Robotic Billiards: A Holistic Approach

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Robotic Billiards: A Holistic Approach

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***Abstract*—The development and testing of a robotic system to play billiards is described in this paper. The last two decades have seen a number of developments in creating robots to play billiards. Although the designed systems have successfully incorporated the kinematics required for gameplay, a system level approach needed for accurate shot-making has not been realized. The current work considers the different aspects, like machine vision, dynamics, robot design and computational intelligence, and proposes, for the first time, a method based on robotic non-prehensile manipulation. High-speed video tracking is employed to determine the parameters of balls dynamics. Furthermore, three-dimensional impact models, involving ball spin and friction, are developed for different collisions. a three degree of freedom manipulator is designed and fabricated to execute shots. The design enables the manipulator to position the cue on the ball accurately and strike with controlled speeds. The manipulator is controlled from a PC via a microcontroller board. For a given table scenario, optimization is used to search the inverse dynamics space to find best parameters for the robotic shot maker. Experimental results show that a 90% potting accuracy and a 100–200 mm post-shot cue ball positioning accuracy has been achieved by the autonomous system.**

***Index Terms*—Intelligent systems, game-playing robots, non-prehensile manipulation, object tracking, computer vision, manipulators, impact dynamics, intelligent robots, educational robots**

I. Introduction

THE NEED FOR autonomous systems designed to play both strategy-based and physical games comes from the quest to model human behavior under tough and competitive environments that require human skill at its best. In the last two decades, and especially after the 1996 defeat of the world chess champion by a chess-playing computer, physical games have been receiving greater attention. Robocup^{TM}, i.e. robotic football, is a well-known example, with the participation of thousands of researchers all over the world. The robots created to play the billiards family of games are placed in this context [1]-[2]. The billiards family has a number of variants of which snooker and pool are the most common. The former is quite popular in Europe and has spread to the Far East, hailing TV audiences of hundreds of millions for major tournaments in the last few years [3], while the latter is widely played throughout North America. Although both the games have different rules for pocketing the balls and slightly varied table and ball sizes, from an autonomous game playing robot's point of view, the challenges in terms of ball manipulation, vision, and strategy are entirely similar. Hence, snooker, to play which a robot is considered presently, is frequently referred to as billiards, in a general sense, in this paper. Snooker, as well as being a game of strategy, also requires accurate physical manipulation skills from the player, and these two aspects qualify it as a potential game for autonomous system development research. Although research into playing strategy in billiards has made considerable progress by using various artificial intelligence methods [4, 5], the physical manipulation part of the game is not fully addressed by the robots created so far. This work looks at the different ball manipulation options snooker players use, like the shots that impart spin to the ball in order to accurately position the balls on the table. In this regard, predicting the ball trajectories under the action of various dynamic phenomena, such as impacts and friction, is a key consideration of this paper.

The paper continues as follows. In Section II, a critical review of the current systems available to play billiards is provided. Then, some machine vision-based experiments on ball tracking to determine the dynamics of billiards are described in Section III. This is followed by Section IV, which outlines the various dynamics of balls on the table, also incorporating the effects of 3-dimensional ball spin. Section V treats the design, fabrication and integration of the proposed manipulator. Afterwards, in Section VI, a new approach to strategic shot selection is proposed by integrating the key aspects described in the previous sections. Section VIII concludes the paper by providing the results from experiments with the robot manipulator.

II. A Review of Billiard Robots

In the mid-late 90s, Bristol University developed a robotic snooker player called the Snooker Machine [1]. The robot consists of a PUMA 560 manipulator arm suspended from an SKF Linear Drive System that is mounted as a gantry system above the table. The PUMA arm carries a cue, powered by pneumatics. The vision system consists of an overhead camera to determine the position of balls on the table and another camera mounted on the cue holder to accurately position the cue on the cue ball. Considerable attention has been on developing a playing strategy for the robot. The robot's performance is not discussed, except for some special shots, like the ones where the motions of the balls in impact are confined to a straight-line. The strategy subsystem treats

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various types of impacts by using primitive collision models, like the principle of conservation of linear momentum [6]. In another work, a prismatic, XYZ overhead gantry robot has been used in conjunction with a revolute end effector to play pool [7]. When compared to that of Chang [1], the main difference is on shot selection based on fuzzy-based reasoning [7]. Other works too have had XYZ gantries attached to the tables themselves, e.g. Cheng et al. [8], although such mounting prevents humans and the robots sharing the same table for any competitive gameplay. Another minimalistic, 2 DOF robot consisting of a motorized cue and its yaw control is developed to experiment a certain machine-learning technique called 'reinforcement learning' with it [9].

A major R&D effort to create a pool playing robot, named 'Deep Green', has been going on for the last decade at Queen's University, Canada [10]. Once again, an overhead industrial gantry robot carries a suspended robot manipulator, to which a cue stick manipulator is attached (Figure 1). A

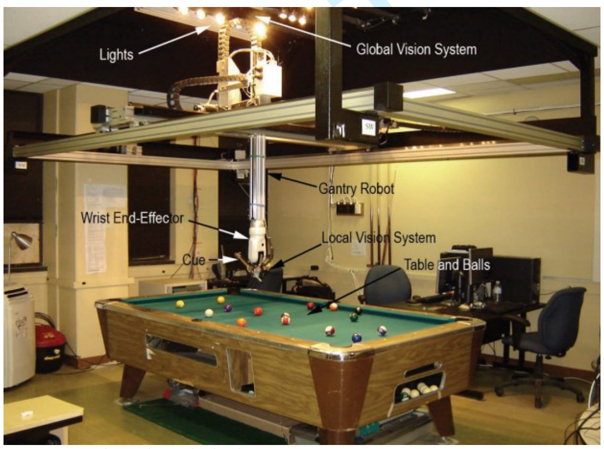


Fig. 1. Gantry-based pool playing system [2]

custom-designed linear manipulator is used to drive the cue stick. The spatial positioning accuracy of the robot is reported to be 0.6 mm and a ball potting accuracy of 67% is claimed for the straight shots [10]. According to their later publication, the robot has pocketed runs of four consecutive balls [2]. However, ball pocketing accuracy and cue ball post-positioning accuracy, which are crucial metrics for a successful and continued gameplay, are not provided. In a recent work an anthropomorphic robot has been developed to play pool [11] The proposed kinematic solution resembles that of a human. However, for the modeling of billiard dynamics the work resorts to primitive impact models such as that of Wallace and Schroeder [12]. More importantly, the sidespin of balls has not been captured in the impact models used.

The robots configurations found in the literature, be it the gantry based configurations or the humanoid, offer perfectly suitable kinematic solutions for shot executions. The vision subcomponents of the described systems appear to be apt for a billiard robot. However, a key aspect of the game is in striking the cue ball by imparting different spins and speeds on it by the cue. The existing billiards systems have handled straight and angled shots whereby the speed and the yaw angle of striking with the cue have been the only parameters controlled with the robot. Whereas, billiard players obtain the required spin and speed combination by choosing the right point on the cue ball to strike. The resulting spin and speed of the cue ball, while potting the object ball, leaves the former at an advantageous spot on the table to maximize the chance of the next strike and so on. This maneuvering is explained in Fig. 2, where depending on the speed and spin imparted, the cue ball ends up in entirely different locations on the table and this ability is vital for any competitive billiard robot of the future.

The reasons for other researchers having not considered the option of spin manipulation of balls is manifold. The spin dynamics of balls itself has not been discussed thoroughly in the physics literature that addresses spherical bodies. Especially, the impact dynamics of spinning spherical bodies is a special problem that has not be treated comprehensively in other literatures for the roboticists to make use of. Moreover, there are a few physical parameters involved in spin dynamics whose values must be known for a robot to work well. A similar situation is found in the non-prehensile robotic manipulation, where the exact dynamic information of the environment is necessary for a robot to manipulate an object without grasping it [13,14,15,16]. This is the background in which this research work is conducted.

III. Vision and High-speed Tracking of Balls

A Riley Renaissance professional snooker table of size 10 × 5 ft^2 is used for all experiments described in this paper. A single overhead camera is sufficient to track the balls as trajectories are confined to the plane of the table. Snooker is

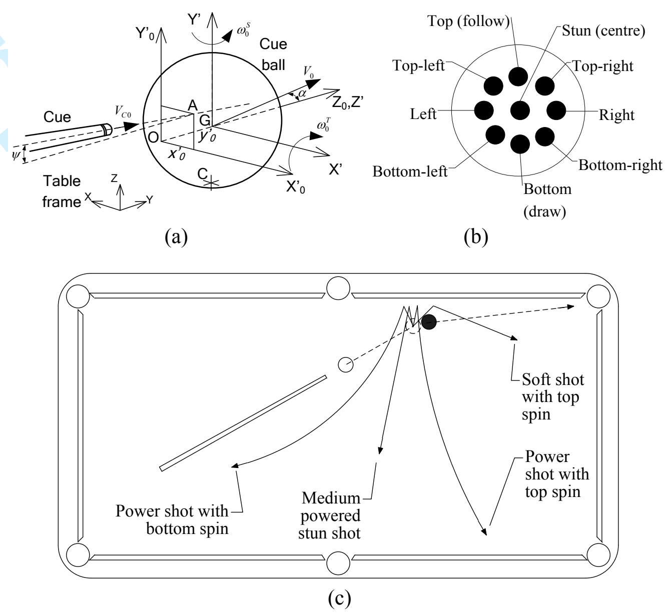


Fig. 2. Geometry in cueing (a), areas to strike on the cue ball and resulting spins – horizontal view of the player [17] (b) and final post-shot cue ball locations after a ball-pot for different top and side spins imparted on the cue ball – top view of the table [22] (c)

played with a cue ball in white and 21 object balls of different, but uniform colors, *sans* any patterns. Hence, a color camera is used. The chosen camera, PixeLINK PL-B776F, is mounted over the table, pointing perpendicularly down. The camera has a resolution of 3.15 megapixels and contains the region of interest (ROI) option with which it can capture up to 1000 frames per second. The camera images the table area of approximately 5 × 6 ft^2 (1.52 x 1.83 m^2) area to a 1mm spatial resolution.

The white cue ball is used for most of the ball tracking experiments. For interactions involving the cue and the cue ball, a part of the cue is also tracked. The cue, of bright-colored wood, is easy to track (see Figure 3). The assumption made in tracking the cue is, it remains close to the horizontal and is at the level of the ball centers. Some of the tracking experiments to measure linear motion parameters are described, in detail, in our earlier paper [17]. The parameters measured were sliding and rolling coefficients of friction, between a ball and the table, and the coefficients of restitution between two snooker balls and a ball and the table side rails. Most of the tracking is performed at 120 frames per second. In addition to the results given in our earlier paper, a circular black pattern is put on the cue ball, as shown in the right of Fig. 3. The ball is spun sideways about the vertical (i.e. sidespin) from a stationary point and is tracked to measure the resistance of the table cloth to sidespin. The resistance to sidespin, in terms of rotational deceleration, is found to be 22 rad/s^2.

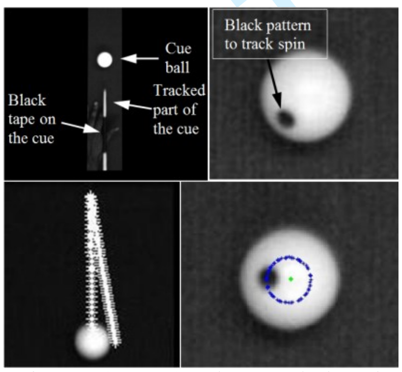


Fig. 3. Tracking features (top row) and two tracked results at 120 and 150 frames per second (bottom row)

IV. DYNAMICS

A number of different dynamic phenomena is involved in billiards. The first one is impulse between the cue and the cue ball. After cueing, the cue ball starts to slide and roll on the table. Then it either impinges another ball or bounces off a cushion (i.e. side rail of the table).

A. Cueing

Referring to Figure 2(a), when the cue inclination ψ is considerable, a significant amount of down force is generated at the interface, denoted by point G. This downward force gives rise to normal forces at C, the point representing the interface between the cue ball and the table. Hence, at G and C, the cue ball will be subject to both normal and frictional impulses making the analysis complicated, as comprehensively worked out by [18]. In addition, there are other phenomena such as cue ball squirt [19], where the ball deviates from its ideal movement direction. This angle is denoted by α in Figure 2(a).

B. Ball motion on the table

A sphere, such as a billiards ball, that rolls on a very hard surface makes a point-wise contact, as shown in Figure 4(a). However, when either the sphere or the rolling surface are deformable the contact is not through a point, but via a surface. In billiards, the balls are hard and the table is laid with a soft cloth, resulting in contact scenario shown in Figure 4(b). Here, no consideration is given to the sidespin of the ball as it is assumed not to affect the forward ball motion. This is known as decoupled motion, where sidespin is considered to be independent of the linear velocity, V or the topspin, ω, of the ball.

A ball is said to be rolling when $V = R\omega$, where R is the radius of the snooker balls (Figure 4a). If the ball and the surface are non-deformable, as in Figure 4(a), the ball will be in perpetual rolling motion, as the friction forces between the surfaces in contact are zero for the condition $V = R\omega$. However, when one of the surfaces is deformable - an example is shown in Figure 4(b) - the ball speeds, V and ω, are gradually reduced in the rolling phase. In billiards, the table cloth that makes up the top surface of the table is deformable. Referring to Figure 4(b), the normal force F_N, from the surface does not go through the center of the ball [20]. The horizontal component of the force F_N, $F_N\sin\beta$, acts against the motion, similar to friction force, reducing the linear speed, V**.** Moreover, the action of F_N also introduces a torque in a direction opposite to that of the direction of rotation of the ball, introducing an angular deceleration. Although the decelerations are not due to friction, an equivalent friction coefficient of μ_r can be defined for the rolling condition of the ball. The linear and rotational decelerations are given by $\mu_r g$ and $\mu_r g/R$, respectively, where g denotes gravitational acceleration.

Rolling usually takes place towards the latter part of ball motion, before it comes to a stop. At the start, the ball slides on the table, in general, and referring to Figure 4, $V \neq R\omega$. Here, the regular friction coefficient, μ_s, affects the ball motion. In snooker and billiards, μ_s is 10 times larger than μ_r [17]. Therefore, to analyze the sliding phase, one can disregard the effects of rolling. In sliding, the rate of change of speeds are given by $\mu_s g$ and $\frac{5\mu_s g}{2R}$, for the linear and angular motions respectively. When $V > R\omega$, the ball has linear deceleration and angular acceleration. Conversely, when $V < R\omega$, the ball is under linearly acceleration and rotational deceleration in its rolling motion, and this phenomenon is called the 'overspinning' of the ball.

Given initial conditions, the above expressions for linear and angular accelerations of balls allow the determination of the instantaneous values of the linear ball speed and top (or bottom) spins.

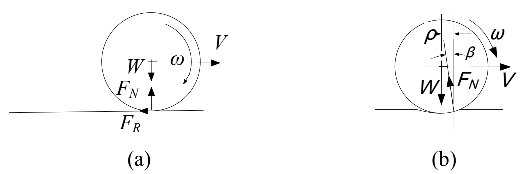
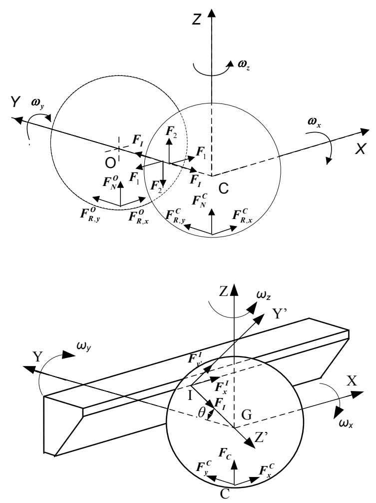


Fig. 4. Ball motion on non-deformable (a) and deformable (b) surfaces

The estimation of the sidespin of the ball, at any time instant, is equally important, mainly from the viewpoint of analyzing ball-ball collisions and ball-cushion impacts, where sidespin is a key parameter. As mentioned in Section III, the table resistance to the sidespin has been measured as 22 rad/s^2. This resistance acts to retard the sidespin, irrespective of whether the ball has left- or right-spin. The measured resistance provides a means to estimate the spin at time instant, given the sidespin value at $t = 0$. Referring to Figure 2(b), for balls under right-spin, the change of sidespin, ω^S will be negative, and for left-spinning balls, the change will be positive (the right-hand notation is used here to measure spins).

C. Collision between two balls

Billiard collisions have been traditionally analyzed without incorporating the effects of ball spin [12,18]. The present authors reported a numerical model considering the 3-dimensional ball spin that set up a number of additional forces



(a)

(b)

Fig. 5. The forces acting on two balls during the impact [21] and the forces setup between a ball and the cushion [22]

as seen in Figure 5(a), when compared to the no-spin models [21]. Furthermore, the curved, sliding trajectories of the two balls, that happen as a consequence of the impacts, are also analyzed [21]. It is also shown, experimentally, that the proposed model performed better when compared to the previous models [21].

D. Collision between a ball and the side rails

Similar to the description in the previous section, the collision between a snooker ball and the side rails is a complex dynamic phenomenon, especially when the ball has 3-dimensional spin impact as shown in Figure 5(b). For the first time, the authors of this paper presented a method to analyze such collisions, under the assumption of negligible cushion deformation during the impact [22]. Similar to collision between two balls, numerical methods are employed.

V. ROBOT MANIPULATOR

A. Robot design

The material and the geometrical shape of the cue and other features such an elastic cue-tip serve a number of purposes including the suppression of vibration at the time of impact with the cue ball. In order to prevent any unwanted dynamics being transferred to the cue ball, a regular snooker cue is proposed to be used with the robot. In addition, to allow the regular transverse vibrations of the cue stick at impact, the holding conditions of the cue, which requires a firm grip with the back hand a guiding through the extended front hand, are kept unchanged. The other robots have had either a modified cue or some other mechanical contraption to strike the ball, e.g. Chang [1], Long et al. [10] and Alian et al. [7]. A rack and pinion system is designed for the linear manipulation of the cue. Based on the requirement of a maximum cue ball velocity of 4 m/s, required peak force, power and rotational speed necessary from the drive motor are estimated [23]. For these requirements a servo system from *SureServo*™ called the "200 W Low Inertia System" is selected (see Figures 7 and 8). The servomotor is connected to the pinion via a 3:1 reduction gearbox from Shimpo Drives®. A planar (i.e. two-axis) stepper drive, AEROTECH® ATS302, is used so that the cue launcher

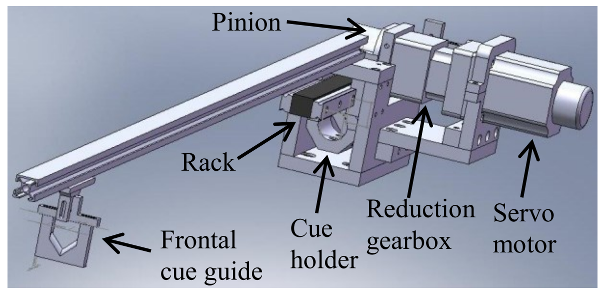


Fig. 6. A SolidWorks rendering of the cue launcher design

can be mounted on it to position on the ball to impart different spins on the ball. The mounting detail is seen in Figure 8(b). Depending on the movement effected by the stepper drive the whole cue launcher moves in the vertical plane normal to the cueing direction. There is also a small tilt of 6.5° introduced in the cue orientation about the horizontal plane.

B. System description

The speed of cueing, thereby the rotational speed of the servo motor, is kept constant throughout the stroke. The speed of the servomotor, hence the cue speed, is controlled by an IENSYS® microcontroller board, based on PIC18F458, via a terminal block called the ZIPLink® kit.

The IENSYS® board's four digital I/Os are programmed to send synchronized, phase shifted pulses to emulate the four output channels of a quadrature encoder thereby setting the speed requested from the servomotor. The rate of the pulses sent out from digital I/Os is set on the microcontroller board from the main program running on a PC by RS-232 communication. By changing the phase sequence of the pulses, the cue launcher is retracted to its original position. Visual Basic® 6.0 (VB) is used as the programming language for the main control program. The vision algorithms are written as M-files in MATLAB®. These M-files are then called from VB using a function procedure called MATLAB® COM component, which is generally used to integrate MATLAB® with other programming environments.

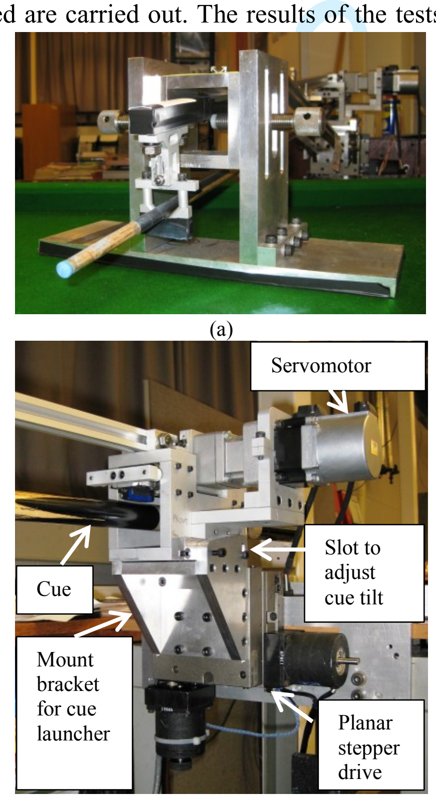
When the pulse rate is at its highest, the linear striking velocity of the cue reaches 2.75 m/s, a typical high-end cue velocity found in a normal game of snooker. The rate at which the pulses are sent out from the microcontroller is selected by a string consisting of a 3 digit number appended with a 'p' from the PC through its serial port to the serial interface of the microcontroller. This 3-digit string, which ranges from'001' to'200', selects the intended pulse rate. String '001p' corresponds to a cue velocity of 2.75 m/s and '200p' achieves 0.3 m/s. This resolution of the cue velocity can position the cue ball, theoretically, to a 15mm spatial accuracy on the table, but the repeatability characteristics of the robot, as described later, will also have an effect on the positioning accuracy.

The stepper drive is driven from the main VB program via the DB-25 connector of the PC. Figure 8 shows the overall hardware configuration of the system. A thin-film force sensor Flexiforce® A201-100 is also integrated in the cue, but is only used for some off-line experiments.

C. Tests with the robot

1) Human and robot shots

In order to compare robot and human cueing performance, a number of high-speed video tests measuring the cue and cue ball speed are carried out. The results of the tests are given



(a)

(b)

Fig. 7. The designed robotic manipulator, with the passive frontal cue support (a) and the stepper drive (b)

in Figure 9(a), where it can be seen that the robot is performing on a par with human shots. The repeatability of the robot for shots up to a cue speed of 2.8 m/s is found to be around ±50 mm, on average (snooker ball diameter is 52.4 mm).

2) Dynamics of cueing

For a given cue manipulation parameters, i.e. cue speed and the position on the cue ball where the cue hits, initial motion parameters such as ball velocity and ball spin must be determined to predict the subsequent motion of different balls on the table. The theoretical cueing model proposed by de la Torre Juarez [18] incorporates all of the effects that are present during cueing. However, the cue tip, being made out of leather or synthetic materials, is highly elastic. Hence, it is very difficult to determine the frictional forces, and their directions, set up between the cue ball and the cue tip. For this reason, it is decided to pursue an experimental approach to determine the dynamics.

The robot manipulator and the cue balls are initially set to have x'_0=0 and y'_0=0, where x'_0 and y'_0 are the coordinates of the point at which the line of strike of the cue intersects

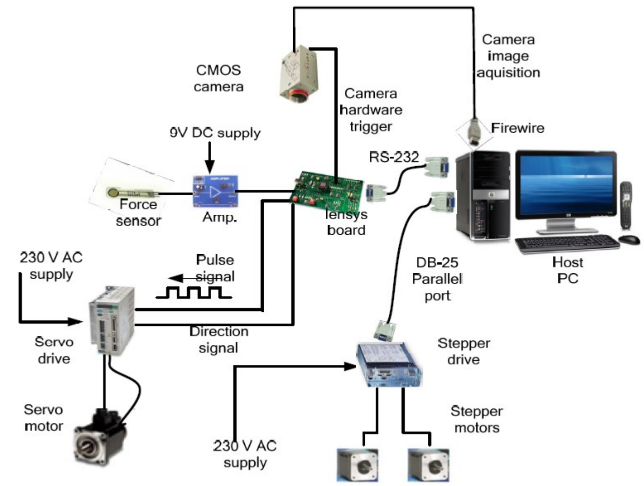
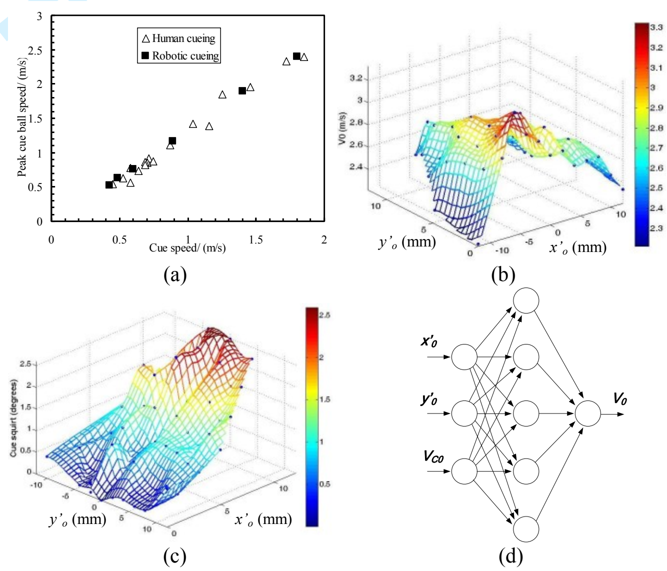


Fig. 8. Overall hardware configuration of the system

the X_o'Y_o' coordinate system, refer to Figure 2(a). Five shots of different speeds at approximately 0.5 m/s intervals (in the range of 0.5 m/s to 2.5 m/s) are executed for the same ball position, by replacing the ball back to the initial



(a) (b)

(c) (d)

Fig. 9. Human and robot cueing comparison (a), experimental plots for the velocity (b) and squirt(c) of the cue ball against cue the position of cue impact for the velocity string '070p' (cue speed ~ 2.0 m/s) and the neural network predicting cue ball speed (d)

position after each shot (a guiding structure is used for this purpose). For each shot, the ball is placed such that the black pattern, as shown in Figure 3, for spin tracking, is seen by the overhead camera and the ball motion is recorded at 180 fps. Then y'_0 is varied from -12 mm to 12 mm in increments of 2 mm and for each y'_0, x'_0 is varied from zero to 12 mm also in 2 mm increments. For each combination of x'_0 and y'_0, 5 shots were played. Only right spin shots are played, as the cueing dynamics have a symmetry about the x'_0=0 line, the results obtained for right spins of the ball can be easily translated to left spins. Spin tracking in the face of linear ball movements is observed to be very unreliable. Hence, only the resulting ball velocities and cue ball squirt, plotted in Figures 9(b) and 9(c) respectively, are utilized for subsequent analysis. As the data for different cue speeds is essentially 3-dimensional in nature, a back-propagation feed-forward neural network (NN) is trained with the data to predict resulting cue ball speed. A 3-5-1 neuron configuration, as seen in Figure 9(d), is found to be suitable for the task. Another similar NN is designed to predict ball squirt as well.

In the absence of any reliable experimental data for initial spins of the cue ball, an alternative way has to be found to estimate ω^T_0 and ω^S_0, i.e. top and side spins, respectively. Researchers have often used the assumption of the cue tip gripping the cue ball during their impact [19,24]. This is largely owing to the fact the cue tip is well chalked before each shot. The tip has good frictional properties, and it is also flexible, hence the cue tip is assumed to grip the ball surface as soon as the cue and the cue ball come into contact. For a cue inclination angle of 6.5°, it is shown that the friction force setup between the cue ball and table during cueing is 2% of the cueing force [23], and is neglected in obtaining ω^T_0 and ω^S_0. Now, for an initial cue ball velocity of V_0, the initial ball spins are,

$$\omega_0^S = \frac{5x_C'V_0}{2R^2} \tag{1a}$$

$$\omega_0^T = \frac{5y_C'V_0}{2R^2} \tag{1b}$$

Where, x_C' and y_C' are the initial point of contact between the cue tip and the ball in the X'Y' coordinate system shown in Figure 2(a). For a given x_O' and y_O', the values of x_C' and y_C' can estimated from the geometry given in Figure 2(a) and using cue tip radius of 10 mm. This concludes the estimation of initial ball velocities and spins immediately after cueing.

VI. Manipulation Problem

The artificial intelligence (AI) part of the system always makes decisions regarding which object ball has to be played next, the pocket in which the object ball must to be potted and where to leave the cue ball in order to make the next shot according to the overall game plan (this is discussed in Section 2.1). Thus, for a given initial cue ball location, C_I, as depicted in Figure 10 (only a part of the table is shown there), the decision to play the ball O_1 into the pocket P_1 and then to leave the cue ball in or very close to the desired ball location C_D has already been taken by the decision-making system. These results are assumed to be readily available, as the system described in this paper is devoid of the AI component.

A. Problem definition

Referring to Figure 10, for a given cue ball location (C_I), targeted object ball (O_1) and pocket (P_1) combination, and to attain a certain desired final cue ball location (C_D), the task is to determine the initial required parameters of the ball motion, given by V_0, ω^T_0, ω^S_0 and θ, and thereby establish the robot's manipulation parameters V_{C0}, x'_0, y'_0 and θ_C, the last being the cueing direction of the cue manipulator. For the proposed robot, θ_C cannot be adjusted automatically, as no swivel DOF is assigned to the robot. Hence, the platform is aligned manually with the feedback from the overhead camera. For the optimal shot parameter selection, both the shot configuration shown in Figure 10 as well as the most basic shot, where no cue ball – cushion impact takes place, are considered. For a billiards robot, there can be other additional constraints such as other balls very close to the general area of trajectories that limit the possible ball trajectories, and hence the solution space, further. However, the fundamental problem, as defined above, is treated here and the proposed methodology can be easily extended to include any additional constraints.

B. Model for forward dynamics

For the ball trajectory shown in Figure 10, the estimation of forward dynamics is as presented in Figure 11.

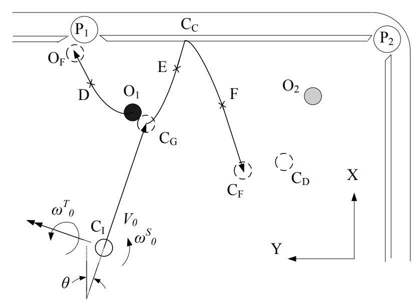


Fig. 10. Typical ball trajectory in snooker

C. Solution methodology

For positioning flat objects (axi-symmetric and polygonal ones) on a plane with the action of sliding friction, Huang *et al.* [25] and Han and Park [26] use inverse numerical algorithms. The flowchart representing the forward dynamics of the ball, given in Figure 11, shows that some of the

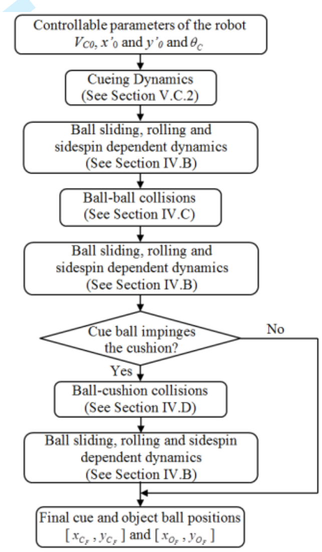


Fig. 11. Forward dynamics

dynamics are not explicitly expressed by equations, but by numerical procedures. Furthermore, there may arise situations where, due to the properties inherent to the dynamics of the system, an inverse solution does not exist. Hence, the direct inverse solution based approach is ruled out.

For a given positioning task, instead of finding a direct inverse solution, the manipulation space can also be searched by using the forward dynamics models. A possible solution can be found by trying to reduce the error in positioning, using a forward motion model of the object, whilst satisfying any possible constraints on the object motion. Various methodologies have been used in this regard. The major types of solutions used by various researchers are nonlinear optimization [13,14,15], iterative learning control [16] and machine learning [27].

An optimization-based approach is proposed here to position the balls on the table. The optimization function will have to be a composition of spatial errors between the actual positions where the balls will end up for a given shot parameters, and the desired ball locations. The conditions to ensure that the object ball is potted are also a part of the problem. This is generally known as nonlinearly constrained optimization, and can be defined as [28], by referring to Figure 10,

For $q \in \Re^4$ and also subject to the conditions $V_{C0} < 2.2$ m/s, $\sqrt{(x'_C)^2 + (y'_C)^2} < 0.5$R (for no mis-cueing to occur) and $\theta_{min} < \theta < \theta_{max}$ with θ_{min} and θ_{max} ensuring that cue ball C_1 will hit the object ball O_1,

$$\text{Minimize } F(q) = \left(x_{C_D} - x_{C_F}\right)^2 + \left(y_{C_D} - y_{C_F}\right)^2,$$
$$\text{subject to } [K(q)] \le [L], \text{ where, } q = [V_{C0}, x'_0, y'_0, \theta_C] \qquad (2)$$

The matrix condition $[K(q)] \le [L]$ consists of two elements. This constraint ensures that the object ball is potted by imposing conditions that the trajectory segment O_1O_F should go up to the pocket P_1 (or go past it) and that the minimum distance between the line segment and the center of P1 must be less than 55 mm (for the ball to fall into the pocket). Since $F(q)$ is not differentiable, a derivative-free optimization method must be used. Under similar conditions, a quasi-Newtonian method has been used by Li and Payandeh [14] for planar sliding objects and by Lynch and Black [15] for a batting manipulator. For the present problem, Genetic Algorithms (GAs) are used. Ball's forward dynamics shown in Figure 11 is coded as a M-function in MATLAB with q, as defined in Equation 6, as input. The GA-based optimization is performed in the Matlab® Optimization Toolbox.

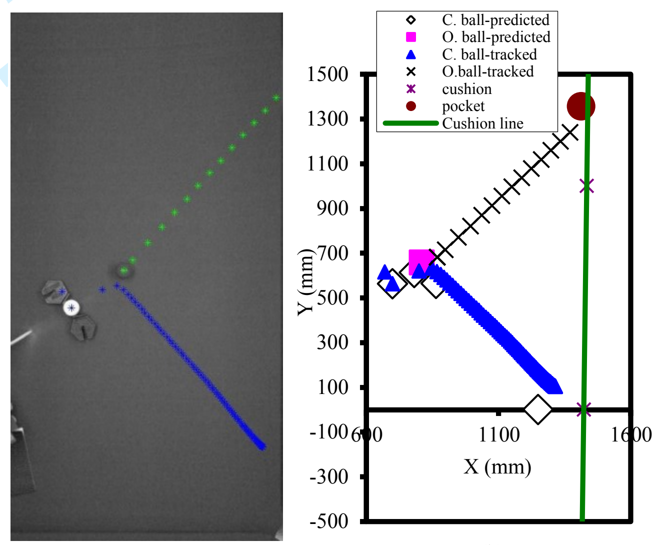
VII. Experimental Results

Using the overhead camera, the pocket locations on the part of the table that is in the field of view of the camera, and the cushion (i.e. side rail) lines are established. The initial cue ball location is determined using the camera. A red ball is used as the object ball and its position on the table is established by processing the R component of the RGB color image sequences obtained by the camera. The two initial ball positions are embedded in the M-function describing the forward dynamics. The desired final cue ball location is also specified. The M-file is then called from the Optimization Toolbox using its function handle and executed to deliver the best value for q.

For the following values of θ_C = 0.515 rad, [x_C, y_C] = [698 mm, 562 mm], [x_O, y_O] = [869 mm, 681 mm], and a desired cue ball location of [x_{C_D}, y_{C_D}] = [1250 mm, 0 mm], the optimization routine has predicted the following parameters for the robot: V_{C0} corresponding to string '073p', x'_0 = -11 mm and y'_0 = 0 (x'_0 and y'_0 are approximated to the nearest millimeter) without any cue ball-cushion collision. The shot that is executed for the above results obtained from the GA optimization is shown in Figure 12(a) (the pocket is not seen in the figure as it is right next to the right side edge of the image). The ball is potted and the cue ball, without any collision with the cushion, is positioned at 110 mm from its desired location.

VIII. Discussion

Experiments on ball positioning are performed within a table area of 5ft x 6ft. Within this area of the table, an object ball potting accuracy of more than 90% is obtained. In addition, the cue ball is positioned to an accuracy within the range of 100-250mm, in general. These are the first reported research efforts on the post-shot positioning of the cue ball. In its early stages of development, the Deep Green system was claimed to have 67% potting accuracy [10]; Deep Green plays on a pool table of size 4 ft x 8ft. However, the Deep Green research has not reported on the issues related to the cue ball positioning. In their latest publication, *Greenspan et al.* [2]



(a) (b)

Fig. 12. Positioning results

state that the robot has pocketed runs of four consecutive balls, but no quantitative figure is given for the ball potting accuracy. Here some facts concerning the pocket and the ball sizes in pool and snooker must also be considered. In snooker, all six pockets are 90 mm in size and the ball diameter is 52.5 mm. If the mid-pocket entry is considered to be ideal for an object ball in snooker, the margin of maximum allowable error for a flawless entry (not touching the pockets) is around 19 mm, on either side of the ideal line of entry. However the way the cushion near the pocket entrance is shaped allows up to a

45 mm deviation for the corner pockets and a 55mm for the middle pockets, in snooker. Pool balls are 52.5 mm in diameter. In pool, the four corner pockets are 114-117 mm in size while the middle pockets measure 127-130 mm [29]. This leaves a robot with the margin of error of 28.5 mm for the corner pockets and 35 mm for the middle pockets, for a non contact-entry of the object ball; thus, the maximum allowable distance offset values can also be expected to be larger than those in snooker. The preceding comparison underlines the fact that the ball potting is difficult in snooker. Hence, if the same robot is employed to play both the games, it will have a higher potting accuracy in pool when compared to that in snooker.

The performance of the current robot must be evaluated in light of other facts concerning the robot and the forward dynamics model for the ball motion. The robot's repeatability in ball positioning is found to be around ±50 mm and this, in turn, will affect the positioning accuracy of the robot. There have been vibration problems with the robot and the servo drive system is proposed to be replaced with a linear servo actuator [23]. In addition, a very basic model is utilized for the spin estimates of the cue ball immediately after cueing, neglecting the friction from the table. Hence, the forward model itself is not perfect and more accurate spin tracking with a higher resolution camera will establish a better model for cueing dynamics.

REFERENCES

[1] S. W. S. Chang, "Automating Skills Using a Robot Snooker Player," Ph.D. dissertation, University of Bristol, Bristol, UK, 1994.

[2] M. Greenspan, J. Lam, M., Godard, I. Zaidi, and S. Jordan, "Toward a Competitive Pool-Playing Robot", *Computer*, vol. 41, no. 1, pp. 46-53, 2008.

[3] World Snooker 2014, Available: http://www.worldsnooker.com/page/NewsArticles/0,,13165~2352434,00.html

[4] J.-P. Dussault, and J.-F. Landry, "AI Optimization of a Billiard Player", *J. of Intell. and Robotic Syst.*, vol. 50, no. 4, pp. 399-417, 2007.

[5] M. Smith, "PickPocket: A computer billiards shark", *Artificial Intell.,* vol. 171, no. 16-17, pp. 1069-1091, 2007.

[6] K. H. L. Ho, T. Martin, and J. Baldwin, "Snooker Robot Player - 20 Years On", in *IEEE Symp. on Computational Intell. and Games (CIG 2007)*, Honolulu, Hawaii, 2007, pp: 1-8.

[7] M. E. Alian and S.B. Shouraki, "A Fuzzy Pool Player Robot with Learning Ability", *WSEAS Trans. on Electronics*, vol. 2, no. 1, pp. 422–425, 2004.

[8] B. -R. Cheng, J. -T. Li, and J.-S. Yang, "Design of the Neural-Fuzzy Compensator for a Billiard Robot", in *IEEE Int. Conference on Networking, Sensing & Control*, Taipei, Taiwan, 2004, pp. 909-913.

[9] A. W. Moore, D. J. Hill, and M. P. Johnson, An Empirical Investigation of Brute Force to choose Features, Smoothers and Function Approximators, in S. Hanson, S. Judd, and T. Petsche, editors, Computational Learning Theory and Natural Learning Systems, Volume 3. MIT Press, 1994.

[10] F. Long, J. Herland, M. -C. Tessier, D. Naulls, A. Roth, G. Roth, and M. Greenspan, "Robotic Pool: An Experiment in Automatic Potting", in *IEEE/RSJ Int. Conference on Intell. Robots and Syst.*, Sendai, Japan, pp. 2520-2525, 2004.

[11] T. Nierhoff, K. Heunisch, and S. Hirche, "Startegic play for a pool-playing robot", in *IEEE Workshop on Advanced Robotics and its Social Impacts*, Munich, Germany, pp. 72-78, 2012.

[12] R. E. Wallace and M. C. Schroeder, "Analysis of Billiard Ball Collisions in Two Dimensions", *Amer. J. of Physics*, vol. 56, no. 9, pp. 815-819, 1988.

[13] M. T. Mason, "Progress in Nonprehensile Manipulation", *Int. J. of Robotics Research*, vol. 18, no. 11, pp. 1129-1141, 1999.

[14] Q. Li and S. Payandeh, "Planning Velocities of Free Sliding Objects as a Free Boundary Value Problem", *Int. J. of Robotics Research* , vol. 23, no. 1, pp. 69-87, 2003.

[15] K. M. Lynch and C. K. Black, "Recurrence, Controllability, and Stabilization of Juggling" *IEEE Trans. on Robotics and Automation*, vol. 17, no. 2, pp. 113-124, 2001.

[16] C. Zhu, Y. Aiyama, T. Arai, and A. Kawamura, "Positioning in Releasing Manipulation by Iterative Learning Control", *J. of Int. and Robotic Syst.*, vol. 46, no. 4, pp. 383-404, 2006.

[17] S. Mathavan, M. R. Jackson, and R. M. Parkin, "Application of high-speed imaging to determine the dynamics of billiards", *Am. J. Physics*, vol. 77, no. 9, 788–794, 2009.

[18] M. de la Torre Juarez, M, "The Effect of Impulsive Forces on a System with Friction: The Example of the Billiard Game", *European J. of Physics* , vol. 15, no. 4, pp. 184-190, 1994.

[19] R. Cross, "Cue and ball deflection (or "squirt") in billiards", *Am. J. of Physics*, vol. 76, no. 3, pp. 205-212, 2008.

[20] J. Hierrezuelo and C. Carnero, "Sliding and Rolling: The Physics of a Rolling Ball", *Physics Educ.*, vol. 30, no. 3, pp. 177-182, 1995.

[21] S. Mathavan. M. R. Jackson, and R. M. Parkin, "Numerical simulations of the frictional collisions of solid balls on a rough surface", *J. of Sports Eng.*, accepted and published online; DOI: 10.1007/s12283-014-0158-y

[22] S. Mathavan. M. R. Jackson, and R. M. Parkin, "A theoretical analysis of billiard ball dynamics under cushion impacts", *Proc. of the IMechE, Part C, J. of Mech. Eng. Sci.,* vol. 224, no. 9, pp. 1863-1873, 2010.

[23] S. Mathavan, "Trajectory Solutions for a Game-Playing Robot Using Non-Prehensile Manipulation Methods and Machine Vision", Ph.D. dissertation, Loughborough University, Loughborough, UK, 2009.

[24] D. G. Alciatore,"Fundamentals", in *The Illustrated Principles of Pool and Billiards*, Sterling Publishing, New York.

[25] W. H. Huang, E. P. Krotkov, and M. T. Mason, "Impulsive Manipulation", In *IEEE Int. Conference on Robotics and Automation*, Nagoya, Japan, pp. 120-125, 1995.

[26] I. Han and S. –U., "Impulsive Motion Planning for Positioning and Orienting a Polygonal Part", *Int. J. of Robotics Research*, vol. 20, no. 3 , pp. 249-262, 2001.

[27] M. Matsushima, T. Hashimoto, M. Takeuchi, and F. Miyazaki, "A Learning Approach to Robotic Table Tennis", *IEEE Trans. on Robotics*, vol. 21, no. 4, pp: 767-771, 2005.

[28] J.-S. R. Jang, C. -T. Sun, and E. Mizutani, "Derivative-based optimization", in *Neuro-Fuzzy and Soft Computing - a Computational Approach to Learning and Machi. Intell.*, 1st Ed., Prentice Hall , New Jersey.

[29] World Pool-Billiard Association 2014, Available: http://www.wpa-pool.com/

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