq. (23) incorporates, in an approximate way, the effects of the actual failure
242 mechanism geometry and any changes through the addition of the roots (through the use of DLO-derived yield
243 acceleration values), soil non-associativity (via ϕ^*) and geometric re-grading (updating of β) into a yield
244 acceleration that can evolve as the slope slips. It requires two initial DLO analyses of the initial geometry, one

245 fallow and one rooted, and subsequently only Eq. (17) and Eq. (23) need to be computed at each time step
246 within an otherwise standard Newmark sliding block analysis. A flowchart, showing the complete procedure, is
247 shown in Fig.5. The effectiveness of this model in quantifying the performance of rooted slopes will be
248 validated against previously reported centrifuge data [8] in the following section.

249

250 3. Validation of sliding block model

251 3.1 Centrifuge modelling

252 Dynamic centrifuge modelling was conducted using the 3.5 m diameter beam centrifuge and servo-hydraulic
253 earthquake simulator (EQS) at the University of Dundee [31]. The modelling and observations from these tests
254 are described in detail in [8]; only a brief summary is given here. The results of four tests from this previously
255 reported programme are utilised herein for validation of the Newmark model, representing identical 1:2 slopes
256 ($\beta \approx 27^\circ$) at model scale, with varied g-level (to model slopes of different prototype height) and motion
257 frequency content as indicated in Table 1. All values presented herein are given at prototype scale, unless
258 specifically noted otherwise. The slope models were constructed within an Equivalent Shear Beam (ESB)
259 container in order to replicate a semi-infinite horizontal boundary condition in the direction of shaking [32],[33].
260 The slopes (at model scales) were prepared using dry HST 95 silica sand at a relative density of 55%-60% to
261 form a model slope of height 240 mm from toe to crest, with a further 80 mm underneath. Based on these
262 dimensions, at 1:10 scale (i.e. in a 10-g test) the prototype slope was 2.4 m high from toe to crest and at 1:30
263 scale and 30-g, the slope was 7.2 m tall. These models are shown in Fig.6. The sand was pluviated in air around
264 suspended model root clusters with realistic 3-D geometry that were fabricated at 1:10 and 1:30 scales using a
265 Stratesys Inc. uPrint SE Acrylonitrile Butadiene Styrene (ABS) prototyper (also known as a 3-D printer)
266 following the procedures outlined in [9], in each case penetrating into the slope to the same rooting depth (1.5
267 m). The ABS plastic root analogues were validated to be highly representative of the mechanical behaviour of
268 real roots (in terms of Young's Modulus and tensile strength) after a series of uniaxial tension and bending tests,
269 reported in [7],[9]. In the out-of-plane direction, model root clusters were uniformly distributed at a spacing of
270 1.4 m. The models were each subjected to eight successive earthquake motions, comprising three different
271 records with distinct peak ground acceleration (PGA), duration and frequency content. The first motion (EQ1)
272 was recorded during the 1995 Aegion earthquake (M_s 6.2) and was predicted to cause only a small amount of
273 slip and predominantly acts to characterize the elastic dynamic behaviour of the slope. This initial motion was
274 followed by three nominally identical stronger motions (EQ2 – EQ4) from the 1994 Northridge earthquake (M_s
275 6.8) and a further three (EQ5 – EQ7) from the 2009 L'Aquila earthquake (M_s 6.3), followed by a final Aegion
276 motion (EQ8). More details about these motions can be found in [7],[8].

277 3.2 Determination of yield accelerations from DLO

278 Before the Newmark-type analysis can be conducted, yield accelerations must be determined for the fallow and
279 rooted cases using DLO. Model layouts are shown in Fig.7 for rooted cases TL 07 (Fig.6(a)) and TL 06
280 (Fig.6(b)). Fallow cases had identical external geometry but without the rooted soil blocks shown in Fig.7. A

281 fine nodal density (1000 nodes) was used in all DLO calculations to accurately describe the geometry of the
282 failure mechanism.

283 The properties of the soil within the slope were determined using the model shown in Fig.3. According to
284 Eq. (2) to Eq. (6), peak friction angle may be evaluated as a function of depth in the two slope models shown in
285 Fig.7 and averaged over the slope height H to obtain mean peak (upper-bound) friction angles of 47° and 44.5° ,
286 for the 2.4 m and 7.2 m slopes, respectively. Considering first the shorter slope, the recorded peak accelerations
287 in EQ1 were 0.124g and 0.144g, for the fallow and rooted slopes, respectively, corresponding to a yield surface
288 with an initial ϕ'_{mob} of 34° and 35° . Compared with the subsequent motions, the peak acceleration of EQ1 was
289 relatively small. A mobilised friction angle of approximately 38° (or 38.5°) can be determined for the
290 subsequent earthquake motions EQ2-EQ4, as shown in Fig.8 (a). Given that the peak accelerations of the
291 remaining motions (EQ5-EQ8) are not higher than those of EQ2 to EQ4, the maximum mobilised dilation has
292 been achieved during motion EQ2, and no further change in mobilised friction angle would be observed for the
293 final motions. In terms of the taller slope (Fig.8 (b)) which is subject to larger motions due to the increased
294 prototype low frequency content that could be simulated by the EQS at the higher scaling factor, the recorded
295 peak acceleration of EQ 1 is 0.196g, which corresponds to a yield surface with an initial ϕ'_{mob} of 38° . For EQ2,
296 the recorded peak acceleration is 0.61g, which is significantly higher than 0.31g (acceleration corresponding to
297 the capping yield surface when $\phi'_{mob} = \phi'_{pk}$), so all subsequent motions will mobilise the full peak friction angle
298 of the soil. These values of ϕ'_{mob} were subsequently converted to equivalent associative values ϕ^* using Eq. (1),
299 with the values shown in Table 2.

300 For the rooted soil, the additional strength contribution ($\Delta\tau$) from the roots used within the smeared rooted
301 zones (see Fig.1 & Fig.7) were input to represent the 3-D model root clusters based on the results of tests in a
302 large direct shear apparatus (DSA) that are reported in [7] and summarised in Fig.9. In the out plane direction,
303 the spacing between the adjacent root clusters was 1.4 m, so the input values were reduced by a factor of 1.4
304 compared to the measured values (this is shown in Fig.9 (b)) to determine an equivalent amount of additional
305 shear strength in the rooted zones per metre length of the (long) slope. For future practical application in the
306 field, $\Delta\tau$ as a function of depth could be determined using new in-situ test methods (e.g. the 'corkscrew' test)
307 currently under development and undergoing field trials at the University of Dundee [34],[35]. The results of the
308 DLO analyses for the different slope heights, vegetation conditions and mobilised friction angles are
309 summarised in Table 2. In addition to the yield acceleration, the static factor of safety (F_s) is also determined in
310 each case for context. Yield accelerations for fallow conditions were also estimated using the limit equilibrium
311 method (Eq. 17) and these results confirm that a reasonable estimation of k_{hy} is made using DLO for the fallow
312 cases in cohesionless soil.

313 The presence of roots is found to improve slope stability both in the static and dynamic condition. From
314 Table 2, an improvement of approximately 8% and 14% is observed for the static safety factor, for the 2.4 m and
315 7.2 m high slopes, respectively. In the dynamic condition, the yield acceleration is increased by 14-21% and 23-
316 39%, for the 2.4 m and 7.2 m slopes, respectively. It is clear therefore that the presence of plant roots increases
317 slope stability and will reduce seismic slip due to increased yield acceleration.

318 A comparison of the failure mechanisms determined for the fallow and vegetated slopes is shown in Fig.10.
319 It is clear that the 1:2 fallow slopes fail in a shallow translational mechanism, with a shear plane located at a
320 depth of 0.25 m and 0.70m, for the 2.4 m and 7.2 m slopes, respectively. This is consistent with visual
321 observations from the centrifuge tests. For the vegetated cases, different failure mechanisms are illustrated
322 between the larger slope (7.2 m) and the smaller slope (2.4 m). For the 7.2 m high slope, the slip plane is
323 observed to move from its fallow position at a depth of 0.7 m below the ground surface, which would have
324 passed (at approximately mid-depth) through the rooted zone, to below the rooted zone. For the 2.4 m high
325 rooted slope, it is subject to a much shallower (0.09m) localised slip failure between the rooted zones. This
326 appears to be a very different ‘buttressing’ mechanism, similar to that identified via FE modelling of a similar
327 slope with much simpler straight vertical rod root analogues in [7]. However, given that the roots penetrate very
328 deeply into the 2.4m slope such that they almost touch the base of the slope (Fig. 7(a) and Fig. 10(b)), it may be
329 that there is a deep mechanism passing beneath the roots as in the 7.2 m high slope with a similar k_{hy} that is
330 suppressed by the closeness of the bottom boundary (as a result of the limited model container size in the
331 centrifuge). In any case, it is apparent that the mechanism by which the roots achieve their stabilising effect is
332 by forcing the slip plane into a less optimal position around the rooted zones, compared to the fallow case.

333 Historically, the contribution of roots within slope stability problems has been considered through the
334 addition of $\Delta\tau$ along the unaltered fallow position of the slip plane, i.e. an increase of strength, rather than a
335 change in mechanism. This would previously have suggested that in order to maximise the effect of the
336 vegetation, species should be selected to have a large root area ratio and the strongest biomechanical strength
337 (i.e. lots of strong roots). The results shown here suggest that knowing the root shear strength contribution is
338 still important, but that (i) it is important to understand how this varies with position (particularly depth) in the
339 soil, rather than just conducting shear box tests of rooted soil block samples at a single depth, as this will affect
340 the optimal position of the shear plane as found using DLO; and (ii) once the roots provide a strong enough
341 contribution to force the slip plane to pass beneath them, there will be little point in targeting further root
342 strength. This suggests that if planting vegetation to improve slope performance, it may not be ideal to limit
343 species choice to the strongest rooting species, but that selection should be made based on rooting depth (and,
344 potentially, lateral root spread, CRZ) to result in the greatest deviation in the position of the slip plane. This will
345 be explored further in a later section.

346 3.3 Prediction of slip via sliding block analyses

347 Sliding-block analyses were subsequently conducted for each of the centrifuge tests, for the complete set of
348 eight successive earthquake motions. The input earthquake motion used was the acceleration record measured
349 at instrument ACC2 in each case (Fig. 6). **The effects of root resistance, geometric re-grading (change in β) and**
350 **non-associativity on the yield acceleration** compared to the fallow slope using the mobilised friction angles for
351 EQ1 (small earthquake) and EQ2 (large earthquake) of TL 06 is shown in Fig. 11 as an example. Only the
352 positive (downslope) accelerations have been shown for clarity. For the dry, cohesionless soil used in the
353 centrifuge tests, Eqs. (17) and (23) become:

354
$$k_{hy(fallow)i} = \frac{\tan \phi^* - \tan \beta_i}{1 + \tan \beta_i \tan \phi^*} \quad (24)$$

355
$$k_{hy(rooted)i} = k_{hy(fallow)}^{DLO} \left[\frac{(\tan \phi^* - \tan \beta_i)(1 + \tan \beta_{i-1} \tan \phi^*)}{(1 + \tan \beta_i \tan \phi^*)(\tan \phi^* - \tan \beta_{i-1})} \right] + (k_{hy(rooted)}^{DLO} - k_{hy(fallow)}^{DLO}) \quad (25)$$

356 It can be seen that the model considering non-associativity via ϕ^* increases the initial yield acceleration
 357 compared with an analysis using a critical state strength model (strain hardening, SH, to ϕ'_{cs}). As a result, a large
 358 portion of EQ1 is below the yield acceleration and this will strongly influence the deformation response (this
 359 will be illustrated later). It is worth noting here that the effect of root resistance on yield acceleration was
 360 constant between EQ1 and EQ2. This is clearly a simplification of the problem because in reality, root
 361 resistance will be mobilised progressively with slip rather than instantaneously reaching peak resistance.
 362 However, given that root-soil interaction will mobilise very rapidly with slip due to the small diameter of the
 363 roots [7], this simplification is considered to be a reasonable approximation in an analysis which is designed to
 364 be practical to use. Geometric re-grading causes the yield acceleration to increase non-linearly throughout the
 365 earthquake with continuing slip (this is most noticeable for the larger motion, EQ2, in Fig.11 (b)), which will
 366 lead to reduced slip velocity and hence reduced permanent slip compared with the case with no geometric
 367 hardening.

368 *3.4 Fallow slopes*

369 Fig.12 and Fig.13 show the cumulative crest settlement across the eight earthquakes as predicted by the new
 370 sliding block model and compare these predictions to the values measured in the centrifuge tests, for the 1:30
 371 scale (7.2 m high) slope and the 1:10 scale (2.4 m high) models, respectively. In Fig.12, predictions are made
 372 based on both Eq. (17), which uses ϕ^* based on ϕ'_{mob} , and also using a previous strain-softening model [13]. As
 373 the earthquake motions were large enough to mobilise ϕ'_{pk} in all but EQ1, this case is a test of the suitability of
 374 using ϕ^* within analyses; this is shown to give a very good match to the centrifuge data. In terms of the 2.4 m
 375 slope, because of the smaller motions in this test, all of the earthquakes have $\phi'_{mob} < \phi'_{pk}$, so this represents a
 376 good test of the new sub-peak slip model (Eq. 11). As the earthquake motions get stronger, the mobilised
 377 friction angle increases from 34° to 36.7° and 38°, for EQ1, EQ2 and the last six motions, respectively. The
 378 match to the centrifuge data is very good, with the new model capturing the accrual of small deformations (of
 379 the order of ~40 mm total, compared to the ~300 mm in Fig. 12). In contrast, the use of the previous model
 380 from [13] predicts no slip in the 2.4 m case, as despite having a sophisticated strain-softening model, the
 381 dynamically-induced shear stresses are never sufficient to exceed the peak strength and thereby trigger slip.

382 *3.5 Rooted slopes*

383 Fig.14 shows the results of simulations of cumulative crest displacement compared with the centrifuge test data,
 384 for the 7.2 m rooted slope. A good match to the total measured crest settlement at the end of the test is presented.
 385 A reduction of 15% in calculated permanent crest settlement is observed compared to the fallow case through
 386 the modified sliding block model results. This reduction is consistent with the reduction in slip observed in the
 387 centrifuge tests (15%). The sliding block model does not quite capture the reduction within each motion

388 perfectly – as observed, the root contribution is mainly mobilised in EQ4 in the centrifuge tests, but this is
389 mobilised from EQ2 progressively in the simulated case.

390 Results for the 2.4 m rooted slope case are presented in Fig.15. Here, four cases were considered: case (a)
391 is a direct comparison to the fallow case, with the only difference being the addition of the rooted zones; case (b)
392 is established to account for the root buttressing behaviour observed in DLO (Fig. 7(a)), and is achieved by
393 adjusting the slope height from 2.4m to 0.4 m in the calculation; case (c) corresponds to the reduction of peak
394 acceleration observed in centrifuge tests (by 10% - 20% at instruments ACC 6, 7, 10 and 11 in Fig.6 (a)) due to
395 the presence of the roots in this particular test and is incorporated by multiplying the input motion by a factor of
396 0.85 to obtain a new input motion [9]; case (d) considers the combined effects of case (b) and case (c). It can
397 clearly be seen that case (a) without consideration of the acceleration reduction effect highly under-estimates the
398 contribution of the roots in reducing the slope crest deformation response. Compared to the fallow case, the
399 inclusion of roots (case (a)) reduced the crest settlement by 61%; accounting for the buttressing effect (case (b))
400 reduced it by 74%; the reduction in acceleration (case (c)) reduces it by 86% and the combined effects (case (d))
401 result in a reduction of 89%, which is a little higher than the reduction observed in the centrifuge tests (85%).
402 The reason for this is associated with the fact that the contribution of roots is mainly mobilized during the first
403 two motions and then has an apparently less significant effect for the last six motions in the centrifuge tests but
404 the simulated case assumes that the root contribution remains constant across the eight earthquakes. Fig.15
405 suggests that the contribution of roots to reducing seismic slip within slopes is a combination of an increase in
406 yield acceleration associated with a change of failure mechanism and a small reduction in accelerations within
407 the slipping mass. Fig.16 summarises the results of all of the predictions at the end of each earthquake motion,
408 from which it can be seen that the new model is effective across the full range of slope heights and motions
409 tested, for both fallow and rooted slopes.

410

411 4. Further insights into rooted slope seismic behaviour

412 In this section, the influence of the root contribution to shear strength is further investigated using the modified
413 sliding-block procedure, particularly to explore the aforementioned feature of the increase in yield acceleration
414 and reduction in slip resulting principally from a change in mechanism rather than the addition of root strength
415 along the fallow slip plane. Starting with the $\Delta\tau$ -depth profiles shown in Fig.9, the values of $\Delta\tau$ were
416 progressively reduced at all depths by a constant factor. This could represent the use of a different species
417 which has a smaller strength contribution (but similar distribution with depth), or a slope with the initial strength
418 distribution considered herein as the vegetation dies and the roots subsequently decay. The variation of yield
419 acceleration with the reduction of root cohesion as determined from DLO is shown in Fig.17 (a). The
420 normalised root contribution is the reduction factor used to multiply the initial $\Delta\tau$ -depth profile (essentially the
421 percentage strength remaining if the roots were decaying); the normalised yield acceleration is $k_{hy(rooted)}$ from
422 DLO, divided by $k_{hy(fallow)}$, also from DLO. Fig.17 (b) shows crest settlements subsequently computed using the
423 Newmark procedure, where the normalised settlement is the crest settlement of the rooted case divided by the
424 crest settlement of the fallow case. Mechanisms for some of the key low strength cases showing transitions in
425 behaviour are given in Fig.18. It can be seen that the yield acceleration remains constant even when the

426 normalised root contribution decreases to 2.5% of its initial strength for the 2.4 m slope (0.4-0.5 kPa of
427 normalised root contribution within the rooted zone). FE model simulations reported in [36] show almost
428 identical reinforcing effect at 100% normalised root contribution, and also suggest that the reinforcing effect can
429 be maintained down to 25% of the initial value of the normalised root contribution (i.e. over a wide range of $\Delta\tau$
430 values) in the 2.4 m case – Fig.18 (b). However a smoother transition to no effect at zero normalised root
431 contribution is shown in the FE compared to the more abrupt change in the approximate DLO-Newmark
432 approach. For the taller slope, there is again a very close match to FE simulations at 100% normalised root
433 contribution. The reduction in yield acceleration as the normalised root contribution is reduced is more sudden
434 for the DLO-Newmark approach in this case reducing once normalised root contribution becomes less than
435 7.5%. The FE simulations again show a more progressive reduction in reinforcing effect compared to the DLO-
436 Newmark approach. However, it is clear in both cases that (i) a substantial component of the reinforcing effect
437 of the roots can be maintained even if the root contribution is only half as strong, which has important
438 implications for vegetation management in allowing new vegetation to establish as older roots decay; and (ii)
439 once the failure mechanism has moved deeper, there is no further increase in yield acceleration with stronger
440 roots, which suggests that for the use of vegetation in engineering practice, species should be selected on the
441 basis of maximum h_r (and CRZ) to alter the failure mechanism as much as possible, rather than selecting for the
442 strongest possible roots.

443

444 5. Conclusions

445 An improved Newmark sliding-block procedure, which can include the effect of plant roots on seismic slope
446 performance, has been developed and validated against dynamic centrifuge data. The procedure consists of two
447 components. Firstly, DLO analysis is used to determine the seismic slope failure mechanism and estimate the
448 corresponding yield accelerations of a given slope in fallow and rooted cases. A rigid perfectly plastic (Mohr–
449 Coulomb) model with associative flow is used to model the soil, but utilises mobilised equivalent friction angles
450 to approximate both the non-associative behaviour of cohesionless slopes and predict small accrued
451 deformations when the earthquake-induced shear stresses are not sufficient to exceed peak strength, but may
452 result in non-linear elasto-plastic behaviour and some plastic straining. The second stage utilises these derived
453 yield accelerations from DLO into a modified Newmark sliding block approach to predict the permanent
454 settlement at the crest of the slope; this also accounts for the geometric hardening (flattening) of the slope with
455 continued slip making the model suitable for whole-life performance estimation. This procedure has been
456 validated to be highly effective in predicting permanent slip for both fallow and vegetated slopes as measured in
457 centrifuge tests and can be easily performed in preliminary design with lower computational effort than Finite
458 Element modelling. Some factors that may influence the seismic performance of root reinforced slopes were
459 also revealed during the development of sliding-block model. The presence of roots increase the slip plane depth
460 and it is this effect which is principally responsible for increasing the yield acceleration and hence reducing
461 deformations within the slope. This is in contrast to previous models which assume roots add additional shear
462 strength onto the pre-existing (fallow) shear plane. This new finding suggests that once the roots provide enough
463 additional shear strength to deviate the shear plane in this manner, the key controlling property of the roots will

464 be the rooting depth (and possibly also spread) rather than the strength of the roots. The potential benefit of
 465 roots appears to vary with the size of the slope. For taller slopes where the root depth is only a small proportion
 466 of the slope height (low h_r/H), roots only increase the yield acceleration of the slope against dynamic loading.
 467 For smaller slopes with higher h_r/H the proportional effect of this increase in yield acceleration appears to be
 468 more significant, and there is some evidence that the roots also reduce the strength of the earthquake motion
 469 within the slipping mass resulting in increased effectiveness and much reduced deformation response at the crest.
 470 Vegetation may therefore be particularly effective in smaller slopes, offering a low cost and low carbon
 471 alternative that could potentially replace more traditional stabilisation methods.

472

473 Appendix A

474 This appendix is from [16]. The primal kinematic problem formulation for the plane strain analysis of a quasi-
 475 statically loaded, perfectly plastic cohesive-frictional body discretised using m nodal connections (slip-line
 476 discontinuities), n nodes and a single load case can be given by

$$477 \quad \min \lambda \mathbf{f}_L^T \mathbf{d} = -\mathbf{f}_D^T \mathbf{d} + \mathbf{g}^T \mathbf{p} \quad (\text{A.1})$$

478 subject to

$$479 \quad \mathbf{B} \mathbf{d} = \mathbf{0} \quad (\text{A.2})$$

$$480 \quad \mathbf{N} \mathbf{p} - \mathbf{d} = \mathbf{0} \quad (\text{A.3})$$

$$481 \quad \mathbf{f}_L^T \mathbf{d} = 1 \quad (\text{A.4})$$

$$482 \quad \mathbf{p} \geq \mathbf{0} \quad (\text{A.5})$$

483 where \mathbf{f}_D and \mathbf{f}_L are vectors containing respectively specified dead and live loads, \mathbf{d} contains displacements
 484 along the discontinuities, where $\mathbf{d}^T = \{s_1, n_1, s_2, n_2, \dots, n_m\}$ and s_i and n_i are the relative shear and normal
 485 displacements between blocks at discontinuity i ; $\mathbf{d}^T = \{c_1 l_1, c_2 l_2, \dots, c_m l_m\}$, where l_i and c_i are respectively the
 486 length and cohesive shear strength of discontinuity i . \mathbf{B} is a suitable $(2n \times 2m)$ compatibility matrix, \mathbf{N} is a
 487 suitable $(2m \times 2m)$ flow matrix and \mathbf{p} is a $(2m)$ vector of plastic multipliers. The discontinuity displacement in \mathbf{d}
 488 and the plastic multipliers in \mathbf{p} are the linear programming variables.

489 For seismic problems, pseudo-static theory may be employed [16]. The imposition of horizontal and vertical
 490 seismic acceleration within the system results in additional work terms in the governing equation that are
 491 analogous to that for self-weight. Here, the contribution made by discontinuity i to the $\mathbf{f}_D^T \mathbf{d}$ term in Eq. (A.1)
 492 can be written as

493
$$\mathbf{f}_{D_i}^T \mathbf{d}_i = \{(1 - k_v)[-W_i \chi_i \quad -W_i \alpha_i] + k_h[-W_i \alpha_i \quad W_i \chi_i]\} \begin{bmatrix} s_i \\ n_i \end{bmatrix} \quad (\text{A.6})$$

494 where k_v and k_h are the vertical and horizontal pseudo-static acceleration coefficients, respectively; W_i is the
 495 total weight of the strip of material laying vertically above discontinuity i ; α_i and χ_i are the horizontal and
 496 vertical direction cosines of the discontinuity in question.

497 The DLO method finds the optimal collapse mechanism for the problem studied. This is achieved through
 498 increasing loading within the system until collapse is achieved, by applying what is termed an ‘adequacy factor’
 499 to a given load. In the case of seismic loading, this factor is applied to the horizontal or vertical acceleration. To
 500 apply live loading to the horizontal and vertical acceleration, the $\mathbf{f}_D^T \mathbf{d}$ term in Eq. (A.1) is not modified, instead
 501 modification is performed on the $\mathbf{f}_L^T \mathbf{d}$ terms, and given by

502
$$\mathbf{f}_{L_i}^T \mathbf{d}_i = \{k_v[-W_i \beta_i \quad -W_i \alpha_i] + k_h[-W_i \alpha_i \quad W_i \beta_i]\} \begin{bmatrix} s_i \\ n_i \end{bmatrix} \quad (\text{A.7})$$

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508

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587

Table 1. Summary of centrifuge models tested

Test identification number	Scale	Slope height (m)	Rooting type	Root cluster quantity	Plant spacing, out-of-plane (m)	Motion frequency content (Hz)
TL 04	1:10	2.4	Fallow	0	1	4-30
TL 05	1:30	7.2	Fallow	0	1	1.33-10
TL 06	1:30	7.2	1:30 scale root cluster	36	1.4	1.33-10
TL 07	1:10	2.4	1:10 scale root cluster	4	1.4	4-30

Table 2. Static and dynamic slope stability data

Model ID	Slope type	Slope height (m)	Motion	ϕ'_{mob}	ϕ^*	F_s (DLO)	z_{slip} (m)	k_{hy} (DLO)
DLO 01	Fallow	2.4	EQ1	34°	29.8°	1.246	0.36	0.057g
DLO 02	Fallow	2.4	EQ2-EQ8	38°	33.6°	1.435	0.25	0.124g
DLO 03	Fallow	2.4	EQ1	35°	30.7°	1.289	0.36	0.073g
DLO 04	Rooted	2.4	EQ1	35°	30.7°	1.399	0.20*	0.089g
DLO 05	Fallow	2.4	EQ2-EQ8	38.5°	34.0°	1.456	0.25	0.132g
DLO 06	Rooted	2.4	EQ2-EQ8	38.5°	34.0°	1.574	0.20*	0.151g
DLO 07	Fallow	7.2	EQ1	38°	33.6°	1.382	0.75	0.125g
DLO 08	Rooted	7.2	EQ1	38°	33.6°	1.58	1.50	0.174g
DLO 09	Fallow	7.2	EQ2-EQ8	44.5°	39.8°	1.724	0.50	0.238g
DLO 10	Rooted	7.2	EQ2-EQ8	44.5°	39.8°	1.976	1.50	0.293g

* These cases showed a local slip between root clusters (buttressing effect of roots – Fig.8)

Figure 1
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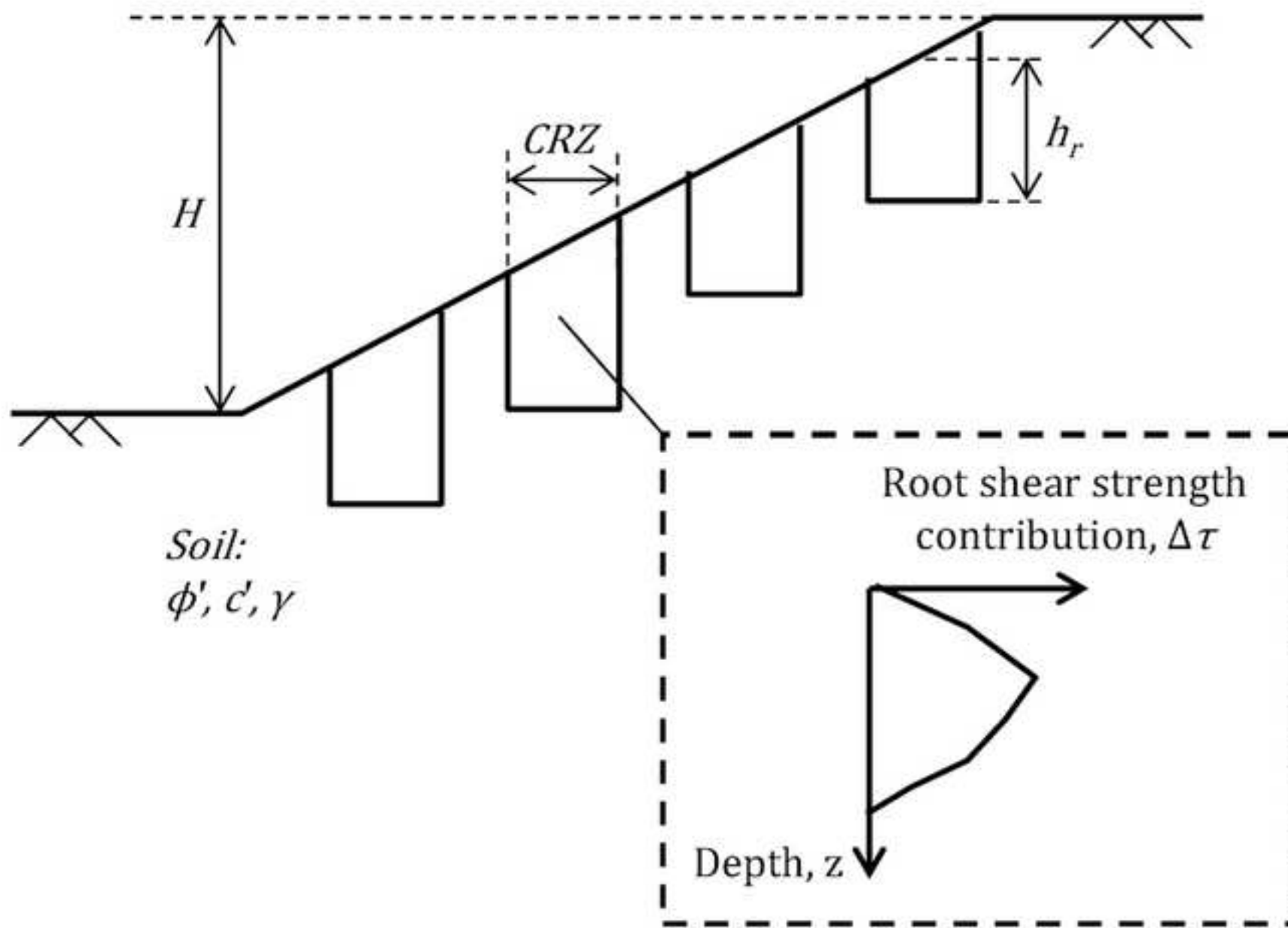


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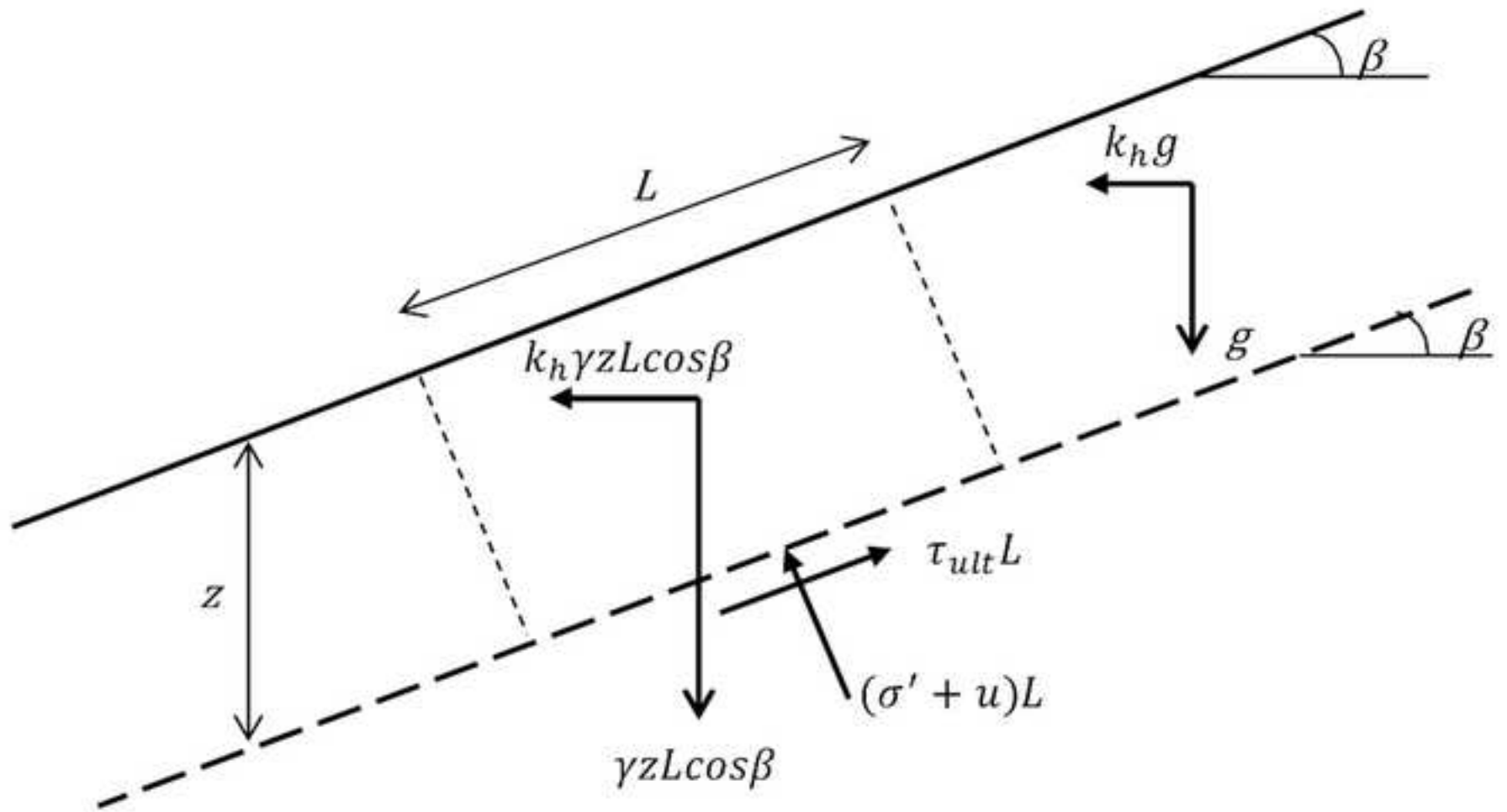


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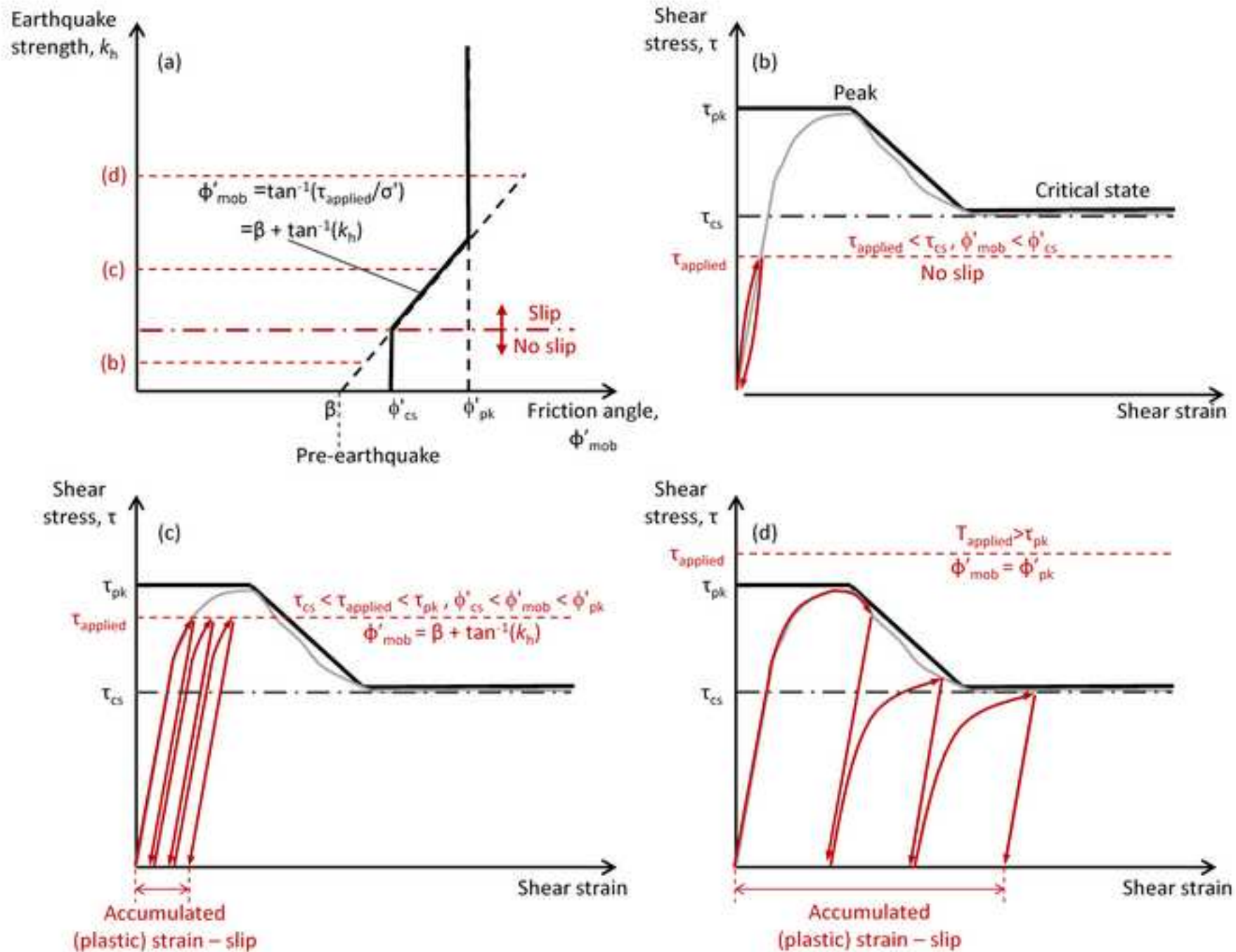


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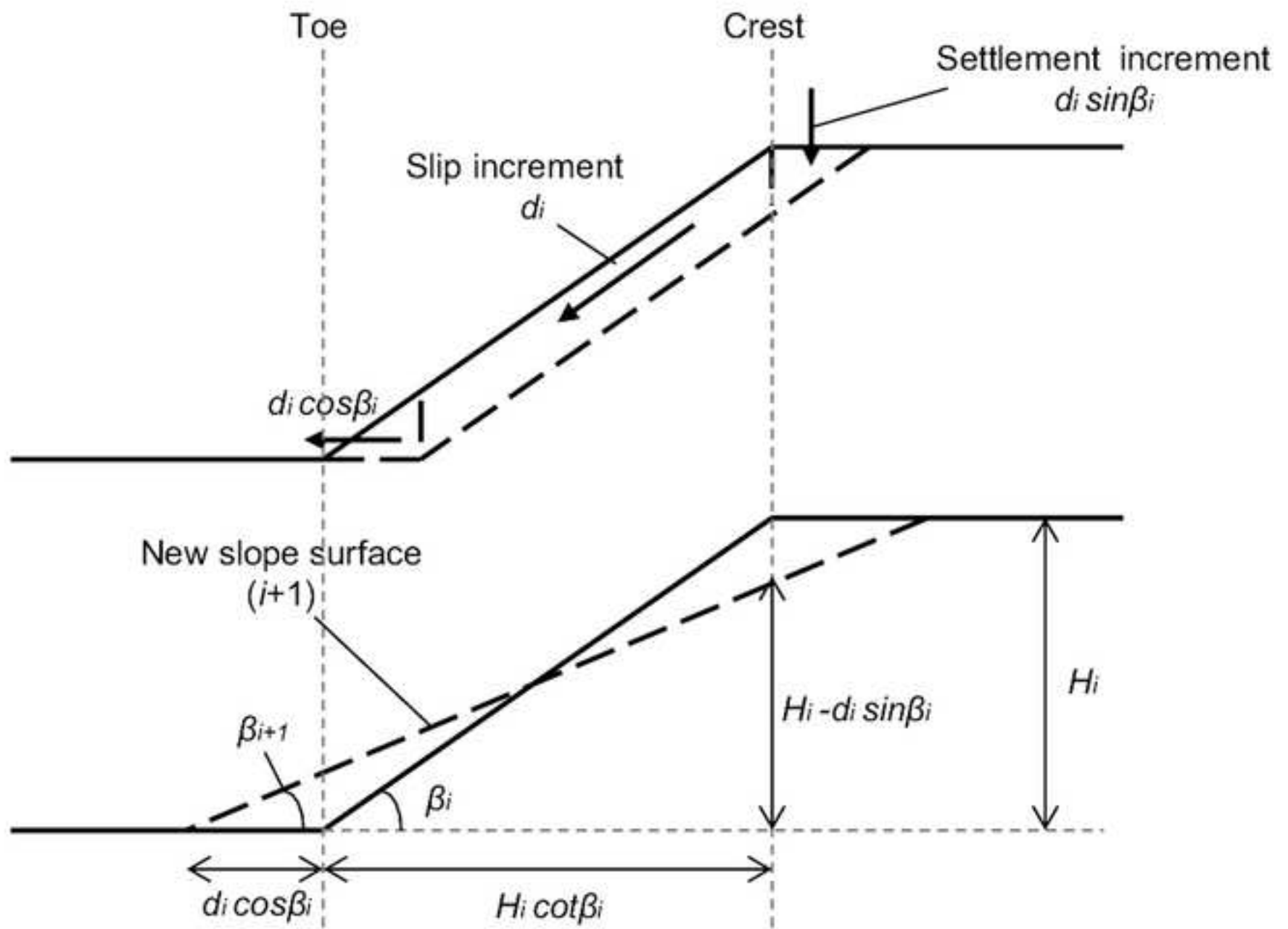


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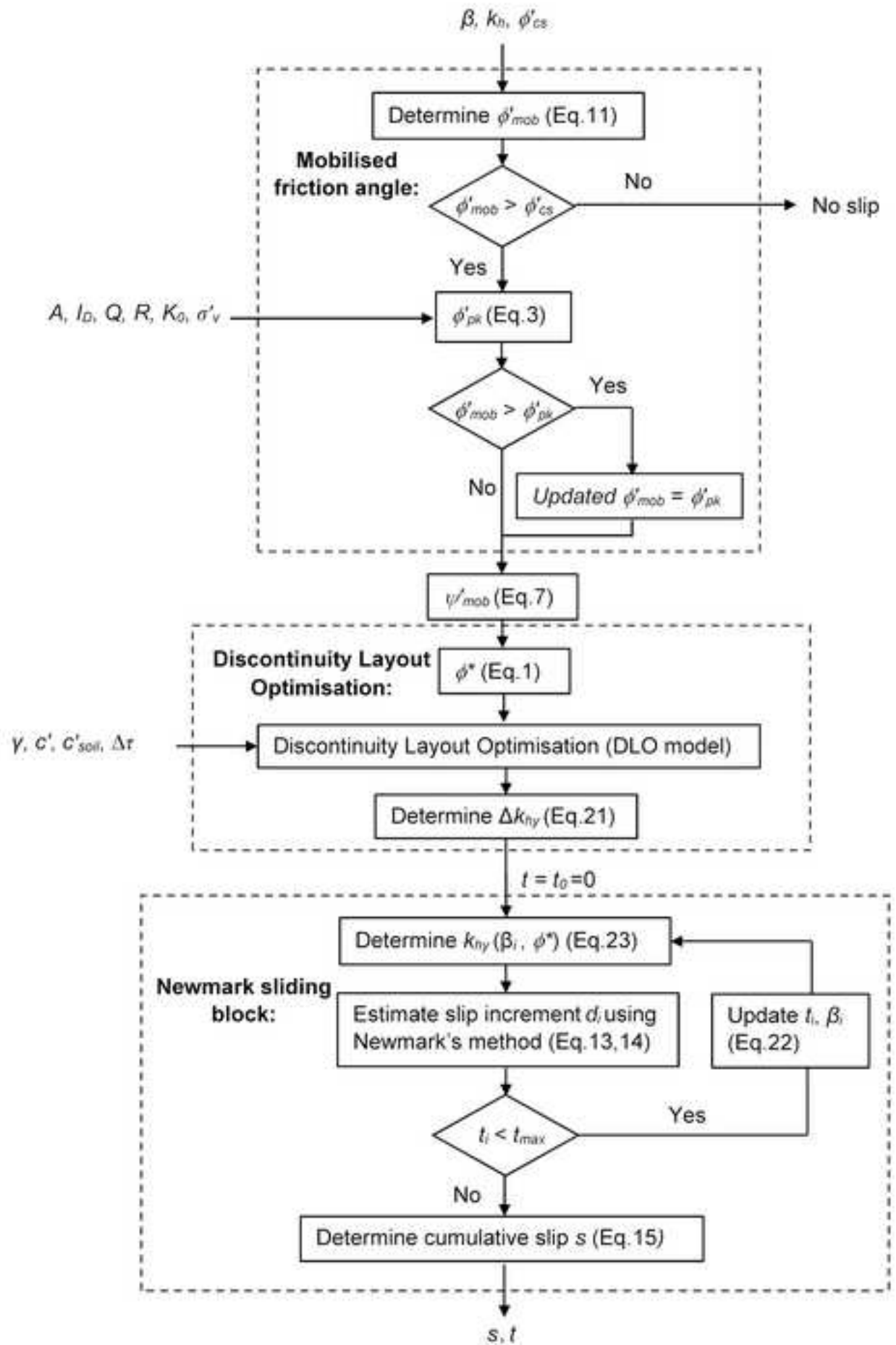
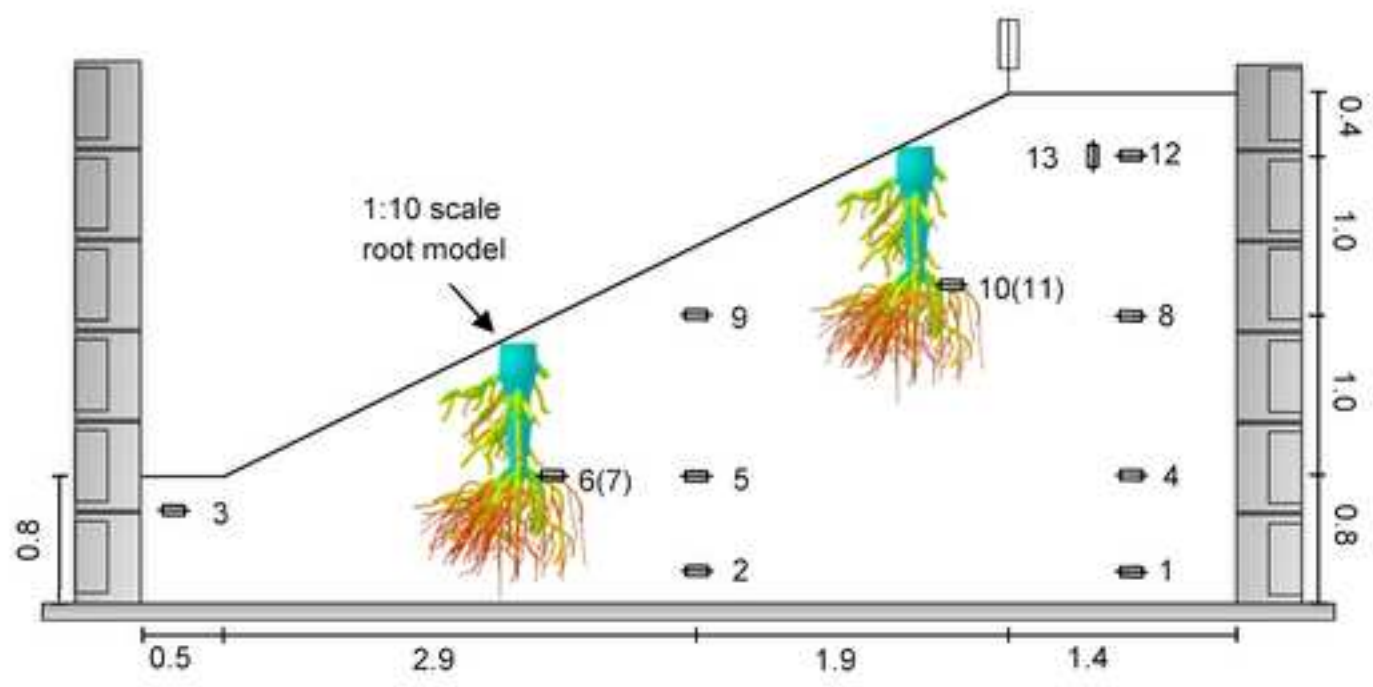
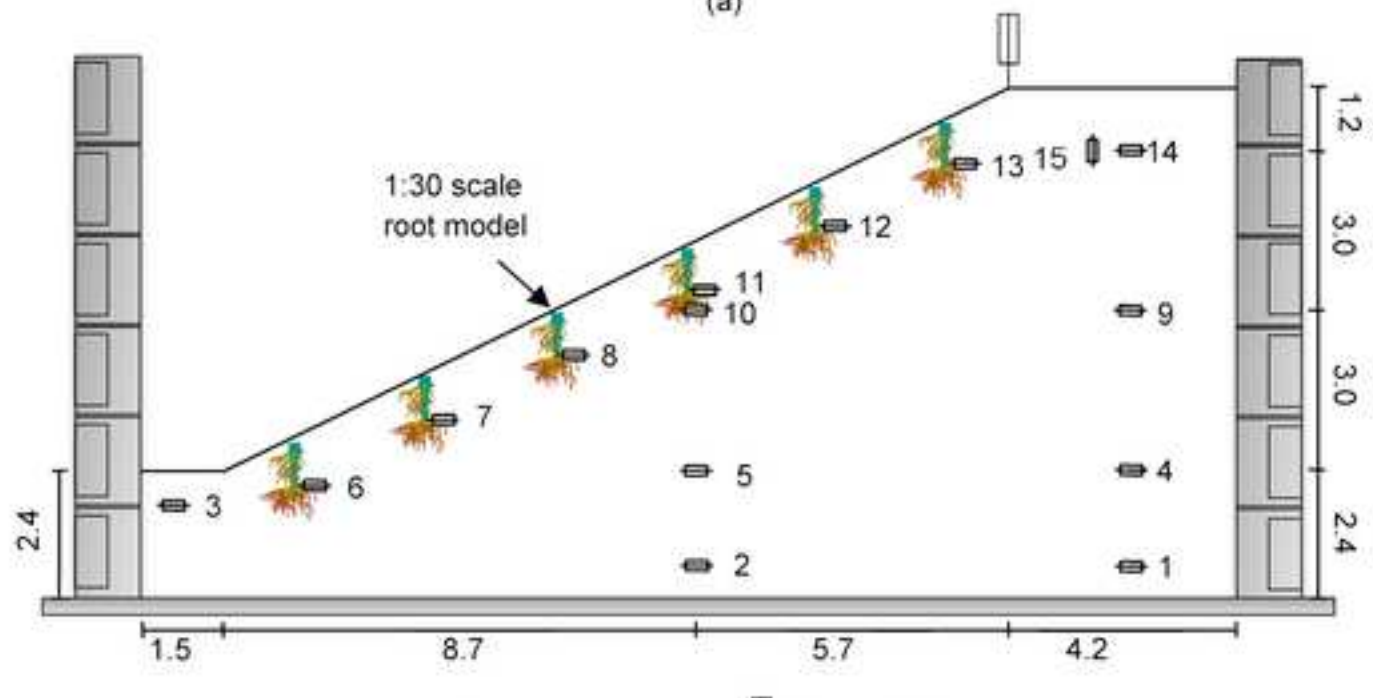


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(a)

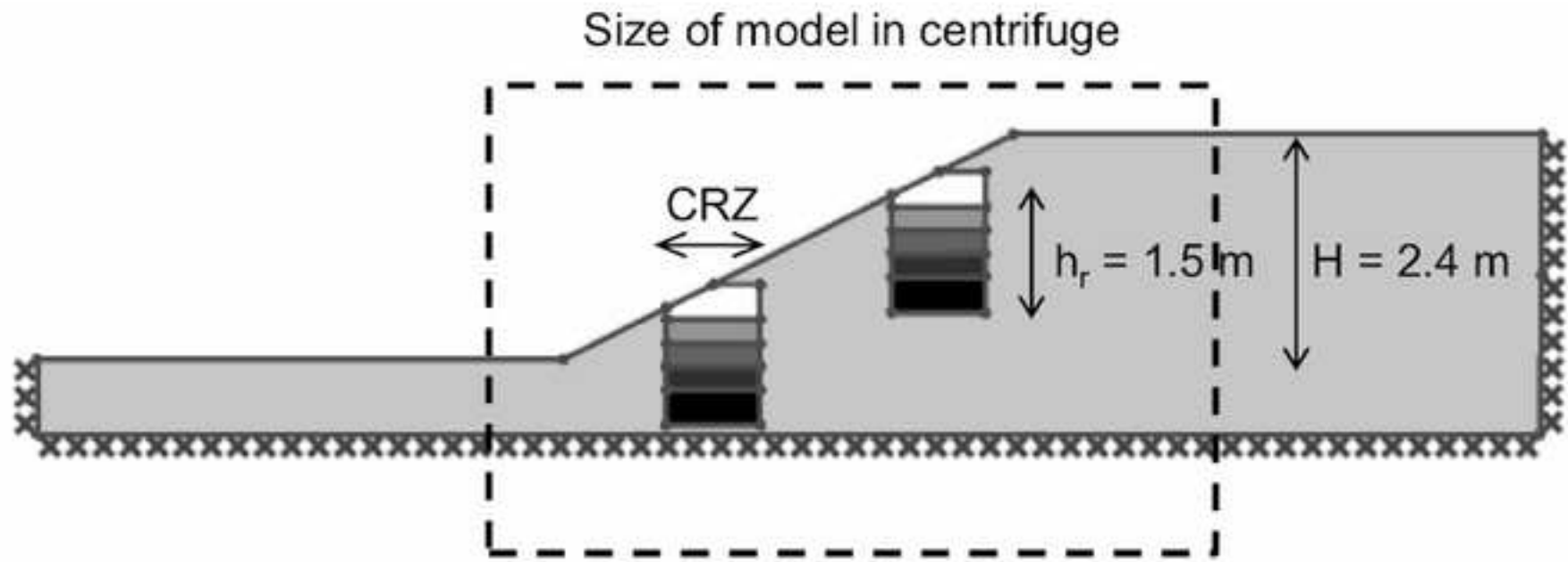


Accelerometer

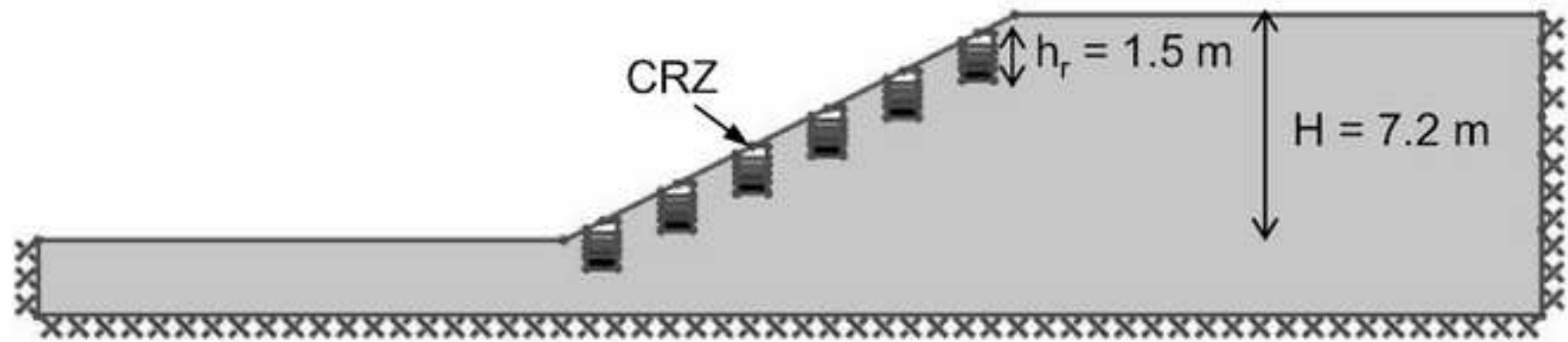
LVDT

(b)

Figure 7
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(a)



(b)

Figure 8
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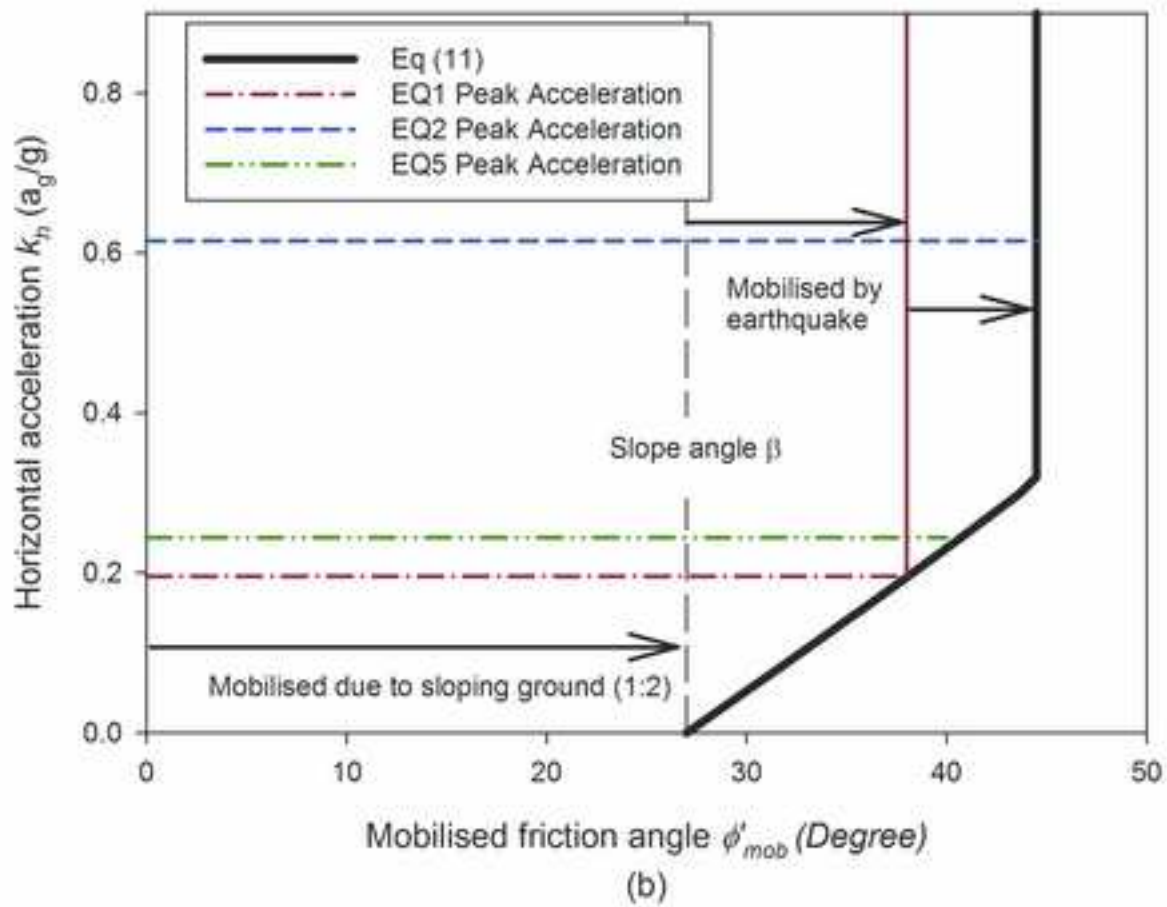
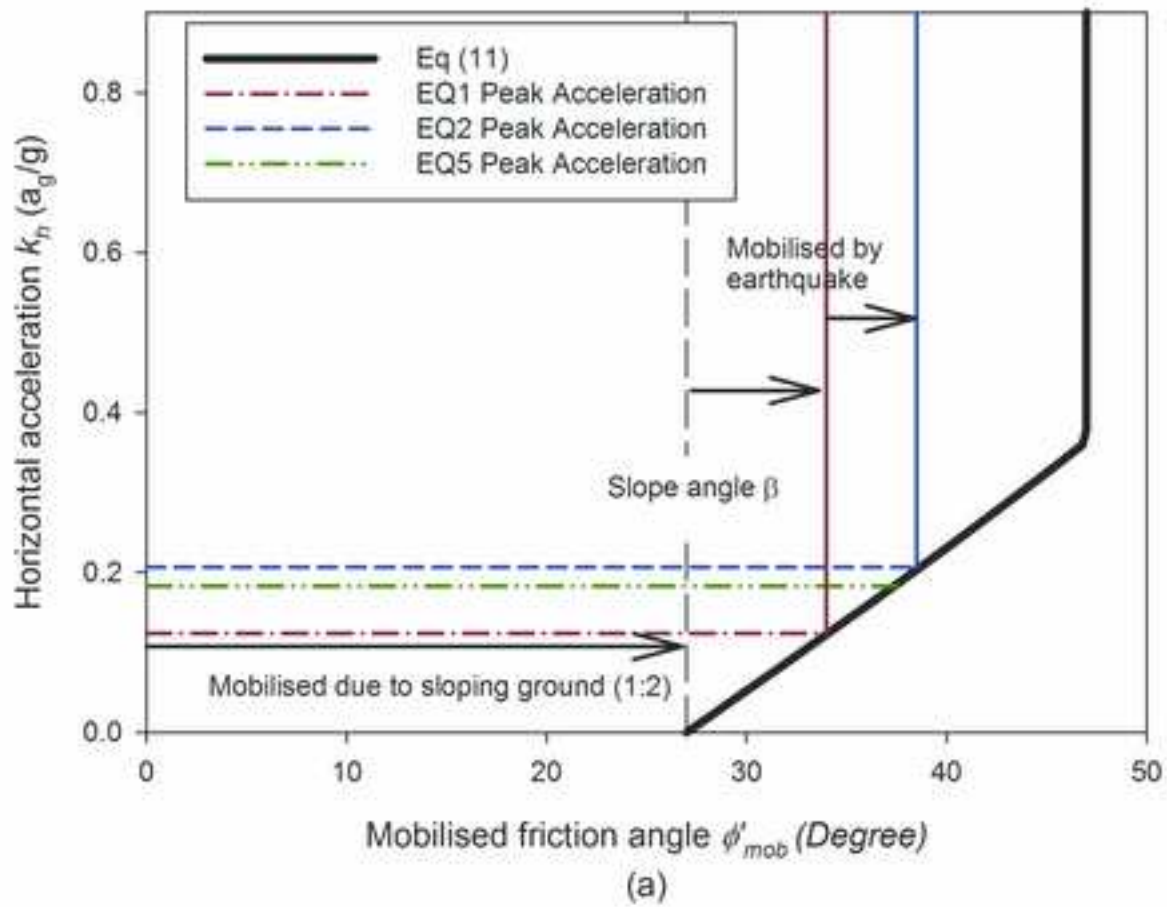
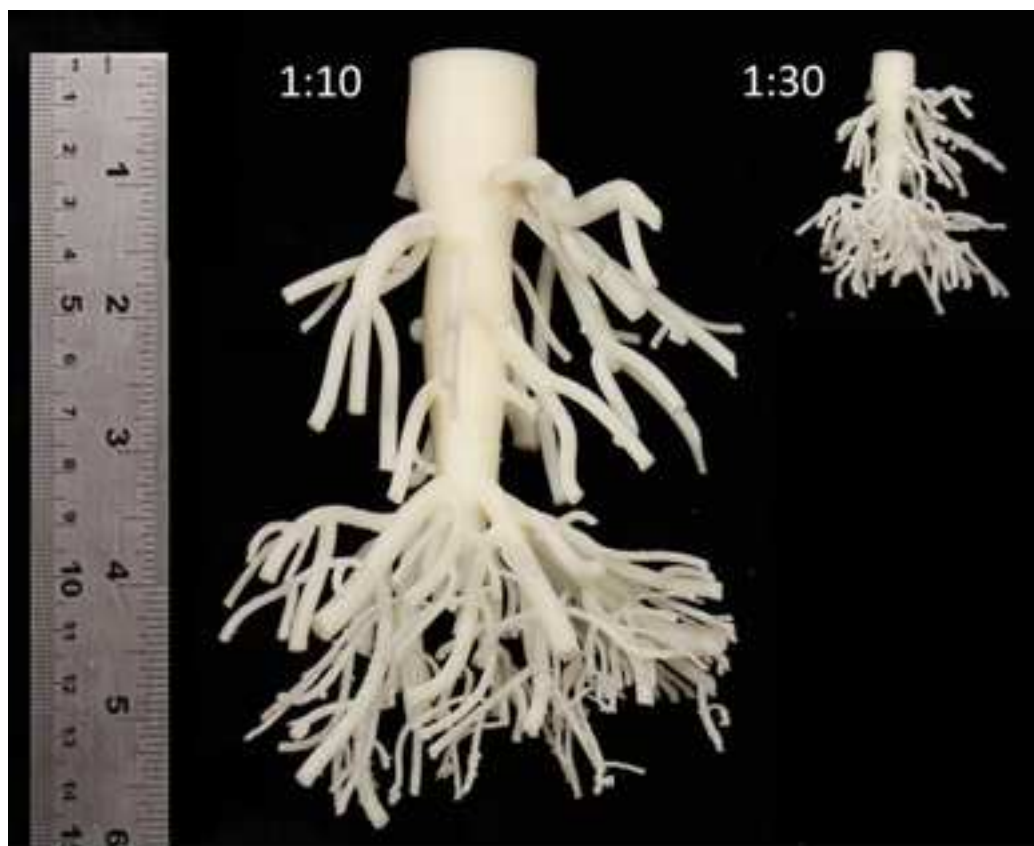
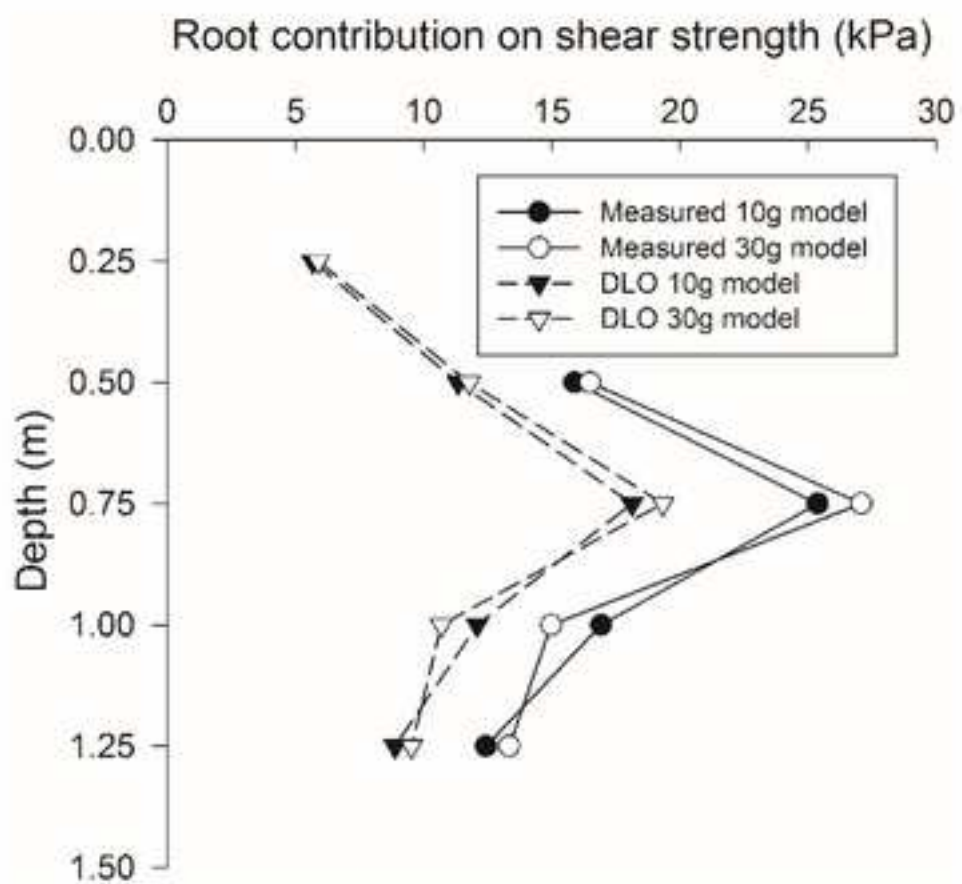


Figure 9

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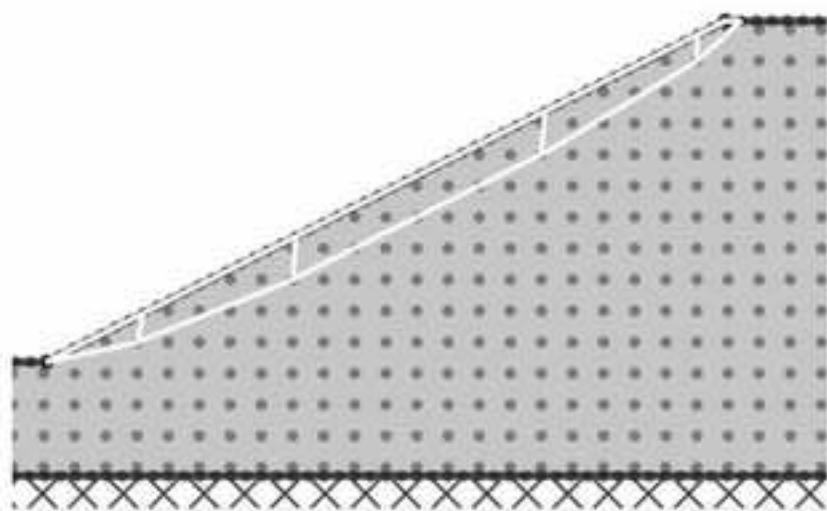


(a)



(b)

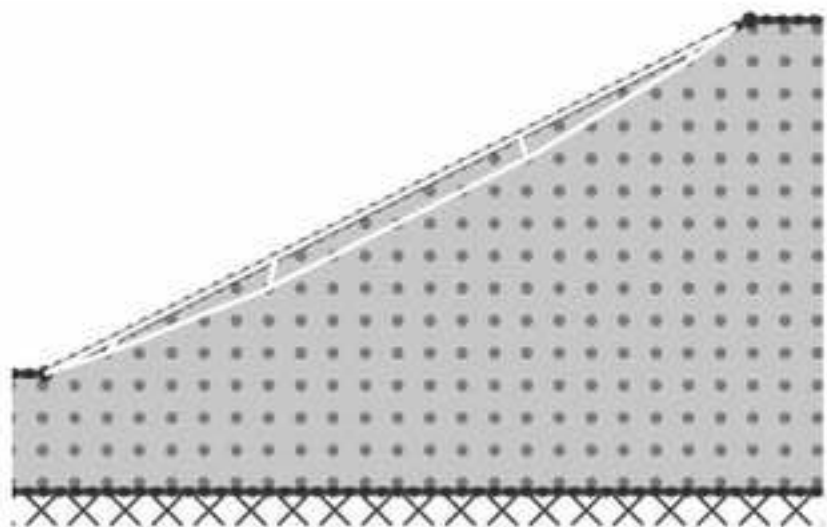
Figure 10
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(a)



(b)



(c)



(d)

Figure 11

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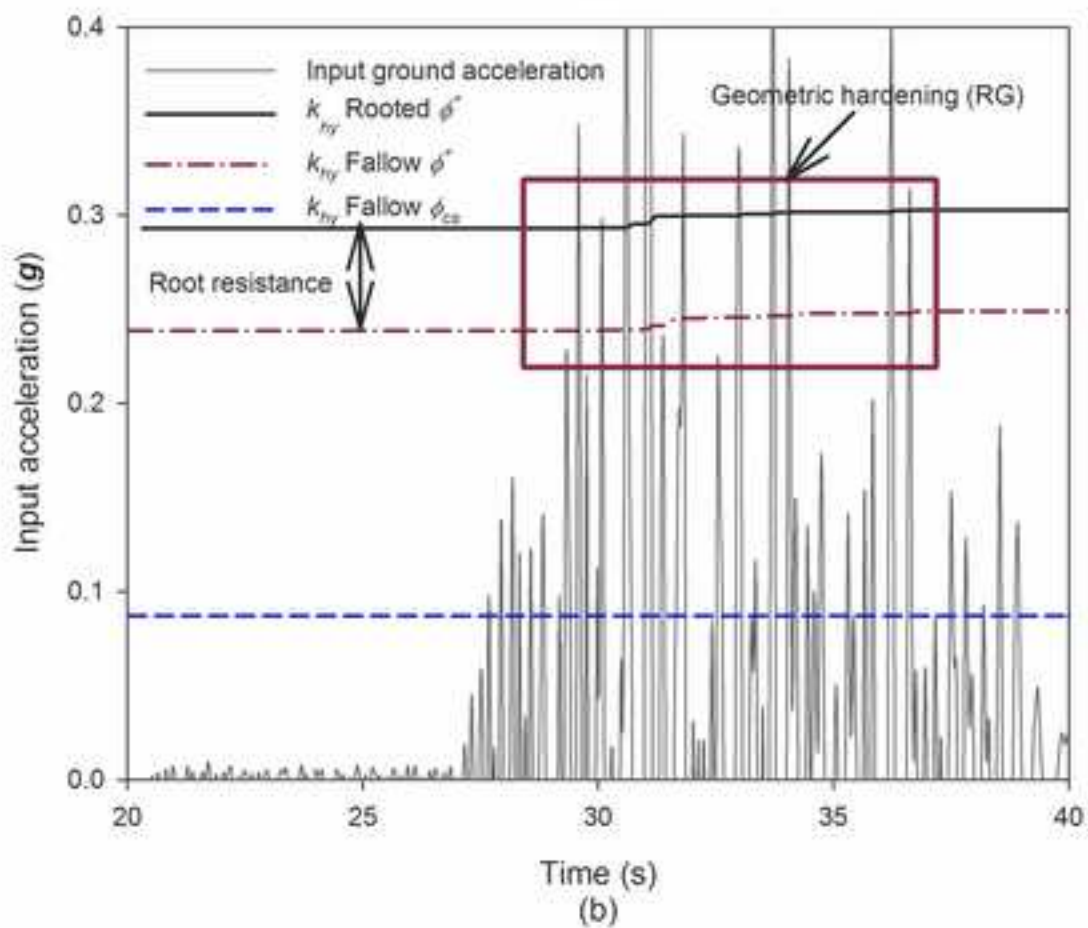
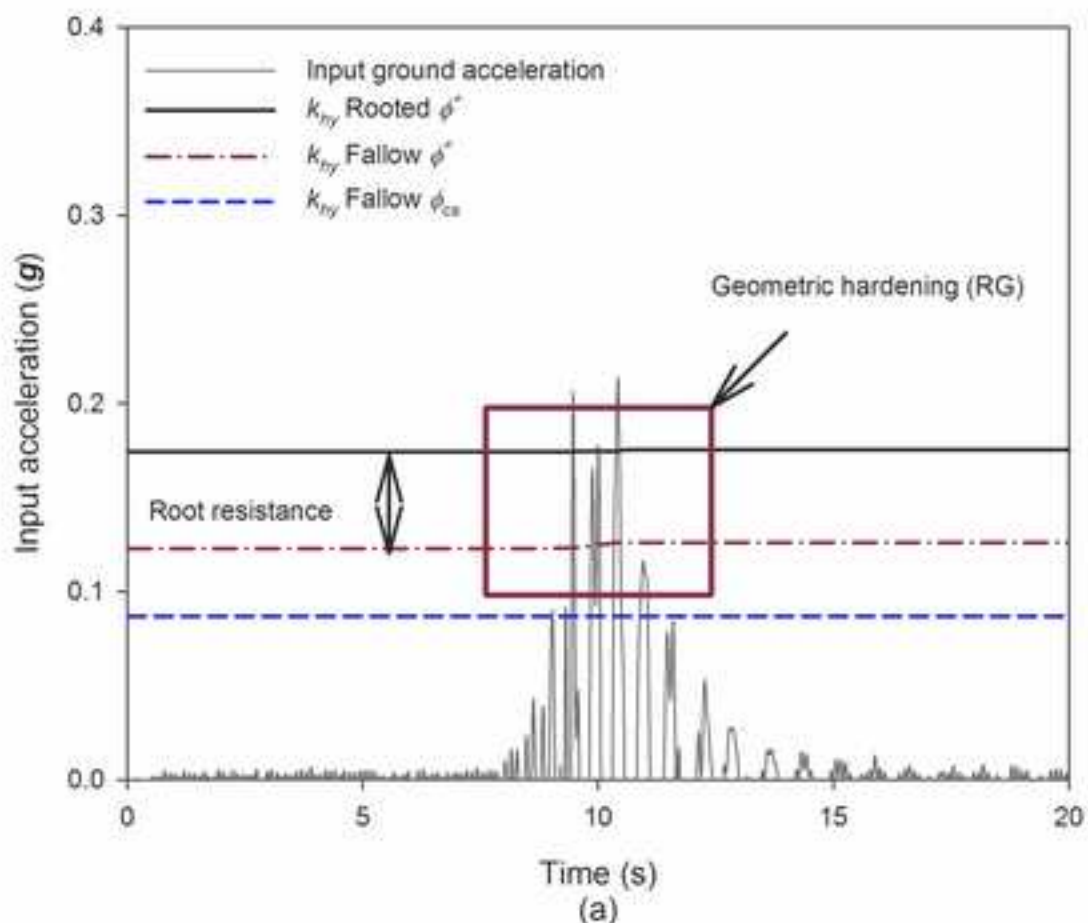


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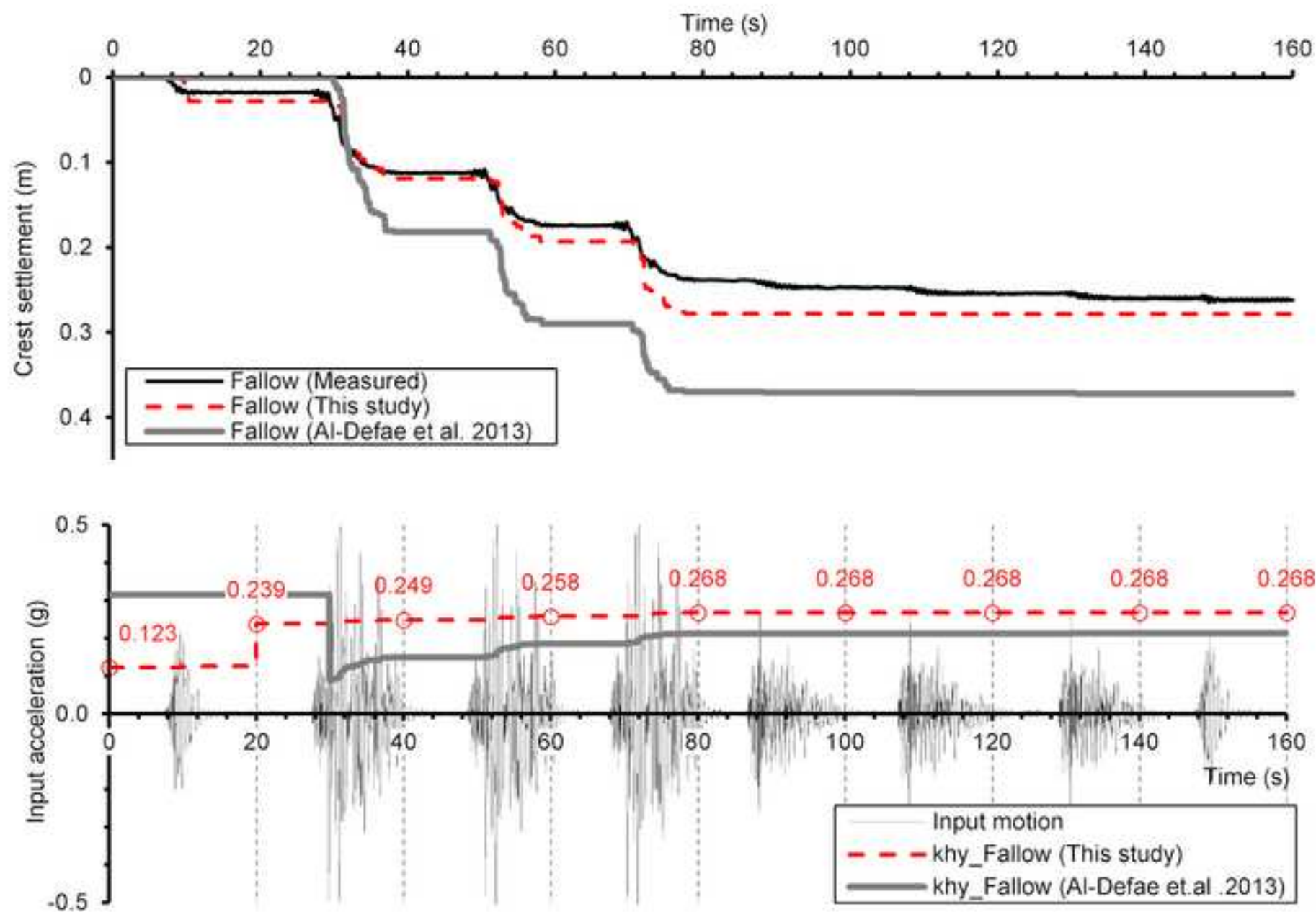


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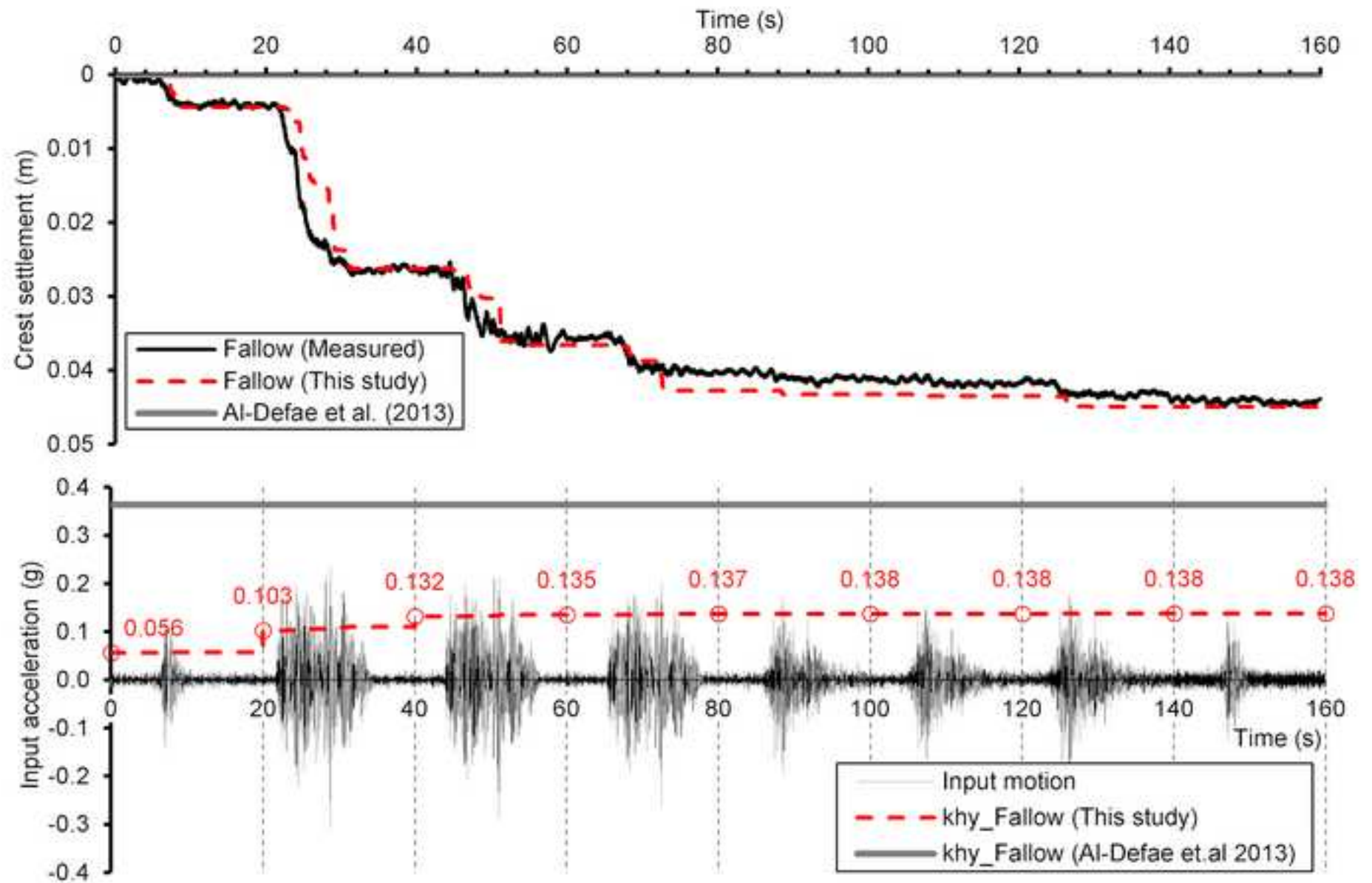


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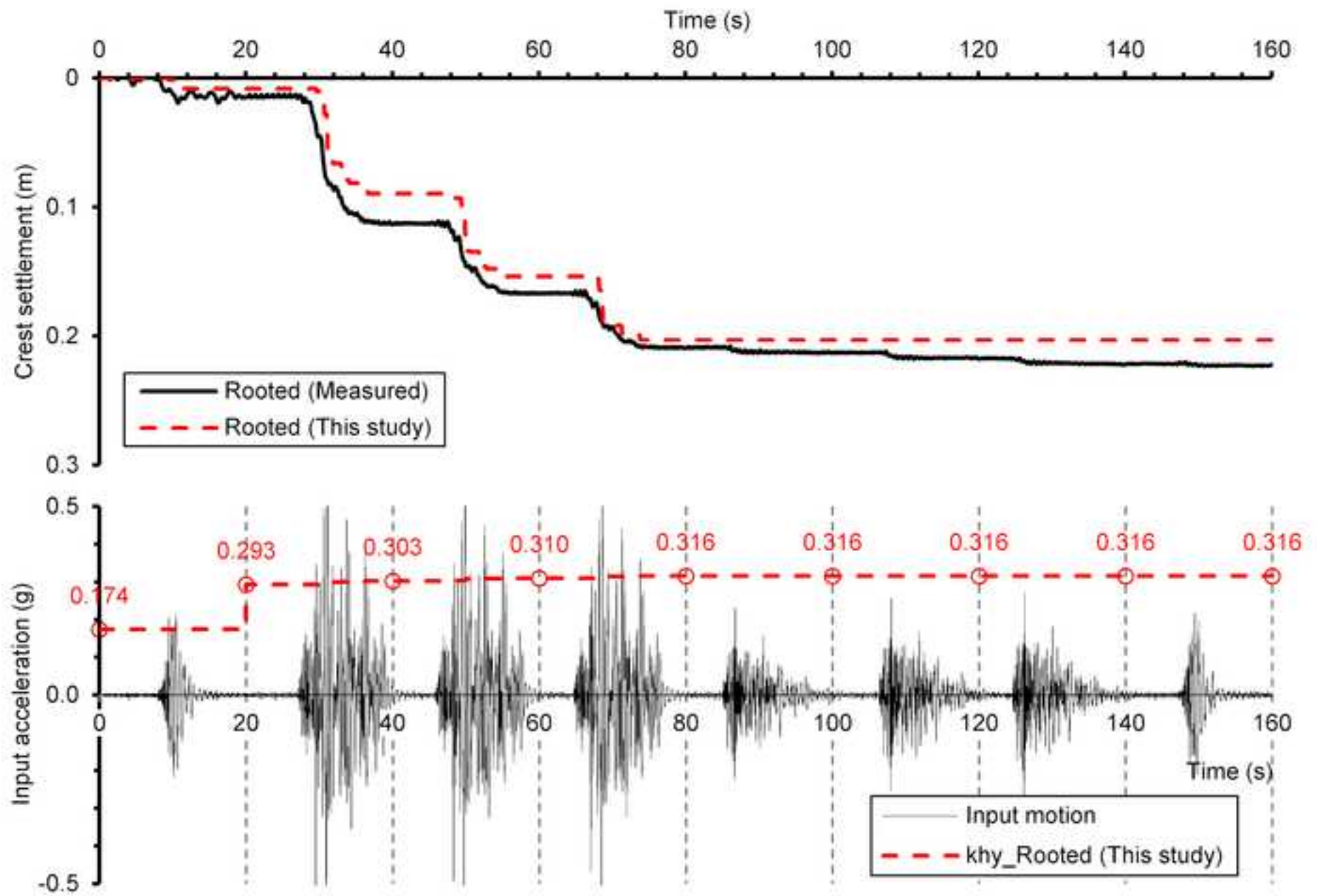


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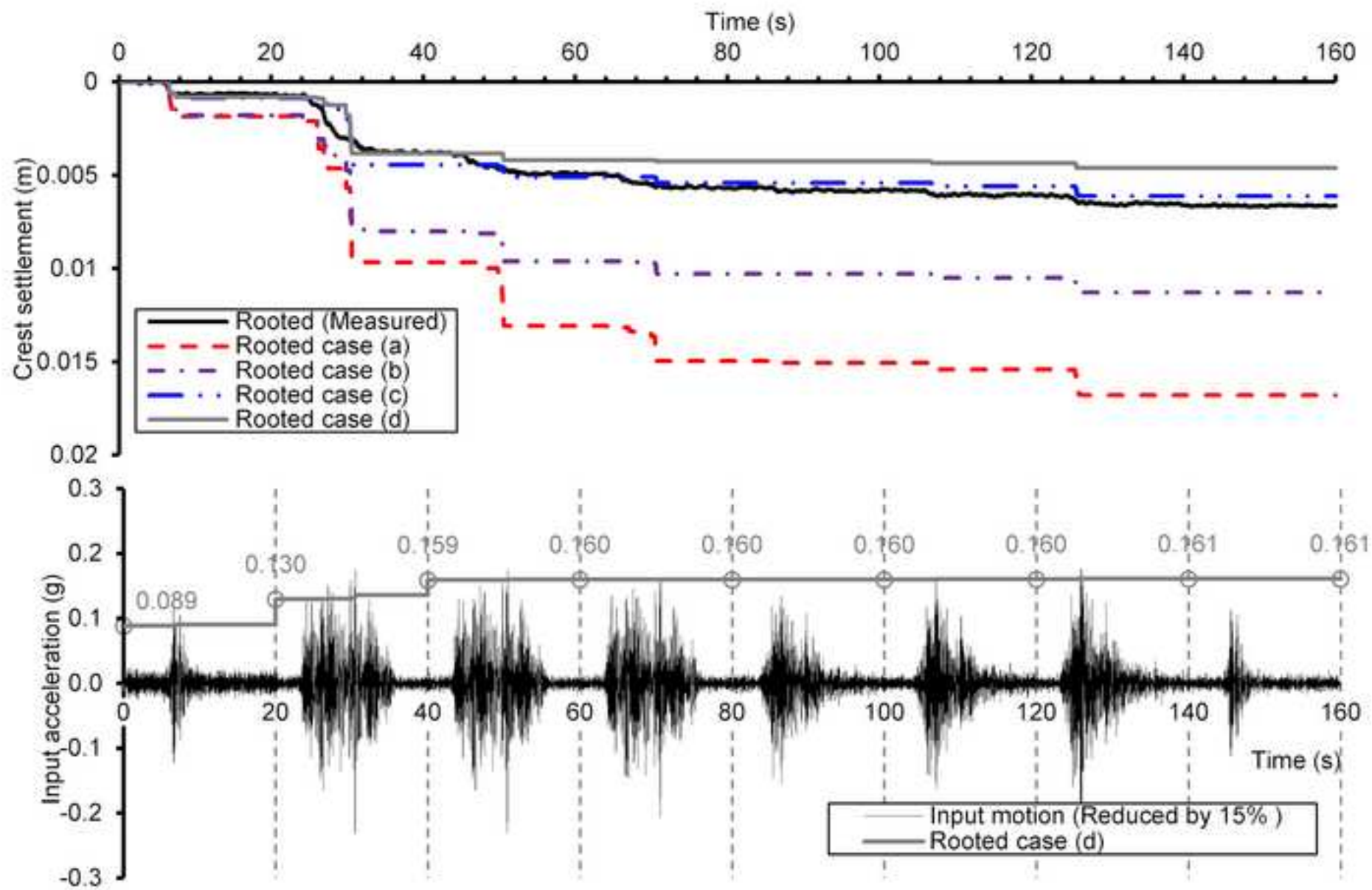


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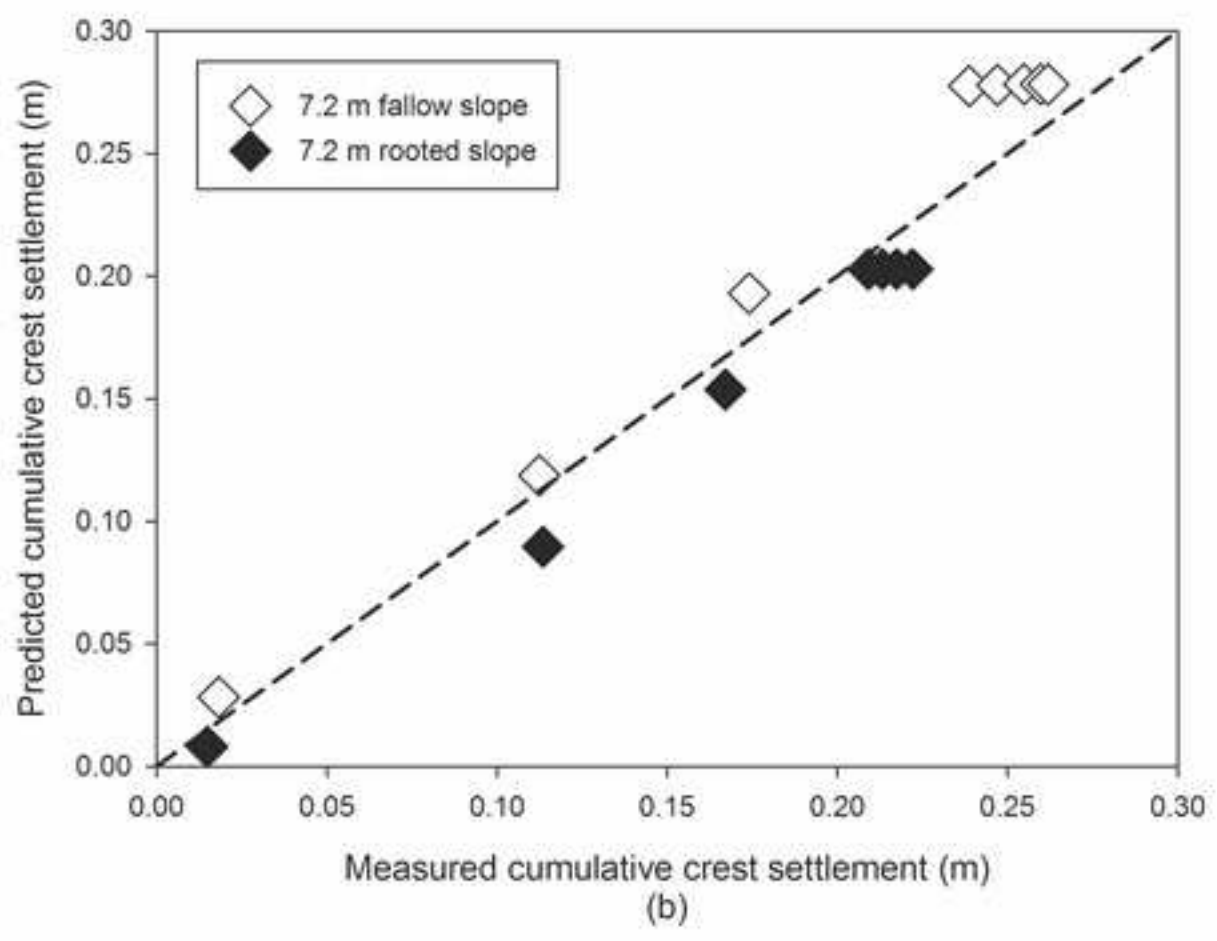
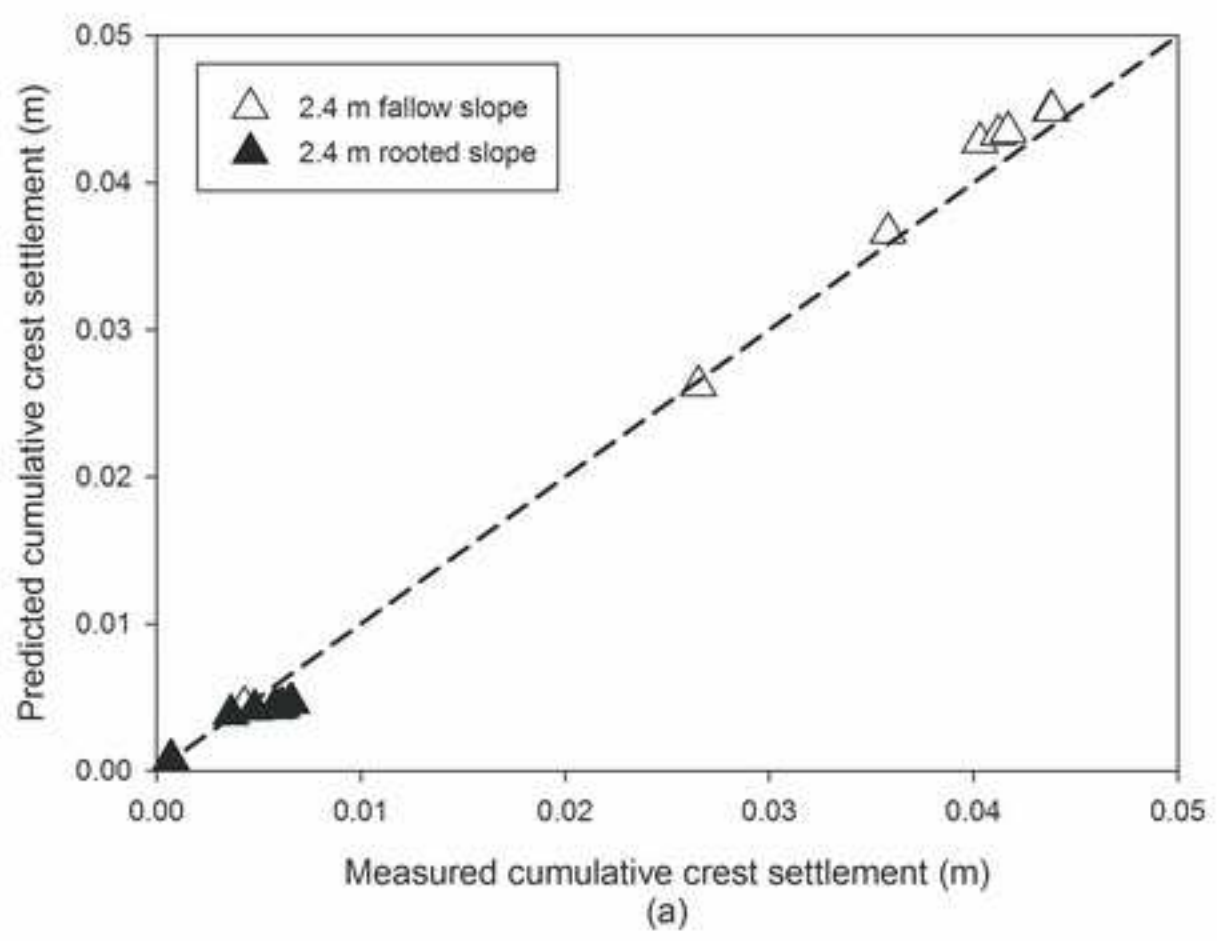
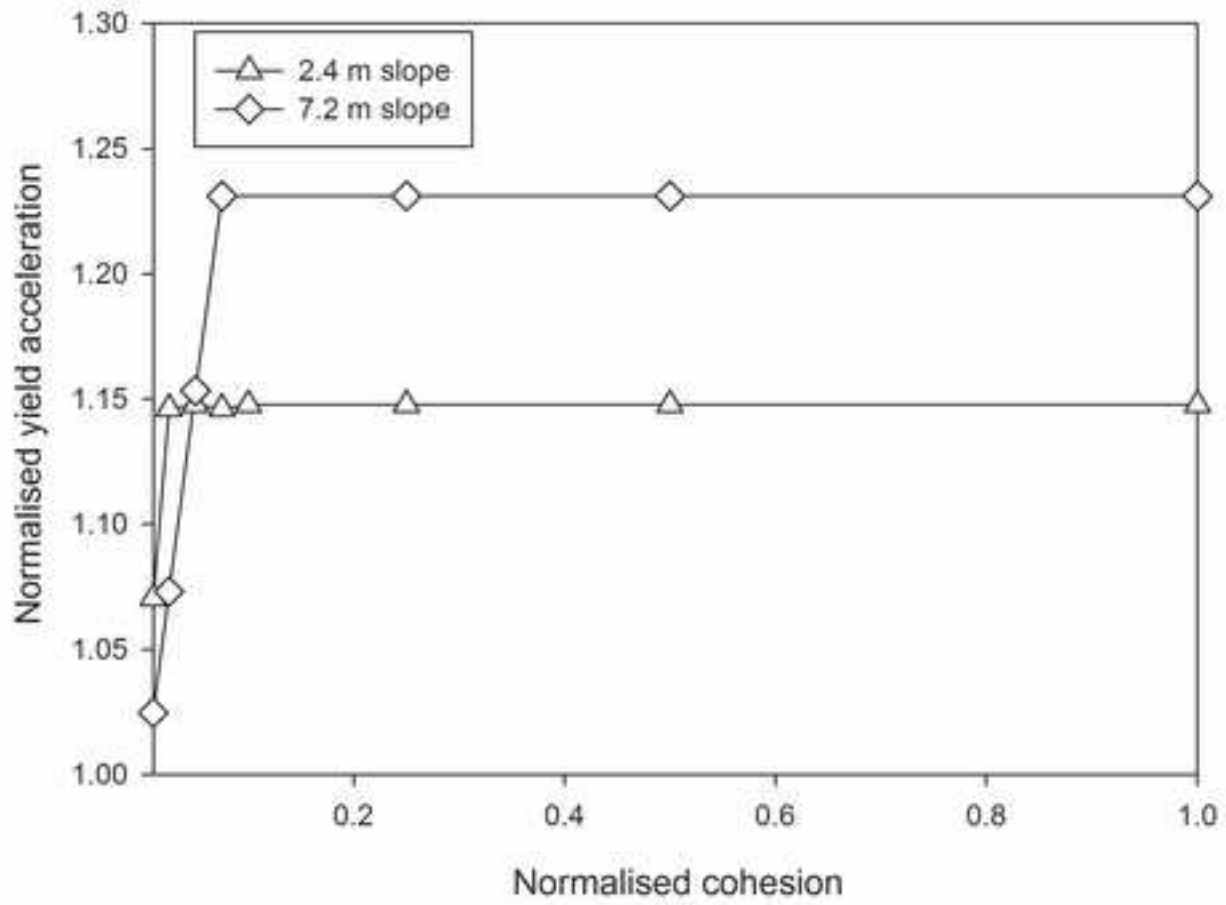
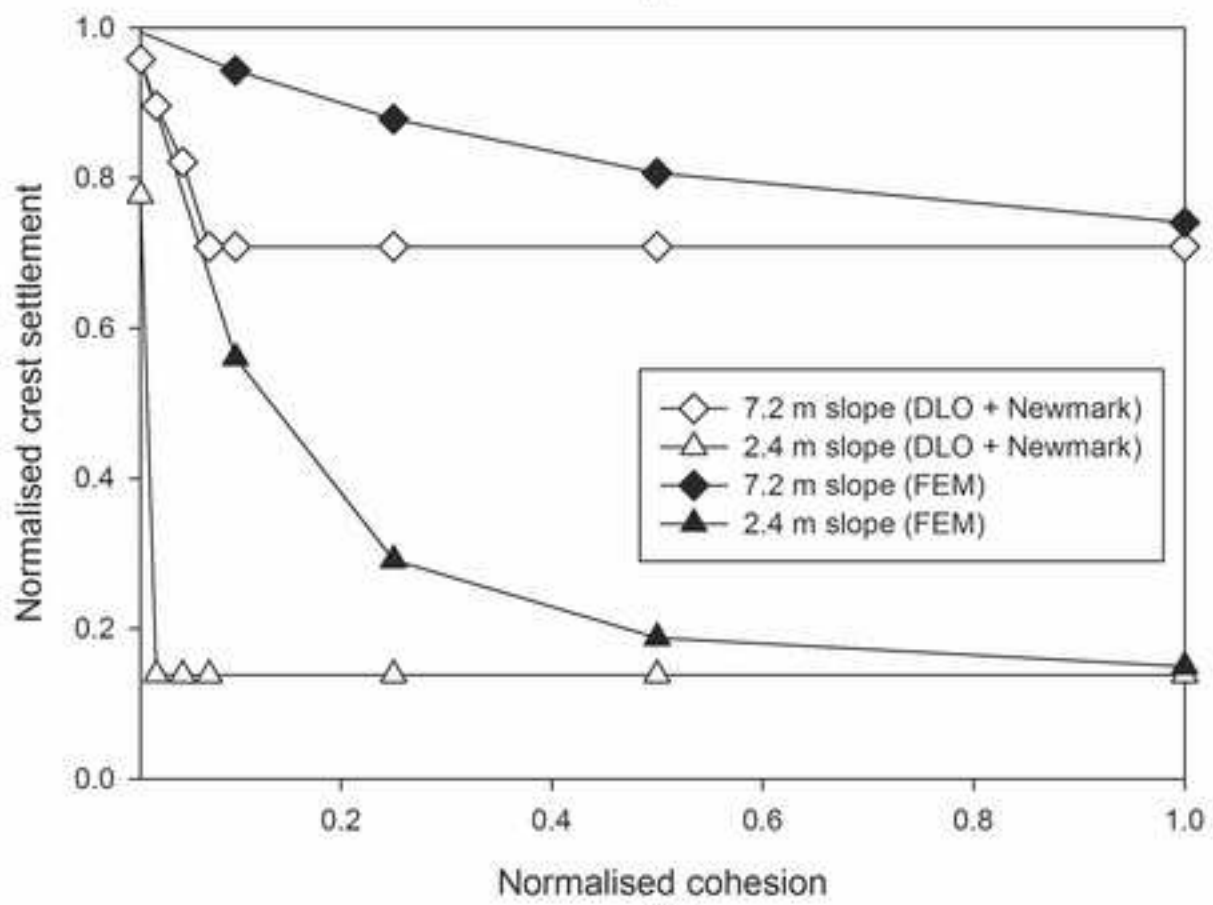


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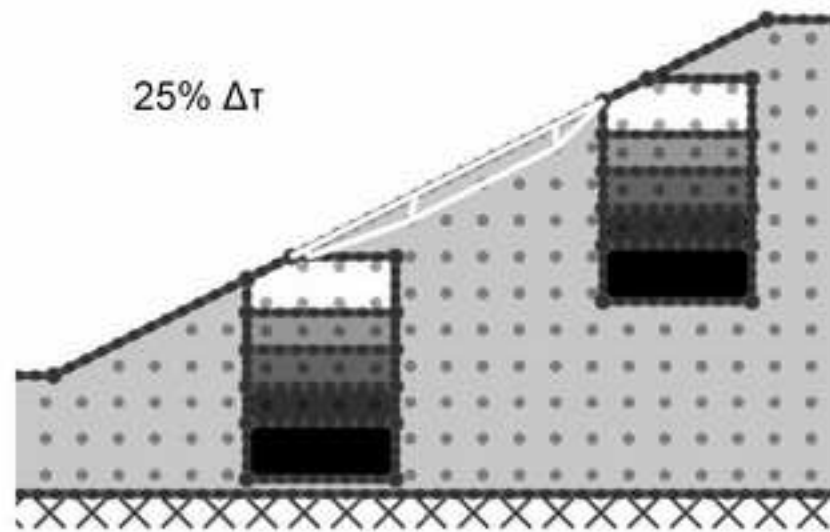
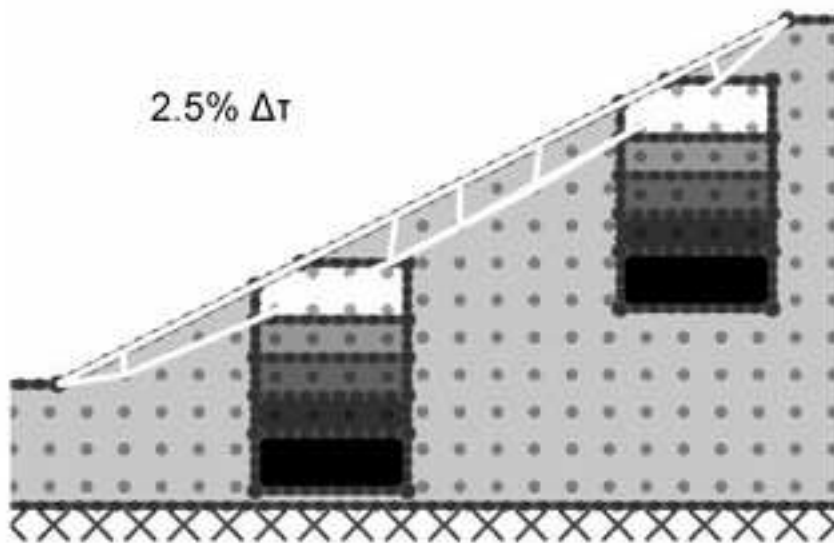


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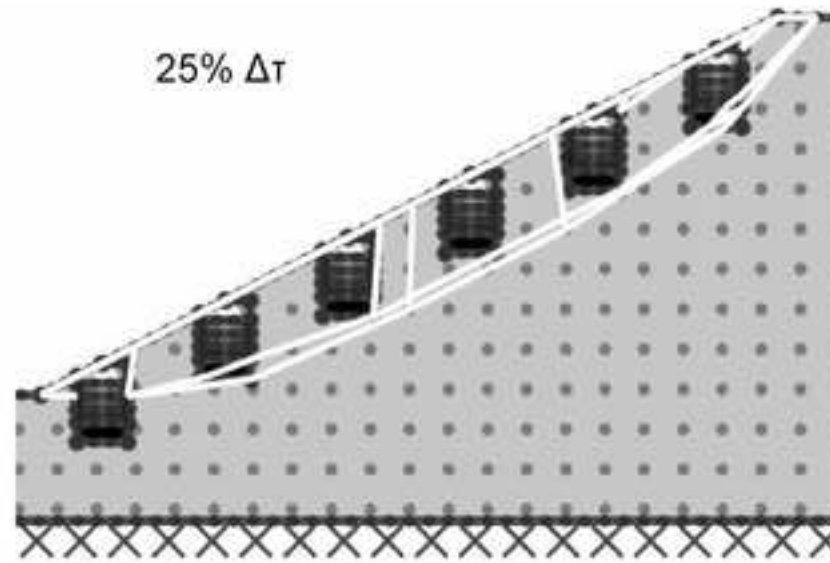
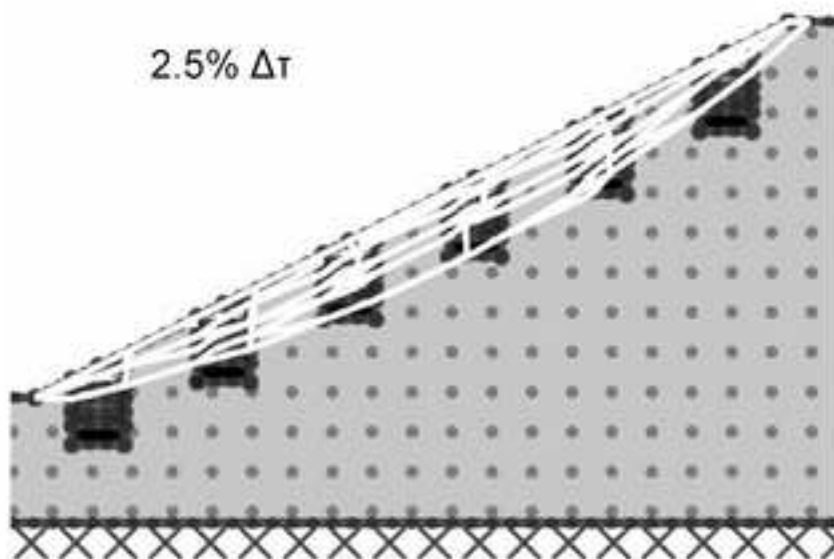


(b)

Figure 18
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(a)



(b)

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